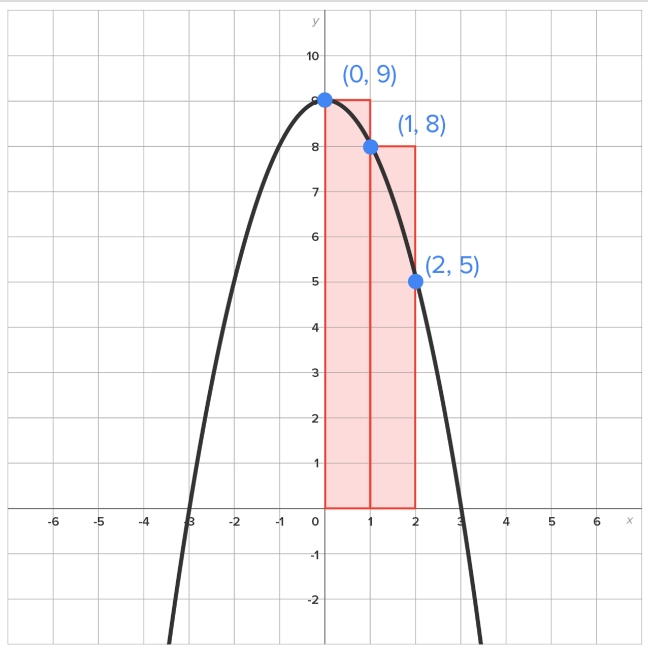
Practice Milestone

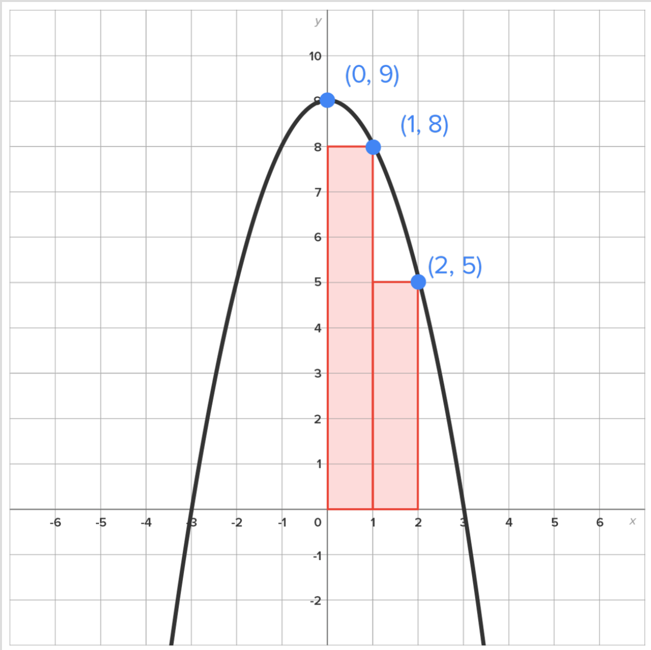
Calculus I — Practice Milestone 5

Taking this practice test is a stress-free way to find out if you are ready for the Milestone 5 assessment. You can print it out and test yourself to discover your strengths and weaknesses. The answer key is at the end of this Practice Milestone.

1.

Approximate the area of the region bounded by $y = -x^2 + 9$**, the x-axis,** $x = 0$**, and** $x = 2$ **by finding the combined area of the rectangles (as shown in each figure) and averaging the results.**



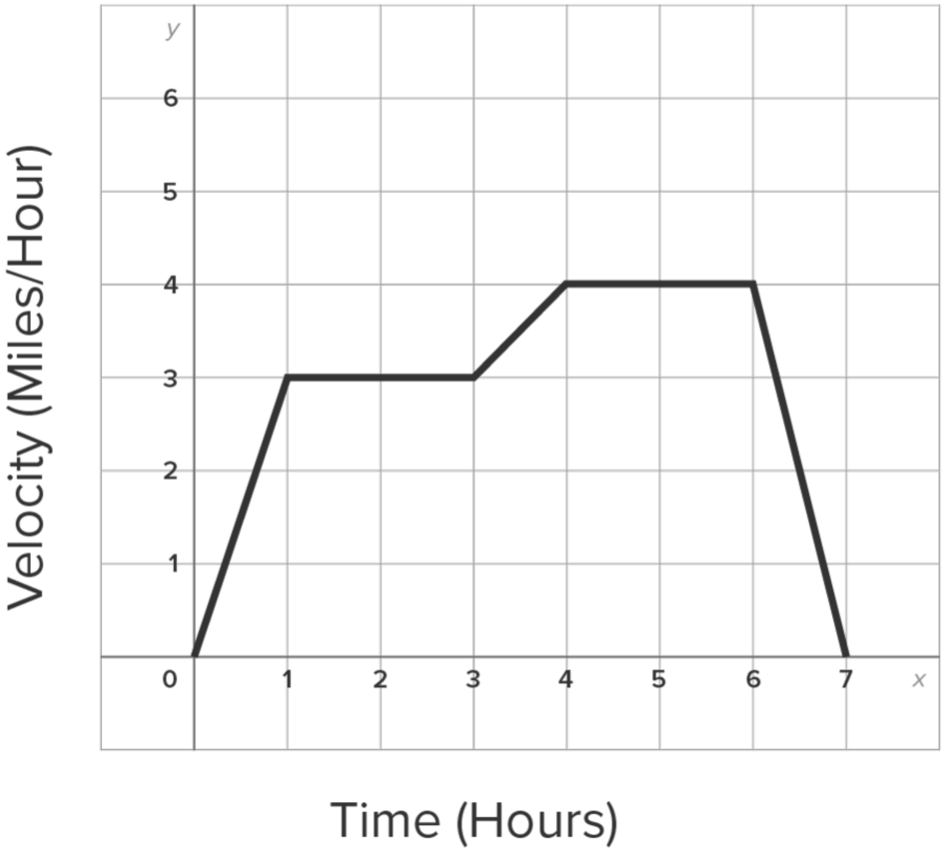


- a.) 2 units^2
- b.) 7.5 units^2
- c.) 22 units^2
- d.) 15 units^2

2.

While hiking with friends, Ashley's velocity varied, as shown on the graph below.

Find Ashley's total distance traveled on the interval $[0,6]$.



- a.) 17.5 miles
- b.) 21 miles
- c.) 20 miles
- d.) 19 miles

3.

Use summation formulas to evaluate the sum: $\sum_{k=1}^{46}(3k^2+17k-11)$.

a.) $\sum_{k=1}^{46}(3k^2+17k-11)=119416$

b.) $\sum_{k=1}^{46}(3k^2+17k-11)=118404$

c.) $\sum_{k=1}^{46}(3k^2+17k-11)=118921$

d.) $\sum_{k=1}^{46}(3k^2+17k-11)=118899$

4.

Use a Riemann sum with 4 rectangles of equal width to approximate the area between $y=2x^2+3$ and the x-axis on the interval $[2,5]$. Use the left-hand endpoint of each subinterval.

a.) $12.5\ units^2$

b.) $95.75\ units^2$

c.) $71.8125\ units^2$

d.) $103.3125\ units^2$

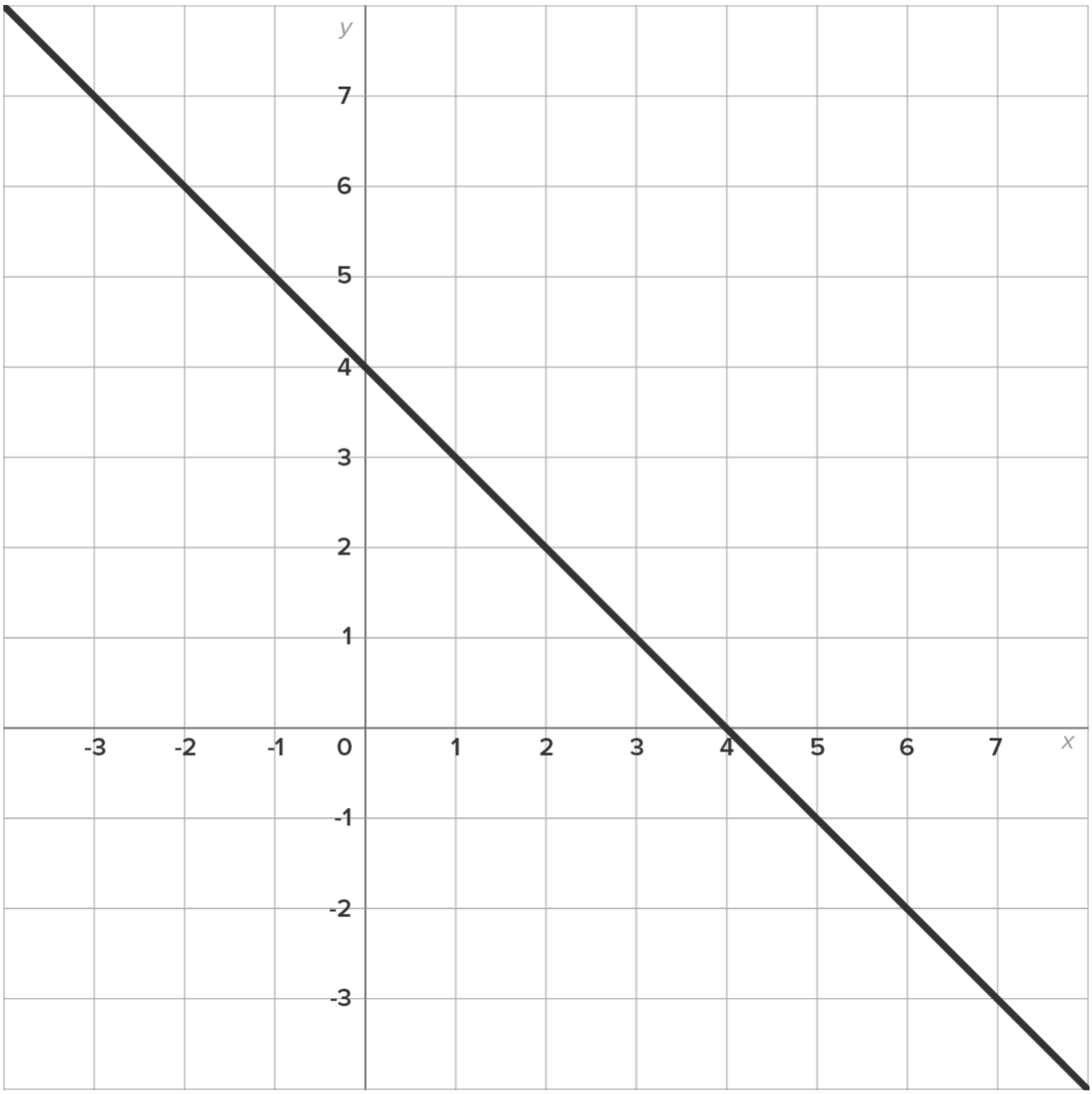
5.

Write $\int_{-1}^{4}(3x-1)\,dx$ as a limit of a Riemann sum and evaluate.

- a.) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{15k}{n}\right)\left(\frac{5}{n}\right)=\frac{95}{2}$
- b.) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-4+\frac{15k}{n}\right)\left(\frac{5}{n}\right)=\frac{35}{2}$
- c.) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+\frac{9k}{n}\right)\left(\frac{3}{n}\right)=\frac{21}{2}$
- d.) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{9k}{n}\right)\left(\frac{3}{n}\right)=\frac{39}{2}$

6.

Evaluate $\int_{-3}^{7}(-x+4)dx$ using the following graph.

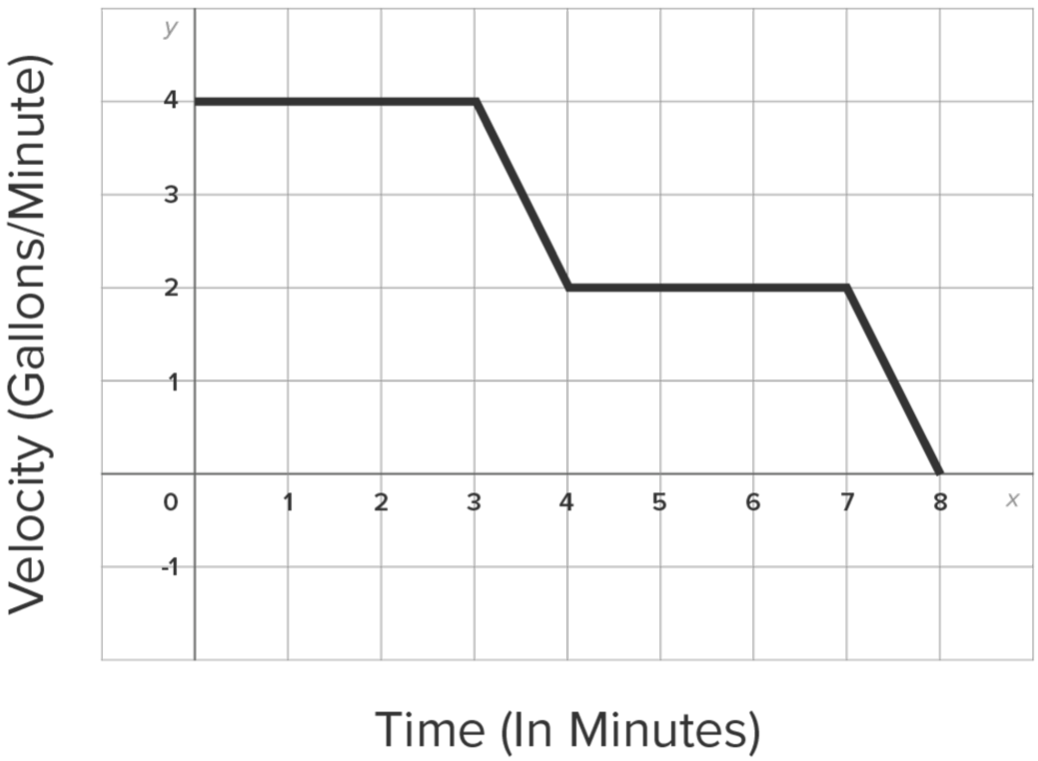


- a.) $\int_{-3}^{7}(-x+4)dx = 20$
- b.) $\int_{-3}^{7}(-x+4)dx = -20$
- c.) $\int_{-3}^{7}(-x+4)dx = -29$
- d.) $\int_{-3}^{7}(-x+4)dx = 29$

7.

Shown below is the graph of the flow rate $f(t)$ of a pipe, in gallons per minute. Here, $t = \textit{the number of minutes}$
.

Using this information, find $\int_0^8 f(t)\,dt$, which gives the total number of gallons of water that flowed through this pipe in 8 minutes.



a.) $\int_0^8 f(t)\,dt = 23\ \textit{gallons}$

b.) $\int_0^8 f(t)\,dt = 22\ \textit{gallons}$

c.) $\int_0^8 f(t)\,dt = 21\ \textit{gallons}$

d.) $\int_0^8 f(t)\,dt = 21.5 \text{ gallons}$

8.

Given $\int_4^{15} f(x)dx = 31$ **and** $\int_4^{15} g(x)dx = -7$**, find the following:**

$\int_4^{15} [5f(x) - 12g(x)]dx$

a.) 239

b.) 38

c.) 24

d.) 71

9.

Evaluate $\int_{-3}^{2} 6x^5 dx$ **using the fundamental theorem of calculus.**

a.) $\int_{-3}^{2} 6x^5 dx = 793$

b.) $\int_{-3}^{2} 6x^5 dx = -665$

c.) $\int_{-3}^{2} 6x^5 dx = 665$

d.) $\int_{-3}^{2} 6x^5 dx = -793$

10.

Find the following indefinite integral:

$$\int\left(-35x^4+14x-3+\frac{10}{x}\right)dx$$

- a.) $-7x^5+7x^2+\frac{10}{x^2}$
- b.) $-7x^5+7x^2-3x+\frac{10}{x^2}+c$
- c.) $-7x^5+7x^2+10\ln|x|$
- d.) $-7x^5+7x^2-3x+10\ln|x|+c$

11.

Find the following indefinite integral:

$$\int x^3(7x-5)^2dx$$

- a.) $\frac{49}{6}x^6+\frac{25}{4}x^4+c$
- b.) $\frac{49}{6}x^6-14x^5+\frac{25}{4}x^4+c$
- c.) $\frac{49}{6}x^6+14x^5+\frac{25}{4}x^4+c$
- d.) $\frac{49}{6}x^6-7x^5+\frac{25}{4}x^4+c$

12.

Find the following indefinite integral:

$\int(2\cos x+9\sec^2 x)dx$

- **a.)** $-2\sin x-9\tan x+c$
- **b.)** $2\sin x+9\tan x+c$
- **c.)** $-2\sin x+9\tan x+c$
- **d.)** $2\sin x-9\tan x+c$

13.

Find the following indefinite integral:

$\int\left(26e^x+\frac{5}{x^4}-10\cos x\right)dx$

- **a.)** $26e^x+\frac{25}{x^5}+10\sin x+c$
- **b.)** $26e^x+\frac{5}{3x^3}+10\sin x+c$
- **c.)** $26e^x-\frac{25}{x^5}-10\sin x+c$
- **d.)** $26e^x-\frac{5}{3x^3}-10\sin x+c$

14.

Find the following indefinite integral:

$\int 12x^5(x^6+32)^{23}dx$

- **a.)** $\frac{1}{12}(x^6+32)^{24}+c$
- **b.)** $\frac{1}{2}u^{24}+c$
- **c.)** $\frac{1}{12}x^6(x^6+32)^{24}+c$
- **d.)** $\frac{1}{2}(x^6+32)^{24}+c$

15.

Find the following indefinite integral:

$\int 15x^4\sin(x^5-19)dx$

- **a.)** $-3x^5\cos\left(\frac{1}{6}x^6-19x\right)+c$
- **b.)** $3x^5\cos\left(\frac{1}{6}x^6-19x\right)+c$
- **c.)** $-3\cos(x^5-19)+c$
- **d.)** $3\cos(x^5-19)+c$

16.

Find the following indefinite integral:

$\int (8x^3+5)e^{2x^4+5x-1}dx$

- a.) $(2x^4+5x)e^{\frac{2}{5}x^5+\frac{5}{2}x^2-x+c}$

- b.) $(2x^4+5x)e^{\frac{2}{5}x^5+\frac{5}{2}x^2-x}+c$

- c.) $(2x^4+5x)e^{2x^4+5x-1}+c$

- d.) $e^{2x^4+5x-1}+c$

17.

Solve $y'=27e^{-3x}-3\sqrt{x}+8\cos x$**, given that the solution passes through the point** $(0,2)$.

- a.) $y=-9e^{-3x}-2x^{\frac{3}{2}}+8\sin x+3$

- b.) $y=-9e^{-3x}-2x^{\frac{3}{2}}+8\sin x+11$

- c.) $y=27e^{-3x}-2x^{\frac{3}{2}}+8\sin x+2$

- d.) $y=27e^{-3x}-2x^{\frac{3}{2}}+8\sin x-33$

18.

Evaluate the following definite integral:

$\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx$

- a.) $\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx = -\frac{3}{8}$
- b.) $\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx = \frac{3}{8}$
- c.) $\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx = \frac{45}{8}$
- d.) $\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx = -\frac{45}{8}$

19.

Find the total area between the x-axis and the curve $f(x) = x^2 - 4$ between $x = -1$ and $x = 4$.

- a.) $\frac{59}{3}$ $units^2$
- b.) $\frac{65}{3}$ $units^2$
- c.) $\frac{17}{3}$ $units^2$
- d.) 21 $units^2$

20.

Find the exact area between the graphs of $f(x) = x^2 - 2$ and $g(x) = -x + 4$.

- a.) $\frac{5}{6}$ $units^2$
- b.) $\frac{125}{3}$ $units^2$
- c.) $\frac{71}{6}$ $units^2$
- d.) $\frac{125}{6}$ $units^2$

21.

Find the total area bounded by the graphs of $f(x) = -x^2 + 4$ and $g(x) = 2x + 1$ on the interval $[-3,2]$.

- a.) $\frac{39}{6}$ $units^2$
- b.) 13 $units^2$
- c.) $\frac{25}{3}$ $units^2$
- d.) 19 $units^2$

22.

Find the average value of $f(x) = 3x^2 + 1$ over the interval $[0,4]$.

- a.) -68
- b.) 17
- c.) 68
- d.) -17

23.

Find the average value and the value(s) of *c* guaranteed by the mean value theorem for integrals if $f(x) = 2x^2 + 1$ over the interval $[0,3]$.

- a.) $f_{avg} = 7$
  $c = \sqrt{3}$
- b.) $f_{avg} = 7$
  $c = -\sqrt{3}$ and $c = \sqrt{3}$
- c.) $f_{avg} = 21$
  $c = -\sqrt{10}$ and $c = \sqrt{10}$
- d.) $f_{avg} = 21$
  $c = \sqrt{10}$

24.

Use the table of integration formulas to identify and use an appropriate formula to find the following indefinite integral:

$$\int \cot^2(12x)dx$$

- a.) $-\frac{1}{12}\cot(12x) - x + c$
- b.) $-\cot(12x) - x + c$
- c.) $-\frac{1}{36}\cot^3(12x) + c$
- d.) $-\frac{1}{3}\cot^3(12x) + c$

25.

Estimate the value of $\int_1^3 \frac{1}{x^3+5}dx$ by using $n=4$ subintervals of equal width using the trapezoidal rule.

Round the final result to five decimal places.

- **a.)** ≈ 0.17188
- **b.)** ≈ 0.34377
- **c.)** ≈ 2.5
- **d.)** ≈ 0.22136

Answer Key

Question	Answer	
1	d	**Concept:** Area **Rationale:** For both figures, each rectangle is 1 unit wide. For the figure on the left: • The first rectangle has a height of 9 units; the area of the first rectangle is $(9)(1)=9\ units^2$. • The second rectangle has a height of 8 units; the area of the second rectangle is $(8)(1)=8\ units^2$. • The combined area is $17\ units^2$. We know this is an overestimate of the actual area since the rectangles are circumscribed. For the figure on the right: • The first rectangle has a height of 8 units; the area of the first rectangle is $(8)(1)=8\ units^2$. • The second rectangle has a height of 5 units; the area of the second rectangle is $(5)(1)=5\ units^2$. • The combined area is $13\ units^2$. We know this is an underestimate of the actual area since the rectangles are inscribed. Since one estimate is an overestimate and one is an underestimate, one way to get a better approximation is to average them: $\frac{17+13}{2}=\frac{30}{2}=15\ units^2$
2	d	**Concept:** Some Applications of "Area" **Rationale:** For the interval $[0,1]$, the region is a triangle with a base of 1 and a height of 3. Thus, the area is $\frac{1}{2}(1)(3)=\frac{3}{2}$ and the distance traveled is 1.5 miles. For the interval $[1,3]$, the region is a rectangle with a base of 2 and a height of 3. Thus, the area is $2(3)=6$ and the distance traveled is 6 miles. For the interval $[3,4]$, the region is a trapezoid with a height of 1 and bases of 3 and 4. Thus, the area is $\frac{1}{2}(1)(3+4)=\frac{7}{2}=3.5$ and the distance traveled is 3.5 miles. For the interval $[4,6]$, the region is a rectangle with a base of 2 and a height of 4. Thus, the area is $2(4)=8$ and the distance traveled is 8 miles. The total distance traveled on $[0,6]$ is $1.5+6+3.5+8=19$, or 19 miles.

3	b	**Concept:** Sigma Notation

Rationale:

Use properties of summations to write the expression as summations of separate terms:

$$\sum_{k=1}^{n} C\cdot a_k = C\cdot \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

Therefore:

$$\sum_{k=1}^{46} (3k^2+17k-11) = 3\sum_{k=1}^{46} k^2 + 17\sum_{k=1}^{46} k - \sum_{k=1}^{46} 11$$

Note that for each, the summation starts at $k=1$; therefore, apply the summation formulas with $n=46$:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} c = nc$$

Therefore:

$$3\sum_{k=1}^{46} k^2 + 17\sum_{k=1}^{46} k - \sum_{k=1}^{46} 11$$

$$= 3\cdot\frac{46(46+1)(2(46)+1)}{6} + 17\cdot\frac{46(46+1)}{2} - 46\cdot 11$$

$$= \frac{3\cdot 46(47)(93)}{6} + \frac{17\cdot 46(47)}{2} - 506$$

$$= 100533 + 18377 - 506$$

$$= 118404$$

4	c	**Concept:** Area Under A Curve –– Riemann Sums

Rationale:

Since each subinterval will have equal width, that width is:

$$\frac{\textit{Width of } [2,5]}{4} = \frac{5-2}{4} = \frac{3}{4} = 0.75$$

Based on the problem, we have the following information:

Subinterval	Width of Subinterval	Value Chosen in Each Subinterval
[2,2.75]	0.75	2
[2.75,3.5]	0.75	2.75
[3.5,4.25]	0.75	3.5
[4.25,5]	0.75	4.25

Written using sigma notation, the Riemann sum is:

		$\sum_{k=1}^{4} f(c_k)\Delta x_k$ Since the rectangles are of equal width, we can write the sum as: $= \Delta x \sum_{k=1}^{4} f(c_k)$ $= 0.75[f(2)+f(2.75)+f(3.5)+f(4.25)]$ Using the function $y = 2x^2+3$, evaluate: $= 0.75[11+18.125+27.5+39.125]$ $= 71.8125\ units^2$
5	b	**Concept:** Definition of The Definite Integral **Rationale:** 1) Find the width of each subinterval: $\Delta x = \frac{4-(-1)}{n} = \frac{5}{n}$ 2) Find the right-hand endpoints: $c_k = a + k\Delta x = -1 + k\left(\frac{5}{n}\right) = -1 + \frac{5k}{n}$ 3) Evaluate the function at c_k: $f(c_k) = 3\cdot\left(-1+\frac{5k}{n}\right) - 1 = -4 + \frac{15k}{n}$ 4) Evaluate the sum and compute the limit: $\int_{-1}^{4}(3x-1)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left(-4+\frac{15k}{n}\right)\left(\frac{5}{n}\right)$ $\frac{5}{n}$ is a constant factor in the summation since k is the index of summation. Therefore, it can be factored out: $= \lim_{n\to\infty}\frac{5}{n}\sum_{k=1}^{n}\left(-4+\frac{15k}{n}\right)$ Use the properties of summations to write the expression as summations of separate terms:

$$\sum_{k=1}^{n} C \cdot a_k = C \cdot \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

Therefore:

$$= \lim_{n \to \infty} \frac{5}{n}\left(\sum_{k=1}^{n} (-4) + \frac{15}{n} \sum_{k=1}^{n} k\right)$$

Use the summation formulas:

$$\sum_{k=1}^{n} c = nc$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Therefore:

$$= \lim_{n \to \infty} \frac{5}{n}\left(-4n + \frac{15}{n} \cdot \frac{n(n+1)}{2}\right)$$

Simplify:

$$= \lim_{n \to \infty}\left(-20 + \frac{75n^2 + 75n}{2n^2}\right)$$

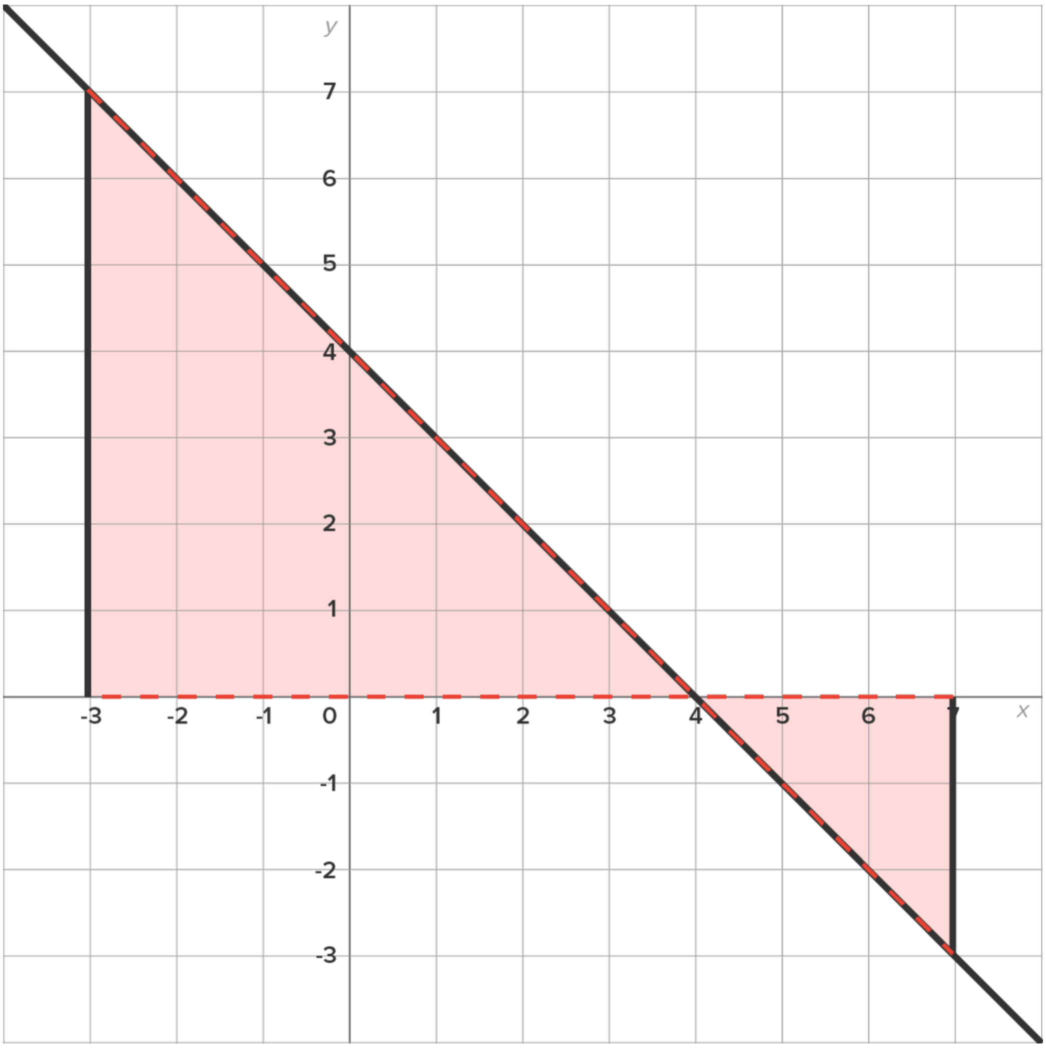
$$= \lim_{n \to \infty}\left(-20 + \frac{75}{2} + \frac{75}{2n}\right)$$

$$= \frac{35}{2}$$

Concept: Definite Integrals of Negative Functions

Rationale:

To use the graph to evaluate the definite integral, begin by shading in the relevant regions:

6	a	 The triangle between $x = -3$ and $x = 4$ has area $\frac{1}{2}(7)(7) = 24.5$ and is above the x-axis. The triangle between $x = 4$ and $x = 7$ has area $\frac{1}{2}(3)(3) = 4.5$ and is below the x-axis. Then, $\int_{-3}^{7}(-x+4)dx = 24.5 + (-4.5) = 20$.
7	b	**Concept:** Units For the Definite Integral **Rationale:** Note that the vertical scale is measured in gallons per minute and the horizontal scale is measured in minutes. Therefore, the area of any region is measured in $\frac{gallons}{minute} \cdot minutes = gallons$. To find the total area, note that the graph of the function is above the horizontal axis on $[0,8]$. This means that we add all positive values. On $[0,3]$, the region is a rectangle. Its area is $(3)(4) = 12$, or 12 gallons.

		On $[3,4]$, the region is a trapezoid, which means its area is $\frac{1}{2}(1)(4+2)=3$, or 3 gallons On $[4,8]$, the region is a trapezoid. Its area is $\frac{1}{2}(2)(4+3)=7$, or 7 gallons. Because the region is above the t-axis throughout the interval, the total area is the number of gallons, so $12+3+7=22$, or 22 gallons. So, $\int_0^8 f(t)\,dt = 22\ gallons$.
8	a	**Concept:** Properties of the Definite Integral **Rationale:** Use properties of definite integrals: $\int_a^b k\cdot f(x)dx = k\int_a^b f(x)dx$ $\int_a^b \lvert f(x)-g(x)\rvert\,dx = \int_a^b f(x)dx - \int_a^b g(x)dx$ Therefore: $=\int_4^{15}[5f(x)dx - \int_4^{15} 12g(x)]dx = 5\int_4^{15} f(x)dx - 12\int_4^{15} g(x)dx$ Given $\int_4^{15} f(x)dx = 31$ and $\int_4^{15} g(x)dx = -7$, evaluate: $=5(31)-12(-7)$ $=155+84$ $=239$
9	b	**Concept:** Areas, Integrals, and Antiderivatives **Rationale:** If $f(x)=6x^5$, then $F(x)=x^6$ is an antiderivative of $f(x)$. Evaluate: $\int_{-3}^{2} 6x^5 dx = F(2)-F(-3)$ $=2^6-(-3)^6$ $=64-729$ $=-665$
		Concept: Indefinite Integrals and Antiderivatives of Polynomial Functions **Rationale:**

10	d	Apply the properties of indefinite integrals. First, apply the sum/difference properties: $\int -35x^4 dx + \int 14x\, dx - \int 3\, dx + \int \frac{10}{x} dx$ Next, apply the constant multiple property: $= -35\int x^4 dx + 14\int x\, dx - \int 3\, dx + 10\int \frac{1}{x} dx$ Then, integrate using the general power rule and the natural logarithm rule. Note: there is only one "+*c*" needed. If a constant were added to each indefinite integral, they could be merged and written as one constant: $= -35 \cdot \left(\frac{1}{5}\right)x^5 + 14\left(\frac{1}{2}\right)x^2 - 3x + 10\ln\|x\| + c$ Simplify: $= -7x^5 + 7x^2 - 3x + 10\ln\|x\| + c$
11	b	**Concept:** Indefinite Integrals of Functions Requiring Rewriting Before Applying Rules **Rationale:** The integrand $x^3(7x-5)^2$ is not a power of *x*, nor is it a sum or difference of powers of *x*. Perform the multiplication to expand: $x^3(7x-5)(7x-5) = 49x^5 - 70x^4 + 25x^3$ We learned how to antidifferentiate this integrand in the last tutorial, so we now have: $\int x^3(7x-5)^2 dx = \int (49x^5 - 70x^4 + 25x^3) dx$ Next, use the sum/difference properties followed by the constant multiple rule: $= 49\int x^5 dx - 70\int x^4 dx + 25\int x^3 dx$ Integrate using the general power rule: $= 49 \cdot \frac{x^6}{6} - 70 \cdot \frac{x^5}{5} + 25 \cdot \frac{x^4}{4} + c$ Simplify: $= \frac{49}{6}x^6 - 14x^5 + \frac{25}{4}x^4 + c$
		Concept: Indefinite Integrals of Trigonometric Functions

12	b	**Rationale:** First, use the sum/difference properties followed by the constant multiple rule: $\int(2\cos x+9\sec^2 x)dx=2\int\cos x dx+9\int\sec^2 x dx$ Then, use the fact that $\int\cos x dx=\sin x+c$ and $\int\sec^2 x dx=\tan x+c$: $=2\sin x+9\tan x+c$
13	d	**Concept:** Indefinite Integrals of Exponential Functions **Rationale:** First, rewrite $\frac{5}{x^4}=5x^{-4}$, then use the sum/difference properties followed by the constant multiple rule: $=26\int e^x dx+5\int x^{-4}dx-10\int\cos x dx$ Then, evaluate. Note: $\int e^x dx=e^x+c$, $\int x^n dx=\frac{x^{n+1}}{n+1}+c$, and $\int\cos x dx=\sin x+c$: $=26e^x+5\cdot\frac{x^{-3}}{-3}-10\sin x+c$ Simplify: $=26e^x-\frac{5}{3x^3}-10\sin x+c$
		Concept: Changing the Variable: u-substitution with Power Rule **Rationale:** The expression x^6+32 is being acted on by the power 23. Because multiplying out a binomial to the power 23 is lengthy and tedious, we will try u-substitution. Make the substitution $u=x^6+32$ and find the differential: $du=6x^5 dx$ Solve the differential for $x^5 dx$: $\frac{1}{6}du=x^5 dx$ Replace x^6+32 with *u*, $x^5 dx$ with $\frac{1}{6}du$:

14	a	$=12\int u^{23}\cdot\frac{1}{6}du$ Use the constant multiple rule and general power rule, where $\int x^n dx=\frac{x^{n+1}}{n+1}+c$: $=\frac{12}{1}\cdot\frac{1}{6}\cdot\frac{u^{24}}{24}+c$ Simplify: $=\frac{1}{12}u^{24}+c$ Back substitute $u=x^6+32$: $=\frac{1}{12}(x^6+32)^{24}+c$
15	c	**Concept:** Changing the Variable: u-Substitution with Trigonometric Functions **Rationale:** Make the substitution $u=x^5-19$ and find the differential: $du=5x^4dx$ Solve for x^4dx: $\frac{1}{5}du=x^4dx$ Replace (x^5-19) with u and x^4dx with $\frac{1}{5}du$ and use the constant multiple rule: $\int 15x^4\sin(x^5-19)dx=15\int \sin u\cdot\frac{1}{5}du=3\int \sin u du$ Then, evaluate. Note: $\int \sin u du=-\cos u+c$: $=3(-\cos u)+c$ Simplify: $=-3\cos u+c$ Back substitute $u=x^5-19$: $=-3\cos(x^5-19)+c$

16	d	**Concept:** Changing the Variable: u-Substitution with Exponential Functions **Rationale:** Note that the $2x^4+5x-1$ is in the exponent of the exponential function and that $D[2x^4+5x-1]=8x^3+5$ is also in the integrand. Make the substitution $u=2x^4+5x-1$ and find the differential: $du=(8x^3+5)dx$ Replace the $2x^4+5x-1$ with *u* and $(8x^3+5)dx$ with *du*: $\int(8x^3+5)e^{2x^4+5x-1}dx=\int e^u du$ Then, evaluate: $=e^u+c$ Back substitute $u=2x^4+5x-1$: $=e^{2x^4+5x-1}+c$
17	b	**Concept:** Solving y' = f(x) **Rationale:** First, find the family of solutions. If $y'=f(x)$, then $y=\int f(x)dx$: $y=\int(27e^{-3x}-3\sqrt{x}+8\cos x)dx$ Next, apply the sum/difference properties followed by the constant multiple rule: $y=27\int e^{-3x}dx-3\int x^{\frac{1}{2}}dx+8\int\cos x dx$ Then, evaluate. Note: $\int e^{kx}dx=\frac{1}{k}e^{kx}+c$, $\int x^n dx=\frac{x^{n+1}}{n+1}+c$, and $\int\cos x dx=\sin x+c$: $y=27\cdot\frac{1}{-3}e^{-3x}-3\cdot\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+8\sin x+c$ To divide by a fraction, multiply the reciprocal:

		$y=27\cdot\frac{1}{-3}e^{-3x}-3\cdot\left(\frac{2}{3}\right)x^{\frac{3}{2}}+8\sin x+c$ Simplify: $y=-9e^{-3x}-2x^{\frac{3}{2}}+8\sin x+c$ Next, find the particular solution by substituting $x=0$, $y=2$, and solving for *c*. $2=-9e^{-3(0)}-2(0)^{\frac{3}{2}}+8\sin(0)+c$ $2=-9(1)-2(0)+8(0)+c$ $c=11$ The final solution is $y=-9e^{-3x}-2x^{\frac{3}{2}}+8\sin x+11$.
18	c	**Concept:** The Fundamental Theorem of Calculus **Rationale:** Begin by finding the indefinite integral. First, rewrite the exponent: $24\cos^3 x\sin x=24(\cos x)^3\cdot\sin x$ To evaluate, let $u=\cos x$ and find *du*. $du=-\sin x dx$ $-du=\sin x dx$ Replace $u=\cos x$ and $-du=\sin x dx$ and apply the constant multiple rule: $\int 24(\cos x)^3\cdot\sin x dx=\int 24u^3(-du)=-24\int u^3 du$ Now, use the power rule: $=-24\cdot\frac{u^4}{4}+c$ Simplify: $=-6u^4+c$ Back substitute $u=\cos x$: $=-6(\cos x)^4+c$ Now evaluate the definite integral using the fundamental theorem of calculus.

Remember "+c" is omitted since we are evaluating a definite integral:

$$\int_0^{\frac{2\pi}{3}} 24\cos^3 x \sin x dx = -6(\cos x)^4 \Big|_0^{\frac{2\pi}{3}}$$

$$= -6\left(\cos\left(\frac{2\pi}{3}\right)\right)^4 - (-6(\cos(0))^4)$$

$$= -6\left(-\frac{1}{2}\right)^4 + 6(1)^4$$

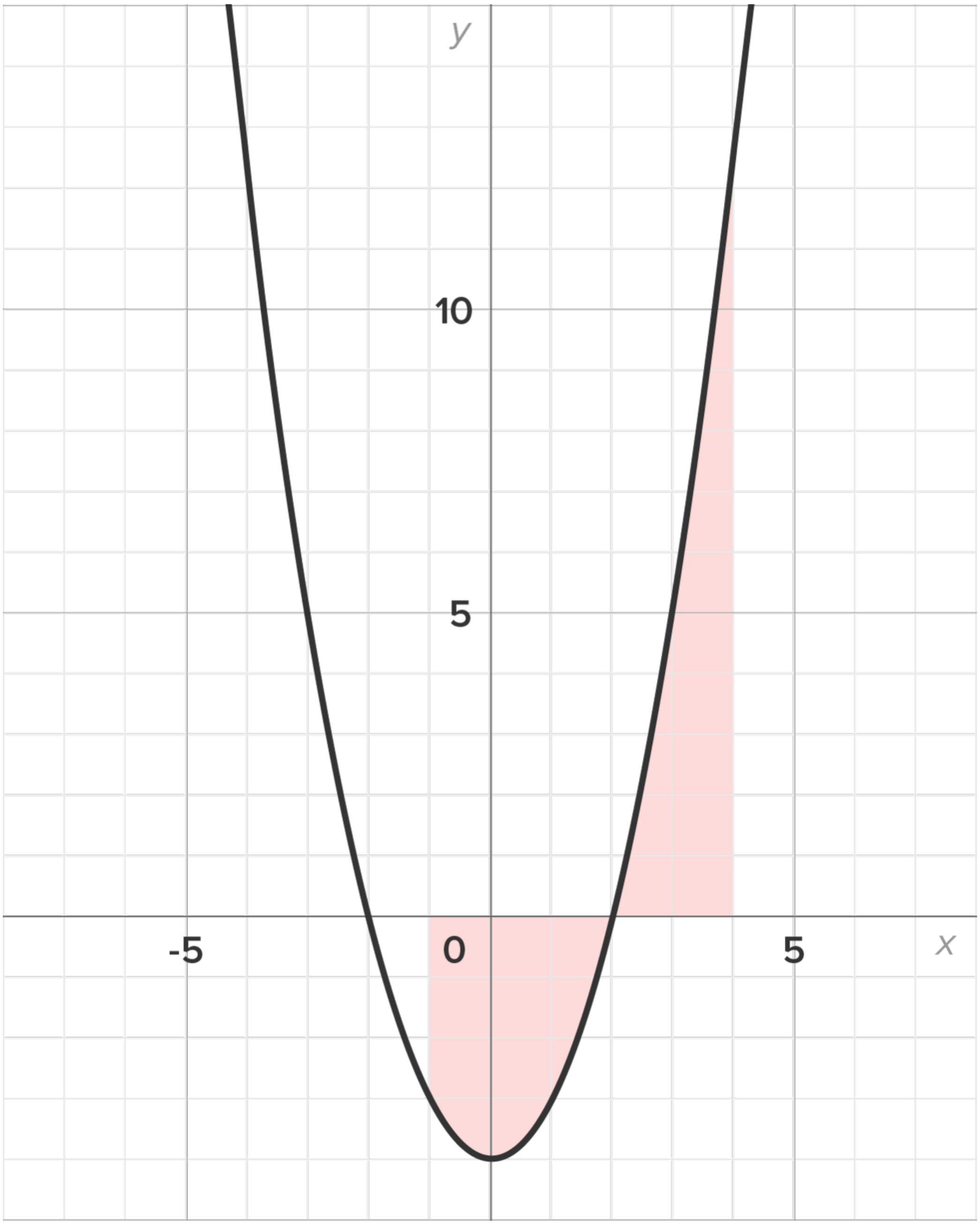
$$= -\frac{3}{8} + 6$$

$$= \frac{45}{8}$$

Concept: Antiderivative Applications

Rationale:

First, graph the function and shade between the function and the x-axis over the interval $[-1,4]$.



19 a

Notice that part of the region is below the x-axis and part of it is above the x-axis.

On the interval $[-1,2]$, the region is below the x-axis. On the interval $[2,4]$, the region is above the x-axis. This means $\int_{-1}^{2}(x^2-4)dx$ will give the negative value of the area and $\int_{2}^{4}(x^2-4)dx$ will give the value of the area.

Find the area of the region on $[-1,2]$:

$$\int_{-1}^{2}(x^2-4)dx=\left(\frac{1}{3}x^3-4x\right)\Big|_{-1}^{2}$$
$$=\left(\frac{1}{3}(2)^3-4(2)\right)-\left(\frac{1}{3}(-1)^3-4(-1)\right)$$
$$=\left(\frac{8}{3}-8\right)-\left(-\frac{1}{3}+4\right)$$
$$=\frac{8}{3}-8+\frac{1}{3}-4$$
$$=-9$$

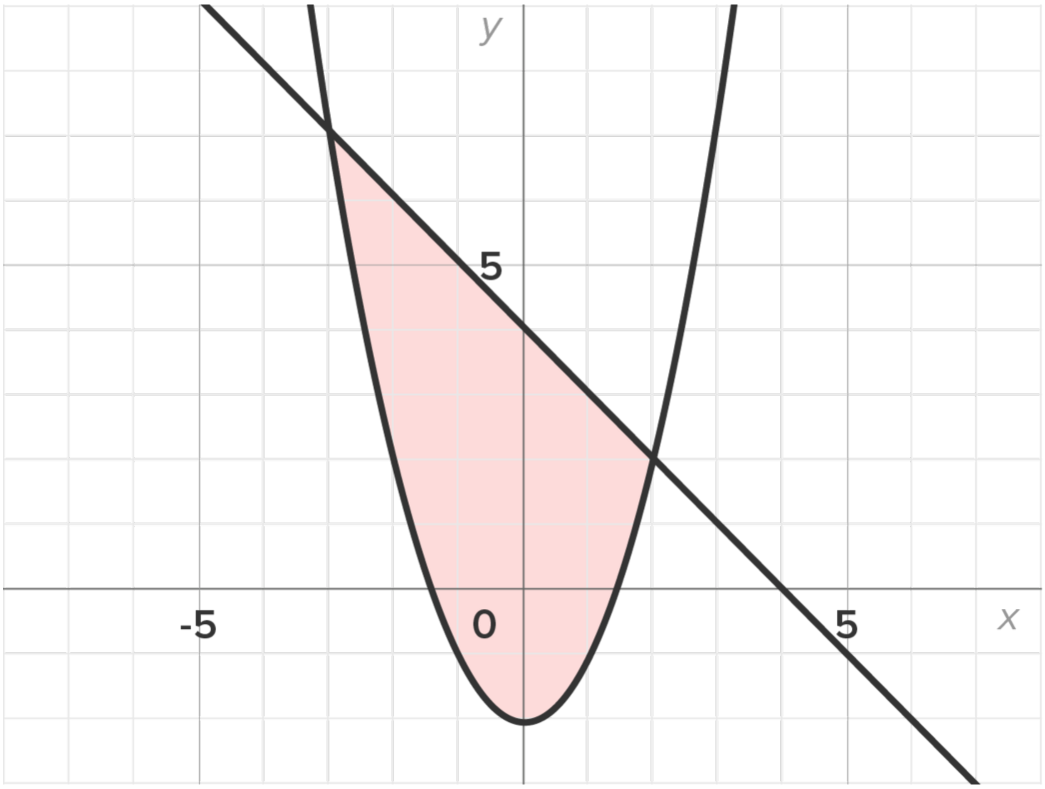
This result actually gives the negative value of the area. Therefore, take the opposite of it to get the area: 9.

Find the area of the region on $[2,4]$:

$$\int_{2}^{4}(x^2-4)dx=\left(\frac{1}{3}x^3-4x\right)\Big|_{2}^{4}$$
$$=\left(\frac{1}{3}(4)^3-4(4)\right)-\left(\frac{1}{3}(2)^3-4(2)\right)$$
$$=\left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-8\right)$$
$$=\frac{64}{3}-16-\frac{8}{3}+8$$
$$=\frac{32}{3}$$

Then, the total area of the region is:

$$9+\frac{32}{3}=\frac{59}{3}\ units^2$$

20	d	**Concept:** The Area Between Two Curves that Do Not Intertwine **Rationale:** First, graph the two functions and shade between them over the specified area:  The figure shows that $g(x) = -x+4$ is higher than $f(x) = x^2-2$ on the entire interval. Then, the area of the region is: $\int_{-3}^{2}((-x+4)-(x^2-2))dx$ Remove parentheses and collect like terms: $= \int_{-3}^{2}(-x^2-x+6)dx$ Now, integrate:

$$= \left(-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x\right)\Bigg|_{-3}^{2}$$

$$= \left(-\frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 + 6(2)\right) - \left(-\frac{1}{3}(-3)^3 - \frac{1}{2}(-3)^2 + 6(-3)\right)$$

$$= \left(-\frac{8}{3} - 2 + 12\right) - \left(9 - \frac{9}{2} - 18\right)$$

$$= -\frac{8}{3} - 2 + 12 - 9 + \frac{9}{2} + 18$$

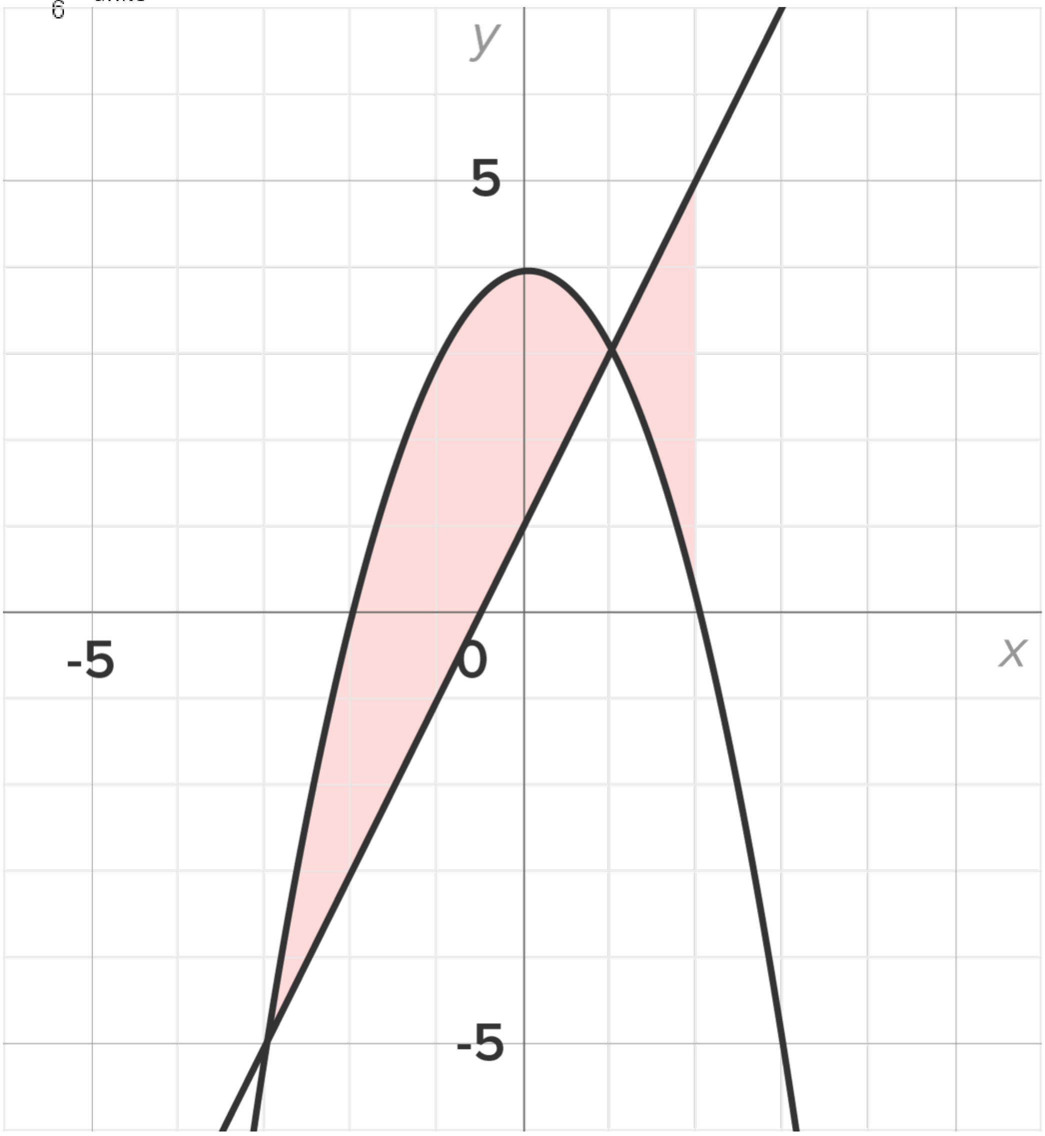
$$= \frac{125}{6}\ units^2$$

21 | b

Concept: The Area Between Two Curves that Intertwine

Rationale:

First, graph the functions and shade in the regions bounded by the graphs over the indicated interval.



Next, find the points where the graphs intersect:

$$-x^2 + 4 = 2x + 1$$
$$-x^2 - 2x + 3 = 0$$
$$-(x^2 + 2x - 3) = 0$$
$$-(x+3)(x-1) = 0$$
$$x = -3,\ x = 1$$

The two regions are on the intervals $[-3, 1]$ and $[1, 2]$. On the interval $[-3, 1]$, the graph

		of $y = -x^2+4$ is above the graph of $y = 2x+1$. On the interval $[1,2]$, the graph of $y = 2x+1$ is above the graph of $y = -x^2+4$. Evaluate each interval, starting with $[-3,1]$: $\int_{-3}^{1}((-x^2+4)-(2x+1))dx = \int_{-3}^{1}(-x^2-2x+3)dx$ $= \left(-\frac{1}{3}x^3-x^2+3x\right)\Big\|_{-3}^{1}$ $= \left(-\frac{1}{3}(1)^3-(1)^2+3(1)\right)-\left(-\frac{1}{3}(-3)^3-(-3)^2+3(-3)\right)$ $= \left(-\frac{1}{3}-1+3\right)-(9-9-9)$ $= -\frac{1}{3}-1+3-9+9+9$ $= \frac{32}{3}$ Next, evaluate the interval $[1,2]$: $\int_{1}^{2}((2x+1)-(-x^2+4))dx = \int_{1}^{2}(x^2+2x-3)dx$ $= \left(\frac{1}{3}x^3+x^2-3x\right)\Big\|_{1}^{2}$ $= \left(\frac{1}{3}(2)^3+(2)^2-3(2)\right)-\left(\frac{1}{3}(1)^3+(1)^2-3(1)\right)$ $= \left(\frac{8}{3}+4-6\right)-\left(\frac{1}{3}+1-3\right)$ $= \frac{8}{3}+4-6-\frac{1}{3}-1+3$ $= \frac{7}{3}$ To get the total area, add the areas of each of the two regions: $= \frac{32}{3}+\frac{7}{3} = 13\ units^2$
22	b	**Concept:** The Average Value of a Continuous Function on a Closed Interval **Rationale:** Recall the formula to find the average value of a continuous function on a closed interval is: $f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$ In this question, $f(x) = 3x^2+1$, $a=0$, and $b=4$. Substitute these values into the formula and evaluate:

		$f_{avg} = \frac{1}{4-0}\int_0^4 (3x^2+1)dx$ $= \frac{1}{4}\left(3\cdot\frac{x^3}{3}+x\right)\Big\vert_0^4$ $= \frac{1}{4}(x^3+x)\Big\vert_0^4$ $= \frac{1}{4}((4^3+4)-(0^4+0))$ $= 17$
23	a	**Concept:** The Mean Value Theorem for Integrals **Rationale:** For a continuous function on a closed interval, for at least one c in $[a,b]$, recall the formula: $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$ First, evaluate the average value: $\frac{1}{3-0}\int_0^3 (2x^2+1)dx = \frac{1}{3}\left(\frac{2}{3}x^3+x\right)\Big\vert_0^3$ $= \left(\frac{1}{3}\left(\frac{2}{3}(3)^3+3\right) - \frac{1}{3}\left(\frac{2}{3}(0)^3+0\right)\right)$ $= \frac{1}{3}\left(\frac{1}{8}+3\right)-0$ $= 7$ Substituting c in the function for the left-hand side of the equation and the average value of the function for the right-hand side of the equation gives: $2c^2+1=7$ $2c^2=6$ $c^2=3$ $c=\pm\sqrt{3}$ However, the values of c must be in the interval $[0,3]$. Therefore, only $c=\sqrt{3}$ should be reported as the answer.
24	a	**Concept:** Using Tables to Find Antiderivatives **Rationale:** Look through the table and find the formula that matches the original integral's form. According to formula #21: $\int \cot^2(ax)dx = -\frac{1}{a}\cot(ax)-x+c$ Substitute 12 for a: $\int \cot^2(12x)dx = -\frac{1}{12}\cot(12x)-x+c$

25	a	**Concept:** Approximating Definite Integrals **Rationale:** First, identify the important components of the definite integral $\int_1^3 \frac{1}{x^3+5}dx$ using $n=4$: $a=1$ $b=3$ $f(x)=\frac{1}{x^3+5}$ To estimate the definite integral using the trapezoidal rule, follow these steps: 1) Find $\Delta x=\frac{b-a}{n}$: $\Delta x=\frac{3-1}{4}=\frac{1}{2}$. 2) Write out the *x* values: $x_0=1,\ x_1=\frac{3}{2},\ x_2=2,\ x_3=\frac{5}{2},\ x_4=3$ 3) Then, by the trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}\cdot\Delta x(f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4))$ $\int_1^3 \frac{1}{x^3+5}dx \approx \frac{1}{2}\cdot\frac{1}{2}(f(1)+2f\left(\frac{3}{2}\right)+2f(2)+2f\left(\frac{5}{2}\right)+f(3))$ $\approx \frac{1}{4}\left(\frac{1}{1^3+5}+2\cdot\frac{1}{\left(\frac{3}{2}\right)^3+5}+2\cdot\frac{1}{2^3+5}+2\cdot\frac{1}{\left(\frac{5}{2}\right)^3+5}+\frac{1}{3^3+5}\right)$ $\approx \frac{1}{4}\left(\frac{1}{6}+\frac{16}{67}+\frac{2}{13}+\frac{16}{165}+\frac{1}{32}\right)$ ≈ 0.1718846219