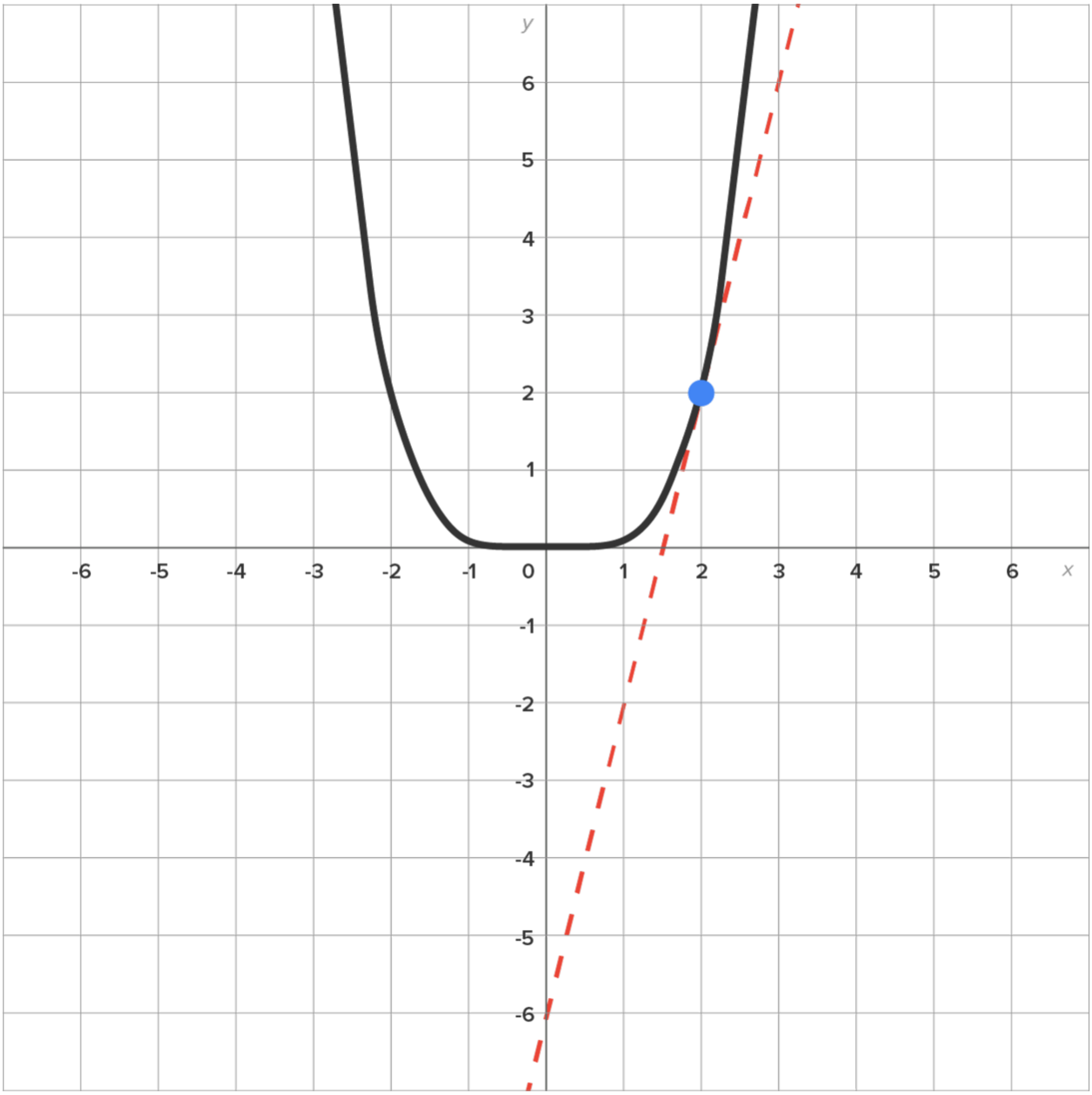
Practice Milestone

Calculus I — Practice Milestone 3

Taking this practice test is a stress-free way to find out if you are ready for the Milestone 3 assessment. You can print it out and test yourself to discover your strengths and weaknesses. The answer key is at the end of this Practice Milestone.

1.



Using the graph of $f(x) = \frac{1}{8}x^4$, estimate the derivative of $f(x)$ when $x = 2$.

- a.) 4
- b.) $-\frac{1}{4}$
- c.) $\frac{1}{4}$
- d.) -4

2.

Use the limit definition of the derivative to find $f'(x)$ when $f(x) = -9x^2$.

- a.) $f'(x) = -18x - 9h$
- b.) $f'(x) = -18x$
- c.) $f'(x) = 2x$
- d.) $f'(x) = 18x$

3.

Find the derivative of $g(x) = \frac{1}{x^{10}}$.

- a.) $g'(x) = \frac{-10}{x^9}$
- b.) $g'(x) = \frac{1}{10x^9}$
- c.) $g'(x) = \frac{-10}{x^{11}}$
- d.) $g'(x) = \frac{10}{x^{11}}$

4.

Write the equation of the line tangent to $f(x) = \frac{1}{x^2}$ **when** $x = -1$.

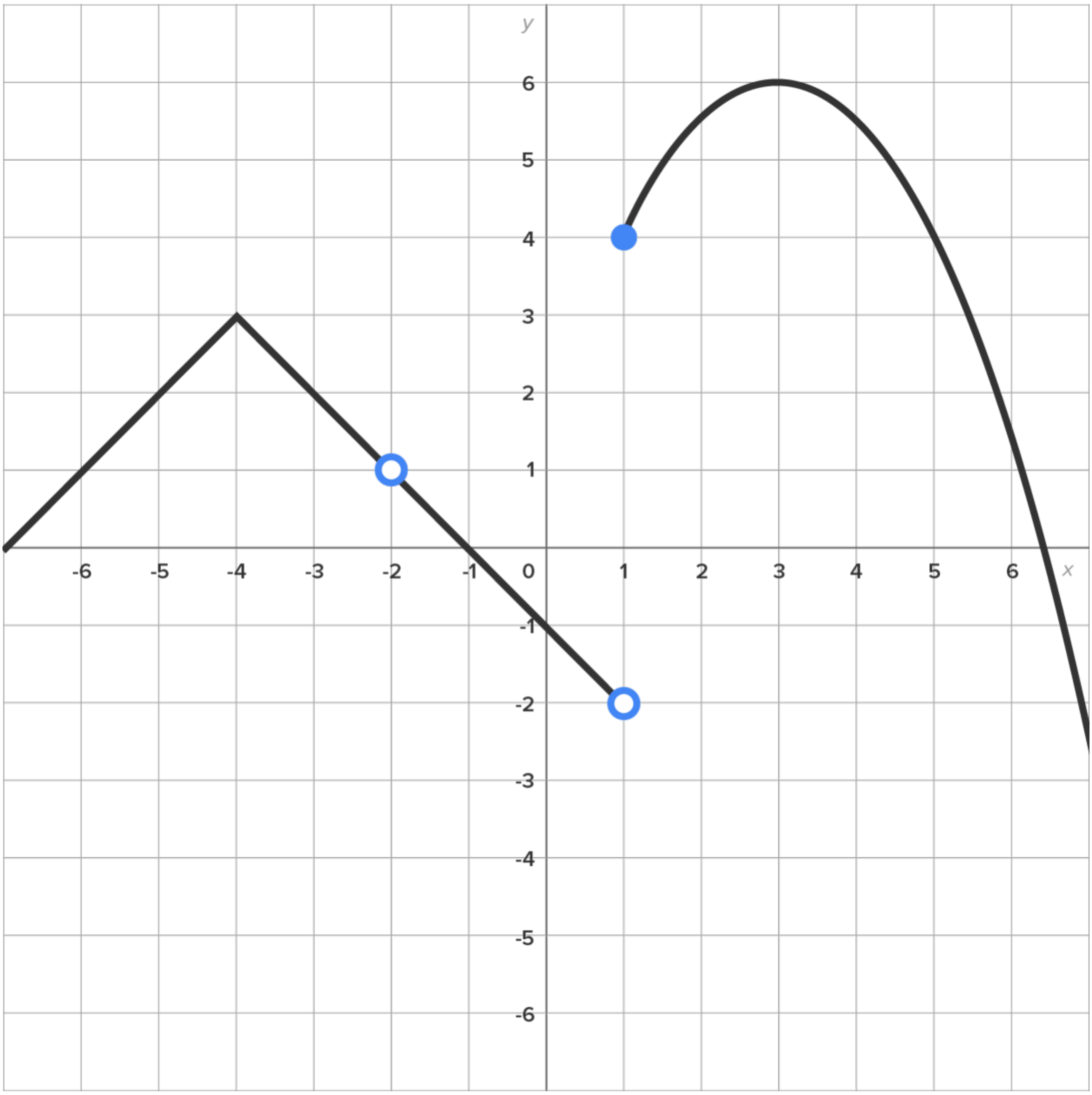
- a.) $y = 2x + 3$
- b.) $y = -2x - 1$
- c.) $y = -\frac{1}{2}x + \frac{1}{2}$
- d.) $y = \frac{1}{2}x + \frac{3}{2}$

5.

Determine if the function $f(x) = \sqrt[3]{x^2}$ **is differentiable at** $x = -64$**. If not, identify why.**

- a.) $f(x) = \sqrt[3]{x^2}$ is not differentiable at $x = -64$ because $f'(-64)$ is not defined.
- b.) $f(x) = \sqrt[3]{x^2}$ is differentiable at $x = -64$.
- c.) $f(x) = \sqrt[3]{x^2}$ is not differentiable at $x = -64$ because $\lim_{x \to -64} \sqrt[3]{x^2}$ does not exist.
- d.) $f(x) = \sqrt[3]{x^2}$ is not differentiable at $x = -64$ because $f(-64)$ is not defined.

6.



For the graph shown below, find all values of x for which $f(x)$ is not differentiable.

- **a.)** $x = -2, x = 1$
- **b.)** $x = -4, x = -2, x = 1$
- **c.)** $x = -4$
- **d.)** $f(x)$ is differentiable for all x-values.

7.

Find the derivative of $f(x) = \frac{3}{x^8} + 4\sqrt[3]{x} - 2x^3 - 17\cos x$.

a.) $f'(x) = -\frac{24}{x^9} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 - 17\sin x$

b.) $f'(x) = -\frac{24}{x^9} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 + 17\sin x$

c.) $f'(x) = -\frac{3}{8x^7} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 + 17\sin x$

d.) $f'(x) = -\frac{3}{8x^7} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 - 17\sin x$

8.

Find the derivative of $f(x) = x^5\cos x - 5\sin x$.

a.) $f'(x) = 5x^4\cos x - x^5\sin x - 5\cos x$

b.) $f'(x) = 5x^4\cos x\sin x + 5\cos x$

c.) $f'(x) = -5x^4\cos x\sin x - 5\cos x$

d.) $f'(x) = 5x^4\cos x + x^5\sin x + 5\cos x$

9.

Find the derivative of $f(x) = \frac{3x-5}{17\sin x}$.

- a.) $f'(x) = \frac{3}{17\cos x}$
- b.) $f'(x) = \frac{-3\sin x + 3x\cos x - 5\cos x}{17\sin^2 x}$
- c.) $f'(x) = \frac{17\sin x + 17x\cos x + 5\cos x}{17\sin^2 x}$
- d.) $f'(x) = \frac{3\sin x - 3x\cos x + 5\cos x}{17\sin^2 x}$

10.

Find the derivative of $f(x) = (4x^2 - 7x + 32)^{11}$.

- a.) $f'(x) = 11(8x-7)^{10}$
- b.) $f'(x) = 88x^{21} - 77x^{10}$
- c.) $f'(x) = 11(8x-7)(4x^2 - 7x + 32)^{10}$
- d.) $f'(x) = 11(4x^2 - 7x + 32)^{10}$

11.

Find the derivative of $y = 8x^3 - \frac{3}{x^8} + 7\tan x$.

- a.) $y' = 24x^2 - \frac{24}{x^7} + 7\sec^2 x$
- b.) $y' = 24x^2 + \frac{24}{x^7} + 7\sec x$
- c.) $y' = 24x^2 + \frac{24}{x^9} + 7\sec^2 x$
- d.) $y' = 24x^2 - \frac{3}{8x^7} + 7\sec^2 x$

12.

Find the third derivative of $f(x) = \frac{3}{x^5} - 9\cos x$.

- a.) $f'''(x) = -\frac{630}{x^8} + 9\sin x$
- b.) $f'''(x) = -\frac{3}{20x^3} - 9\sin x$
- c.) $f'''(x) = -\frac{630}{x^8} - 9\sin x$
- d.) $f'''(x) = -\frac{60}{x^3} - 9\sin x$

13.

Find the derivative of $y = \cos(5x^3 + 2x - 1)$.

- a.) $\frac{dy}{dx} = \sin(15x^2 + 2)$
- b.) $\frac{dy}{dx} = -\sin(15x^2 + 2)$
- c.) $\frac{dy}{dx} = (15x^2 + 2)\sin(5x^3 + 2x - 1)$
- d.) $\frac{dy}{dx} = -(15x^2 + 2)\sin(5x^3 + 2x - 1)$

14.

Find the derivative of $y = 3x^4e^{8x}$.

- a.) $\frac{dy}{dx} = 12x^3e^{8x} + 24x^4e^{8x}$
- b.) $\frac{dy}{dx} = 12x^3e^{8x}$
- c.) $\frac{dy}{dx} = 96x^3e^{8x}$
- d.) $\frac{dy}{dx} = 12x^3e^{8x} + 3x^4e^{8x}$

15.

Find the derivative of $f(x)=\sqrt{15^{6x+2}+9}$.

- a.) $f'(x)=\dfrac{3}{\sqrt{6\cdot 15^{6x+2}\ln 15}}$
- b.) $f'(x)=\dfrac{3}{\sqrt{6\cdot 15^{6x+2}}}$
- c.) $f'(x)=\dfrac{3}{\sqrt{15^{6x+2}+9}}\cdot 15^{6x+2}\ln 15$
- d.) $f'(x)=\dfrac{3}{\sqrt{15^{6x+2}+9}}\cdot 15^{6x+2}$

16.

Find the derivative of $f(x)=\ln\left(\dfrac{x^9}{8x^3-13}\right)$.

- a.) $f'(x)=\dfrac{9}{x}+\dfrac{24x^2}{8x^3-13}$
- b.) $f'(x)=\dfrac{9}{x}-\dfrac{24x^2}{8x^3-13}$
- c.) $f'(x)=\dfrac{9}{x}+\dfrac{1}{8x^3-13}$
- d.) $f'(x)=\dfrac{9}{x}-\dfrac{1}{8x^3-13}$

17.

Find the derivative of $y = \log_4\left(\frac{5x^2-2}{\sqrt{4x+7}}\right)$.

- a.) $\frac{dy}{dx} = \frac{10x}{5x^2-2} - \frac{2}{4x+7}$
- b.) $\frac{dy}{dx} = \frac{10x}{(5x^2-2)\ln 4} - \frac{2}{(4x+7)\ln 4}$
- c.) $\frac{dy}{dx} = \frac{10x}{(5x^2-2)\ln 4} + \frac{2}{(4x+7)\ln 4}$
- d.) $\frac{dy}{dx} = \frac{1}{5x^2-2} - \frac{1}{4x+7}$

18.

A penny is tossed upward with a velocity of 3.4 meters per second from a height of 26.5 meters. Its height in meters after t seconds is given by $h(t) = -4.9t^2 + 3.4t + 26.5$.

Find the velocity of the penny after 2.5 seconds.

- a.) 21.1 meters per second
- b.) -4.375 meters per second
- c.) -21.1 meters per second
- d.) 4.375 meters per second

19.

Find the linear approximation of $f(x)=\sqrt[5]{x}$ **when** $x=32$.

- a.) $L(x)=2-\frac{16}{5}(x-32)$
- b.) $L(x)=2+\frac{1}{80}(x-32)$
- c.) $L(x)=2+\frac{16}{5}(x+32)$
- d.) $L(x)=2+\frac{1}{80}(x+32)$

20.

Consider the function $f(x)=\sqrt[3]{x}$ near $x=27$.

Find the linear approximation error when using the linear approximation to estimate $\sqrt[3]{27.45}$.

- a.) -0.00010
- b.) -3.01667
- c.) 0.00010
- d.) 3.01667

21.

Let $y = \sin(5x^6 + x^2 - 17)$.

Find the differential, dy.

- a.) $dy = -\cos(5x^6 + x^2 - 17)dx$
- b.) $dy = \cos(5x^6 + x^2 - 17)dx$
- c.) $dy = (30x^5 + 2x)\cos(5x^6 + x^2 - 17)dx$
- d.) $dy = -(30x^5 + 2x)\cos(5x^6 + x^2 - 17)dx$

22.

A square is to be designed with sides of length 15 cm.

Use differentials to estimate the maximum error when measuring the area of the square if the possible error in measuring the side is 0.01 cm.

- a.) $0.3\ cm^2$
- b.) $0.0225\ cm^2$
- c.) $2.25\ cm^2$
- d.) $30\ cm^2$

23.

Write Newton's formula used to approximate $\sqrt[3]{27.5}$, a solution of the equation $x^3 - 27.5 = 0$ and find the third iteration value. Be sure to verify both parts of the answer are correct when making your selection.

- a.) $x_{n+1} = x_n - \dfrac{x_n^3 - 27.5}{3x_n^2}$; $x_3 \approx 3.018405368$
- b.) $x_{n+1} = 3 - \dfrac{x_n^3 - 27.5}{3x_n^2}$; $x_3 \approx 3.018633066$
- c.) $x_{n+1} = 3 + \dfrac{x_n^3 - 27.5}{3x_n^2}$; $x_3 \approx 2.943154442$
- d.) $x_{n+1} = x_n - \dfrac{x_n^3 - 27.5}{3x_n^2}$; $x_3 \approx 2.867900252$

24.

Given $7x^3 - 7\cos x = y^5 - 5$, compute $\dfrac{dy}{dx}$.

- a.) $\dfrac{dy}{dx} = \dfrac{21x^2 + 7\sin x - 5}{5y^4}$
- b.) $\dfrac{dy}{dx} = \dfrac{21x^2 + 7\sin x}{5y^4}$
- c.) $\dfrac{dy}{dx} = \dfrac{21x^2 - 7\sin x}{5y^4}$
- d.) $\dfrac{dy}{dx} = \dfrac{21x^2 - 7\sin x - 5}{5y^4}$

25.

Using logarithmic differentiation, find the derivative of $y = x^{4x^3+7}$.

- a.) $\frac{dy}{dx} = x^{4x^3+7}\left(12x^2\ln x + 4x^2 + \frac{7}{x}\right)$
- b.) $\frac{dy}{dx} = 12x^2\ln x + 4x^2 + \frac{7}{x}$
- c.) $\frac{dy}{dx} = \frac{1}{x^{4x^3+7}}\left(12x^2\ln x + 4x^2 + \frac{7}{x}\right)$
- d.) $\frac{dy}{dx} = x^{4x^3+7}(12x)$

26.

Find the exact value of $\cos^{-1}(-1)$.

- a.) $\cos^{-1}(-1) = \frac{3\pi}{2}$
- b.) $\cos^{-1}(-1) = \pi$
- c.) $\cos^{-1}(-1) = \frac{3\pi}{4}$
- d.) $\cos^{-1}(-1) = -\frac{\pi}{2}$

27.

Find the derivative of $f(x) = \arctan(4x^2)$.

- a.) $\dfrac{dy}{dx} = \dfrac{32x^3}{1+16x^4}$
- b.) $\dfrac{dy}{dx} = \dfrac{8x}{1+16x^4}$
- c.) $\dfrac{dy}{dx} = \dfrac{1}{1+16x^4}$
- d.) $\dfrac{dy}{dx} = \dfrac{8x}{\sqrt{1+16x^4}}$

28.

The radius of a circular oil spill is increasing at the rate of 1.2 meters per hour.

Find the rate at which the area of the spill is increasing when the radius is 55 meters.

- a.) $\dfrac{dA}{dt} = 110\pi \dfrac{m^2}{hr}$
- b.) $\dfrac{dA}{dt} = 66\pi \dfrac{m^2}{hr}$
- c.) $\dfrac{dA}{dt} = 132 \dfrac{m^2}{hr}$
- d.) $\dfrac{dA}{dt} = 132\pi \dfrac{m^2}{hr}$

29.

Gravel is falling on a conical pile at the rate of $28 \frac{ft^3}{min}$.

If the radius of the pile is always $\frac{1}{3}$ the height, find the rate at which the height is changing when the pile is 11.5 feet high.

- a.) $\frac{dh}{dt} = \frac{336\pi}{529} \frac{ft}{min}$
- b.) $\frac{dh}{dt} = \frac{1008\pi}{529} \frac{ft}{min}$
- c.) $\frac{dh}{dt} = \frac{1008}{529\pi} \frac{ft}{min}$
- d.) $\frac{dh}{dt} = \frac{336}{529\pi} \frac{ft}{min}$

Answer Key

Question	Answer	
1	a	**Concept:** Derivatives and Graphs **Rationale:** Consider the graph that shows $f(x)=\frac{1}{8}x^4$ (solid) and the tangent line (dashed) when $x=2$. The tangent line touches the graph at the point (2,2) and also goes through the point (1,-2). Using the slope formula, the slope of the tangent line appears to be 4 when $x=2$. Another way to say this is the derivative of $f(x)=\frac{1}{8}x^4$ when $x=2$ is 4.
2	b	**Concept:** Definition of Derivative **Rationale:** The limit definition of the derivative is $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$. First, compute $f(x+h)$: $f(x+h)=-9(x+h)^2$ $f(x+h)=-9(x^2+2xh+h^2)$ $f(x+h)=-9x^2-18xh-9h^2$ Then, replace $f(x+h)$ with $-9x^2-18xh-9h^2$ and $f(x)$ with $-9x^2$: $f'(x)=\lim\limits_{h\to 0}\frac{-9x^2-18xh-9h^2-(-9x^2)}{h}$ Combine like terms in the numerator: $f'(x)=\lim\limits_{h\to 0}\frac{-18xh-9h^2}{h}$ Separate the fractions: $f'(x)=\lim\limits_{h\to 0}\left(\frac{-18xh}{h}+\frac{-9h^2}{h}\right)$ Remove the common factor of "*h*" in each fraction: $f'(x)=\lim\limits_{h\to 0}(-18x-9h)$ Evaluate the limit as *h* approaches 0 by substituting 0 for *h*. $f'(x)=-18x$ Thus, if $f(x)=-9x^2$, then $f'(x)=-18x$.
		Concept: Basic Derivative Rules

3	c	**Rationale:** First, rewrite the function without an exponent in the denominator: $g(x) = x^{-10}$ Now apply the power rule: $g'(x) = -10x^{-10-1} = -10x^{-11}$ Since there is a negative exponent in the answer, this is not considered to be in simplest form. Use properties of exponents to rewrite without negative exponents: $g'(x) = \frac{-10}{x^{11}}$
4	a	**Concept:** Equations of Tangent Lines **Rationale:** First, the line is tangent to graph at the point $(-1, f(-1))$, or (-1, 1). Next, find the slope of the line by evaluating $f'(-1)$. You will need to write $f(x) = \frac{1}{x^2}$ with a single exponent, or $f(x) = x^{-2}$. By the power rule, $f'(x) = -2x^{-3} = \frac{-2}{x^3}$. Then, the slope of the tangent line at $x = -1$ is $f'(-1) = \frac{-2}{(-1)^3} = 2$. Now, use the tangent line formula $y = f(a) + f'(a)(x-a)$ and plug in the values $a = -1$, $f(-1) = 1$, $f'(-1) = 2$: $y = f(-1) + f'(-1)(x-(-1))$ $y = 1 + 2(x-(-1))$ Simplify the double negative: $y = 1 + 2(x+1)$ Distribute: $y = 1 + 2x + 2$ Combine like terms: $y = 2x + 3$ The equation of the tangent line is $y = 2x + 3$.
		Concept: Differentiability **Rationale:** $f(x) = \sqrt[3]{x^2}$ is a radical function with $n = 3$, which is odd, so it is continuous $(-\infty, \infty)$. Next, check the derivative. Rewrite the function with the fractional exponent:

5	b	$f(x) = x^{\frac{2}{3}}$ Then, find the derivative: $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$ Write without a negative exponent: $f'(x) = \frac{2}{3x^{\frac{1}{3}}}$ This answer is defined for all real numbers except $x = 0$. This means $f'(64)$ is defined. Because $f(x) = \sqrt[3]{x^2}$ is continuous at $x = -64$ and $f'(-64)$ is defined, $f(x)$ is differentiable at $x = -64$.
6	b	**Concept:** Determining Differentiability Graphically **Rationale:** Since the continuity requirement isn't met at any discontinuity, it follows that a function is not differentiable at any x-value where $f(x)$ is discontinuous. For this graph, the function is not continuous at $x = -2$ and $x = 1$, and, therefore, is not differentiable there. When the slope suddenly changes at $x = a$, then we say $f(x)$ is not differentiable when $x = a$. This is sometimes referred to as a sharp corner. For this graph, the function has a sharp corner at $x = -4$ and, therefore, is not differentiable there.
		Concept: Derivative of Elementary Combinations of Functions **Rationale:** Since this is a sum and difference of functions, use the sum and difference rules. First, evaluate $D\left[\frac{3}{x^8}\right]$ by rewriting without a power in the denominator: $D\left[\frac{3}{x^8}\right] = D[3x^{-8}]$ Then, use the constant multiple rule: $D\left[\frac{3}{x^8}\right] = 3D[x^{-8}]$ Now, use the power rule: $D\left[\frac{3}{x^8}\right] = 3(-8x^{-9})$ Simplify: $D\left[\frac{3}{x^8}\right] = -24x^{-9}$

Rewrite using positive exponents:
$D\left[\frac{3}{x^8}\right] = -\frac{24}{x^9}$

Second, find $D[4\sqrt[3]{x}]$. Rewrite the expression using rational exponents:
$D[4\sqrt[3]{x}] = D\left[4x^{\frac{1}{3}}\right]$

Then, use the constant multiple rule:
$D[4\sqrt[3]{x}] = 4D\left[x^{\frac{1}{3}}\right]$

7 b

Now, use the power rule:
$D[4\sqrt[3]{x}] = 4\left(\frac{1}{3}x^{-\frac{2}{3}}\right)$

Rewrite using positive exponents:
$D[4\sqrt[3]{x}] = \frac{4}{3x^{\frac{2}{3}}}$

Third, evaluate $D[2x^3]$ by applying the constant multiple rule:
$D[2x^3] = 2D[x^3]$

Now, apply the power rule:
$D[2x^3] = 2(3x^2)$

Simplify:
$D[2x^3] = 6x^2$

Third, evaluate $D[17\cos x]$ by using the constant multiple rule:
$D[17\cos x] = 17D[\cos x]$

Now, find the derivative of $\cos x$:
$D[17\cos x] = 17(-\sin x)$

Simplify:
$D[17\cos x] = -17\sin x$

Now combine all three parts:
$$D\left[\frac{3}{x^8} + 4\sqrt[3]{x} - 2x^3 - 17\cos x\right] = -\frac{24}{x^9} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 - (-17\sin x) = -\frac{24}{x^9} + \frac{4}{3x^{\frac{2}{3}}} - 6x^2 + 17\sin x$$

		Thus, $f'(x) = -\dfrac{24}{x^9} + \dfrac{4}{3x^{\frac{2}{3}}} - 6x^2 + 17\sin x$.
8	a	**Concept:** The Product Rule **Rationale:** Since $f(x)$ is a difference of two functions, the difference rule should be applied first: $f'(x) = D[x^5\cos x] - D[5\sin x]$ Next, apply the constant multiple and product rules: $f'(x) = D[x^5]\cdot\cos x + x^5 D[\cos x] - 5D[\sin x]$ Recall $D[x^5] = 5x^4$, $D[\cos x] = -\sin x$, and $D[\sin x] = \cos x$: $f'(x) = 5x^4\cdot\cos x + x^5(-\sin x) - 5\cos x$ Simplify and remove excess symbols: $f'(x) = 5x^4\cos x - x^5\sin x - 5\cos x$
9	d	**Concept:** The Quotient Rule **Rationale:** This is a quotient of two functions that cannot be simplified prior to differentiating. Apply the quotient rule formula and simplify: $f'(x) = \dfrac{17\sin x\cdot D[3x-5] - (3x-5)\cdot D[17\sin x]}{(17\sin x)^2}$ Recall $D[3x-5] = 3$ and $D[17\sin x] = 17\cos x$: $f'(x) = \dfrac{17\sin x(3) - (3x-5)\cdot 17\cos x}{(17\sin x)^2}$ Simplify by removing the parentheses: $f'(x) = \dfrac{51\sin x - 51x\cos x + 85\cos x}{289\sin^2 x}$ In the numerator, factor out a common factor of 17: $f'(x) = \dfrac{17(3\sin x - 3x\cos x + 5\cos x)}{289\sin^2 x}$ Remove the common factor of 17 from the numerator and the denominator: $f'(x) = \dfrac{3\sin x - 3x\cos x + 5\cos x}{17\sin^2 x}$ This is the simplest form.
		Concept: The General Power Rule for Functions **Rationale:** This is a function raised to a power. By the power rule, we have the following: $f'(x) = 11(4x^2 - 7x + 32)^{10}\cdot D[4x^2 - 7x + 32]$

10	c	Next, evaluate $D[4x^2-7x+32]$: $f'(x)=11(4x^2-7x+32)^{10}\cdot(8x-7)$ Rearrange factors: $f'(x)=11(8x-7)(4x^2-7x+32)^{10}$ Thus, $f'(x)=11(8x-7)(4x^2-7x+32)^{10}$.
11	c	**Concept:** Derivatives of Trigonometric Functions **Rationale:** First, rewrite $\frac{3}{x^8}$ as $3x^{-8}$ so that the power rule can be used: $y=8x^3-3x^{-8}+7\tan x$ Then, apply sum/dIfference rules: $y'=D[8x^3]-D\left[3x^{-8}\right]+D[7\tan x]$ Now, apply the constant multiple rules: $y'=8D[x^3]-3D\left[x^{-8}\right]+7D[\tan x]$ Recall $D[x^3]=3x^2$, $D\left[x^{-8}\right]=-8x^{-9}$, and $D[\tan x]=\sec^2 x$: $y'=8(3x^2)-3(-8x^{-9})+7(\sec^2 x)$ Simplify: $y'=24x^2+24x^{-9}+7\sec^2 x$ Write the middle term with a positive exponent: $y'=24x^2+\frac{24}{x^9}+7\sec^2 x$
		Concept: Higher-Order Derivatives **Rationale:** First, rewrite using negative exponents to make use of the power rule: $f(x)=\frac{3}{x^5}-9\cos x=3x^{-5}-9\cos x$ Now, take appropriate derivatives. To find the first derivative, apply the difference and power rules and recall that $D[\cos x]=-\sin x$: $f'(x)=3(-5x^{-6})-9(-\sin x)$ Simplify: $f'(x)=-15x^{-6}+9\sin x$

12	c	Since we are finding more derivatives, there is no need to rewrite with positive exponents just yet. We will save this for when all derivatives are taken. To find the second derivative, use the sum and power rules, and recall that $D[\sin x] = \cos x$: $f''(x) = -15(-6x^{-7}) + 9(\cos x)$ Simplify: $f''(x) = 90x^{-7} + 9\cos x$ To find the third derivative, use sum, constant multiple, and power rules, and recall that $D[\cos x] = -\sin x$: $f'''(x) = 90(-7x^{-8}) + 9(-\sin x)$ Simplify: $f'''(x) = -630x^{-8} - 9\sin x$ Rewrite with positive exponents: $f'''(x) = -\frac{630}{x^8} - 9\sin x$
13	d	**Concept:** The Chain Rule **Rationale:** This is a composite function. Let $u = 5x^3 + 2x - 1$, the inner function: $y = \cos u$ Apply the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$: $\frac{dy}{dx} = -\sin u \cdot D[u]$ Substitute $5x^3 + 2x - 1$ in for u to find the derivative: $\frac{dy}{dx} = -\sin u \cdot D[5x^3 + 2x - 1]$ Evaluate $D[5x^3 + 2x - 1]$: $\frac{dy}{dx} = -\sin u \cdot (15x^2 + 2)$ Write $(15x^2 + 2)$ in front and replace all other instances of u with $(5x^3 + 2x - 1)$: $\frac{dy}{dx} = -(15x^2 + 2)\sin(5x^3 + 2x - 1)$
		Concept: Derivative of $y = e^x$ **Rationale:** This is a product of two functions and the composition of functions.

14	a	First, use the product rule: $\frac{dy}{dx} = D[3x^4] \cdot e^{8x} + 3x^4 D[e^{8x}]$ Next, evaluate $D[3x^4]$: $\frac{dy}{dx} = 12x^3 \cdot e^{8x} + 3x^4 D[e^{8x}]$ Then, use the chain rule to evaluate $D[e^{8x}]$ with $u = 8x$: $\frac{dy}{dx} = 12x^3 \cdot e^{8x} + 3x^4(8e^{8x})$ Simplify: $\frac{dy}{dx} = 12x^3 e^{8x} + 24x^4 e^{8x}$
15	c	**Concept:** Derivative of y = a^x **Rationale:** Rewrite the radical using a fractional exponent: $f(x) = \left(15^{6x+2} + 9\right)^{\frac{1}{2}}$ This is a composition of functions with two layers. First, use the power rule where $D[u^{\frac{1}{2}}] = \frac{1}{2} u^{-\frac{1}{2}} \cdot u'$ with $u = 15^{6x+2} + 9$: $f'(x) = \frac{1}{2}\left(15^{6x+2} + 9\right)^{-\frac{1}{2}} \cdot D[15^{6x+2} + 9]$ Next, use the chain rule where $D[15^u + 9] = 15^u \cdot \ln 15 \cdot u'$ with $u = 6x + 2$: $f'(x) = \frac{1}{2}\left(15^{6x+2} + 9\right)^{-\frac{1}{2}} \cdot 15^{6x+2} \ln 15 \cdot D[6x + 2]$ Evaluate $D[6x + 2]$: $f'(x) = \frac{1}{2}\left(15^{6x+2} + 9\right)^{-\frac{1}{2}} \cdot 15^{6x+2} \ln 15 \cdot 6$ Rewrite multiplying numerical factors and without negative exponents: $f'(x) = \frac{3}{\sqrt{15^{6x+2} + 9}} \cdot 15^{6x+2} \ln 15$
		Concept: Derivatives of Natural Logarithmic Functions **Rationale:** First, use properties of logarithms to rewrite the function as $\ln\frac{a}{b} = \ln a - \ln b$: $f(x) = \ln(x^9) - \ln(8x^3 - 13)$

16	b	Next, use the property of logarithm $\ln(a^b) = b \cdot \ln a$ to rewrite the function: $f(x) = 9\ln(x) - \ln(8x^3 - 13)$ Now, evaluate the derivative: $f'(x) = D[9\ln(x)] - D[\ln(8x^3 - 13)]$ Find each derivative, and use $D[\ln u] = \frac{u'}{u}$ with $u = 8x^3 - 13$: $f'(x) = 9 \cdot \frac{1}{x} - \frac{24x^2}{8x^3 - 13}$ Simplify: $f'(x) = \frac{9}{x} - \frac{24x^2}{8x^3 - 13}$
17	b	**Concept:** Derivatives of Non-Natural Logarithmic Functions **Rationale:** First, use properties of logarithms to rewrite the function as $\log_4 \frac{a}{b} = \log_4 a - \log_4 b$: $y = \log_4(5x^2 - 2) - \log_4\sqrt{4x + 7}$ Write the radical as a fractional exponent: $y = \log_4(5x^2 - 2) - \log_4(4x + 7)^{\frac{1}{2}}$ Next, use the property of logarithms $\log_4 a^p = p \cdot \log_4 a$ to rewrite the function: $y = \log_4(5x^2 - 2) - \frac{1}{2}\log_4(4x + 7)$ Now, evaluate the derivative: $\frac{dy}{dx} = D[\log_4(5x^2 - 2)] - D[\frac{1}{2}\log_4(4x + 7)]$ Recall that $D[\log_a u] = \frac{1}{u \cdot \ln a} \cdot u' = \frac{u'}{u \cdot \ln a}$: $\frac{dy}{dx} = \frac{10x}{(5x^2 - 2)\ln 4} - \frac{1}{2} \cdot \frac{4}{(4x + 7)\ln 4}$ Simplify: $\frac{dy}{dx} = \frac{10x}{(5x^2 - 2)\ln 4} - \frac{2}{(4x + 7)\ln 4}$
		Concept: Applications of Rates of Change **Rationale:** The instantaneous velocity function is the derivative of the height function: $v(t) = h'(t) = -9.8t + 3.4$

18	c	Evaluate the velocity at $t=2.5$: $v(2.5) = -9.8(2.5)+3.4$ Simplify: $v(2.5) = -21.1$ meters per second
19	b	**Concept:** Linear Approximation **Rationale:** Remember that this is really the equation of the tangent line at $x=32$. To find the derivative, first rewrite as a power of *x*: $f(x)=\sqrt[5]{x}=x^{\frac{1}{5}}$ Apply the power rule: $f'(x)=\frac{1}{5}x^{-\frac{4}{5}}$ Rewrite with a positive exponent, then the radical: $f'(x)=\frac{1}{5x^{\frac{4}{5}}}=\frac{1}{5\sqrt[5]{x^4}}=\frac{1}{5(\sqrt[5]{x})^4}$ Substitute 32 for *x* and simplify: $f'(32)=\frac{1}{5(\sqrt[5]{32})^4}=\frac{1}{5(2)^4}=\frac{1}{5(16)}=\frac{1}{80}$ Now, form the linear approximation $L(x)=f(a)+f'(a)(x-a)$ when $a=32$: $L(x)=f(32)+f'(32)(x-32)$ Evaluate $f(32)$ and $f'(32)$: $f(32)=2$ $f'(32)=\frac{1}{80}$ Substitute values into the linear approximation: $L(x)=2+\frac{1}{80}(x-32)$ The linear approximation at $x=32$ is $L(x)=2+\frac{1}{80}(x-32)$. This means that $L(x)$ approximates the value of $f(x)=\sqrt[5]{x}$ near $x=32$.
		Concept: The Linear Approximation Error | f(x) – L(x) | **Rationale:** To find the linear approximation error, first find the linear approximation at a compatible *x* value near 27.45. Since 27 is close to 27.45, and $\sqrt[3]{27}$ is easy to calculate, write the linear approximation equation at $x=27$. Remember that this is really the equation of the tangent line at $a=27$.

20	c	To find the linear approximation, first find the derivative: $f(x)=x^{\frac{1}{3}}$ $f'(x)=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3x^{\frac{2}{3}}}$ Now, form the linear approximation $L(x)=f(a)+f'(a)(x-a)$ when $a=27$: $L(x)=f(27)+f'(27)(x-27)$ Evaluate $f(27)$ and $f'(27)$: $f(27)=3$ $f'(27)=\frac{1}{27}$ Substitute into the linear approximation: $L(x)=3+\frac{1}{27}(x-27)$ The linear approximation to 5 decimal places tells us that $\sqrt[3]{27.45}\approx L(27.45)=3+\frac{1}{27}(27.45-27)=3.01667$. The actual value of $\sqrt[3]{27.45}$ is 3.01657 (to 5 decimal places). Then, the linear approximation error is: $\lvert f(27.45)-L(27.45)\rvert=\lvert 3.01657-3.01667\rvert=0.00010$
21	c	**Concept:** The Differential of f **Rationale:** First, find the derivative. This is a composite function, so use the chain rule with $D[\sin u]=\cos u\cdot u'$ where $u=5x^6+x^2-17$: $y'=(5x^6+x^2-17)\cdot(30x^5+2x)$ Rewrite: $y'=(30x^5+2x)\cos(5x^6+x^2-17)$ Thus, the differential is $dy=y'dx$: $dy=(30x^5+2x)\cos(5x^6+x^2-17)dx$
22	a	**Concept:** Approximation of Measurement Error Using Differentials **Rationale:** The area of a square is $A(s)=s^2$, which has the derivative $A'(s)=2s$. Thus, the differential is $dA=2s\ ds$. Now, let $s=15$ and $ds=0.01$. Then, the maximum error in estimating the area when $s=15$ and $ds=0.01$ is:

		$dA = 2(15)(0.01) = 0.3\ cm^2$
23	a	**Concept:** The Algorithm for Newton's Method **Rationale:** Since the equation, which is solved by $\sqrt[3]{27.5}$, is written with 0 on one side, this gives $f(x) = x^3 - 27.5$. The derivative is $f'(x) = 3x^2$. To find the initial guess, graph the function and note the graph crosses the *x*-axis near 3. Choose $x_0 = 3$. For Newton's method, use the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Substituting the function and the derivative, we have: $x_{n+1} = x_n - \frac{x_n^3 - 27.5}{3x_n^2}$ Using $x_0 = 3$, the iterations are: $x_1 = 3 - \frac{3^3 - 27.5}{3(3)^2} \approx 3.018518519$ $x_2 = 3.0185185519 - \frac{(3.0185185519)^3 - 27.5}{3(3.0185185519)^2} \approx 3.018405373$ $x_3 = 3.018405373 - \frac{(3.018405373)^3 - 27.5}{3(3.018405373)^2} \approx 3.018405368$
24	b	**Concept:** Implicit Differentiation **Rationale:** Apply the derivative to each term (sum/difference rule): $D[7x^3] - D[7\cos x] = D[y^5] - D[5]$ Evaluate each term: $D[7x^3] = 21x^2$ $D[7\cos x] = 7 \cdot (-\sin x) = -7\sin x$ $D[y^5] = 5y^4 \cdot \frac{dy}{dx}$ $D[5] = 0$ Substitute each value: $21x^2 - (-7\sin x) = 5y^4 \frac{dy}{dx}$ Simplify: $21x^2 + 7\sin x = 5y^4 \frac{dy}{dx}$

		Solve for $\frac{dy}{dx}$ by dividing both sides by $5y^4$: $\frac{dy}{dx}=\frac{21x^2+7\sin x}{5y^4}$
25	a	**Concept:** Logarithmic Differentiation **Rationale:** Since both the base and power are variables, logarithmic differentiation will be useful. Apply natural logarithm to both sides: $\ln y=\ln x^{4x^3+7}$ Use the property of logarithms, $\ln a^b=b\ln a$: $\ln y=(4x^3+7)\ln x$ Take the derivative of both sides: $D[\ln y]=D[(4x^3+7)\ln x]$ Evaluate each term and use the product rule: $D[\ln y]=\frac{1}{y}\cdot\frac{dy}{dx}$ $D\left[(4x^3+7)\cdot\ln x\right]=D[4x^3+7]\cdot\ln x+(4x^3+7)\cdot D[\ln x]=12x^2\ln x+(4x^3+7)\frac{1}{x}$ Substitute each value: $\frac{1}{y}\cdot\frac{dy}{dx}=12x^2\ln x+(4x^3+7)\frac{1}{x}$ Remove the parentheses on the right hand side of the equation: $\frac{1}{y}\cdot\frac{dy}{dx}=12x^2\ln x+4x^2+\frac{7}{x}$ To solve for $\frac{dy}{dx}$, multiply both sides by y: $\frac{dy}{dx}=y\left(12x^2\ln x+4x^2+\frac{7}{x}\right)$ Because $y=x^{4x^3+7}$, substitute x^{4x^3+7} for y: $\frac{dy}{dx}=x^{4x^3+7}\left(12x^2\ln x+4x^2+\frac{7}{x}\right)$
26	b	**Concept:** The Inverse Trigonometric Functions **Rationale:** To find the value of $\cos^{-1}(-1)$, first note that the range for $\cos^{-1}x$ is $[0,\pi]$. Then think what angle in the range from 0 to π has a cosine ratio of -1. Thus, $\cos^{-1}(-1)=\pi$ because $\cos(\pi)=-1$ and π is in $[0,\pi]$.

27	b	**Concept:** Derivatives of Inverse Trigonometric Functions **Rationale:** The original function is $f(x)=\arctan(4x^2)$. Use the chain rule to find the derivative of $\arctan u$ where $u=4x^2$ and $u'=8x$: $D[\arctan u]=\frac{u'}{1+u^2}=\frac{8x}{1+(4x^2)^2}$ Use this to find $\frac{dy}{dx}$ and simplify the denominator: $\frac{dy}{dx}=\frac{8x}{1+16x^4}$
28	d	**Concept:** Related Rates Problems Using Geometric Formulas **Rationale:** First, identify the geometrical formula to use. Since the ripples are circles and we are asked to find the rate of change of the area, use the formula for the area of a circle, $A=\pi r^2$. Now identify all known quantities and what we are looking for: Given: $\frac{dr}{dt}=1.2\frac{m}{hr}$ Want to know: $\frac{dA}{dt}$ when $r=55m$ Since the information we have (and need) involves rates, we use implicit differentiation to take the derivative with respect to t: $1\cdot\frac{dA}{dt}=\pi\cdot 2r\cdot\frac{dr}{dt}$ Simplify: $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ Substitute the known quantities: $\frac{dA}{dt}=2\pi(55m)(1.2\frac{m}{hr})$ Simplify: $\frac{dA}{dt}=132\pi\frac{m^2}{hr}$ This is the value of $\frac{dA}{dt}$ when $r=55m$ and $\frac{dr}{dt}=1.2\frac{m}{hr}$.
		Concept: Related Rates Problems Using Proportional Reasoning and Trigonometry **Rationale:** We are given $\frac{dV}{dt}=28\frac{ft^3}{min}$, and we want $\frac{dh}{dt}$ when $h=11.5$ feet. From the geometry formulas, we know that $V=\frac{\pi}{3}r^2h$, where r is the radius of the gravel at the base of the cone and h is the height of the gravel of the cone.

29	c	Notice that there isn't any numerical information given about the radius of the conical shape. However, we are given that $r = \frac{1}{3}h$. Using this information, substitute $\frac{1}{3}h$ for *r* in the formula before differentiating: $V = \frac{\pi}{3}(\frac{1}{3}h)^2(h)$ Simplify: $V = \frac{\pi}{27}h^3$ Now that the equation has been written in terms of the variables we know or want to know, take the derivative of both sides with respect to *t*: $1 \cdot \frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \frac{dh}{dt}$ Simplify: $\frac{dV}{dt} = \frac{\pi}{9}h^2 \frac{dh}{dt}$ Substitute the given information: $28\frac{ft^3}{min} = \frac{\pi}{9}(11.5\ ft)^2 \frac{dh}{dt}$ Simplify: $28\frac{ft^3}{min} = \frac{132.25\pi}{9}ft^2 \frac{dh}{dt}$ $\frac{252}{132.25\pi}\ \frac{ft}{min} = \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{1008}{529}\ \frac{ft}{min}$