Type-2 Fuzzy Logic in Decision Support

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Outline

Philosophical

On traditional logic

All traditional logic assumes that precise symbols are being employed. It is therefore not applicable to this terrestial life, but only to an imaginary celestial existence.
Everything is vague to a degree you do not realise until you have tried to make it precise

Bertrand Russell
The Australasian Journal of Psychology and Philosophy, 1 (June 1923): 84–92

Philosophical

On Measurement

The indeterminacy which is characteristic in vagueness is present also in all scientific measurement.
Vagueness is a feature of scientific as other discourse.

Black, Max (1937) "Vagueness: An exercise in logical analysis". Philosophy of Science 4: 427–455

Practical

If only computers were like human beings

...the remarkable human capability to perform a wide variety of physical and mental tasks without any measurements and any computations. Familiar examples of such tasks are parking a car; driving in heavy traffic; playing golf; understanding speech, and summarizing a story. Underlying this remarkable ability is the brain's crucial ability to manipulate perceptions - perceptions of size, distance, weight, speed, time, direction, smell, color, shape, force, likelihood, truth and intent, amongst others.

Zadeh (1996) "Fuzzy Logic=Computing with Words"
IEEE Transactions on Fuzzy Systems, 4(2), 103 - 111

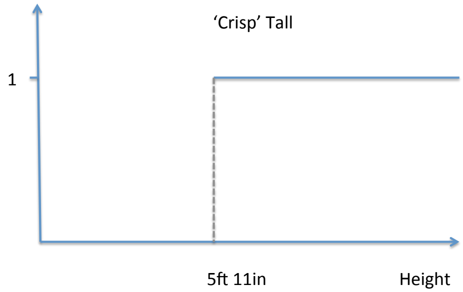
Motivation - Summary

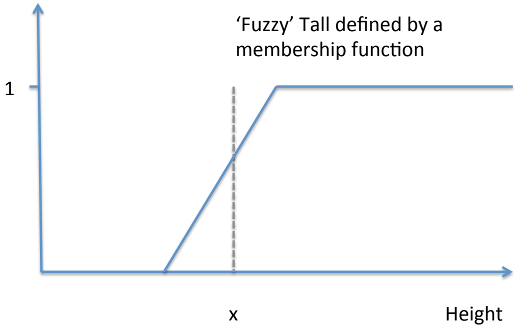
- The world is imprecise and vague.
- Mathematical and Statistical techniques often unsatisfactory.
- Experts make decisions with imprecise data in an uncertain world.
- Working with knowledge that is rarely defined mathematically or algorithmically but uses vague terminology with words.

What it's good for..

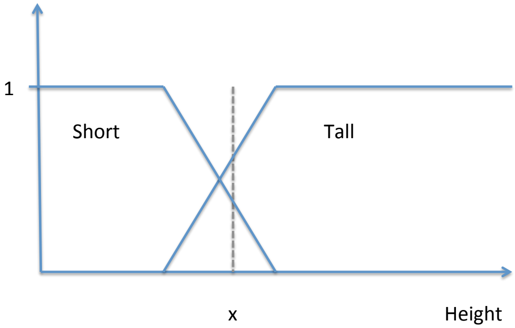
- is particularly good at handling uncertainty, vagueness and imprecision.
- especially useful where a problem can be described linguistically (using words)
- where there is data and you are looking for relationships or patterns within that data.
- uses imprecision to provide robust, tractable solutions to problems.

(Type-1) Fuzzy Sets




'Fuzzy' Tall defined by a membership function
1
x
Height

You can be 'tall' and 'short' at the same time:



A Mathematical Definition

Original (Zadeh 1965)

For any fuzzy set, A, the function μ_A represents the membership function for which $\mu_A(x)$ indicates the degree of membership that x, of the universal set X, belongs to set A and is, usually, expressed as a number between 0 and 1:

$$\mu_A(x) : X \rightarrow [0, 1]$$

Discrete sets are written as:

$$A = \mu_1/x_1 + \mu_2/x_2 + \ldots\ldots\ldots + \mu_n/x_n$$

or

$$A = \sum_{i=1,n} \mu_i/x_i$$

where $x_1, x_2 \ldots. x_n$ are members of the set A and $\mu_1, \mu_2, \ldots., \mu_n$ are their degrees of membership. A continuous fuzzy set A is written as:

$$A = \int_X \mu(x)/x.$$

A Mathematical Definition

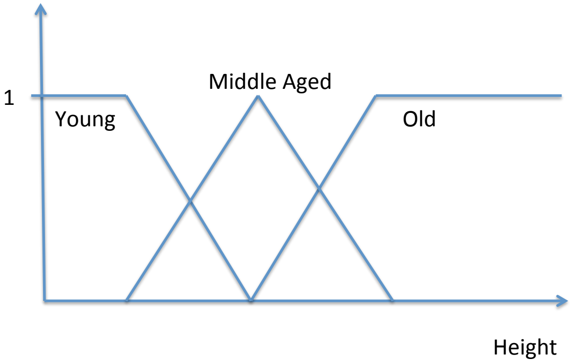
Modern

A type-1 fuzzy set , A, over X is a defined by the following function

$$A : X \rightarrow [0, 1]$$

Let, $F(X)$, be the set of all type-1 fuzzy sets on X.

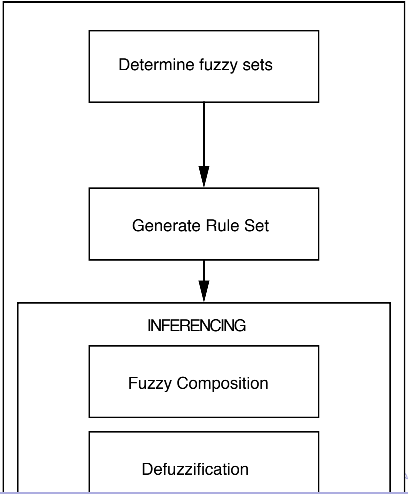
The concept of fuzzy age



- Fuzzy logic has a well-established theoretical base.
- practical implementations require a relatively small number of operations
- Two particularly important operations are intersection and union which correspond to ‘AND’ and ‘OR’ respectively.
- the building blocks for us to be able to compute with fuzzy if-then rules.

Type-1 Fuzzy Systems

- The ‘base’ fuzzy sets that describe the problem;
- The if-then rules;
- Rule Composition;
- Defuzzification.



Mmmm.... But....

- Where do the membership functions come from?
- Where do the rules come from?
- What operators should we use?
- The operations are crisp

Type-2 Fuzzy Sets

That man Zadeh again!

A fuzzy set is of type $n, n = 2, 3, \ldots$ if its membership function ranges over fuzzy sets of type $n-1$. The membership function of a fuzzy set of type-1 ranges over the interval [0,1].

Zadeh, L.A., The Concept of a Linguistic Variable and its Application to Approximate Reasoning - I, Information Sciences, 8,199–249, 1975

Type-2 Fuzzy Sets

'Traditional Notation'

A type-2 fuzzy set, $\tilde{A}$, is characterised by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

Mendel, J.M. and John, R.I., Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems,10(2),117–127

Type-2 Fuzzy Sets

'Modern Notation'

A type-2 fuzzy set, $\widetilde{A}$, over X is a defined by the following function

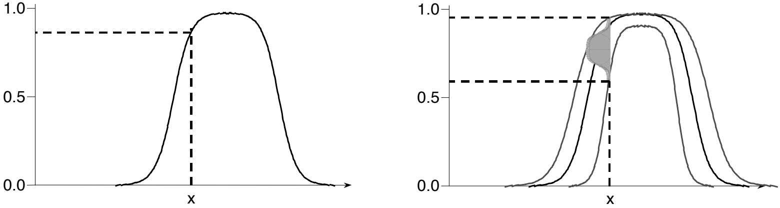
$$\begin{aligned} \widetilde{A} : X &\rightarrow \boldsymbol{F([0,1])} : \\ x &\mapsto \widetilde{A}(x) \equiv A_x \end{aligned} \tag{1}$$

where

$$\begin{aligned} A_x : [0,1] &\rightarrow [0,1] : \\ u_x &\mapsto A_x(u_x) \end{aligned}$$

Let, $\widetilde{F}\boldsymbol{(X)}$, be the set of all T2FSs on X.

Drawing them is difficult



The FootPrint of Uncertainty

The footprint of uncertainty (FOU) is the union of all primary memberships:

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x$$

This and related definitions/results:

John, R , Towards a Higher Level of Linguistic Granulation, IPMU Proceedings, 2, 1129-1133,2001

Type-1 cf Type-2

- Type-1 fuzzy sets have an x axis representing the *domain*
- Type-2 fuzzy sets do not require a domain
- Type-1 fuzzy sets two dimensional
- Type-2 fuzzy sets three dimensional
- We can have linguistic grades with type-2

Appealing

> *Type-2 fuzzy sets are fuzzy sets whose grades of membership are themselves fuzzy. They are intuitively appealing because grades of membership can never be obtained precisely in practical situations.*

Dubois, D. and Prade, H.,Fuzzy Sets and Systems: Theory and Applications, Academic Press,1980

Interval Valued Fuzzy Sets

An interval valued type-2 fuzzy set (IVFS), $\tilde{A}^{iv}$, is characterised by a type-2 membership function $\mu_{\tilde{A}^{iv}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$

$$\tilde{A}^{iv} = \{((x, u), 1) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

Union of Type-2 Fuzzy Sets

The union ($\cup$) of two type-2 fuzzy sets ($\tilde{A}$, $\tilde{B}$) corresponding to $\tilde{A}$ OR $\tilde{B}$ is given by:

$$\begin{aligned} A \cup B \Leftrightarrow \mu_{\tilde{A} \cup \tilde{B}}(x) &= \mu_{\tilde{A}}(x) \sqcup \mu_{\tilde{B}}(x) \\ &= \sum_{i,j} (f(u_i) \wedge g(w_j))/(u_i \vee w_j) \end{aligned}$$

Intersection of Type-2 Fuzzy Sets

The intersection ($\cap$) of two type-2 fuzzy sets ($\tilde{A}$, $\tilde{B}$) corresponding to $\tilde{A}$ AND $\tilde{B}$ is given by:

$$\begin{aligned} \tilde{A} \cap \tilde{B} \Leftrightarrow \mu_{\tilde{A} \cap \tilde{B}}(x) &= \mu_{\tilde{A}}(x) \sqcap \mu_{\tilde{B}}(x) \\ &= \sum_{i,j} (f(u_i) \wedge g(w_j))/(u_i \wedge w_j) \end{aligned}$$

Type-2 Fuzzy Inferencing System: Diagram

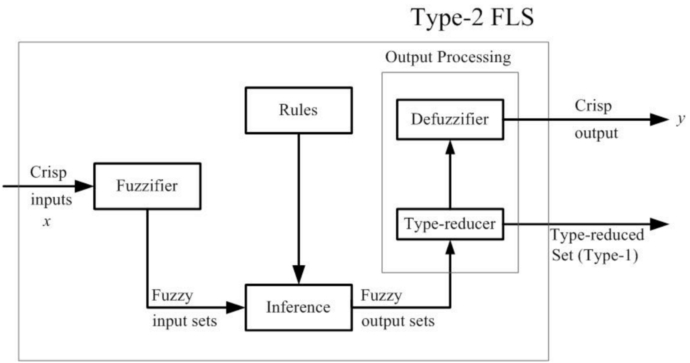


Figure : Type-2 FIS (from Mendel).

Why it’s important and difficult

- the output of inferencing is a fuzzy sets
- most applications require a single number as an output
- defuzzifying a type-1 fuzzy set is easy (although there are many methods)
- defuzzifying type-2 less straightforward
- but.... there are now a number of methods available as type-2 fuzzy representation has improved
  - α-planes, z-slices (Wagner and Hagras)
  - α-cuts (Hamrawi et al)
- and new methods
  - Sampling (Greenfield et al)
  - Collapsing (Greenfield et al)

Type-Reduction

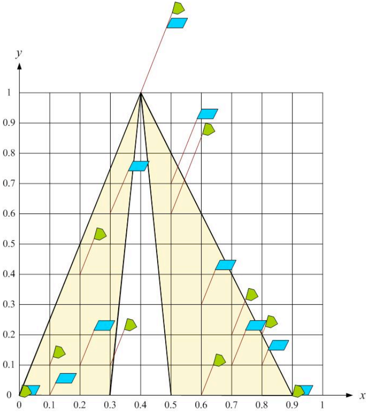
Conventionally, defuzzification of a type-2 fuzzy sets is a two stage process:

1. Type-reduction, creating a type-1 fuzzy set, and
2. defuzzification of the resultant type-1 fuzzy set.

The second stage is easily implemented; the first stage is more problematic.

Type-reduction is reliant on the concept of an *embedded set*.

Embedded Sets: Diagram



Embedded Set: Definition

An embedded set is contained in a type-2 fuzzy set in such a way that for every value of x, there is only one value of y, (the secondary membership grade being determined by x and y.)

Mendel and John's **representation theorem** states that a *type-2 fuzzy set can be represented as the union of its embedded sets*.

Algorithm for Type-Reduction

- All possible type-2 embedded sets are enumerated.
- For each embedded set the minimum secondary membership grade is found.
- For each embedded set the domain value of the type-1 centroid of the type-2 embedded set is calculated.
- For each embedded set the secondary grade is paired with the domain value to produce a set of ordered pairs (x, z). It is likely that some values of x will corresponding to more than one value of z.
- For each domain value, the maximum secondary grade is selected. This creates a smaller set of ordered pairs $(x, zMax)$ such that there is a one-to-one correspondence between x and $zMax$. This ‘type-reduces’ the type-2 fuzzy set to a type-1 fuzzy set, known as the *type-reduced set* (TRS).

Collapsing Method

Greenfied, John, Coupland & Herrera

Type-reduction suffers from high computational complexity, as embedded sets tend to be extremely prolific. (When a prototype type-2 FIS performed an inference using sets which had been discretised into 51 slices across both the *x* and *y*-axes, the number of embedded sets in the aggregated set was calculated to be about 2.9×10^{63}.)

The collapsing method of defuzzification was developed

1. as a response to the challenge of reversing blurring,
2. to be an efficient algorithm, and
3. to give a highly accurate approximation.

We believe that we have been successful in all three objectives.

The Representative Embedded Set: Diagram

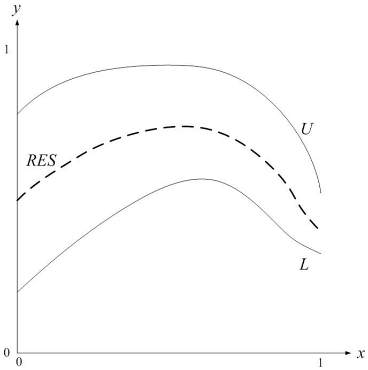


Figure : The region between L and U is the FOU. The dashed line depicts a representative embedded set.

The Representative Embedded Set: Definition

Let $\tilde{F}$ be an interval type-2 fuzzy set with defuzzified value $X_{\tilde{F}}$. Then type-1 fuzzy set R is a *representative embedded set (RES)* of $\tilde{F}$ if its defuzzified value (X_R) is equal to that of $\tilde{F}$, i.e. $X_R = X_{\tilde{F}}$, *and* its membership function lies within the FOU of $\tilde{F}$.

The Representative Embedded Set Approximation (RESA)

A discretised type-2 interval fuzzy set $\tilde{F}$, having defuzzified value $X_{\tilde{F}}$, lower membership function L, and upper membership function U, may be collapsed into a type-1 fuzzy set R, known as the *representative embedded set*, whose defuzzified value X_R is approximately equal to $X_{\tilde{F}}$, with membership function such that:

$$\mu_R(x_i) = \mu_L(x_i) + \frac{\left(\|L\| + \sum_{j=1}^{i-1} r_j\right) b_i}{2\left(\|L\| + \sum_{j=1}^{i-1} r_j\right) + b_i},$$

where $b_i = \mu_U(x_i) - \mu_L(x_i)$.

Future work in this area

- More work on defuzzification of general type-2 fuzzy sets
- Different interoperation of type-2 fuzzy output?
- Maybe using words?
- Better understanding of the structure of type-2 fuzzy sets?

Supply Chain Modelling

Miller, John & Gongora

Aims

Improve resource planning in a Supply Chain using Computational Intelligence techniques.

The work is part of a larger Technology Strategy Board funded project concerned with using Computational Intelligence techniques to improve demand forecasting in a supply chain.

Problem

Resource planning

Resource planning involves making decisions about how much stock to hold at each node of a supply chain. This is difficult due to the uncertain nature of supply chain operation.

Poor allocation of stock can lead to stock outs and holding surplus stock.

Methods

In this research we use the following methods:

- **Genetic Algorithm** - Search the large solution space presented by a typical SC.
- **Interval Type-2 Fuzzy Logic** - Modelling uncertainty inherent in SCs, such as:
  - Customer demand
  - Inventory levels
  - Costs

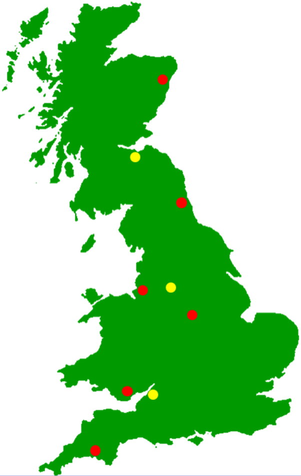
Model

Using a forecast, costs, transport distances and a required service level the model calculates the total cost and service level of a given solution.

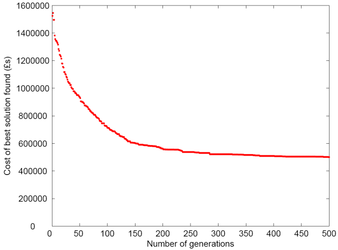
Experiments

Model has been tested in a variety of scenarios:

- *Multiple Tiers, Improved IT2FS, Improved GA*
- *GA configurations, Multiple Tiers, IT2FS*
- *Optimisation methods, Multiple Tiers, IT2FS*
- *2 Tiers, IT2FS*
- *Initial tests, Crisp model*



Results



Through experimentation, discoveries include:

- Users can represent the uncertainty present in their SC.
- System will discover good/sensible resource plans that meet a specified service level.
- Randomness in mutation and crossover improves search results.
- Combining the GA with Simulated Annealing can improve the speed of search.

Other type-2 work

- Geometric Fuzzy Systems
- A novel alpha-cut representation for type-2 fuzzy sets
- Type-1 and Type-2 OWA Operators
- Tuning Fuzzy Systems using Simulated Annealing

Other type-2 work

- Interval Valued Fuzzy Decision Trees
- Constructing Parsimonious Type-2 Fuzzy Logic Systems
- Modelling nursing intuition
- Modelling uncertainty in clinical diagnosis

OK So...

- we can build type-1 fuzzy systems
- type-1 fuzzy systems are successful in control.
- they have not moved well into more fuzzy situations because..
- they are crisp
- so lets use type-2 fuzzy sets
- but they are complex..
- but we now have tools to use them

The type-2 Research Community

- Type-2 fuzzy sets only started to be researched seriously in the late 90s
- Now a large(ish) community
- IEEE Special Symposium, Special sessions at Fuzz-IEEE and WCCI
- Best paper awards in IEEE Transactions on Fuzzy Systems
- Nottingham a centre of excellence

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