| Please check the examination deta               | ails below            | before ente | ering your candidate information |
|---|-----------------------|-------------|----------------------------------|
| Candidate surname                               |                       |             | Other names                      |
| Pearson Edexcel International Advanced Level    | Centro                | e Number    | Candidate Number                 |
| Thursday 9 Ja                                   | anu                   | ary         | 2020                             |
| Afternoon (Time: 1 hour 30 minu                 | ıtes)                 | Paper Re    | Reference WME03/01               |
| Mathematics                                     |                       |             |                                  |
| International Advance<br>Mechanics M3           | d Sul                 | osidiar     | ry/Advanced Level                |
| You must have:<br>Mathematical Formulae and Sta | tistical <sup>-</sup> | Tables (Blu | Total Marks                      |

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
   there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take g = 9.8 m s<sup>-2</sup>, and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







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2. A particle, P, of mass 0.5 kg is moving along the positive x-axis. At time t seconds,  $t \ge 0$ , P is x metres from the origin O and is moving away from O with velocity  $v \text{ m s}^{-1}$ , where  $v = \frac{1}{(4x+3)}$ 

When t = 0, P is at O.

(a) Find the distance of P from O when t = 2

**(5)** 

(b) Find the magnitude of the resultant force acting on P when t = 2

(5)

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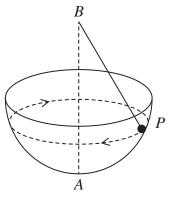


Figure 1

Figure 1 shows a hemispherical bowl of internal radius a fixed with its open plane face uppermost and horizontal. The lowest point of the bowl is A. A light inextensible string of length  $a\sqrt{3}$  has one end fixed to the point B, where B is vertically above A and AB = 2a. A particle, P, of mass m is attached to the other end of the string.

The particle moves in a horizontal circle on the smooth inner surface of the bowl with constant angular speed  $\omega$ . The string remains taut and the particle remains in contact with the bowl throughout the motion.

(a) Find, in terms of m, a,  $\omega$  and g, the tension in the string.

**(7)** 

(b) Show that 
$$\omega \geqslant \sqrt{\frac{2g}{3a}}$$

**(4)** 

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4. A light elastic string has modulus of elasticity 2mg and natural length l. One end of the string is fixed to a point A on a rough plane inclined to the horizontal at angle  $\alpha$ , where  $\sin \alpha = \frac{3}{5}$ . A particle, P, of mass m is attached to the other end of the string. Initially P is held at rest on the plane at the point B, where B is above A and  $AB = \frac{1}{2}l$ . The string lies along a line of greatest slope of the plane.

The particle P is released from rest and moves down the plane along the line of greatest slope. The coefficient of friction between P and the plane is  $\mu$ , where  $\mu < \tan \alpha$ .

Given that P comes to instantaneous rest at the point C, where AC = l + e,

(a) show that

$$\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)} \tag{6}$$

Given that e = l

| (b) find the ma |      | C |  |
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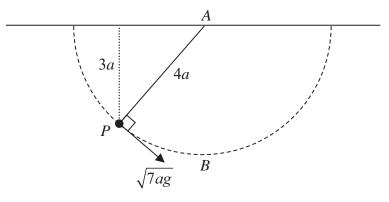


Figure 2

One end of a light inextensible string of length 4a is attached to a fixed point A on a horizontal ceiling. A particle, P, of mass m is attached to the other end of the string. The particle is held in equilibrium at a vertical distance 3a below the ceiling, with the string taut. The particle is then projected with speed  $\sqrt{7ag}$ , in the direction perpendicular to the string, in the vertical plane containing A and the string, as shown in Figure 2. In the subsequent motion the string remains taut.

(a) Find the speed of P at the instant before it hits the ceiling.

**(4)** 

The point B is the lowest point of the path of P. The first time P passes through B the tension in the string is  $T_1$  and the second time P passes through B the tension in the string is  $T_2$ 

Given that the coefficient of restitution between P and the ceiling is  $\frac{1}{2}$ 

(b) find the ratio  $T_1$ :  $T_2$  in its simplest form.

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| 6. | A particle, $P$ , of mass 0.4kg is attached to the midpoint of a light elastic spring of natulength 0.8 m and modulus of elasticity 20 N. The ends of the spring are attached to fixed points $A$ and $B$ on a smooth horizontal table, where $AB = 1.2$ m. Initially $P$ is at r at the midpoint $O$ of $AB$ where $AOB$ is a straight line. The particle $P$ now receives impulse of magnitude 2 Ns so that $P$ starts to move directly towards $B$ . | the<br>est         |
|----|---|--------------------|
|    | (a) Prove that <i>P</i> moves with simple harmonic motion.  | (4)                |
|    | (b) Write down, in terms of $\pi$ , the period of the motion.   | (1)                |
|    | (c) Find the amplitude of the motion.   | (3)                |
|    | (d) Find the length of time in each complete oscillation for which $AP$ is greater than 0.5   | m.<br>( <b>5</b> ) |
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7.

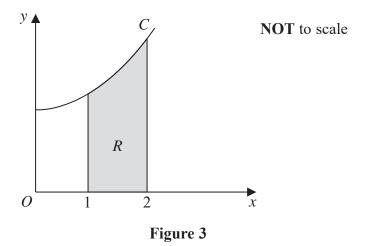


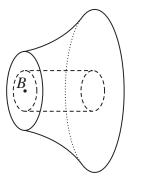
Figure 3 shows part of the curve C with equation  $y = x^2 + 4$ . The shaded region R is bounded by C, the line with equation x = 1, the x-axis and the line with equation x = 2

The unit of length on each axis is one centimetre.

A uniform wooden solid, S, is made in the shape formed by rotating the region R through  $360^{\circ}$  about the x-axis.

- (a) Using algebraic integration,
  - (i) show that the volume of S is  $\frac{613\pi}{15}$  cm<sup>3</sup>
  - (ii) find, to 3 significant figures, the distance of the centre of mass of S from O.

**(8)** 



NOT to scale

Figure 4

A solid,  $S_1$ , is formed by removing a solid cylinder of radius 3 cm and length 1 cm from S. A metal cylinder, of radius 3 cm and length 1 cm is placed in the resulting hole to form a new solid T, as shown in Figure 4. The axis of the metal cylinder coincides with the axis of symmetry of  $S_1$ . The point B is the centre of the smaller plane face of T. The mass per unit volume of  $S_1$  is M and the mass per unit volume of the metal cylinder is S.

(b) Find the distance of the centre of mass of T from B.

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