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Autonomous Robotic Systems

Lecture 6

Sensor and Behaviour Fusion

Last week...

- Localisation issues
 - general scheme
 - sources of uncertainty
 - sensors
 - actuators
- Alternatives to localisation
 - behaviour-based navigation
- Sensor models
 - Bayesian Sonar Sensor Model
 - HIMM

- **Representation**
 - belief representation
 - map representation



Completed
today

First – some housekeeping...

Thank you for all the module feedback. I sent an email capturing adjustments in response to the feedback, summarised below:

- **Lab sheets:** as discussed, I accept that by adopting a looser approach to marking lab sheets (i.e. not marking them in the same session), we have provided less structure, which may result in groups working for longer on the lab sheets, thus having a higher risk of falling behind. Thus, the final lab sheet (number 6) will only be released after the assignment deadline, i.e. in week 11, commencing 4th December, rather than this week, to allow you to focus fully on the assignment and completing any outstanding lab sheets.
Please note that marking of all lab sheets will need to be completed in week 11 and any unmarked lab sheets will be assigned a mark of 0% after week 11 – i.e. please make sure to get all your lab sheets marked.
- **Assignment demos:** the demos for the assignment will be held during regular lab sessions (where possible) in week 10. A schedule for the demos will be published on Moodle in due course.
- **Guest lecture:** Dr Barbara Bruni, a roboticist from the University of Genova, Italy, is visiting the university and has kindly offered to give a guest lecture during the standard lecture slot next week (Nov. 16th). More details of this in a minute....
- **Reading Week:** while the reading week was initially scheduled for week 8, this will be rescheduled to week 9 to allow the guest lecture to go ahead.
- **Additional lab time:** in response to the request for more lab time, and the fact that a number of you encountered problems with the robots/computers/network outside your control, the lab will be open during the lecture slot of the reading week, i.e. 23rd November, 9-11am. The first hour will be dedicated to Group A, and the second hour to Group B. Please be mindful that this will be the final time prior to the assignment deadline that you can work on the robots, thus I strongly recommend to use it only for final testing and slight tuning, respectively to run final experiments for your reports.

If you have concerns or any questions regarding the above, please let me know.



Updated Weekly Topics - 1

Week	Lecture
2	Introduction & Overview (Autonomous) Robots =? Foundations of Robotic Systems Architectures & Behaviours
3	Brooks' Subsumption Architecture - Theory Robot Hardware
4	DARPA Grand Challenge PID Control
5	Fuzzy Control
6	Ultrasonic Sensor Models Localisation and Mapping

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5

Updated Weekly Topics - 2

Week	Lecture
7	Sensor Fusion
8	Guest Lecture: Dr Barbara Bruni, University of Genova
9	Reading week (Ethics and Robotics) Lab will be open in lecture slot
10	Kalman Filters
11	Particle Filters
12	Revision

Note:

Weekly topics may change subject to timing and new material.
All lecture notes will be available online.

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6

This week...

- Sensor/Data Fusion
 - Average
 - Weighted Average
 - Ordered Weighted Average
 - Induced Ordered Weighted Average
- Behaviour Fusion
- Darpa Grand Challenge – Follow on

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7

A different context...

Babies better than adults at knowing where they're being touched

If adults who can see are touched on each hand in quick succession while their hands are crossed, they can find it hard to name which hand was touched first. Adults who have been blind from birth don't have this difficulty, but people who become blind later in life have the same trouble as those who can still see.

"That suggests that early on in life, something to do with visual experience is crucial in setting up a typical way of perceiving touch," says Andrew Bremner at Goldsmiths, University of London.

To investigate how this develops in infancy, Bremner and his colleagues compared how babies reacted to having one foot tickled.

With their legs crossed over, babies aged 6 months moved the foot being tickled half of the time. But 4-month-olds did better, moving the tickled foot 70 per cent of the time – as often as they did with their legs uncrossed.

Separate worlds

The team concludes that at 4 months, *babies haven't yet learned to relate what they touch to the physical space that their body occupies*. For many adults, the concept might be difficult to envision. "It's like imagining that you feel a touch on your body, but not really knowing how that's related to what you're looking at," says Bremner. "It's almost like you have multiple sensory worlds: a visual world, an auditory world and a tactile world, which are separate and not combined in space."

Bremner thinks these worlds start to combine when babies touch objects at the same time as seeing them. It's at about 5 months that babies typically start to successfully reach for objects. "Getting those kinds of coordinated experiences between what they're seeing and what they're feeling is going to give babies this sense that touches are related to the visual spatial fields," he says. "That's going to lead them into these kinds of errors, so when they feel a touch on their right foot, they expect it to be on the right side of space."

Tobias Heed at the University of Hamburg, Germany, is sceptical of the findings, saying he doubts that such young infants give coordinated responses to foot stimulation. "My own experience is that 4-month-olds do not respond coherently to touch on the feet," he says. *His research suggests that children don't automatically integrate mental representations of space until the age of 5.*



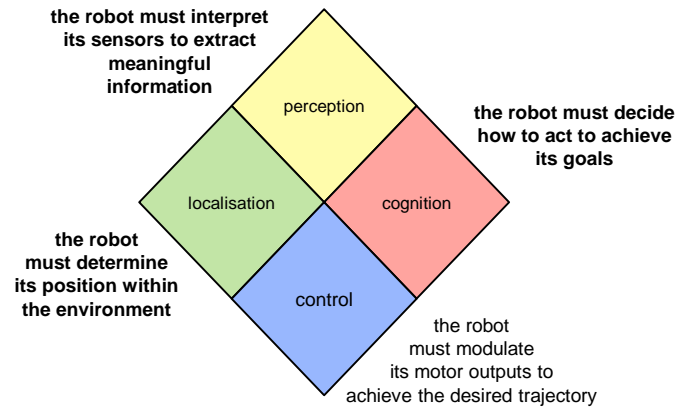
Source: New Scientist, 19th October 2015
Journal reference: [Current Biology, DOI: 10.1016/j.cub.2015.08.055](https://doi.org/10.1016/j.cub.2015.08.055)

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8

Autonomous Mobile Robots - Navigation



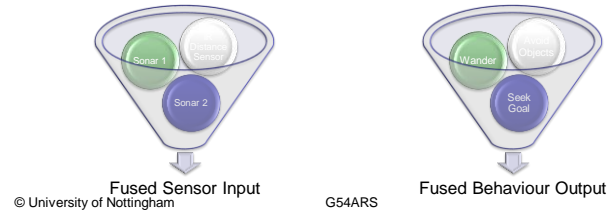
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9

Fusion = ?

- “the process or result of joining two or more things together to form one” (Oxford Dictionary)
 - i.e. a (usually sensible) combination of individual sources
- Fusion is very common in robotics, we look at two distinct types – behaviour and sensor fusion
- Later in the module, we will see Kalman and Particle Filters – which are further examples of information fusion.



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Fusing different sources

Some notation

- Consider a non-empty, finite set of n sources (e.g., sensors):

$$X = \{x_1, \dots, x_n\}$$

- The information (or evidence) h contributed by each source x_i is $h(x_i)$
 - For example, for a sonar sensor x_1 , and a given reading, we may have: $h(x_1) = 30cm$
- The ‘worth’ of a given source x_i is given by $g(x_i)$

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11

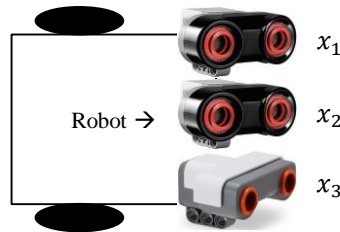
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12

Basic fusion

- Consider three sources (sonar sensors), mounted on the front of a robot :



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13

Basic fusion

- Consider that during a single iteration, we capture three measurements, e.g.: $h(x_1) = 31cm$, $h(x_2) = 33cm$, $h(x_3) = 29cm$
- If we consider the data from all sensors to have *equal 'worth'*, we may want to directly combine the measurements into one single measurement $h(X)$.
- A number of simple operators can be useful, e.g.:

$$MAX(X) = \max_{i=1}^n (h(x_i)) \quad \text{e.g.: } MAX(X) = \max(31, 33, 29) = 33$$

$$MIN(X) = \min_{i=1}^n (h(x_i)) \quad \text{e.g.: } MIN(X) = \min(31, 33, 29) = 29$$

$$AVG(X) = \frac{\sum_{i=1}^n (h(x_i))}{n} \quad \text{e.g.: } AVG(X) = (31 + 33 + 29) / 3 = 31$$

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14

Beyond basic fusion?

- Commonly, we do not consider the information from all sensors to be of equal "worth". Reasons include:
 - Sensor type
 - E.g., sonar vs laser range finder
 - Sensing context
 - We have knowledge about which sensor may perform better than another (e.g. in a glasshouse...)
 - Sensor properties
 - Position
 - Model accuracy, age, potential for interference, etc.
 - Sensor reading
 - E.g., different sonar sensors provide different levels of accuracy at different distances

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15

Different 'worths' - different weights

- A variety of fusion operators exist which combine different 'worths' (weights) with the actual *information arising* from individual sources.
 - I.e., different weights for different sensors.
- This enables assigning higher importance or 'impact' on the final fused result to individual sources.
- Commonly, the resulting combinations are so-called *convex combinations*, i.e., all the weights sum up to one:

$$\sum_{i=1}^n (g(x_i)) = 1$$

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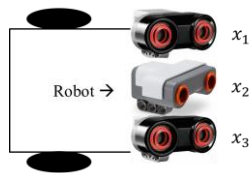
16

The Weighted Average

- More formally – the weighted arithmetic mean:

$$WA(X) = \frac{\sum_{i=1}^n (g(x_i)h(x_i))}{\sum_{i=1}^n (g(x_i))}$$

- Note, if the weights sum up to one, the above simplifies to $WA(X) = \sum_{i=1}^n (g(x_i)h(x_i))$.



Example:

Consider the following case:

$$g(x_1) = 0.4, g(x_2) = 0.2, g(x_3) = 0.4$$

$$h(x_1) = 31, h(x_2) = 33, h(x_3) = 29$$

$$WA(X) = 0.4 * 31 + 0.2 * 33 + 0.4 * 29 = 30.6$$

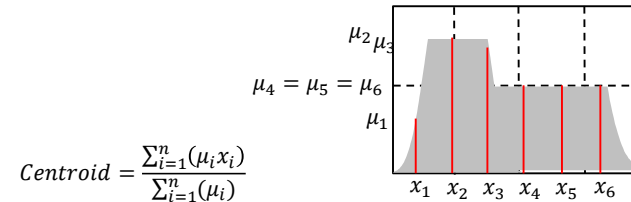
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A side-note on Fuzzy Logic

- Remember the centroid computation?
- It is a weighted average where the weights are the degrees of membership μ and the evidence is the x for a given discretisation.



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18

Challenge

- The weighted average associates ‘worth’ directly with a given source of information.
- In practice however, it is not always the source which tells us how important a given (sensor) input is.
- For example:
 - For cautious obstacle avoidance, we may want to assign the highest weight to the reading/source indicating the shortest distance.

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19

The Ordered Weighted Average

- The **classical** Ordered Weighted Average (OWA) assigns weights based on the size of each source’s contribution (input), rather than assigning them directly to a given source.
- Consider:
 - A set of sources $X = \{x_1, \dots, x_n\}$, contributing scalar evidence (information) $h(x_i)$.
 - A permutation π of X , such that $h(x_{\pi(1)}) \geq h(x_{\pi(2)}) \geq \dots \geq h(x_{\pi(n)})$.
 - a vector of weights $G = (g_1, \dots, g_n)$, where $g_i \in [0,1]$ and $\sum_{i=1}^n g_i = 1$.
- Then, the classical OWA can be written as:

$$OWA(X) = \sum_{i=1}^n (g_i h(x_{\pi(i)}))$$

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20

OWA Example

- Consider a vector of weights as follows:
 $g_1 = 0.2, g_2 = 0.3, g_3 = 0.5$
- And the evidence/information contributed by out three sensors:
 $h(x_1) = 31, h(x_2) = 33, h(x_3) = 29$
- We have:
 - Permutation: $\pi(1) = 2, \pi(2) = 1$ and $\pi(3) = 3$, such that $h(x_{\pi(1)}) \geq h(x_{\pi(2)}) \geq h(x_{\pi(3)})$
 - $OWA(X) = 0.2 * 33 + 0.3 * 31 + 0.5 * 29 = 30.4$
- The OWA enables a non-linear combination of the information. Note that depending of the weight vector, it can replicate a number of other operators, incl. *min*, *max* and the *average*.

Induced Ordered Weighted Average

- The classical OWA assigns weights based on the size of the evidence, i.e. the sources are sorted largest to smallest before the (fixed) vector of weights is applied.
- However, we may have other reasons (beyond largest first) which may influence our ordering and thus assignment of weights.
 - For example: a combination of sensor type and distance
 - Other examples?
- The Induced Ordered Weighted Average (IOWA) enables this.

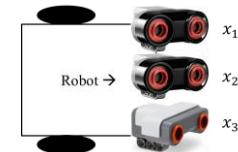
Induced Ordered Weighted Average ctd.

- Consider $u(x_i)$ an additional source of information encoding information regarding the desired ordering of the information $h(x_i)$, such that we have a vector of pairs $(u(x_i), h(x_i))$.
- Let ϖ be a permutation of X , such that $u(x_{\varpi(1)}) \geq u(x_{\varpi(2)}) \geq \dots \geq u(x_{\varpi(n)})$.

$$IOWA(u(x_i), h(x_i)) = \sum_{i=1}^n (g_i h(x_{\varpi(i)}))$$

IOWA Example

- Consider the same vector of weights as of the OWA example:
 $g_1 = 0.2, g_2 = 0.3, g_3 = 0.5$
- And the same evidence/information contributed by out three sensors: $h(x_1) = 31, h(x_2) = 33, h(x_3) = 29$



- Also, consider the (example) ordering encoded in:
 $u(x_1) = \frac{h(x_1)}{0.4} = 77.5, u(x_2) = \frac{h(x_2)}{0.4} = 82.5$ and $u(x_3) = \frac{h(x_3)}{0.2} = 145$
We choose to associate these weights based on the sensor type as before.
- Thus, we have:
 Permutation: $\varpi(1) = 3, \varpi(2) = 2$ and $\varpi(3) = 1$, such that $u(x_{\varpi(1)}) \geq u(x_{\varpi(2)}) \geq u(x_{\varpi(3)})$
 $IOWA(u(x_i), h(x_i)) = 0.2 * 29 + 0.3 * 33 + 0.5 * 31 = 31.2$

Fusing over time

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Fusing over time

- In autonomous robotics, single sensor readings are often not reliable enough.
- One approach to increasing the reliability of the information sensed, is to fuse the information over time.
- In principle, this is very similar to the fusion of different sources.
- The problem here is commonly to incorporate how recent (and thus relevant) information is.
- Example: sliding window approach


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26

Sliding Window Approach

- Example: a sliding window of *size=3* from *t-2* until now (*t*).

	...	t-5	t-4	t-3	t-2	t-1	t	t+1	...	time reading
	...	15	16	16	18	21	20	?	...	

	...	t-5	t-4	t-3	t-2	t-1	t	t+1	...	time reading
	...	15	16	16	18	21	20	?	...	

	...	t-5	t-4	t-3	t-2	t-1	t	t+1	...	time reading
	...	15	16	16	18	21	20	?	...	

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Sliding Window Approach ctd.

- At each time step, several sensor readings (per sensor) are fused to produce an overall reading.
- As for multi-source fusion, many ways of fusing the evidence are possible, the same algorithms can be employed, e.g.:
 - **Average**: to fuse all readings within the window with equal “worth” for all readings.
 - **Weighted average**: e.g., to fuse the readings, but to assign higher “worth” to more recent readings
 - The **Ordered Weighted Average**: e.g., to fuse by considering the actual readings.
 - The **Induced Ordered Weighted Average**: e.g., to fuse by taking an external (or designed) criteria into account, for example the superiority of given sensors in different environmental conditions, recency, ...

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28

Worksheet

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29

Behaviour Fusion

- Remember: there are two main approaches to behaviour coordination.
- Note – in the labs, we use Behaviour Arbitration



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Behaviour Arbitration



30

Behaviour Fusion ctd.

- In practice, behaviour fusion is simply the combination of the outputs of the individual behaviours.
- Averages (WA, OWA, IOWA) are all used and often learning strategies are employed to learn the weights.
- Commonly, behaviour fusion is used in parallel with arbitration, with the latter focussing on high-level behaviours
- Emerging behaviour* can be a challenge, in particular with behaviour fusion.
 - E.g., averaging positive and negative behaviour outputs.

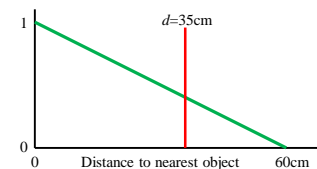
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31

Behaviour Fusion ctd.

- Beyond fusion operators, aspects from Fuzzy Logic also provide an intuitive approach to behaviour fusion.
- For example, consider the case of an obstacle (OA) avoidance and a goal seeking (GS) behaviour:



Intuitively, as an obstacle comes closer, we would like to focus more on obstacle avoidance than on goal seeking and vice-versa.

$$\text{Output} = OA * \mu(d) + GS * (1 - \mu(d))$$

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32

Video

- DARPA Grand Challenge
 - Behind the scenes
 - <https://www.youtube.com/watch?v=TDqzyd7fDRc>



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33

Summary

- Sensor/Data Fusion
 - Average
 - Weighted Average
 - Ordered Weighted Average
 - Induced Ordered Weighted Average
- Behaviour Fusion
- Darpa Grand Challenge – Follow on

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34