Please check the examination details below	before entering your candidate information
Candidate surname	Other names
Pearson Edexcel International Advanced Level	e Number Candidate Number
Friday 15 Januar	ry 2021
Morning (Time: 1 hour 30 minutes)	Paper Reference <b>WFM02/01</b>
Mathematics	
International Advanced Sub Further Pure Mathematics F	·
You must have: Mathematical Formulae and Statistical	Total Marks

Candidates may use any calculator permitted by Pearson regulations.

Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
   there may be more space than you need.
- You should show sufficient working to make your methods clear.
   Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1. The transformation T from the z-plane, where z = x + iy, to the w-plane, where w = u + iv, is given by

$$w = \frac{z + pi}{iz + 3} \qquad z \neq 3i \quad p \in \mathbb{Z}$$

The point representing  $i(1 + \sqrt{3})$  is invariant under T.

Determine the value of p.

_		

Question 1 continued	blank
Question I continued	
	Q1
(Total 3 marks)	



2. (a) Show that, for r > 0

$$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)}$$

**(2)** 

(b) Hence show that

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{n(an+b)}{c(n+1)(n+2)}$$

where a	a, b	and a	are	integers	to	be	determined.
---------	------	-------	-----	----------	----	----	-------------

	( A	`
- 1	4	h
٠,	т	,


	L b
estion 2 continued	



Question 2 continu	ed		

Question 2 continued	blank
Question 2 continued	
	02
	<b>Q2</b>
(Total 6 marks)	



**(7)** 

3. Use algebra to obtain the set of values of $x$ for which	h
---	---

$$|x^2 + x - 2| < \frac{1}{2}(x + 5)$$

ı	2 \	,		

nestion 3 continued	



Question 3 continued		

Question 3 continued	Leave
	02
	Q3
(Total 7 marks)	



4. (a) Show that the substitution  $y^2 = \frac{1}{z}$  transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 3xy^3 \qquad y \neq 0 \tag{I}$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 4z = -6x\tag{II}$$

(3)

(b) Obtain the general solution of differential equation (II).

**(5)** 

(c) Hence obtain the general solution of differential equation (I), giving your answer in the form  $y^2 = f(x)$ 

**(1)** 



	I
	1
uestion 4 continued	



Question 4 continued		

Q4 (Total 9 marks)	Question 4 continued	blank
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
		_
(Total 9 marks)		_ <b>Q4</b>
	(Total 9 mark	s)



5. Given that

$$(2 - x^2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5x \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 3y$$

(a) show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \frac{1}{(2-x^2)} \left( 2x \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \left( 1 - 5 \frac{\mathrm{d}y}{\mathrm{d}x} \right) - 5 \left( \frac{\mathrm{d}y}{\mathrm{d}x} \right)^2 + 3 \frac{\mathrm{d}y}{\mathrm{d}x} \right)$$

**(5)** 

Given also that y = 3 and  $\frac{dy}{dx} = \frac{1}{4}$  at x = 0

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in  $x^3$ 

**(4)** 

estion 5 continued	



Question 5 continued		

	blank
	Q5
(Total 9 marks)	



(a) Determine the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = 6\cos x$$



(b) Find the particular solution for which y = 0 and  $\frac{dy}{dx} = 0$  at

_	= 0	aı	x = 0	
r				
				(5)



	Leav
	blan
Question 6 continued	



Question 6 continued	
Question o continueu	

Question 6 continued	blank
Question o continueu	
	Q6
(Total 12 marks)	



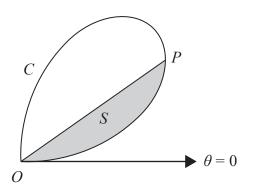


Figure 1

Figure 1 shows a sketch of curve C with polar equation

$$r = 3\sin 2\theta$$
  $0 \leqslant \theta \leqslant \frac{\pi}{2}$ 

The point P on C has polar coordinates  $(R, \phi)$ . The tangent to C at P is perpendicular to the initial line.

(a) Show that  $\tan \phi = \frac{1}{\sqrt{2}}$ 

**(4)** 

(b) Determine the exact value of R.

**(2)** 

The region S, shown shaded in Figure 1, is bounded by C and the line OP, where O is the pole.

(c) Use calculus to show that the exact area of S is

$$p \arctan \frac{1}{\sqrt{2}} + q \sqrt{2}$$

where p and q are constants to be determined.

Solutions relying entirely on calculator technology are not acceptable.

**(7)** 



estion 7 continued	



Question 7 continued	
Question / continued	

Question 7 continued	blank
	<b>Q7</b>
(Total 13 marks)	
(Total 15 marks)	



- **8.** Given that  $z = e^{i\theta}$ 
  - (a) show that  $z^n + \frac{1}{z^n} = 2\cos n\theta$

where n is a positive integer.

**(2)** 

(b) Show that

$$\cos^6\theta = \frac{1}{32} \left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$$

**(5)** 

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0$$
  $0 \le \theta \le \pi$ 

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} (32\cos^6\theta - 4\cos^2\theta) d\theta$$

Solutions relying entirely on calculator technology are not acceptable.

**(5)** 



nestion 8 continued	
estion o continueu	



Question 8 continued		

	L b
uestion 8 continued	0
acstron o continued	
	1



		Q
	(Total 16 marks)	
END	TOTAL FOR PAPER: 75 MARKS	