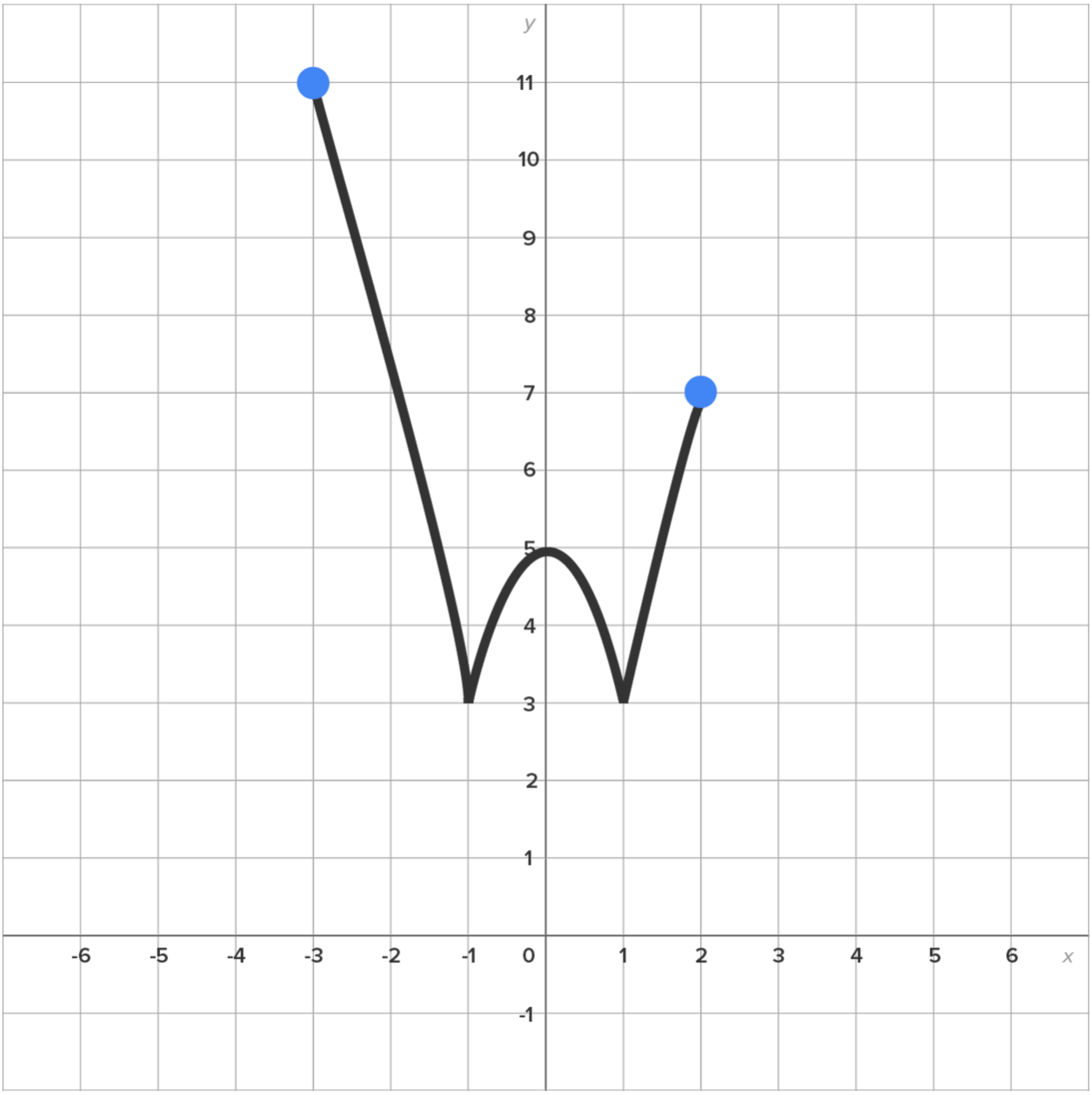
Practice Milestone

Calculus I — Practice Milestone 4

Taking this practice test is a stress-free way to find out if you are ready for the Milestone 4 assessment. You can print it out and test yourself to discover your strengths and weaknesses. The answer key is at the end of this Practice Milestone.

1.

Identify all of the global and local extrema of the graph.



- **a.)** 11 is a global maximum of $f(x)$ at $x = -3$.
  3 is a global and local minimum of $f(x)$ at $x = -1$ and $x = 1$.
  5 is a local maximum at $x = 0$.

- **b.)** 11 is a global and local maximum of $f(x)$ at $x = -3$.
  3 is a local minimum of $f(x)$ at $x = -1$ and $x = 1$.
  5 is a global and local maximum at $x = 0$.

- **c.)** 11 is a global maximum of $f(x)$ at $x = -3$.
  3 is a global and local minimum of $f(x)$ at $x = -1$ and $x = 1$.

  5 is a global and local maximum at $x = 0$.

- **d.)** 11 is a global maximum of $f(x)$ at $x = -3$.

3 is a global and local minimum of $f(x)$ at $x = -1$ and $x = 1$.

2.

Find all the critical numbers of $f(x) = x^4 - 2x^2 + 5$.

- **a.)** $x = 0$ and $x = 1$
- **b.)** $x = -1$ and $x = 1$
- **c.)** $x = -1$ and $x = 0$
- **d.)** $x = -1$, $x = 0$, and $x = 1$

3.

Find all the critical numbers of $y = -x^4 - 4x^3 - 4x^2 + 4$, then determine the local minimum and maximum points by using a graph.

- **a.)** Critical numbers: $x = -2$, $x = 0$, and $x = 1$

  $(-2,4)$ and $(0,4)$ are local maximums and $(1,-5)$ is a local minimum.

- **b.)** Critical numbers: $x = -2$ and $x = -1$

  $(-2,4)$ is a local maximum and $(-1,3)$ is a local minimum.

- **c.)** Critical numbers: $x = -2$, $x = -1$, and $x = 0$

  $(-2,4)$ and $(0,4)$ are local maximums and $(-1,3)$ is a local minimum.

- **d.)** Critical numbers: $x = 0$, $x = 1$, $x = 2$

  $(0,4)$ is a local maximum, $(2,-60)$ is a local minimum, and $(1,-5)$ is a local minimum.

4.

Find the global maximum and minimum points of the function $f(x) = 2x^3 - 54x + 12$ on the interval $[0, 7]$.

- a.) The global minimum is 12 at $x = 0$. The global maximum is 320 at $x = 7$.
- b.) The global minimum is -96 at $x = 3$. The global maximum is 320 at $x = 7$.
- c.) The global minimum is -96 at $x = 3$. The global maximum is 12 at $x = -3$.
- d.) The global minimum is 12 at $x = -3$. The global maximum is 320 at $x = 7$.

5.

Determine if the requirements for Rolle's theorem are met by the function $f(x) = x^2 - 3x + 10$ on the interval $[0, 3]$. If so, find the values of *c* in $(0, 3)$ guaranteed by the theorem.

- a.) $f(x)$ is not continuous on $[0, 3]$ so Rolle's Theorem cannot be applied.
- b.)
  $f(x)$ is a polynomial so it is continuous on $[0, 3]$ and differentiable on $(0, 3)$. When evaluated, $f(0) = 10$ and $f(3) = 10$. Therefore, $f(a) = f(b)$ and the conditions of Rolle's theorem are met.

  The value guaranteed by Rolle's theorem is $c = \frac{3}{2}$.
- c.)
  $f(x)$ is a polynomial so it is continuous on $[0, 3]$ but $f'(x)$ is not differentiable on $(0, 3)$; therefore, Rolle's Theorem cannot be applied.
- d.) $f(x)$ a polynomial so it is continuous on $[0, 3]$ and differentiable on $(0, 3)$
  .

  By Rolle's theorem, the value is $c = -3$.

6.

Determine if the conditions of the mean value theorem are met by the function $f(x) = 2x^3 - 4x + 5$ on $[0, 3]$. If so, find the values of *c* in $(0, 3)$ guaranteed by the theorem.

- **a.)** $f(x)$ is a polynomial and therefore continuous on $[0, 3]$ and differentiable on the interval $(0, 3)$.

  The value guaranteed by the mean value theorem is $c = \frac{\sqrt{6}}{3}$.

- **b.)** $f(x)$ is a polynomial and therefore continuous on $[0, 3]$ and differentiable on the interval $(0, 3)$.

  The values guaranteed by the mean value theorem are $c = -\sqrt{3}$ and $c = \sqrt{3}$.

- **c.)** $f(x)$ is a polynomial and therefore continuous on $[0, 3]$ and differentiable on the interval $(0, 3)$.

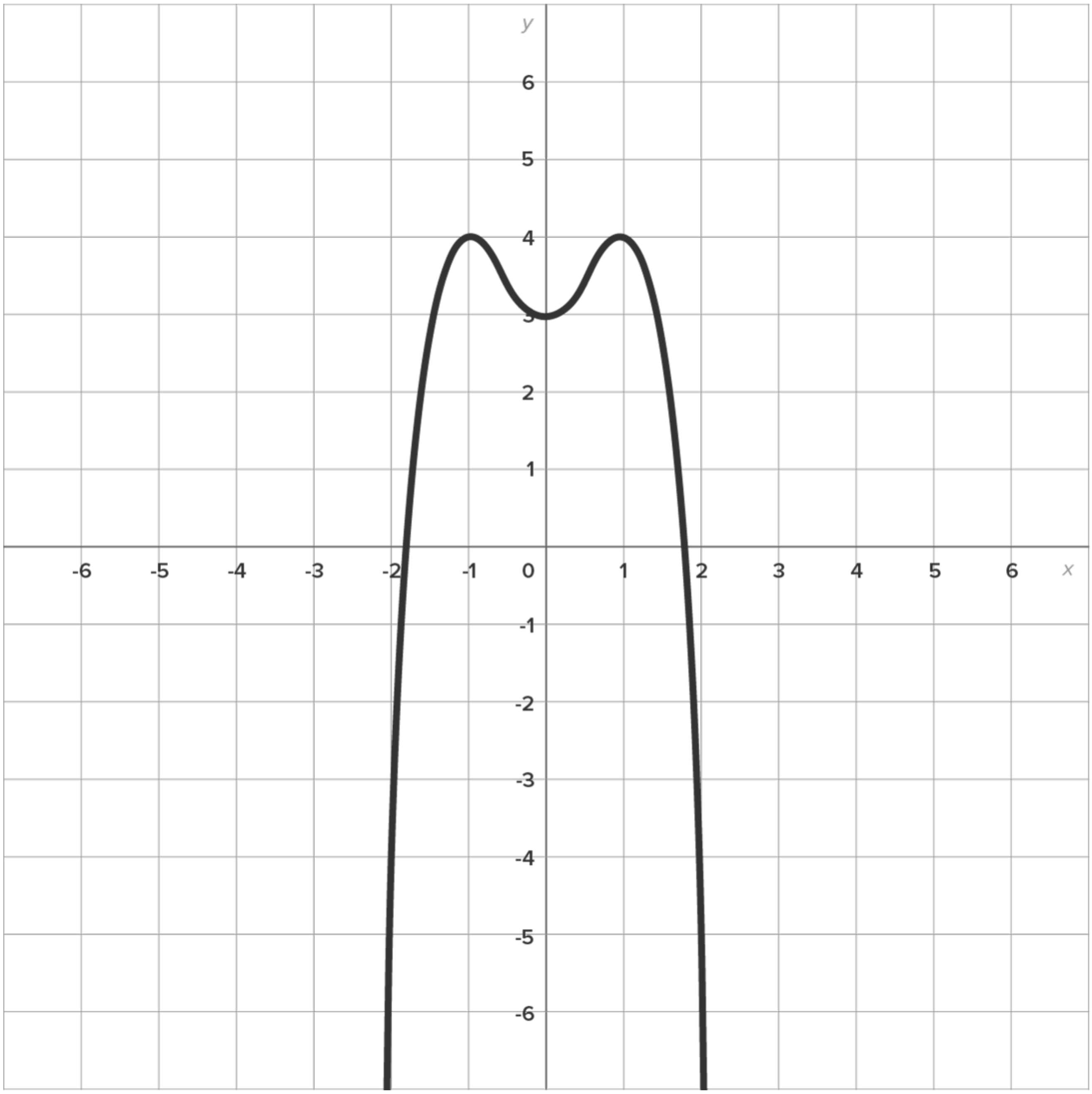
  The values guaranteed by the mean value theorem are $c = -\frac{\sqrt{6}}{3}$ and $c = \frac{\sqrt{6}}{3}$.

- **d.)** $f(x)$ is a polynomial and therefore continuous on $[0, 3]$ and differentiable on the interval $(0, 3)$.

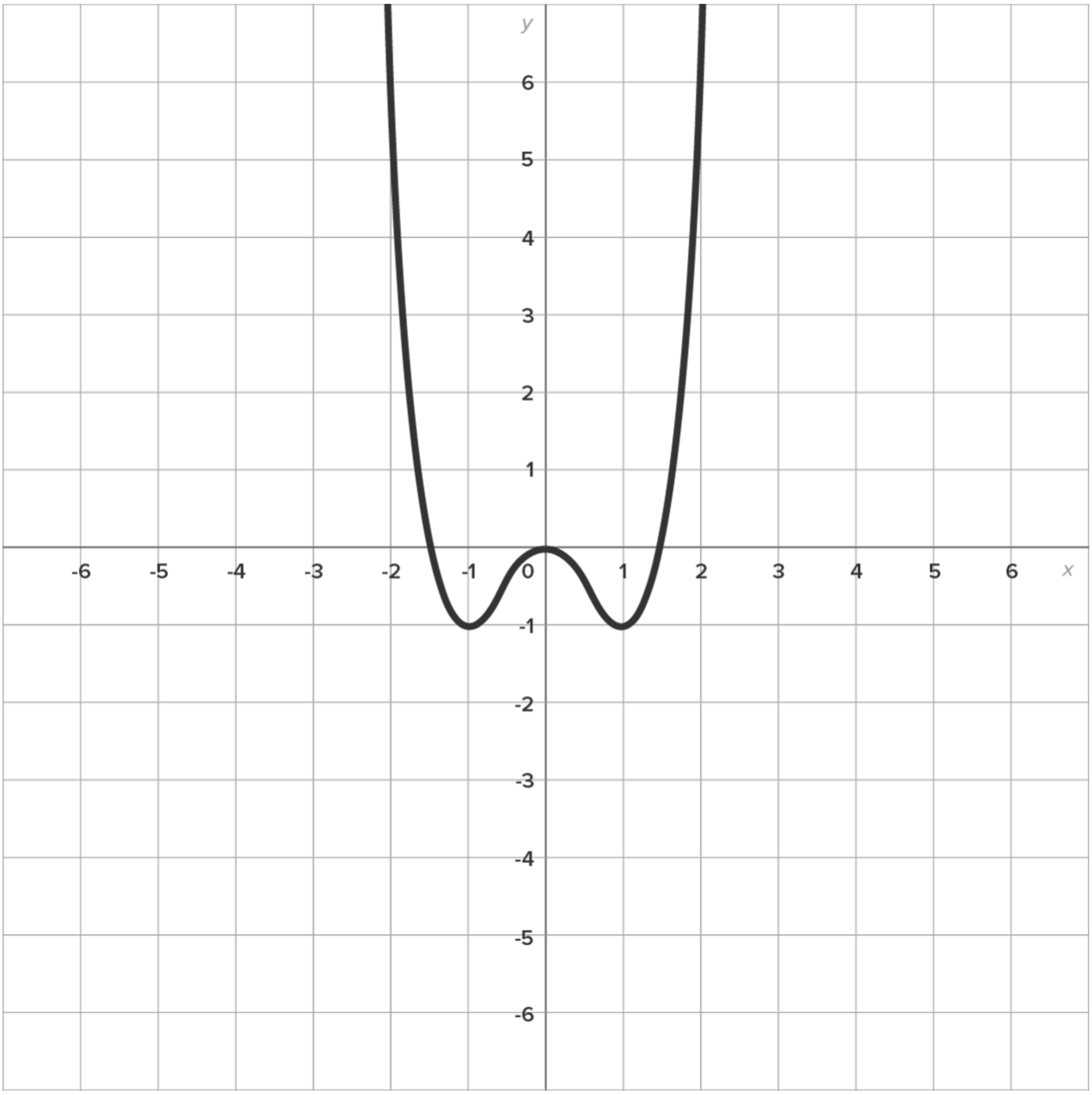
  The value guaranteed by the mean value theorem is $c = \sqrt{3}$.

7.

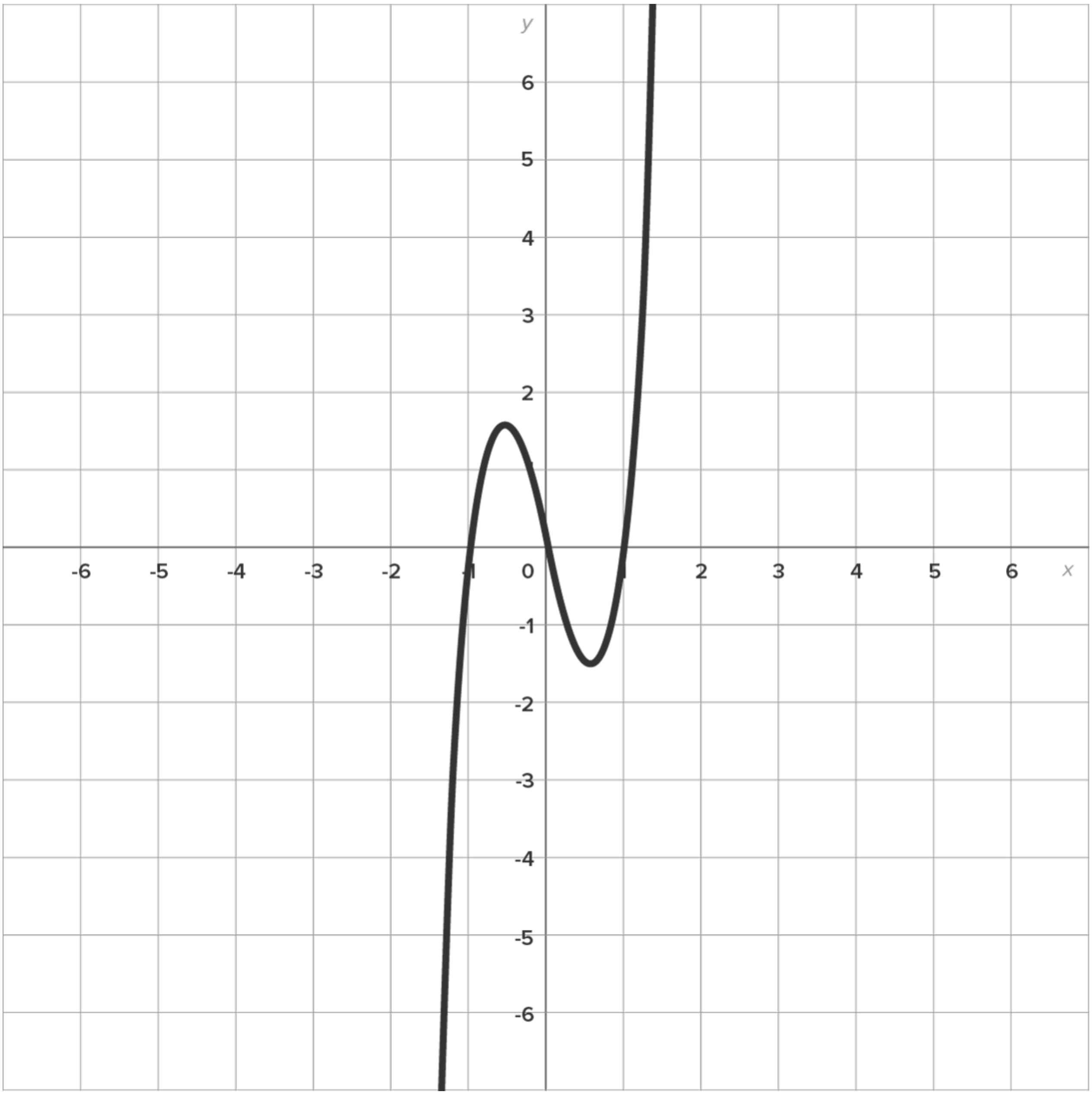
From the graph of $f(x)$, determine the graph of $f'(x)$.



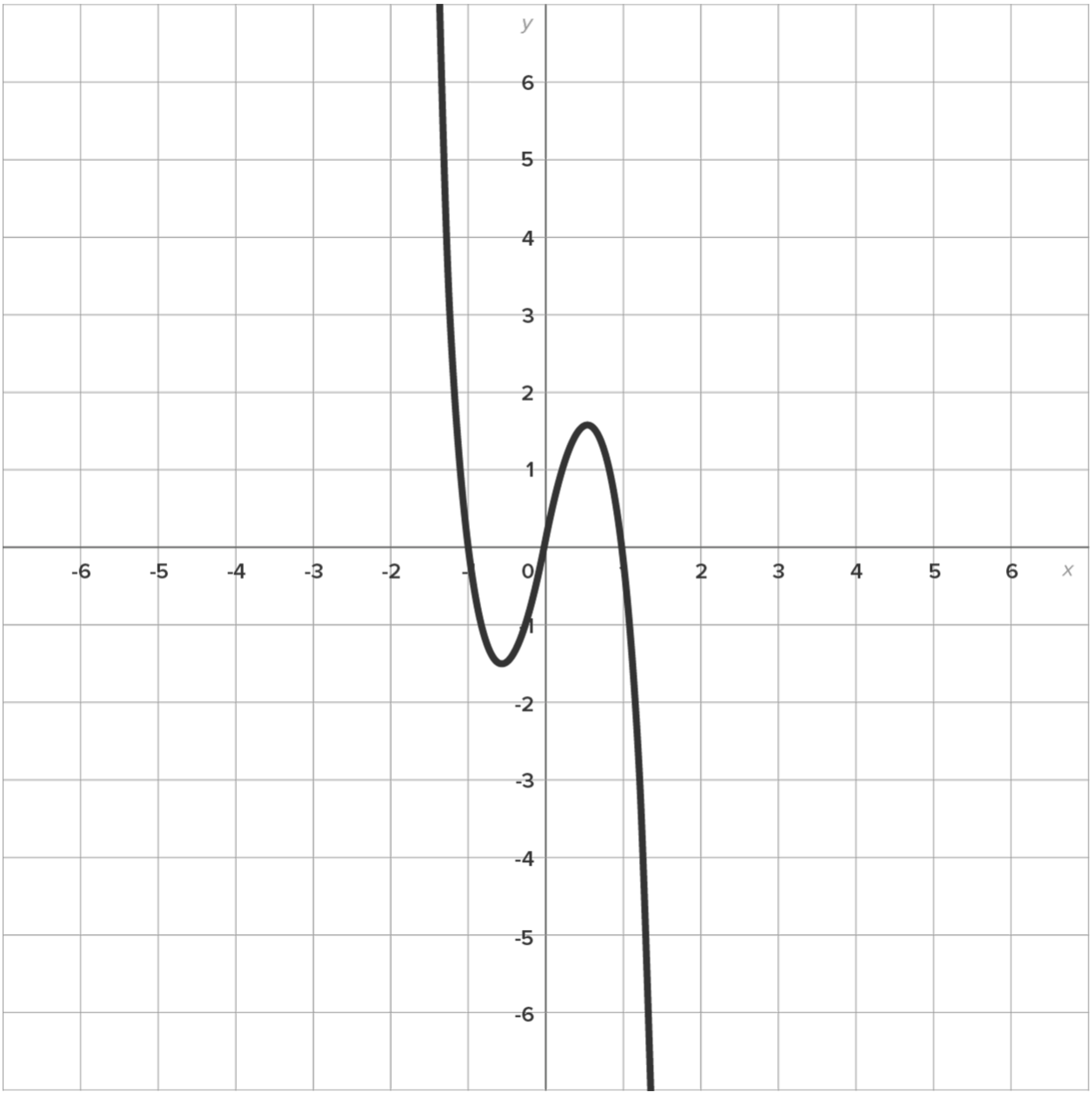
○ **a.)**



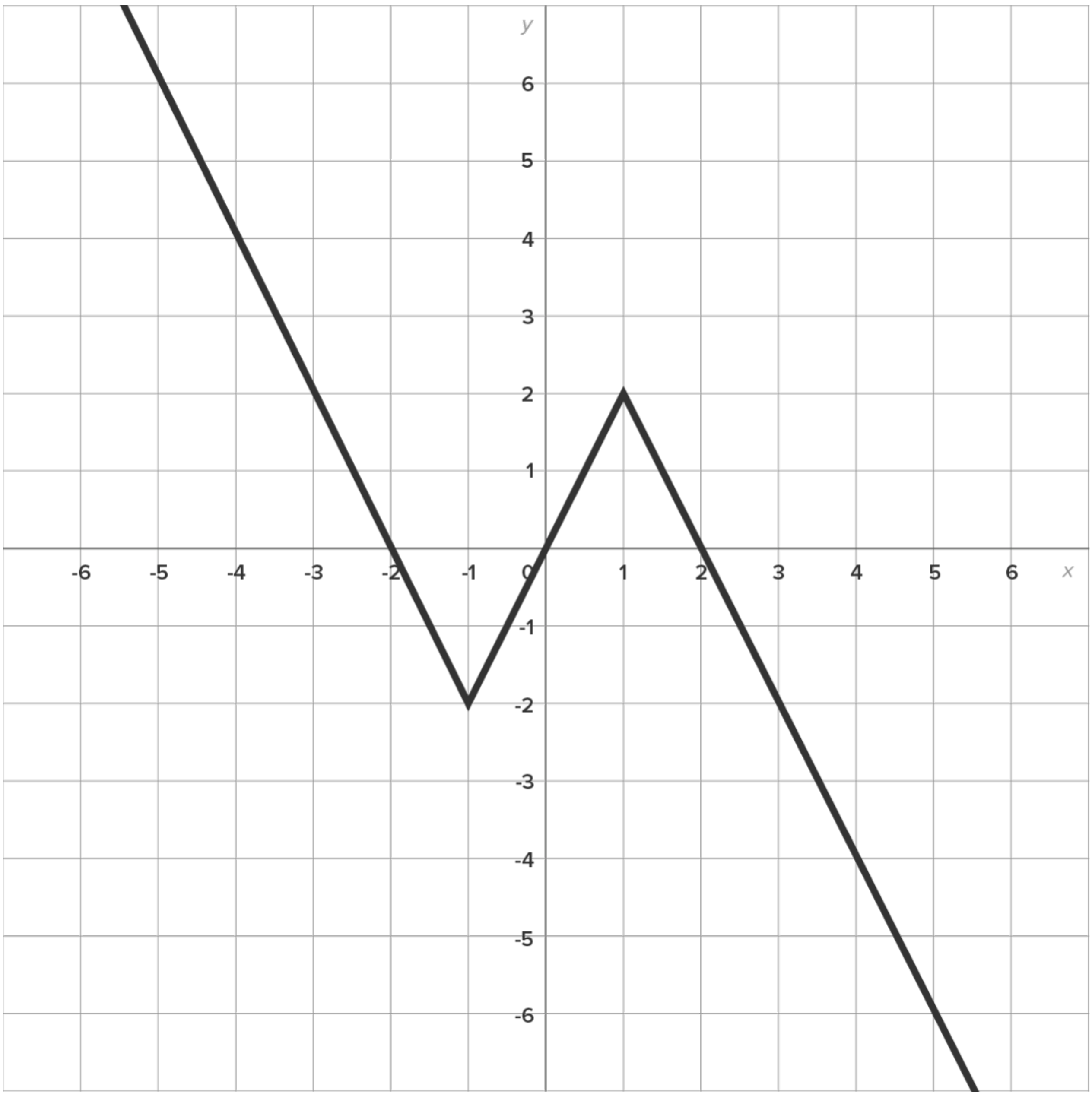
b.)



c.)



d.)


y
6
5
4
3
2
1
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
x
-1
-2
-3
-4
-5
-6

8.

Use the first derivative test to determine all local minimum and maximum points of the function $y = -2x^3 - 12x^2 + 7$.

a.) Local minimum at $(-4, -57)$
No local maximum

b.) Local minimum $(4, -313)$
Local maximum at $(0, 7)$

c.) Local minimum $(4, -313)$
No local maximum

d.) Local minimum at $(-4, -57)$
Local maximum at $(0, 7)$

9.

Determine the interval(s) over which the graph of $f(x) = x^7 + x^6 - 25x + 18$ **is concave up or concave down.**

a.) Concave up on $\left(-\frac{5}{7}, 0\right) \cup (0, \infty)$
Concave down on $\left(-\infty, -\frac{5}{7}\right)$

b.) Concave up on $\left(-\infty, -\frac{5}{7}\right) \cup (0, \infty)$
Concave down on $\left(-\frac{5}{7}, 0\right)$

c.) Concave up on $\left(-\frac{5}{7}, 0\right)$
Concave down on $\left(-\infty, -\frac{5}{7}\right) \cup (0, \infty)$

d.) Concave up on $\left(-\infty, -\frac{5}{7}\right)$
Concave down on $\left(-\frac{5}{7}, \infty\right)$

10.

Find the inflection point(s) for the function $f(x) = -x^4 + 4x^3 + 10x + 5$.

- a.) $(0,5)$
- b.) $(0,5)$ and $(2,41)$
- c.) $(0,0)$ and $(2,0)$
- d.) $(0,10)$ and $(2,26)$

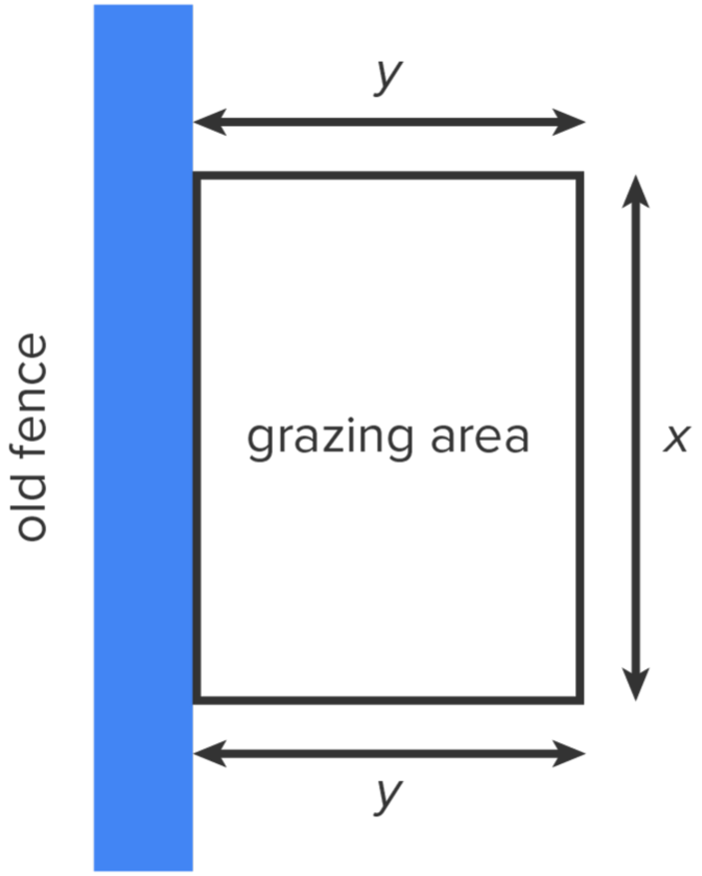
11.

Determine the local maximum and minimum values of $f(x) = -2x^3 + 3x^2 + 12x + 3$ **using the second derivative test when it applies.**

- a.) Local maximum value is -4 at $x = -1$.
  Local minimum value is 23 at $x = 2$.
- b.) Local maximum value is 18 at $x = -1$.
  Local minimum value is -18 at $x = 2$.
- c.) Local minimum value is 18 at $x = -1$.
  Local maximum value is -18 at $x = 2$.
- d.) Local minimum value is -4 at $x = -1$.
  Local maximum value is 23 at $x = 2$.

12.

A rancher plans to fence a rectangular grazing area adjacent to an existing fence. The rancher has 3,000 meters of fencing to be used to make the enclosure and no new fencing is needed on the side with the old fence.



What dimensions should be used so that the enclosed area will be maximized?

- **a.)** 750 meters by 750 meters
- **b.)** 1,500 meters by 1,500 meters
- **c.)** 750 meters by 1,500 meters
- **d.)** 800 meters by 1,400 meters

13.

Evaluate $\lim_{x \to -\infty} \frac{-12x^4 - 8x - 10}{7x^4 + 3x^2 - 5}$ **analytically.**

- **a.)** $-\infty$
- **b.)** $\frac{12}{7}$
- **c.)** $-\frac{12}{7}$
- **d.)** 0

14.

Evaluate $\lim_{x \to 4} \frac{-3x + 8}{x - 4}$ **by graphing.**

- **a.)** ∞
- **b.)** -3
- **c.)** Does not exist
- **d.)** $-\infty$

15.

Determine all the vertical asymptotes of $f(x) = \frac{x^2 + x - 2}{x^2 + 7x + 10}$.

- **a.)** $x = 2$
- **b.)** $x = -5, x = -2$
- **c.)** $x = 2, x = 5$
- **d.)** $x = -5$

16.

Determine the slant or nonlinear asymptote of $f(x)=\dfrac{2x^3+x^2+3x+9}{x+4}$.

- a.) $y=2$
- b.) $y=2x+1$
- c.) $y=2x^2-7x+31$
- d.) $y=-4$

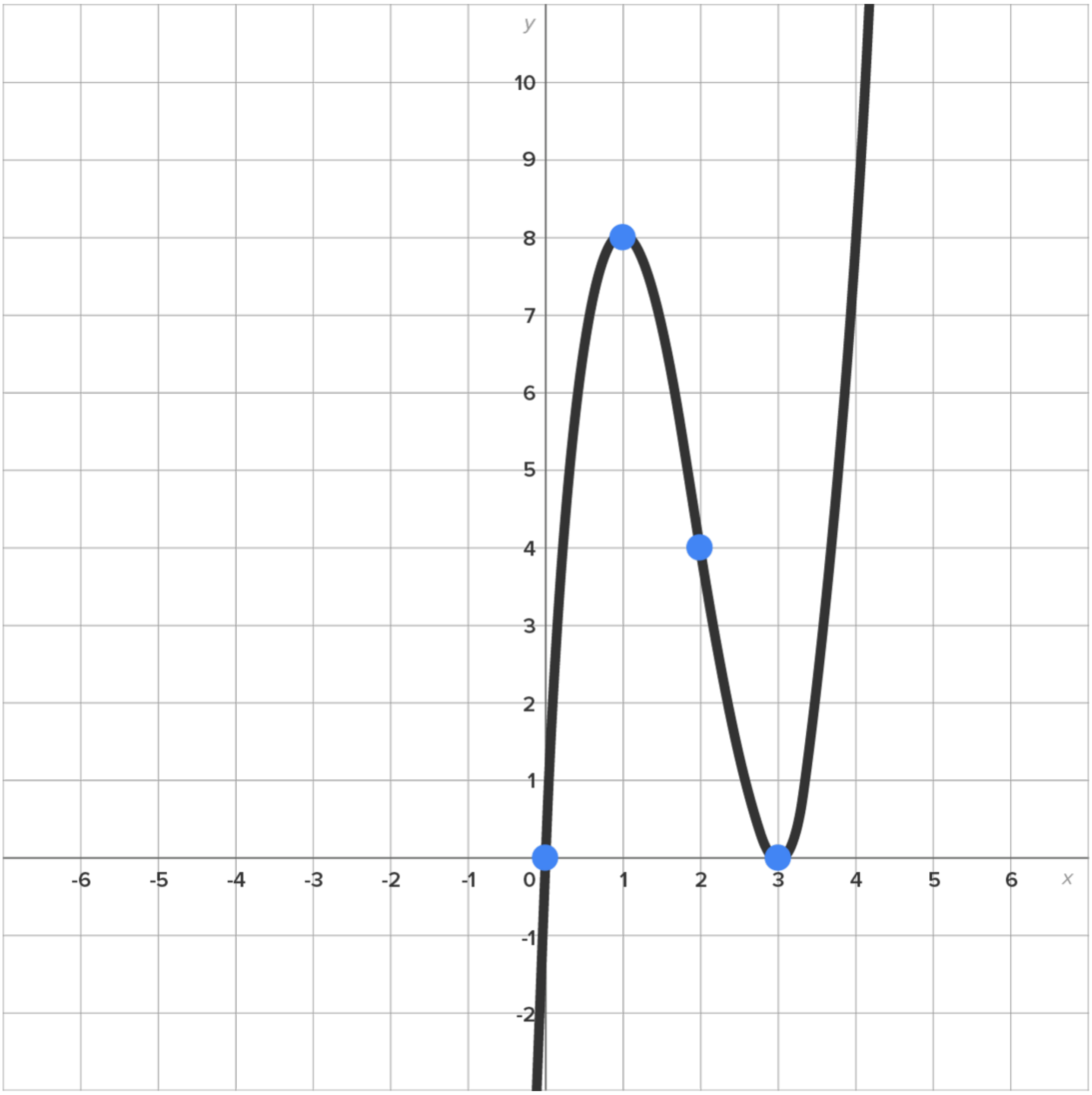
17.

For $f(x)=-2x^3-12x^2-18x$**, find the following:**

A. Domain
B. Asymptotes
C. y- and x-intercepts
D. Where the function is increasing or decreasing
E. Concave down or concave up
F. Relative extrema
G. Inflection points
H. Graph of the curve

Carefully verify all of the information is correct when making your selection.

- a.)
  A) Domain is all reals
  B) No asymptotes
  C) y-intercept $(0,0)$, x-intercepts are $(3,0)$, $(3,0)$, $(0,0)$
  D) Increasing on $(-\infty,-3)\cup(-1,\infty)$, decreasing on $(-3,-1)$
  E) Concave up on $(-\infty,-1)$, concave down on $(-1,\infty)$
  F) Relative min of 0 at $x=-3$, relative max of 4 at $x=-1$ and max of 8 at $x=-1$
  G) No inflection points
  H)



b.)

A) Domain is all reals

B) No asymptotes

C) y-intercept $(0,0)$, x-intercepts are $(3,0)$, $(3,0)$, $(0,0)$

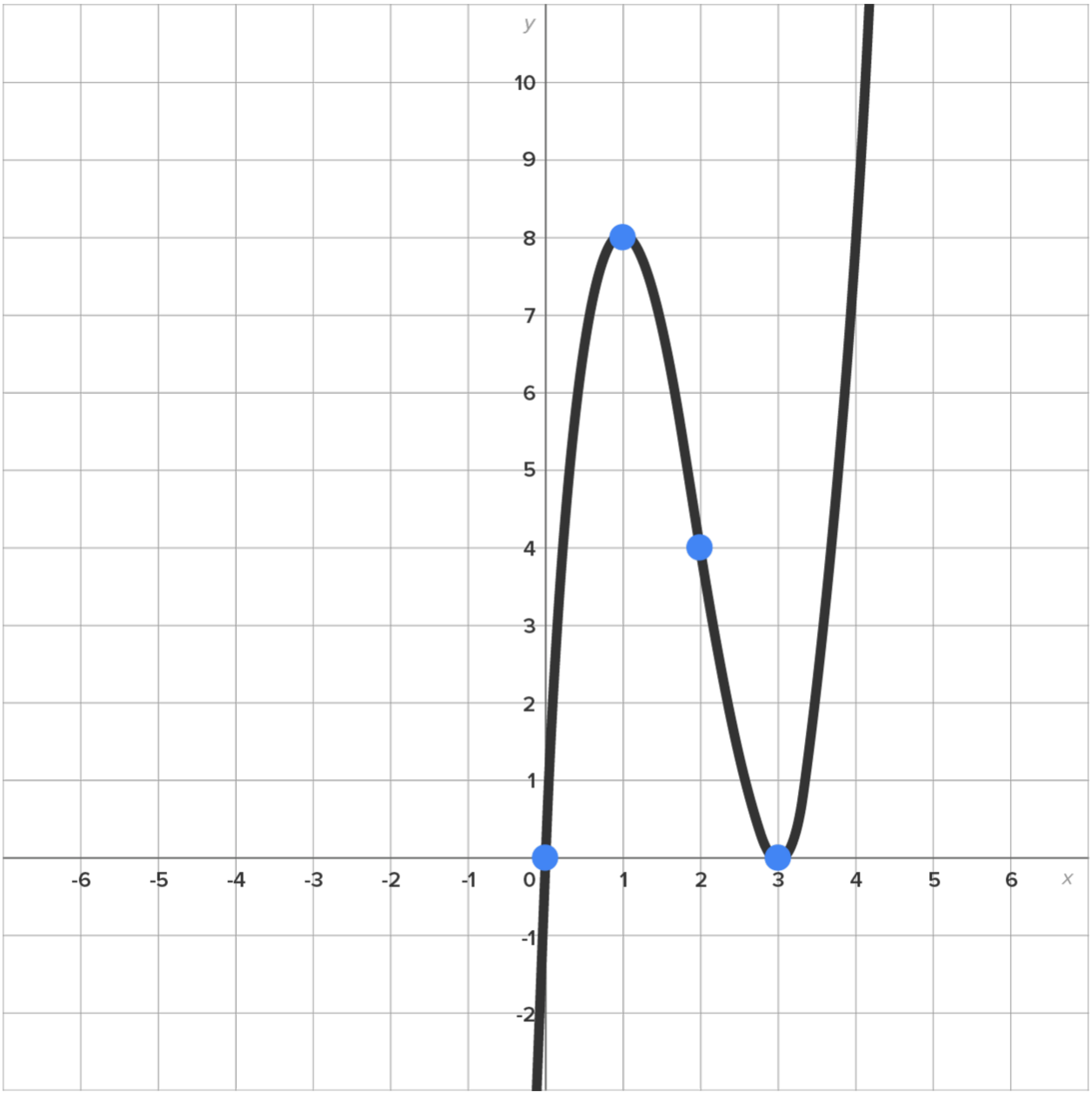
D) Decreasing on $(-\infty,-3)\cup(-1,\infty)$, increasing on $(-3,-1)$

E) Concave up on $(-\infty,-2)$, concave down on $(-2,\infty)$

F) Relative min of 0 at $x=-3$, relative max of 8 at $x=-1$

G) Inflection point at $(-2,4)$

H)



○ c.)

A) Domain is all reals

B) No asymptotes

C) y-intercept $(0,0)$, x-intercepts are $(-3,0)$, $(-3,0)$, $(0,0)$

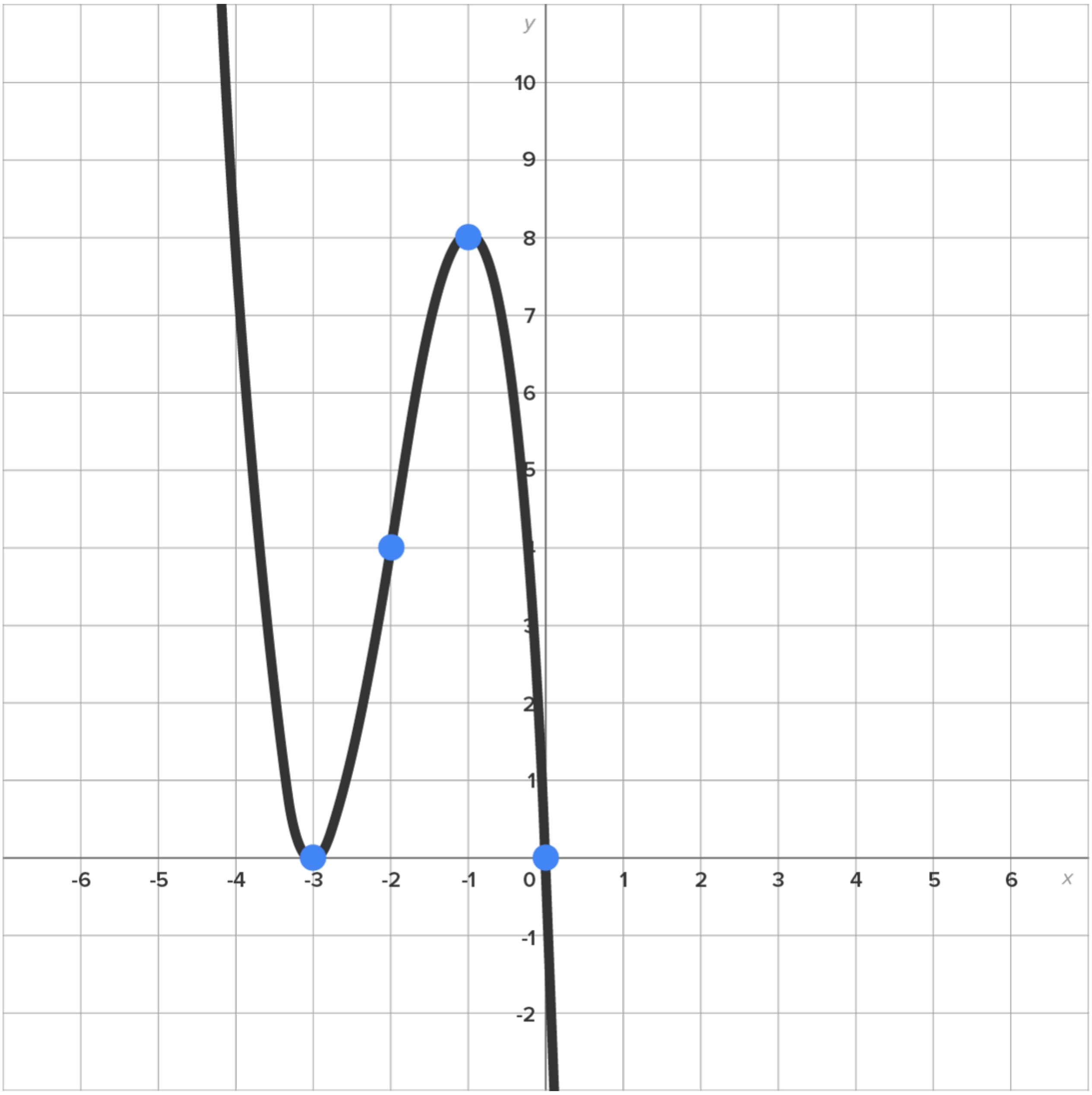
D) Increasing on $(-\infty,-3)\cup(-1,\infty)$, decreasing on $(-3,-1)$

E) Concave up on $(-\infty,-1)$, concave down on $(-1,\infty)$

F) Relative min of 0 at $x=-3$, relative max of 4 at $x=-1$ and max of 8 at $x=-1$

G) No inflection points

H)



○ **d.)**

A) Domain is all reals

B) No asymptotes

C) y-intercept $(0,0)$, x-intercepts are $(-3,0)$, $(-3,0)$, $(0,0)$

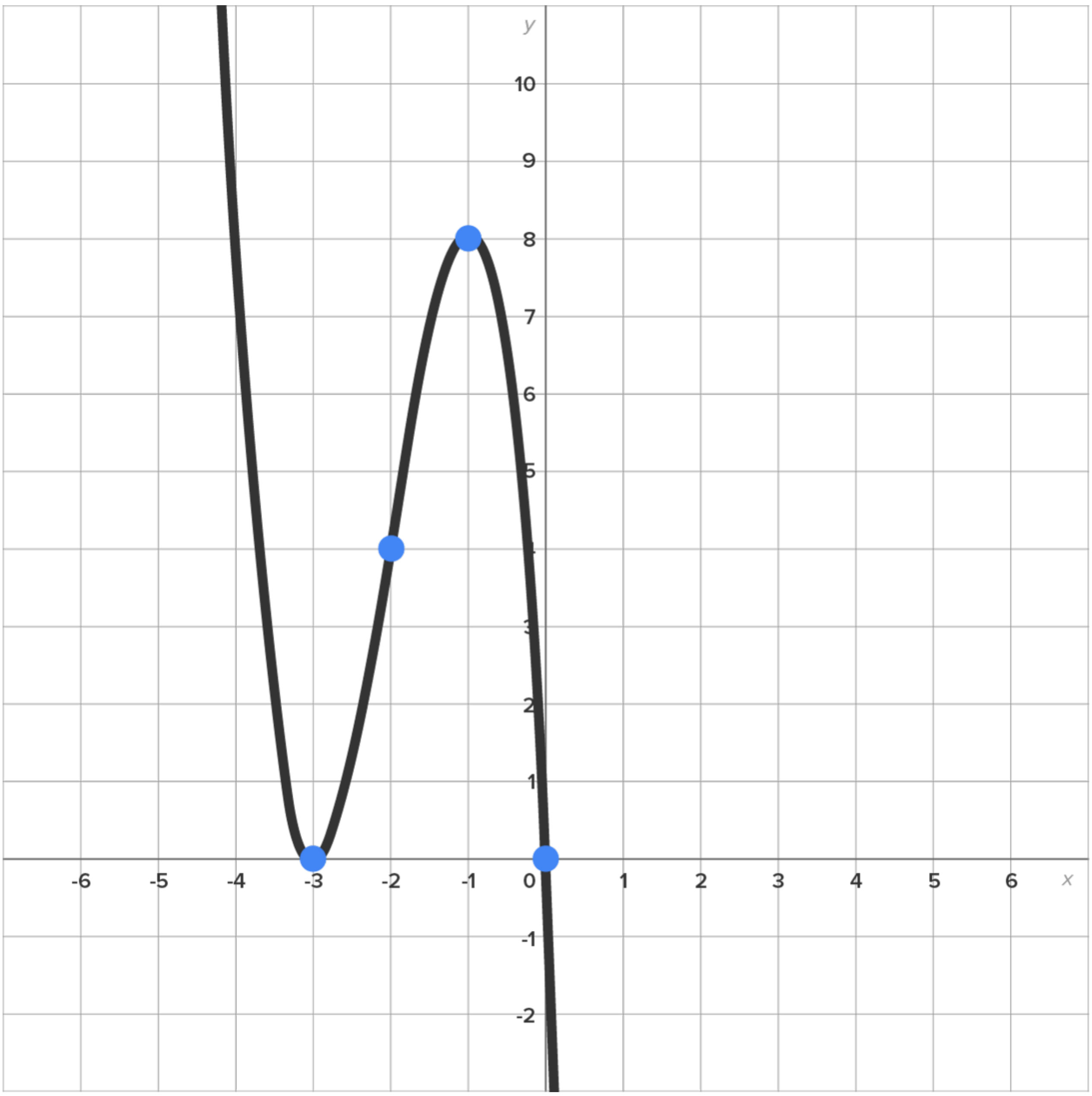
D) Decreasing on $(-\infty,-3)\cup(-1,\infty)$, increasing on $(-3,-1)$

E) Concave up on $(-\infty,-2)$, concave down on $(-2,\infty)$

F) Relative min of 0 at $x=-3$, relative max of 8 at $x=-1$

G) Inflection point at $(-2,4)$

H)


y
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0
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2
3
4
5
6
x
-1
-2

18.

Evaluate the following limit: $\lim_{x \to 0} \frac{2\sin(5x)}{11x}$.

- a.) $\lim_{x \to 0} \frac{2\sin(5x)}{11x} = -\frac{10}{11}$
- b.) $\lim_{x \to 0} \frac{2\sin(5x)}{11x} = \frac{2}{11}$
- c.) $\lim_{x \to 0} \frac{2\sin(5x)}{11x} = -\frac{2}{11}$
- d.) $\lim_{x \to 0} \frac{2\sin(5x)}{11x} = \frac{10}{11}$

19.

Evaluate the following limit: $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2)$.

- a.) $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2) = 9$
- b.) $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2) = 18$
- c.) $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2) = 0$
- d.) $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2) = 2$

20.

Evaluate the following limit: $\lim_{x \to \infty} \left(\frac{2x-1}{2x} \right)^{3x}$.

- a.) $\lim_{x \to \infty} \left(\frac{2x-1}{2x} \right)^{3x} = e^{\frac{3}{2}}$
- b.) $\lim_{x \to \infty} \left(\frac{2x-1}{2x} \right)^{3x} = \frac{3}{2}$
- c.) $\lim_{x \to \infty} \left(\frac{2x-1}{2x} \right)^{3x} = \frac{\sqrt{e}}{e^2}$
- d.) $\lim_{x \to \infty} \left(\frac{2x-1}{2x} \right)^{3x} = -\frac{3}{2}$

Answer Key

Question	Answer	
1	a	**Concept:** What is a Maximum or a Minimum? **Rationale:** The highest point on the graph is $(-3,11)$. Therefore, we say that $f(x)$ has a global maximum value of 11 at $x=-3$. Notice that the ordered pair $(-3,11)$ is at an endpoint, so 11 is not considered a local maximum value. This is because there is no graph on the other side of the point to compare it to. The points $(-1,3)$ and $(1,3)$ are the lowest of the entire graph. Therefore, 3 is a global minimum of $f(x)$ at $x=-1$ and $x=1$. Notice that these points are also the lowest points compared to other points around them and therefore are also local minimums. The point $(0,5)$ is not the highest of the entire graph and therefore not a global maximum. It is the highest compared to other points around it, therefore, 5 is a local maximum at $x=0$. The point $(2,7)$ is neither the highest or lowest point on the graph and therefore not a global extrema. It is not a local extrema since it is an endpoint, so there is no graph on the other side of the point to compare it to. Looking at the graph, there are no other points that are local extrema.
2	d	**Concept:** Critical Numbers **Rationale:** Critical numbers are values in the domain of $f(x)$ for which the derivative is 0 or undefined. The original function is a polynomial and therefore has a domain of all real numbers. Find the derivative of $f(x)=x^4-2x^2+5$: $f'(x)=4x^3-4x$ The derivative is a polynomial and therefore is never undefined. Find the value(s) of *x* for which the derivative is 0: $4x^3-4x=0$ $4x(x^2-1)=0$ $4x(x+1)(x-1)=0$ $4x=0 \text{ or } x+1=0 \text{ or } x-1=0$ $x=0 \text{ or } x=-1 \text{ or } x=1$ The critical numbers of $f(x)$ are $x=-1$, $x=0$, and $x=1$.

Concept: Finding Maximums and Minimums of a Function
Rationale:
Critical numbers are values in the domain of y for which the derivative is 0 or undefined.

The original function is a polynomial and therefore has a domain of all real numbers.

Find the derivative of $y = -x^4 - 4x^3 - 4x^2 + 4$:

$$y' = -4x^3 - 12x^2 - 8x$$

The derivative is a polynomial and therefore is never undefined.

Find the value(s) of x for which the derivative is 0:

$$-4x^3 - 12x^2 - 8x = 0$$

Factor out the common factor of $-4x$:

$$-4x(x^2 + 3x + 2) = 0$$

Factor the trinomial:

$$-4x(x+1)(x+2) = 0$$

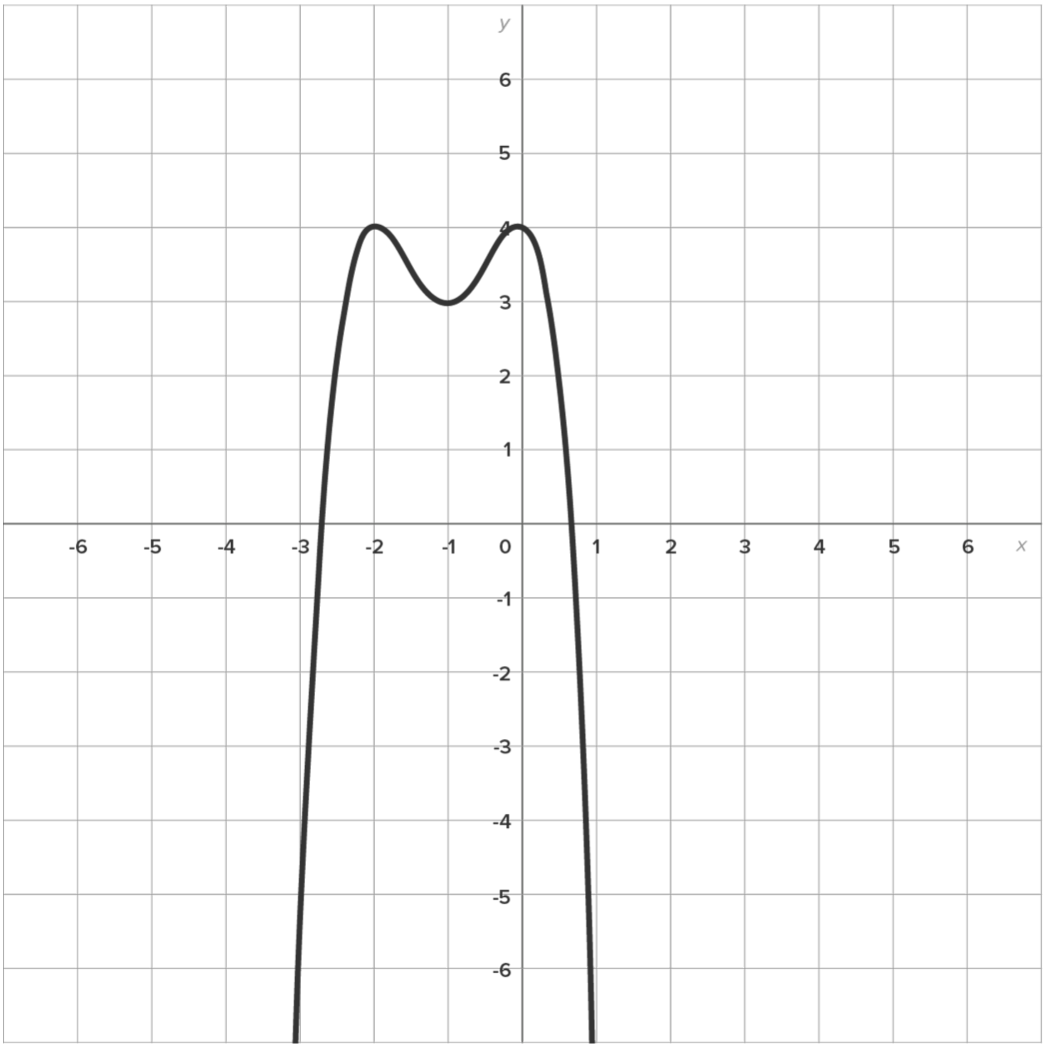
Set each factor equal to 0 and solve for x:

$$-4x = 0 \textit{ or } x + 1 = 0 \textit{ or } x + 2 = 0$$
$$x = 0 \textit{ or } x = -1 \textit{ or } x = -2$$

The critical numbers of y are $x = -2$, $x = -1$, and $x = 0$.

Next, look at the graph of $f(x)$ to determine if there is an extrema at a critical number, and if so, identify the type of extrema.

3	c	



$(-2,4)$ and $(0,4)$ are local maximums and $(-1,3)$ is a local minimum.

Concept: Extreme Value Theorem - Endpoint Extremes
Rationale:
First, find the critical numbers.

The original function is a polynomial and therefore has a domain of all real numbers.

Find the derivative of $f(x) = 2x^3 - 54x + 12$:

$f'(x) = 6x^2 - 54$

The derivative is a polynomial and therefore is never undefined.

Find the value(s) of *x* for which the derivative is 0:

$6x^2 - 54 = 0$

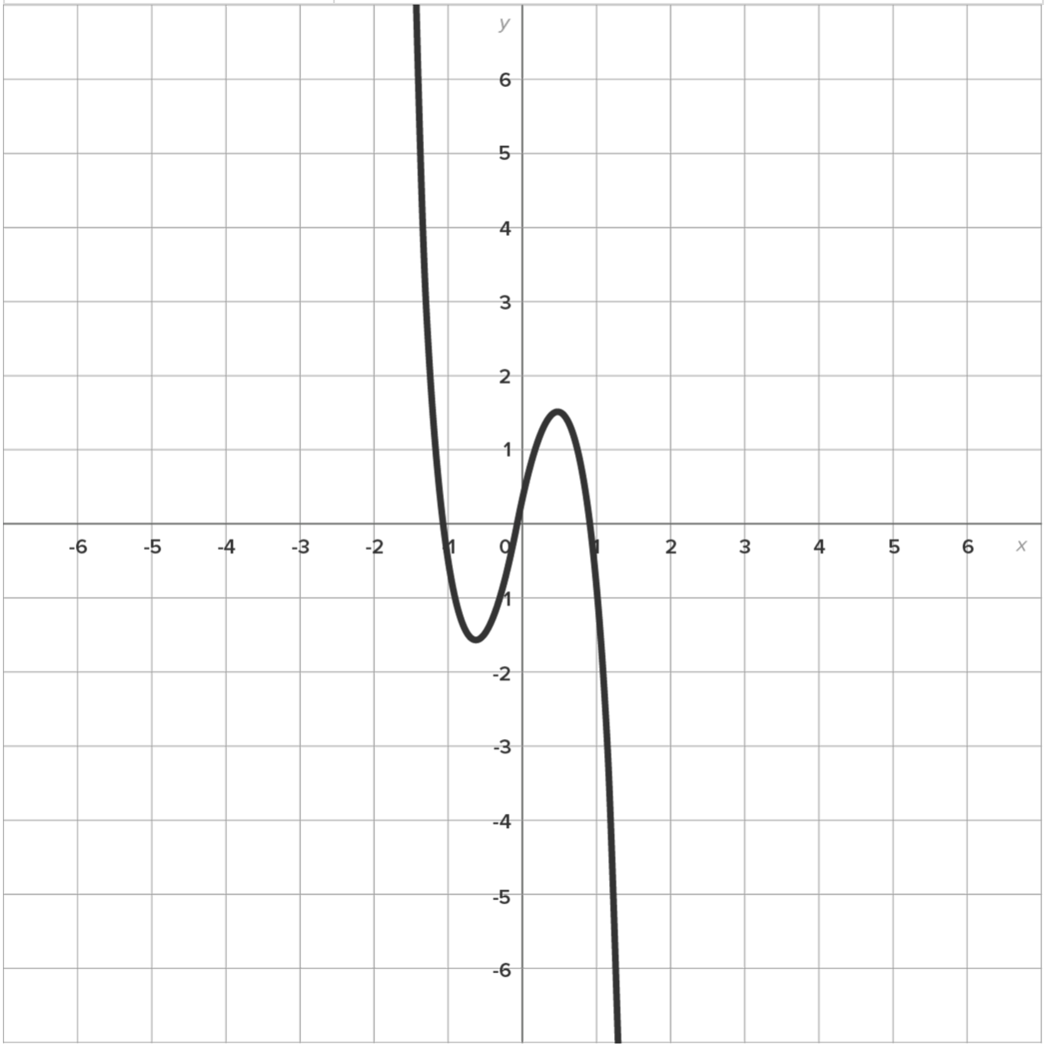
Add 54 to both sides of the equation:

4	b	$6x^2 = 54$ Divide both sides by 6: $x^2 = 9$ Take the square root of both sides of the equation: $\sqrt{x^2} = \pm\sqrt{9}$ Simplify the radical: $x = \pm 3$ The critical numbers of $f(x)$ are $x = -3$ and $x = 3$. Note that -3 is not in the interval $[0, 7]$ but 3 is in the interval $[0, 7]$. Evaluate the function at each critical number in the interval $[0, 7]$ and at the endpoints: $f(0) = 2(0)^3 - 54(0) + 12 = 12$ $f(3) = 2(3)^3 - 54(3) + 12 = -96$ $f(7) = 2(7)^3 - 54(7) + 12 = 320$ The global minimum is -96 at $x = 3$. The global maximum is 320 at $x = 7$.
5	b	**Concept:** Rolle's Theorem **Rationale:** First, check the requirements for Rolle's theorem. $f(x)$ is a polynomial so it is continuous on any interval and, therefore, it is continuous on $[0, 3]$. Also, $f'(x)$ is a polynomial and is differentiable on any interval so it is differentiable on $(0, 3)$. Evaluate: $f0 = (0)^2 - 3(0) + 10 = 10$ $f(3) = (3)^2 - 3(3) + 10 = 10$ Therefore, $f(a) = f(b)$. Thus, the conditions of Rolle's theorem have been met and there is at least one value of *c* between 0 and 3 such that $f'(c) = 0$. To find all values of *c*, take the derivative, then set equal to 0, then solve: $f'(x) = 2x - 3$

		Substitute *c* for *x* and set the derivative equal to 0: $2c-3=0$ Solve for c: $2c=3$ $c=\frac{3}{2}$ Since we want all values on the interval $(0,\ 3)$, the value guaranteed by Rolle's theorem is $c=\frac{3}{2}$.
6	d	**Concept:** Mean Value Theorem for Derivatives **Rationale:** Note that $f(x)$ is a polynomial, and therefore, it is continuous on $[0,\ 3]$ and differentiable on the interval $(0,\ 3)$. We are guaranteed to find a value of *c* between 0 and 3 where $f'(c)=\frac{f(3)-f(0)}{3-0}$. First, evaluate $f(0)$ and $f(3)$: $f(0)=2(0)^3-4(0)+5=0-0+5=5$ $f(3)=2(3)^3-4(3)+5=54-12+5=47$ Next, compute $\frac{f(3)-f(0)}{3-0}$: $\frac{f(3)-f(0)}{3-0}=\frac{47-5}{3}=14$ Now, we want to find the value of *c* guaranteed by the mean value theorem. Find the derivative: $f'(x)=6x^2-4$ Substitute *c* for *x* and set the derivative equal to $\frac{f(3)-f(0)}{3-0}$, or 14: $6c^2-4=14$ Solve for *c*: $6c^2=18$ $c^2=3$ $c=-\sqrt{3}\ and\ c=\sqrt{3}$

		Since we want all values on the interval $(0, 3)$, the value guaranteed by the mean value theorem is $c = \sqrt{3}$.
7	c	**Concept:** First Shape Theorem **Rationale:** We will first determine the behavior of $f'(x)$ and summarize it in a table at several points. We will then graph a sketch representing that behavior. Remember that $m_{\tan}$ is the value of $f'(x)$ at any point.

x	$f'(x)$ behavior
-2	$f'(x) > 0$
-1	$f'(x) = 0$
-0.5	$f'(x) < 0$
0	$f'(x) = 0$
0.5	$f'(x) > 0$
1	$f'(x) = 0$
2	$f'(x) < 0$



8	d	**Concept:** Second Shape Theorem **Rationale:** Find the first derivative: $y' = -6x^2 - 24x$ y' can only change sign when $y' = 0$ or undefined. It is never undefined since y is a polynomial. Set $y' = 0$ and solve: $-6x^2 - 24x = 0$ $-6x(x+4) = 0$ $x = 0 \text{ or } x = -4$ The critical numbers are $x = -4$ and $x = 0$. Now we move to the first derivative test, which means making a sign graph, determining the intervals of increase and decrease, then observing which critical numbers produce a local maximum or local minimum. Break the real number line into three intervals, $(-\infty, -4)$, $(-4, 0)$, $(0, \infty)$:

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, \infty)$
Test Value	-5	-3	1
Value y'	-30	18	-30
Sign of y'	Negative	Positive	Negative
Behavior of y	Decreasing	Increasing	Decreasing

Now let's look at what is happening before and after each critical value:

At $x = -4$, the graph of y transitions from decreasing to increasing, indicating that there is a local minimum value when $x = -4$. There is a local minimum at $(-4, y(-4))$, or $(-4, -57)$.

At $x = 0$, the graph of y transitions from increasing to decreasing, indicating that there is a local maximum value when $x = 0$. There is a local maximum at $(0, y(0))$, or $(0, 7)$.

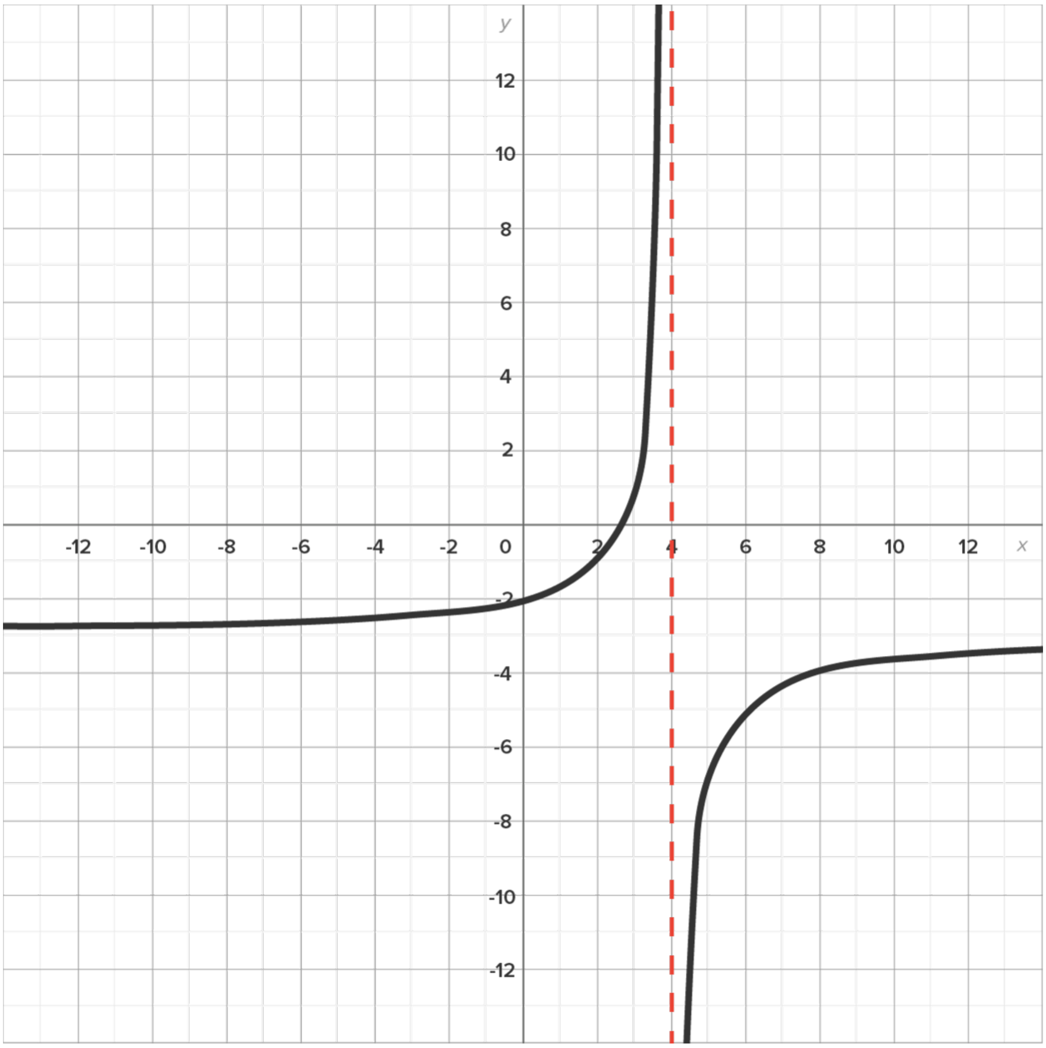
		Concept: Concavity **Rationale:** Find the first and second derivatives: $f'(x) = 7x^6 + 6x^5 - 25$ $f''(x) = 42x^5 + 30x^4$

9	a	$f''(x)$ can only change sign when $f''(x) = 0$ or undefined. It is never undefined since $f(x)$ is a polynomial. Set $f''(x) = 0$ and solve: $42x^5 + 30x^4 = 0$ $6x^4(7x+5) = 0$ $x = 0 \; or \; x = -\frac{5}{7}$ Break the real number line into three intervals, $\left(-\infty, -\frac{5}{7}\right)$, $\left(-\frac{5}{7}, 0\right)$, $(0, \infty)$: Interval: $\left(-\infty, -\frac{5}{7}\right)$ | $\left(-\frac{5}{7}, 0\right)$ | $(0, \infty)$ Test Value: -1 | -0.5 | 1 Value f'': -12 | 0.5625 | 72 Sign of f'': negative | positive | positive Behavior of $f(x)$: Concave down | Concave up | Concave up The graph is concave up on $\left(-\frac{5}{7}, 0\right) \cup (0, \infty)$ and concave down on $\left(-\infty, -\frac{5}{7}\right)$.
10	b	**Concept:** Inflection Points **Rationale:** Find the first and second derivatives: $f'(x) = -4x^3 + 12x^2 + 10$ $f''(x) = -12x^2 + 24x$ $f''(x)$ can only change sign when $f''(x) = 0$ or undefined. It is never undefined since $f(x)$ is a polynomial. Set $f''(x) = 0$ and solve: $-12x^2 + 24x = 0$ $-12x(x-2) = 0$ $x = 0 \; or \; x = 2$ Break the real number line into three intervals, $(-\infty, 0)$, $(0, 2)$, $(2, \infty)$: Interval: $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ Test Value: -1 | 1 | 3 Value f'': -36 | 12 | -36 Sign of f'': Negative | Positive | Negative Behavior of $f(x)$: Concave down | Concave up | Concave down

		Thus, $f(x)$ is concave down on the interval $(-\infty, 0)$ and concave up on the interval $(0, 2)$. A point of inflection occurs when $x = 0$. On the graph of $f(x)$, an inflection point is located at $(0, f(0))$, which is $(0,5)$. Also, $f(x)$ is concave up on the interval $(0, 2)$ and concave down on the interval $(2, \infty)$. A point of inflection occurs when $x = 2$. On the graph of $f(x)$, an inflection point is located at $(2, f(2))$, which is $(2,41)$.
11	d	**Concept:** f'' and Extreme Values of f **Rationale:** First, find the critical numbers. Critical numbers are values in the domain of $f(x)$ for which the derivative is 0 or undefined. The original function is a polynomial and therefore has a domain of all real numbers. Find the derivative: $f'(x) = -6x^2 + 6x + 12$ The derivative is a polynomial and therefore is never undefined. Find the value(s) of *x* for which the derivative is 0: $-6x^2 + 6x + 12 = 0$ $-6(x+1)(x-2) = 0$ $x = -1 \text{ or } x = 2$ The critical numbers of $f(x)$ are $x = -1$ and $x = 2$. Now, take the second derivative using $f'(x) = -6x^2 + 6x + 12$ and substitute $x = -1$ and $x = 2$. Find the second derivative: $f''(x) = -12x + 6$ Evaluate the second derivative at each critical value, starting with $x = -1$: $f''(-1) = -12(-1) + 6 = 18$ The second derivative is positive at the critical value $x = -1$. By the second derivative test, $f(-1)$ is a local minimum. A local minimum value is: $f(-1) = -2(-1)3 + 3(-1)2 + 12(-1) + 3$ $f(-1) = -4$ Do the same with the second critical value, $x = 2$:

		$f''(2) = -12(2)+6 = -18$ The second derivative is negative at the critical value $x = 2$. By the second derivative test, $f(2)$ is a local maximum. A local maximum value is: $f(2) = -2(2)^3+3(2)^2+12(2)+3$ $f(2) = 23$ Therefore, a local minimum value is $f(-1) = -4$ and a local maximum value is $f(2) = 23$.
12	c	**Concept:** Applied Maximum and Minimum Problems **Rationale:** We want to maximize the amount of area of the grazing region, which means our primary equation is $A = xy$, but this equation has too many variables for us to use calculus just yet. Thus, there should be a secondary equation we can use from information in the problem. For the secondary equation, we also know there is 3,000 meters of fencing, which means $x+2y = 3{,}000$. This can be solved for *x*, the equation can be written $x = 3000-2y$. Now, substitute $3000-2y$ for *x* in the primary equation: $A(y) = (3000-2y)y$ The function to optimize (maximize) is $A(y) = 3000y-2y^2$. The next thing we should look at is the domain of the function. Since *y* is a side of the rectangle, it must be nonnegative and can be no more than 1,500 since the total amount of fencing is 3,000 meters. Thus, the domain is $0 \le y \le 1500$. To determine the maximum value, we first take the derivative and find critical points: $A'(y) = 3000-4y$ The derivative is never undefined. Next, set the derivative equal to 0 and solve: $0 = 3000-4y$ $4y = 3000$ $y = 750$ The critical number is $y = 750$, which is inside the interval $[0, 3000]$. To determine if it is a maximum, use the second derivative test: $A''(y) = -4$ Evaluate the second derivative at $x = 750$:

		$A''(750) = -4$ Since $A''(750)$ is negative, there is a maximum when $y = 750$. To find the dimensions, substitute 750 for *y* in the secondary equation to find *x*: $x + 2y = 3{,}000$ $x + 2(750) = 3{,}000$ $x = 1{,}500$ The dimensions of the grazing area are 750 meters by 1,500 meters.
13	c	**Concept:** Limits As x Becomes Arbitrarily Large ("Approaches Infinity") **Rationale:** Divide the numerator and denominator by the highest power of *x* in the denominator, which is x^4: $\lim_{x \to -\infty} \dfrac{\dfrac{-12x^4 - 8x - 10}{x^4}}{\dfrac{7x^4 + 3x^2 - 5}{x^4}}$ Separate the fractions: $\lim_{x \to -\infty} \dfrac{\dfrac{-12x^4}{x^4} - \dfrac{8x}{x^4} - \dfrac{10}{x^4}}{\dfrac{7x^4}{x^4} + \dfrac{3x^2}{x^4} - \dfrac{5}{x^4}}$ Simplify: $\lim_{x \to \infty} \dfrac{-12 - \dfrac{8}{x^3} - \dfrac{10}{x^4}}{7 + \dfrac{3}{x^2} - \dfrac{5}{x^4}}$ Apply the property $\lim_{x \to -\infty} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to -\infty} f(x)}{\lim_{x \to -\infty} g(x)}$: $\dfrac{\lim_{x \to -\infty}\left(-12 - \dfrac{8}{x^3} - \dfrac{10}{x^4}\right)}{\lim_{x \to -\infty}\left(7 + \dfrac{3}{x^2} - \dfrac{5}{x^4}\right)}$ Evaluate each limit:

		$\lim_{x \to -\infty}(-12) = -12$, $\lim_{x \to -\infty}\left(\frac{8}{x^3}\right) = 0$, $\lim_{x \to -\infty}\left(\frac{10}{x^4}\right) = 0$ $\lim_{x \to -\infty}(7) = 7$, $\lim_{x \to -\infty}\left(\frac{3}{x^2}\right) = 0$, $\lim_{x \to -\infty}\left(\frac{5}{x^4}\right) = 0$ Substitute values: $\frac{-12-0-0}{7+0-0}$ Simplify: $-\frac{12}{7}$ Therefore, $\lim_{x \to -\infty} \frac{-12x^4-8x-10}{7x^4+3x^2-5} = -\frac{12}{7}$
14	c	**Concept:** The Limit is Infinite **Rationale:** First, graph the function $f(x) = \frac{-3x+8}{x-4}$: 

		Notice the behavior of the graph near $x=4$. As x gets closer to 4 from the left side, the graph increases in value very quickly. As a limit, this is written as $\lim_{x\to 4^-}\frac{-3x+8}{x-4}=\infty$. As x gets closer to 4 from the right side, the graph decreases in value very quickly. As a limit, this is written as $\lim_{x\to 4^+}\frac{-3x+8}{x-4}=-\infty$. Since the unbounded behavior is not the same as we approach 4 from the left and the right, we say the limit of $\lim_{x\to 4}\frac{-3x+8}{x-4}$ does not exist and cannot be written as unbounded behavior.
15	d	**Concept:** Horizontal and Vertical Asymptotes **Rationale:** First, find all values of x for which the denominator is 0: $x^2+7x+10=0$ $(x+5)(x+2)=0$ $x=-5 \text{ and } x=-2$ Thus, the possible vertical asymptotes are $x=-5$ and $x=-2$. To determine which are vertical asymptotes, we need to evaluate a one-sided limit for each x-value. For this work, we'll choose right-sided limits. Is $x=-5$ a vertical asymptote? Check by first factoring: $\lim_{x\to -5^+}\frac{x^2+x-2}{x^2+7x+10}=\lim_{x\to -5^+}\frac{(x-1)(x+2)}{(x+5)(x+2)}$ Remove common factors: $=\lim_{x\to -5^+}\frac{x-1}{x+5}$ Direct substitution does not work. As x approaches -5 from the right, $x-1$ is around -6, and $x+5$ is a small positive number: $\frac{\text{around } -6}{\text{small positive number}}=\text{increasingly negative number}$ So the limit is represented by $-\infty$. There is a vertical asymptote at $x=-5$. Is $x=-2$ a vertical asymptote?

		Follow the same beginning steps as above, and remember to look at the limit as x approaches -2 from the right. Check by first factoring: $\lim_{x\to -2^+} \frac{x^2+x-2}{x^2+7x+10} = \lim_{x\to -2^+} \frac{(x-1)(x+2)}{(x+5)(x+2)}$ Remove common factors: $= \lim_{x\to -2^+} \frac{x-1}{x+5}$ Use direct substitution: $= \frac{-2-1}{-2+5}$ Simplify: $= -1$ Since the limit is not $\pm\infty$, there is no vertical asymptote at $x = -2$.
16	c	**Concept:** Other Asymptotes as x Approaches ∞ and -∞ **Rationale:** Note the degree of the numerator is larger than the degree of the denominator. Performing the division, we have: $f(x) = 2x^2 - 7x + 31 - \frac{115}{x+4}$ As $x \to \pm\infty$, $-\frac{115}{x+4} \to 0$, which means the graph of $f(x)$ gets closer to the graph of $y = 2x^2 - 7x + 31$. Thus, the nonlinear asymptote is $y = 2x^2 - 7x + 31$.
		Concept: Putting It All Together: Sketching a Graph **Rationale:** The function is a polynomial. The domain is all reals, and there are no asymptotes. Let's find the intercepts: For y-intercept, set $x = 0$: $y = -2(0)^3 - 12(0)^2 - 18(0)$ $y - intercept: (0,0)$ For x-intercept, set $y = 0$:

$0 = -2x^3 - 12x^2 - 18x$
$0 = -2x(x^2 + 6x + 9)$

Factor:

$0 = -2x(x+3)(x+3)$
$x = 0, x = -3, x = -3$
$x - intercepts: (0, 0), (-3, 0), (-3, 0)$

Now find the critical values. Take the derivative:

$f'(x) = -6x^2 - 24x - 18$

The derivative is never undefined. Set the derivative equal to zero and solve by factoring:

$0 = -6x^2 - 24x - 18$
$0 = -6(x+1)(x+3)$
$x = -1, x = -3$

Find the values in the domain of $f(x)$ for which the second derivative is zero or undefined:

$f''(x) = -12x - 24$

The second derivative is never undefined. Find the value(s) that make it zero:

$0 = -12x - 24$
$x = -2$

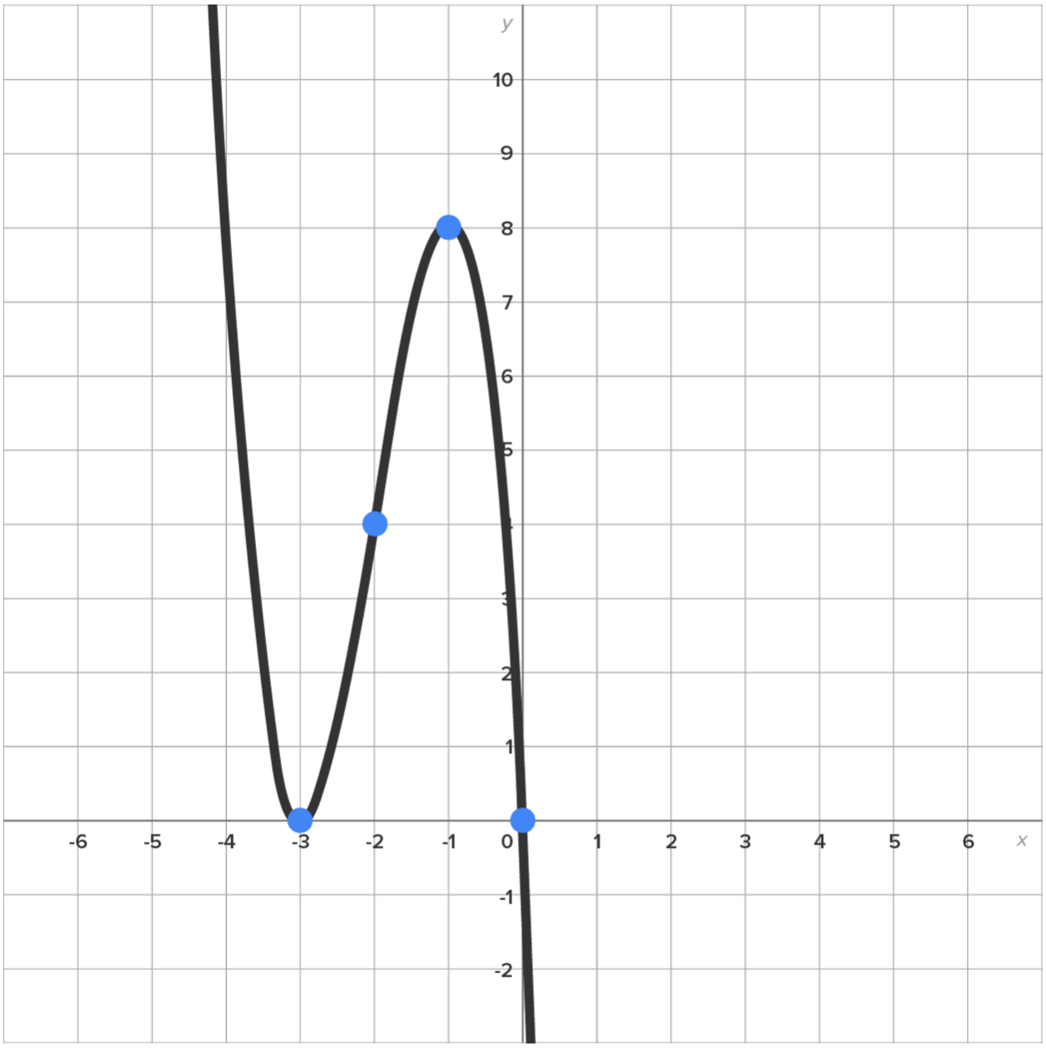
Break the real number line into four intervals, $(-\infty, -3), (-3, -2), (-2, -1), (-1, \infty)$:

17 d

Interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, -1)$	$(-1, \infty)$
Test Value	-4	-2.5	-1.5	0
Value $f'(x)$	-18	4.5	4.5	-18
Sign of $f'(x)$	Negative	Positive	Positive	Negative
Value of $f''(x)$	24	6	-6	-24
Sign of $f''(x)$	Positive	Positive	Negative	Negative
Behavior	Decreasing, Concave Up	Increasing, Concave Up	Increasing, Concave Down	Decreasing, Concave Down

$f(x)$

There is a relative min of 0 at $x = -3$, inflection point at $(-2, 4)$, and a relative max of 8 at $x = -1$. Here is the graph:



Concept: What is L'Hopital's Rule?

18	d	**Rationale:** First, check the requirements: The numerator and denominator both approach 0 as $x \to 0$. The numerator and denominator are both differentiable on any interval containing $x = 0$. This means L'Hopital's rule can be used to evaluate the limit. Take the derivative of the numerator and denominator: $D[2\sin(5x)] = 10\cos(5x)$ $D[11x] = 11$ Substitute values: $= \lim_{x \to 0} \frac{10\cos(5x)}{11}$ This is no longer an indeterminate form; evaluate the limit and simplify: $= \frac{10\cos(5(0))}{11}$ $= \frac{10\cos(0)}{11}$ $= \frac{10}{11}$
19	b	**Concept:** Apply L'Hopital's Rule to the Indeterminate Forms "∞ - ∞" and "∞ * 0" **Rationale:** If we look at each factor separately, we see that $x^{-1} \to \infty$ and $(2e^{9x} - 2) \to 0$ as $x \to 0^+$. Thus, this limit has the form $\infty \cdot 0$. To rewrite, consider the fact that $x^{-1} = \frac{1}{x}$, which means $\lim_{x \to 0^+} x^{-1}(2e^{9x} - 2) = \lim_{x \to 0^+} \frac{2e^{9x} - 2}{x}$, which now has the form $\frac{0}{0}$. To evaluate, use L'Hopital's rule. Since $(2e^{9x} - 2)$ and x are differentiable and the limit has the form $\frac{0}{0}$, L'Hopital's rule is used. Take the derivative of the numerator and denominator: $D[2e^{9x} - 2] = 18e^{9x}$ $D[x] = 1$ Substitute values and simplify: $= \lim_{x \to 0+} \frac{18e^{9x}}{1}$ $= \lim_{x \to 0+} 18e^{9x}$

		This is no longer an indeterminate form; evaluate the limit and simplify: $= 18e^{9(0)}$ $= 18(1)$ $= 18$
20	c	**Concept:** Limits with Variable Bases and Exponents **Rationale:** Note that this is a limit of the form 1^{∞}, which will need our new strategy. Take the natural logarithm of $\left(\frac{2x-1}{2x}\right)^{3x}$: $\ln\left(\frac{2x-1}{2x}\right)^{3x} = 3x\ln\left(\frac{2x-1}{2x}\right)$ Now find the limit: $\lim\limits_{x\to\infty} 3x\ln\left(\frac{2x-1}{2x}\right)$ This has the form $\infty\cdot 0$, which is a form that can be rewritten to an indeterminate form: $\lim\limits_{x\to\infty} \frac{3\ln\left(\frac{2x-1}{2x}\right)}{\frac{1}{x}}$ The limit has the form $\frac{0}{0}$ and both numerator and denominator are differentiable, so L'Hopital's rule can be used. First, use properties of logarithms to rewrite the numerator before differentiating: $3\ln\left(\frac{2x-1}{2x}\right) = 3\ln(2x-1) - 3\ln(2x)$ Take the derivative of the numerator and denominator: $D[3\ln\left(\frac{2x-1}{2x}\right)] = D[3\ln(2x-1) - 3\ln(2x)] = \frac{3\cdot 2}{2x-1} - \frac{3\cdot 2}{2x} = \frac{6}{2x-1} - \frac{3}{x}$ $D\left[\frac{1}{x}\right] = D\left[x^{-1}\right] = -x^{-2} = \frac{-1}{x^2}$ Substitute values: $= \lim\limits_{x\to\infty} \frac{\frac{6}{2x-1} - \frac{3}{x}}{\frac{-1}{x^2}}$

Combine the fractions in the numerator:

$$= \lim_{x \to \infty} \frac{\frac{6x - 6x + 3}{(2x-1)x}}{\left(-\frac{1}{x2}\right)}$$

Simplify the numerator and perform the indicated division:

$$= \lim_{x \to \infty} \frac{3}{(2x-1)x} \cdot \frac{x^2}{-1}$$

Simplify:

$$= \lim_{x \to \infty} -\frac{3x^2}{(2x-1)x}$$
$$= \lim_{x \to \infty} \frac{-3x}{(2x-1)}$$
$$= -\frac{3}{2}$$

Then, the limit of the original function is $e^{-\frac{3}{2}} = \frac{1}{e\sqrt{e}} = \frac{\sqrt{e}}{e^2}$. Thus,

$$\lim_{x \to \infty} \left(\frac{2x-1}{2x}\right)^{3x} = \frac{\sqrt{e}}{e^2}.$$