Linear and Discrete Optimization (G54LDO)

Semester 1 of Academic Session 2017-2018

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Lecture 1 – Basics of Computational Optimization

- Overview of the Module

  To understand purpose, scope and administration of the module

- An Insight into Operations Research

  To describe origins, purpose and scope of operations research

- Modelling Optimization Problems

  To identify elements and types of mathematical optimization models

- Solving Optimization Problems

  To identify software tools for solving models of optimization problems

Additional Reading

Information about module assessment and past feedback posted in Moodle.

Chapters 1-2 of (Hillier and Lieberman, 2015)

Chapter 1 of (Ragsdale, 2015)

Chapter 1 of (Williams, 2013) for a sample of the linear algebra knowledge used in the module.

The Origin of Operational Research. Harold Larnder. *Operations Research*, Vol. 32, No. 2, pp. 465-475, 1984

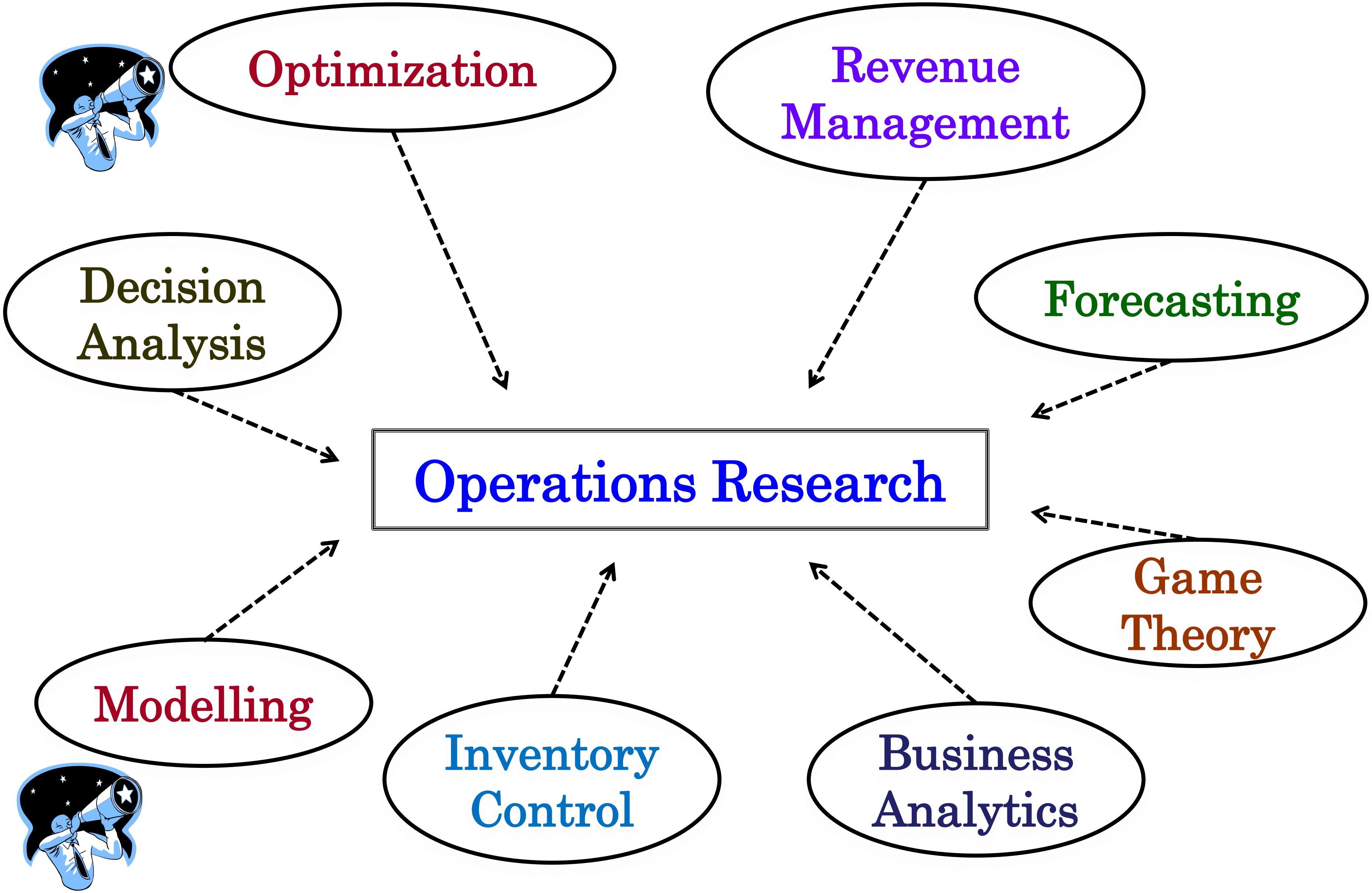
AI and OR in Management of Operations:History and Trends. K.A.H. Kobbacy, S. Vadera, M.H. Rasmy, *Journal of the Operational Research Society*, Vol. 58, No. 1, pp. 10-28, 2007

Overview of the Module

Aim of the Module

Understand linear and discrete optimization techniques with emphasis in developing mathematical programming models and spreadsheet optimization models.

Operations Research (OR) is a fascinating discipline that uses modelling techniques , analytical methods and algorithms to analyse and solve complex real-world problems. OR is the application of modelling, statistics, programming and other general problem-solving skills in improving operations and business processes. Using OR techniques decision-makers can make more effective decisions and build more productive systems.


Optimization
Revenue Management
Decision Analysis
Forecasting
Operations Research
Game Theory
Modelling
Inventory Control
Business Analytics

Module Contents

Various techniques for modelling and solving optimization problems are explained and illustrated with examples, case studies and software.

Learning Activities and Assessment

- 2 hrs Lecture, 2 hrs Workshop, 1 hr Computing
- Module managed through Moodle
  https://moodle.nottingham.ac.uk/
- Examination 50% + Coursework 25% + In-class Tests 25%

Module Resources

- Reading list
- Software: Excel Solver, LP-Solve, etc.
- Articles from journals and magazines

Expected Workload

Activity	Per Week	Hours
Lecture – delivery key material	2×10	20
Workshops – problem modelling and solving	2×10	20
Computing – take online In-Class Test	1×10	10
Self-study – understand lecture material	2×10	20
Coursework (25%) – modelling and optimization solvers to solve a real-world scenario and demonstration video		50
In class Test (25%) – revise workshop assignments to prepare for online tests	4×10	40
Examination (50%) – revise and further study		40
Total (20 credits)		**200**

Feedback Provision

Various mechanisms are used to provide feedback and students' involvement in these will help to make the feedback more effective.

- In-class Tests (weekly online through Moodle)
- Workshops Weekly Feedback
- Coursework Feedback (before examination)
- Questions and Answers in Moodle
- Topic Dedicated Forums

Feedback to the module convenor is welcome via electronic or verbal communication in addition to Early Module Feedback and SET/SEM surveys.

Topics Covered

Linear and Discrete Optimization

- Linear Programming
- Network Flow Optimization
- Post-optimality Analysis
- Routing and Selection Problems
- Product-mix Optimization
- Simplex Algorithm
- Packing Optimization Problems
- Selection and Assignment Problems
- Branch and Bound Algorithm

Origins of OR and Optimization

- During WWII, trends not going well in Europe against Germany
- Limited resources for defense against Germany’s bomber fleet
- Cities in England highly vulnerable to bomb raids
- The Committee for the Scientific Survey of Air Defence (CSSAD) was formed
- CSSAD was also known as the Tizard committee

Examples of Optimization Problems Solved by CSSAD

- Bombing tactics
  - Problem: three bombing (sighting) techniques:
    1. drop on group leader
    2. drop on squadron leader
    3. independent sighting
  - Question: what is the 'best' technique?
  - Operations Research Findings

- Aircraft attack against submarines
  - Problem: submarines are escaping aircraft attacks because coastal command's bombs explode at 100 feet depth
  - Question: what is the 'best' depth to set charges?
  - Operations Research Findings

Origins of OR : Research on Military Operations

The analytical study of military problems is undertaken in order to provide responsible commanders and staff agencies with a scientific basis for decisions on actions to improve military operations.

Scientific Method
Principle of Science
Multi-disciplinary Teams

Applied To

Problems of Military Operational Importance

Data and information to decision-makers
Techniques to analyse and compare possible courses of action
Assist in finding the best alternative

Operations Research is a Multi-disciplinary Field

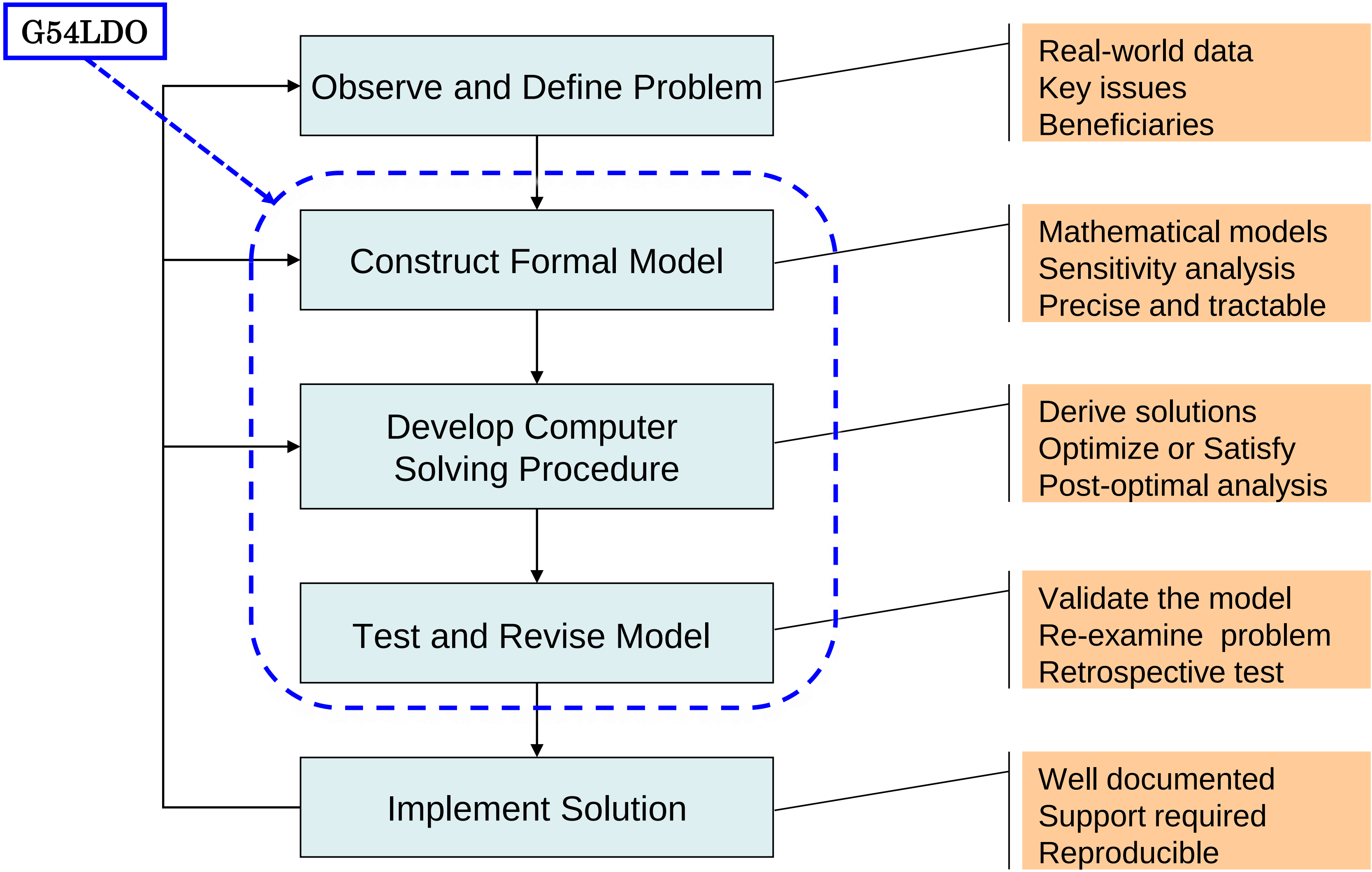
- A scientific approach to improve operations for better management of business and other organisations

- Operational Research or Operations Research , one discipline.

- Related to Management Science , Industrial Engineering, Optimization Science and more recently Analytics.

- Progress and Applicability of OR
  - Improvement in OR techniques
  - Advances in computing technology
  - Development of powerful algorithms
  - Applicable to all sorts of businesses and industries

Phases for Tackling an Optimization Problem



Modelling Optimization Problems

Computational Optimization

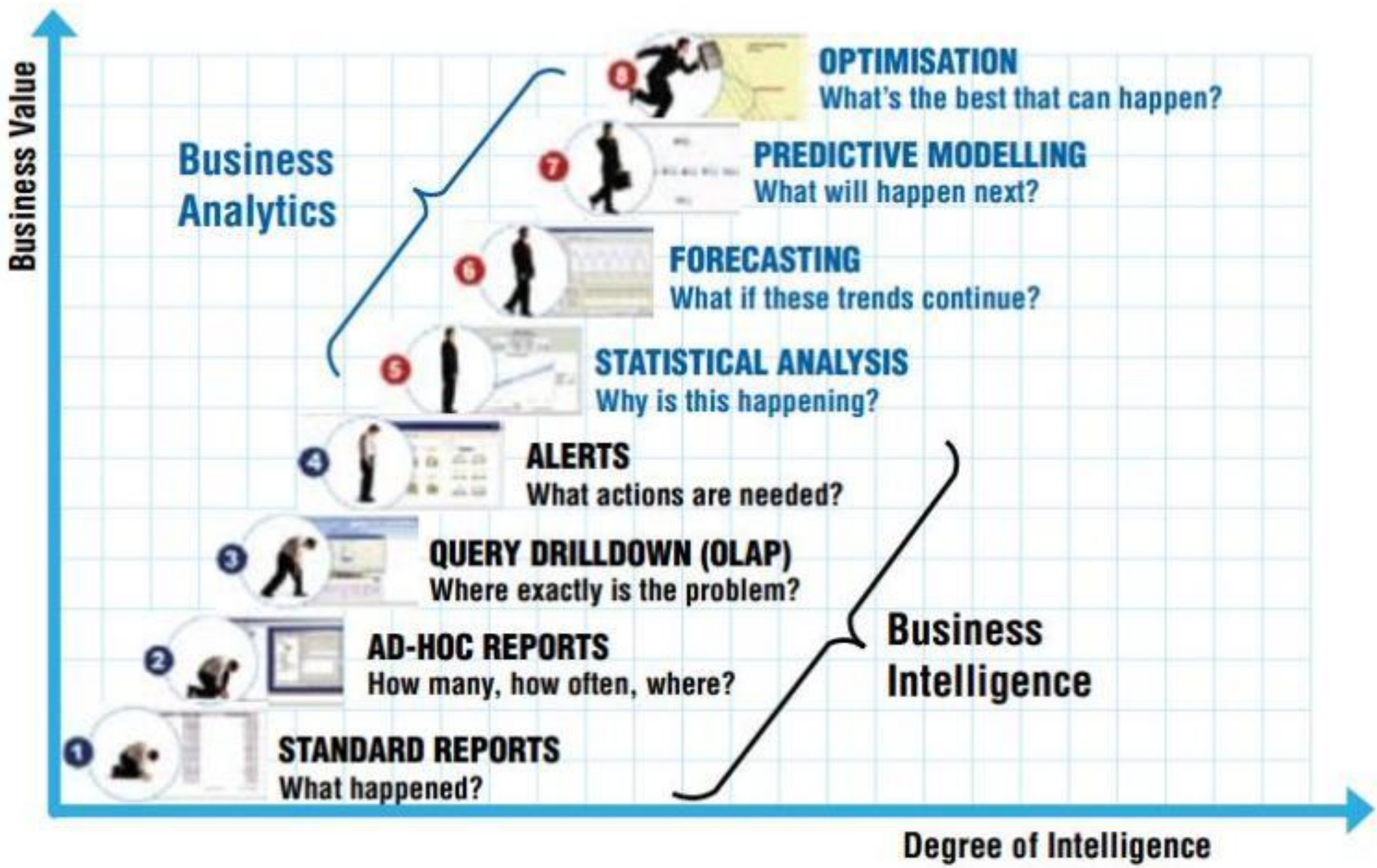
Often, optimization problems arise in industry and businesses, particularly in operational scenarios for which decision makers typically need an optimal solution.

Formal problem modelling is required in an optimization context and this provides several benefits.

Recently, analytics has been associated to operations research and management science.

According to SAS, a provider of Analytics Software and Solutions, there are 8 levels of analytics.

Optimization is the Highest Level of Analytics



A Simple Optimization Problem

The BANK ABC offers 4 types of loans to their customers at the annual interest rates shown below.

1. First mortgage at 14%
2. Second mortgage at 20%
3. Home improvement at 20%
4. Personal overdraft at 10%

The bank has a maximum lending capability of £250 million. It also has the following policies:

1. First mortgages must be at least 55% of all mortgages issued and at least 25% of all loans issued.
2. Second mortgages cannot exceed 25% of all loans issued.
3. To avoid a new windfall tax, the overall interest earned over all loans must not exceed 15% of the total amount of loans issued.

The Bank wants to maximise the interest income while satisfying all the above policies. The problem is to determine the amount that should be allocated to each type of loan in order to maximize the interest income, which is calculated simply as IR1×X1 + IR2×X2 + IR3×X3 + IR4×X4, where IR is the interest rate and X is the amount loaned.

Modelling the Optimization Problem

- Identify: data, decision variables, objective function and constraints.

Data in the Bank ABC Optimization Problem

Interest rates: 14% , 20% , 20% , 10%

Lending capability: £250 million.

Data from policies:

1. First mortgages must be at least 55% of all mortgages issued and at least 25% of all loans issued.
2. Second mortgages cannot exceed 25% of all loans issued.
3. To avoid a new windfall tax, the overall interest earned over all loans must not exceed 15% of the total amount of loans issued.

Decision Variables in the Bank ABC Optimization Problem

The problem is to determine the amount that should be allocated to each type of loan.

1. First mortgage: X1
2. Second mortgage: X2
3. Home improvement: X3
4. Personal overdraft: X4

Objective Function in the Bank ABC Optimization Problem

To maximize the interest income, which is calculated simply as IR1×X1 + IR2×X2 + IR3×X3 + IR4×X4, where IR is the interest rate and X is the amount loaned.

$$\text{Maximize} \quad Z = 0.14X1 + 0.20X2 + 0.20X3 + 0.10X4$$

Constraints in the Bank ABC Optimization Problem

The bank has a maximum lending capability of £250 million

$$X1 + X2 + X3 + X4 \leq 250$$

First mortgages must be at least 55% of all mortgages issued

$$X1 \geq 0.55\ (X1 + X2)$$

and at least 25% of all loans issued

$$X1 \geq 0.25\ (X1 + X2 + X3 + X4)$$

Second mortgages cannot exceed 25% of all loans issued

$$X2 \leq 0.25\ (X1 + X2 + X3 + X4)$$

the overall interest earned over all loans must not exceed 15% of the total amount of loans issued

$$Z \leq 0.15\ (X1 + X2 + X3 + X4)$$

Mathematical Programming Model

$$
\begin{array}{lll}
\text{Maximize:} & Z = 0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4 & (1) \\
\text{Subject to:} & x_1 + x_2 + x_3 + x_4 \leq 250 & (2) \\
& x_1 \geq 0.55(x_1 + x_2) & (3) \\
& x_1 \geq 0.25(x_1 + x_2 + x_3 + x_4) & (4) \\
& x_2 \leq 0.25(x_1 + x_2 + x_3 + x_4) & (5) \\
& 0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4 \leq 0.15(x_1 + x_2 + x_3 + x_4) & (6) \\
& x_1, x_2, x_3, x_4 \geq 0 & (7)
\end{array}
$$

$$
\begin{array}{lll}
\text{Maximize:} & Z = 0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4 & (1) \\
\text{Subject to:} & x_1 + x_2 + x_3 + x_4 \leq 250 & (2) \\
& 0.45x_1 - 0.55x_2 \geq 0 & (3) \\
& 0.75x_1 - 0.25x_2 - 0.25x_3 - 0.25x_4 \geq 0 & (4) \\
& -0.25x_1 + 0.75x_2 - 0.25x_3 - 0.25x_4 \leq 0 & (5) \\
& -0.01x_1 + 0.05x_2 + 0.05x_3 - 0.05x_4 \leq 0 & (6) \\
& x_1, x_2, x_3, x_4 \geq 0 & (7)
\end{array}
$$

Variants of Linear Programming Models

IP (Integer Programming) Model – All decision variables are restricted to take only 'general' integer values.

BIP (Binary Integer Programming) Model – All decision variables are restricted to take only binary values. These models are also called 0-1 IP models.

MIP (Mixed Integer Programming) Model – Some decision variables are restricted to take integer values and the others can take both integer or fractional values.

BIP and IP can be Pure or Mixed models depending on whether or not some decision variables can take fractional values.

What type (LP, IP, BIP, MIP) are these models?

Model I

Maximise: $Z = 30x_1 + 40.5x_2$

Subject to: $x_1 + x_2 \geq 6$

$2x_1 + 3x_2 \leq 20$

$200x_1 + 100x_2 \leq 7500$

$x_1 \geq 0$ is number of TVs

$x_2 \geq 0$ is number of PCs

Model II

Maximise: $Z = 30x_1 + 40.5x_2 + 10x_3 + 8.3x_4$

Subject to: $x_1 + x_2 \geq 1$

$x_1 + x_3 \leq 1$

$x_3 - x_4 \leq 0$

$200x_1 + 100x_2 \leq 80$

$x_i \in \{0,1\}$ to buy or not item i

Model III

Minimise: $Z = 2.5x_1 + 2.1x_2 + 10y_1 + 4y_2$

Subject to: $x_1 + x_2 = 40$

$x_1 \leq 200y_1$

$x_2 \leq 200y_2$

$x_i \geq 0$ is number of trucks type i

$y_i \in \{0,1\}$ to use or not trucks type i

Model IV is like to Model III but...

$x_1 \geq 0$ is liters of petrol

$x_2 \geq 0$ is liters of diesel

$y_1 \in \{0,1\}$ is use or not of petrol

$y_2 \in \{0,1\}$ is use or not of diesel

Solving Optimization Problems

- Identify: data, decision variables, objective function and constraints.
- The problem should be well-defined with a clear and precise statement, but it is likely that some assumptions will need to be made.
- If the problem is ill-defined, exploration of the scenario might be required to define the problem well and then develop the model and optimization technique.
- Option 1: develop the spreadsheet model 'intuitively', the mathematical programming (algebraic) model can be 'extracted' from the spreadsheet model.
- Option 2: write the mathematical programming model, then implement it to solve the problem.

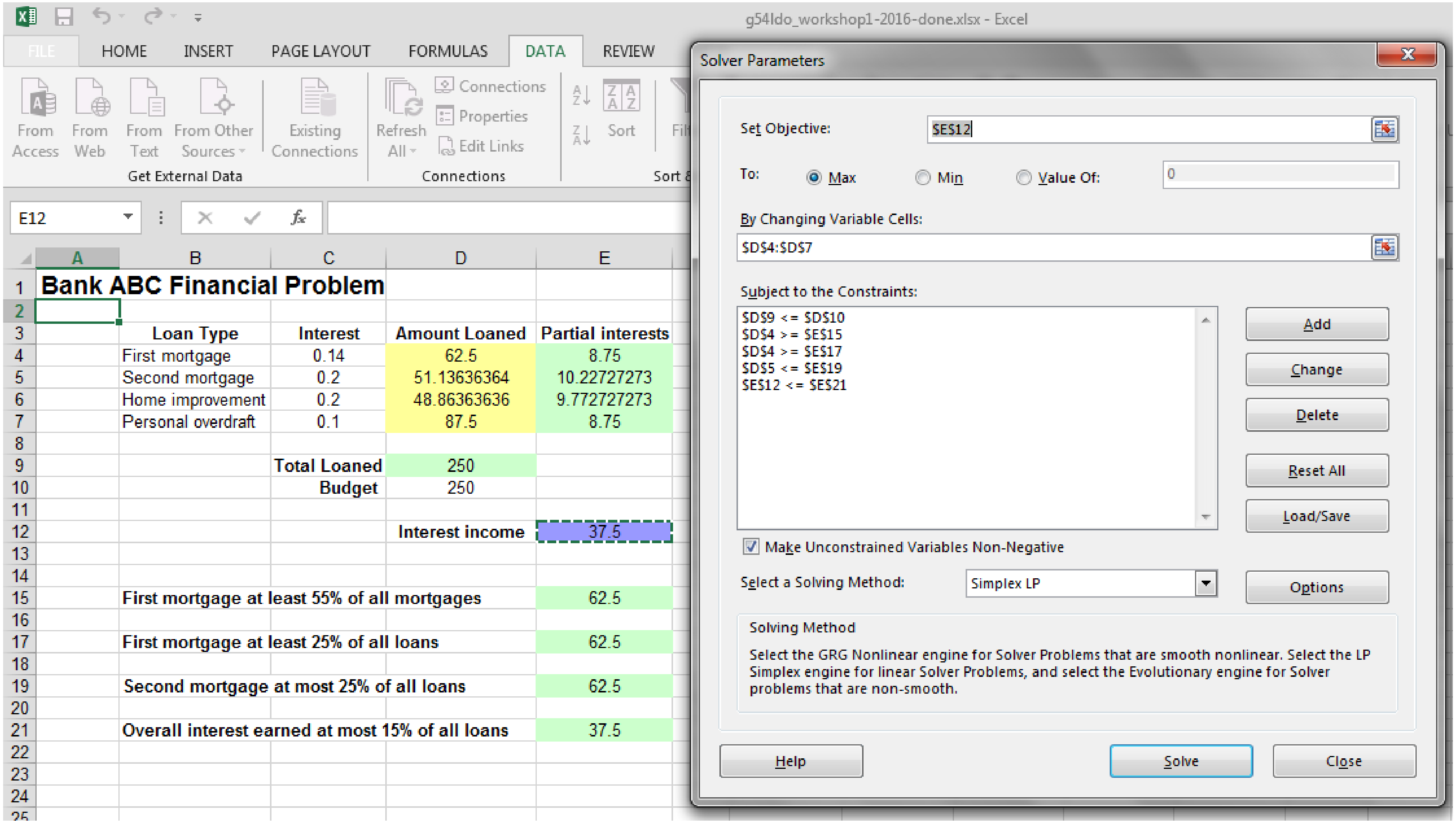
MS Excel Spreadsheet Solver

Excel is one of the modelling and optimization software tools more widely used in industry and businesses.

The Excel skills required as a basis for data analysis and modelling are the following ones:

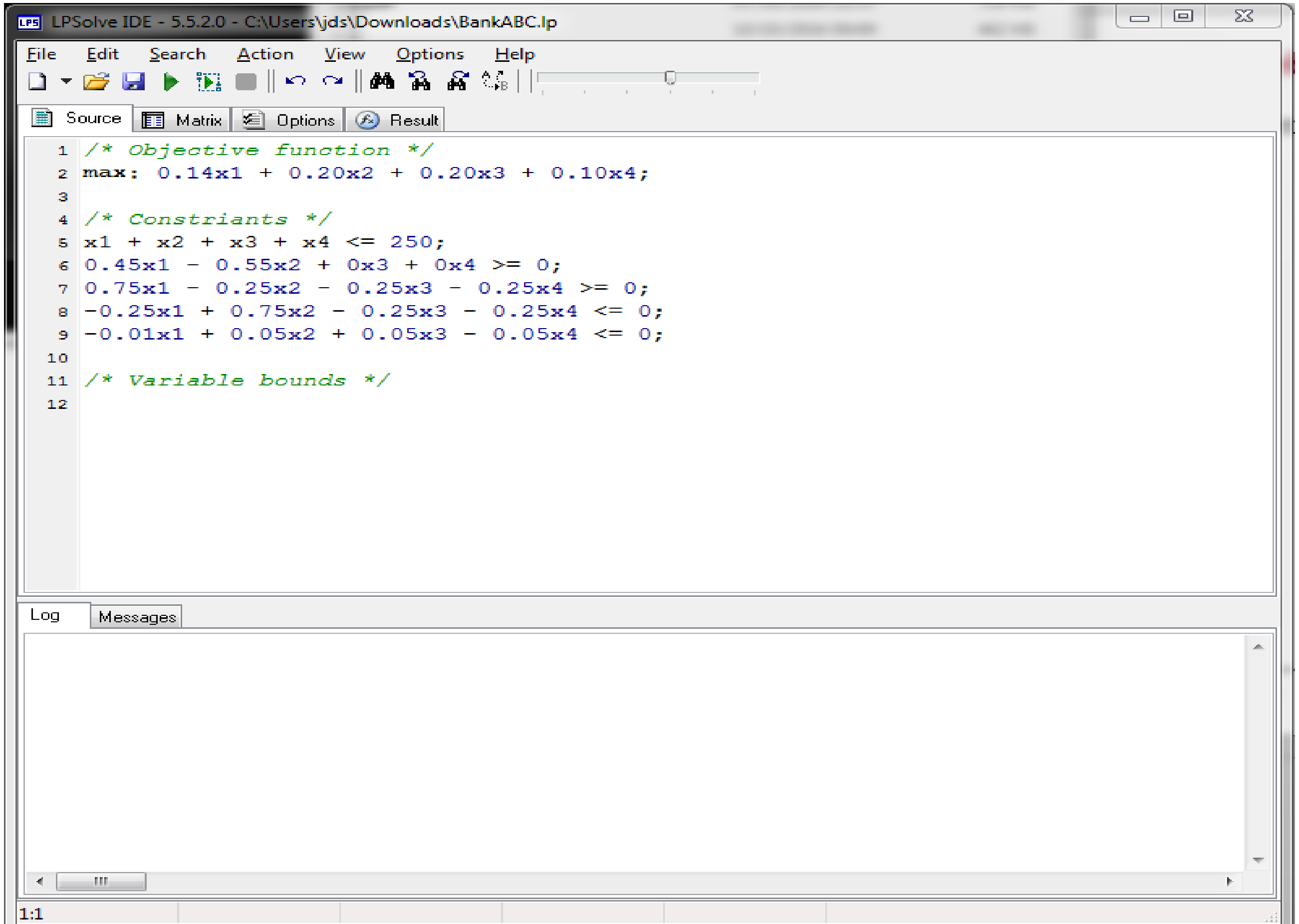
- Navigate around the spreadsheet
- Enter data (text, numbers, etc.)
- Format cells for data calculations
- Editing the content of cells
- Inserting/deleting rows, columns, group of cells, etc.
- Use of basic formulas and functions

Spreadsheet model for the Bank ABC optimization problem



LP-Solve Mathematical Programming (algebraic) Solver

LP-Solve is free (GNU lesser general public license) solver for algebraic optimization models.



Other Mathematical Programming Solvers

Many other optimization solvers available (free and commercial) including: AIMMS, GAMS, Gurobi, Lindo/Lingo, Cplex, Xpress, etc.

TORA System (Taha, 2007) and IOR Tutorial (Hillier and Lieberman, 2010) are basic tutorial and solver software tools for optimization.

All the above solvers take as input the algebraic formulation of the model to be solved. Some solvers have a more specific modelling language associated to them.

Various computational intelligence techniques are applied to tackle optimization problems.

What Skills Do I Need For This Module?

General Analytical

Basic Excel

Basic Algebra

What Skills Will I Learn /Practice in This Module?

Modelling

Algebraic

Spreadsheet

Communication

Problem Solving