G53FUZ
Fuzzy Sets and Systems

Module Introduction

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Overview

- Module outline
  - module delivery
    - website
  - aims and objectives
  - weekly topics
  - assessment
  - resources
- Classical (Boolean) logic and set theory
  - refresher of basic concepts
  - deficiencies / weaknesses

Module Outline

Module Delivery

- Lectures
  - theoretical / conceptual issues
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- Labs
  - a single ONE hour lab per week, **in A32 (CS)**
  - practical issues
    - creating a fuzzy system in R
    - coursework assistance

Aims and Objectives

- Aim
  - present how the fuzzy method can be used to represent knowledge and perform reasoning, in the presence of uncertainty, in a principled manner
- Objectives
  - to introduce the theory and principles of fuzzy logic, fuzzy sets and systems
  - to explain how fuzzy methods can be used to model uncertainty in real world examples
  - to convey the properties and concepts underlying fuzzy inference systems and their applications
  - to provide practical experience on the design and implementation of fuzzy logic systems

Weekly Topics

Week	Lecture	Lab
1	Module Introduction	No Lab
2	Fuzzy Concepts	MATLAB Fuzzy Toolbox
3	Linguistic Variables	Basic R
4	Mamdani Inference and Defuzzification	R Fuzzy Toolkit
5	TSK Inference and Fuzzy Control	R Fuzzy Toolkit; CW Issue
6	Fuzzy Modelling and Tuning	CW Assistance
7	Learning using ANFIS	CW Assistance
8	Real-world Applications	CW Assistance
9	Non-Standard Inference Systems	CW Hand-In
	EASTER HOLIDAYS	
10	Coursework Feedback	
11	Module Revision and Exam Guidance	

Module Assessment

- Coursework
  - 40%
  - implement a fuzzy inference system (in R)
  - write report describing your system
- Exam
  - 60%
  - 1 hour
  - one compulsory and one from two

Resources

- Fuzzy sets and systems
  - *Fuzzy Sets, Uncertainty and Information*, Klir and Folger
    - Prentice Hall PTR, 1988, ISBN 0133459845
  - *The Fuzzy Systems Handbook*, Cox
    - Academic Press, 1994, ISBN 0121942708
  - *Artificial Intelligence*, Negnevitsky
    - Addison Wesley, 2002, ISBN 0201711591
- MATLAB / R
  - MATLAB Fuzzy Logic Toolbox
  - R (www.r-project.org)
    - loads of online tutorials, docs, help, examples
    - R Fuzzy toolbox (`www.cs.nott.ac.uk/~jmg/fuzzy-v0_7.r`)

Classical (Boolean) Logic

Classical (Crisp) Logic

- Origins in Ancient Greece
  - Aristotle
  - Plato
- Two truth values
  - *true*, *false*
- Connectives
  - *not*, *and*, *or*

Propositional Logic

- All concepts (statements) are *true* or *false*
  - represented by symbols *T* and *F*
- Symbols used to represent connectives
  - *not* $\neg$ (negation / complement)
  - *and* $\wedge$ (conjunction / intersection)
  - *or* $\vee$ (disjunction / union)
  - *implies* $\Rightarrow$ (implication)
- Truth table defines meanings of connectives

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$
F	*F*	*T*	*F*	*F*	*T*
T	*F*	*F*	*F*	*T*	*F*
F	*T*	*T*	*F*	*T*	*T*
T	*T*	*F*	*T*	*T*	*T*

Boolean Algebra (Logic)

a	***b***	**NOT *a***	**NOT *b***	***a* AND *b***	***a* OR *b***
0	0	1	1	0	0
1	0	0	1	0	1
0	1	1	0	0	1
1	1	0	0	1	1

- Some properties
  - associativity $a \vee (b \vee c) = (a \vee b) \vee c$
  - commutativity $a \vee b = b \vee a$
  - distributivity $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
  - identity for $\vee$ $a \vee 0 = a$
  - identity for $\wedge$ $a \wedge 1 = a$
  - annihilator for $\wedge$ $a \wedge 0 = 0$

Sets

- A set is a collection elements, from some *universe of discourse*
- The set of all elements in the universe of discourse is the *universal set*
  - denoted by X
- The set that contains no elements is the *empty set*
  - denoted by $\varnothing$
- Set theory and propositional logic are isomorphic
  - every theorem in one has a counterpart in the other

Complement

- The *complement* of a set A is the set of all members of the universal set, X, that are not in A
  - denoted by A^c or $\overline{A}$
- So, if

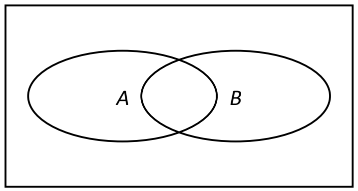
  $X = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$

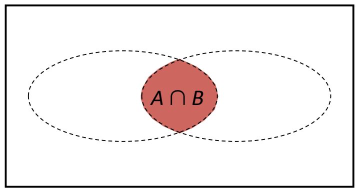
  $A = \{ 0, 2, 4, 6, 8 \}$
- Then

  $A^c = \{ 1, 3, 5, 7, 9 \}$
- Note that $X^c = \varnothing$, $\varnothing^c = X$, and $(A^c)^c = A$
- Complement is equivalent to logical *negation*

Intersection

- The *intersection* of sets A and B is the set containing all the elements that belong to both set A and set B
  - denoted by $A \cap B$
- Note that $A \cap X = A$, $A \cap \varnothing = \varnothing$, and $A \cap A^c = \varnothing$

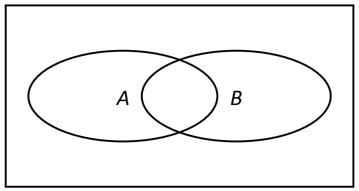


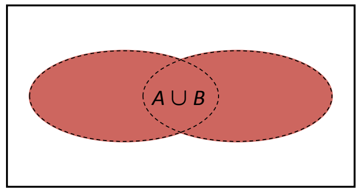


- Intersection is equivalent to logical *and*

Union

- The *union* of sets A and B is the set containing all the elements that belong to set A alone, to set B alone, or to both set A and set B
  - denoted by $A \cup B$
- Note that $A \cup X = X$, $A \cup \varnothing = A$, and $A \cup A^c = X$





- Union is equivalent to logical *or*

Deficiencies of Classical Logic

Aristotle, Zeno and Others

- Aristotle himself
  - future events: what is the truth of
    - it will rain tomorrow?
- Zeno
  - paradoxes (concerning infinities)
  - consider grains of sand
    - when have you got a 'heap'?

Cantor, Russell et al

- Let *R* be the set of all sets that are not members of themselves
  - if *R* qualifies as a member of itself, it would contradict its own definition as *a set containing all sets that are not members of themselves*
  - if *R* is not a member of itself, it would qualify as a member of itself by the same definition
- There are other such problems with classical set theory, including ones identified by Cantor

Who is Tall?

Who is Tall?

Defining Everyday Concepts

- How would you define (in formal terms) membership of the set of:
  - children?
  - old people?
  - celebrities?
- Write down the age at which you would consider someone to be 'old'

Fuzzy Logic – An Introduction

IEEE Computational Intelligence Society

- This first-prize-winning video was sponsored by the IEEE Computational Intelligence Society (CIS) in its 2011 Fuzzy Logic Video Competition
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Fuzzy Logic – Boiling Eggs

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Summary

- Classical two-valued logic has short-comings when representing the real-world
  - *classical*, *Boolean*, *two-valued*, ***crisp***
- Fuzzy logic relaxes the requirement to be either TRUE or FALSE
  - allows real-valued degrees of truth
- Logic and set theory are isomorphic
  - true/false statements can be mapped to set membership / non-membership