Practice Milestone

Calculus I — Practice Milestone 1

Taking this practice test is a stress-free way to find out if you are ready for the Milestone 1 assessment. You can print it out and test yourself to discover your strengths and weaknesses. The answer key is at the end of this Practice Milestone.

1.

Calculate the exact distance between the points (8, -3) and (12, 2).

- a.) 9
- b.) $\sqrt{17}$
- c.) $\sqrt{221}$
- d.) $\sqrt{41}$

2.

Write the equation of a circle whose center is (3, -7) and which has a radius of 9.

- a.) $(x+3)^2+(y-7)^2=81$
- b.) $(x-3)^2+(y+7)^2=81$
- c.) $(x-3)^2+(y+7)^2=9$
- d.) $(x-3)^2+(y+7)^2=18$

3.

Calculate the slope of the line that passes through (13, -8) and (2, 8).

- a.) $-\frac{11}{16}$
- b.) $-\frac{7}{2}$
- c.) $-\frac{16}{11}$
- d.) 0

4.

Write the equation of the line that contains the point (-6, 2) and has slope $\frac{5}{3}$ in slope-intercept form.

- a.) $y=\frac{5}{3}x+12$
- b.) $y=\frac{5}{3}x-12$
- c.) $y=\frac{5}{3}x+2$
- d.) $y=\frac{5}{3}x-8$

5.

Which of the following is a function?

- a.) $6x-3y=21$
- b.) $x^2+y^2=121$
- c.) $59y^2-14x^2=23$
- d.) $y^2+7x^2=12x$

6.

Consider the function $f(x)=-2x^2+7x+5$.

Use it to find *f*(3), *f*(-4), and *f*(a – 6).

- a.) $f(3)=62$
  $f(-4)=9$
  $f(a-6)=-2a^2+7a-109$
- b.) $f(3)=8$
  $f(-4)=-55$
  $f(a-6)=-2a^2+31a-109$
- c.) $f(3)=14$
  $f(-4)=41$
  $f(a-6)=-2a^2+7a-109$
- d.) $f(3)=62$
  $f(-4)=65$
  $f(a-6)=-2a^2-5a+35$

7.

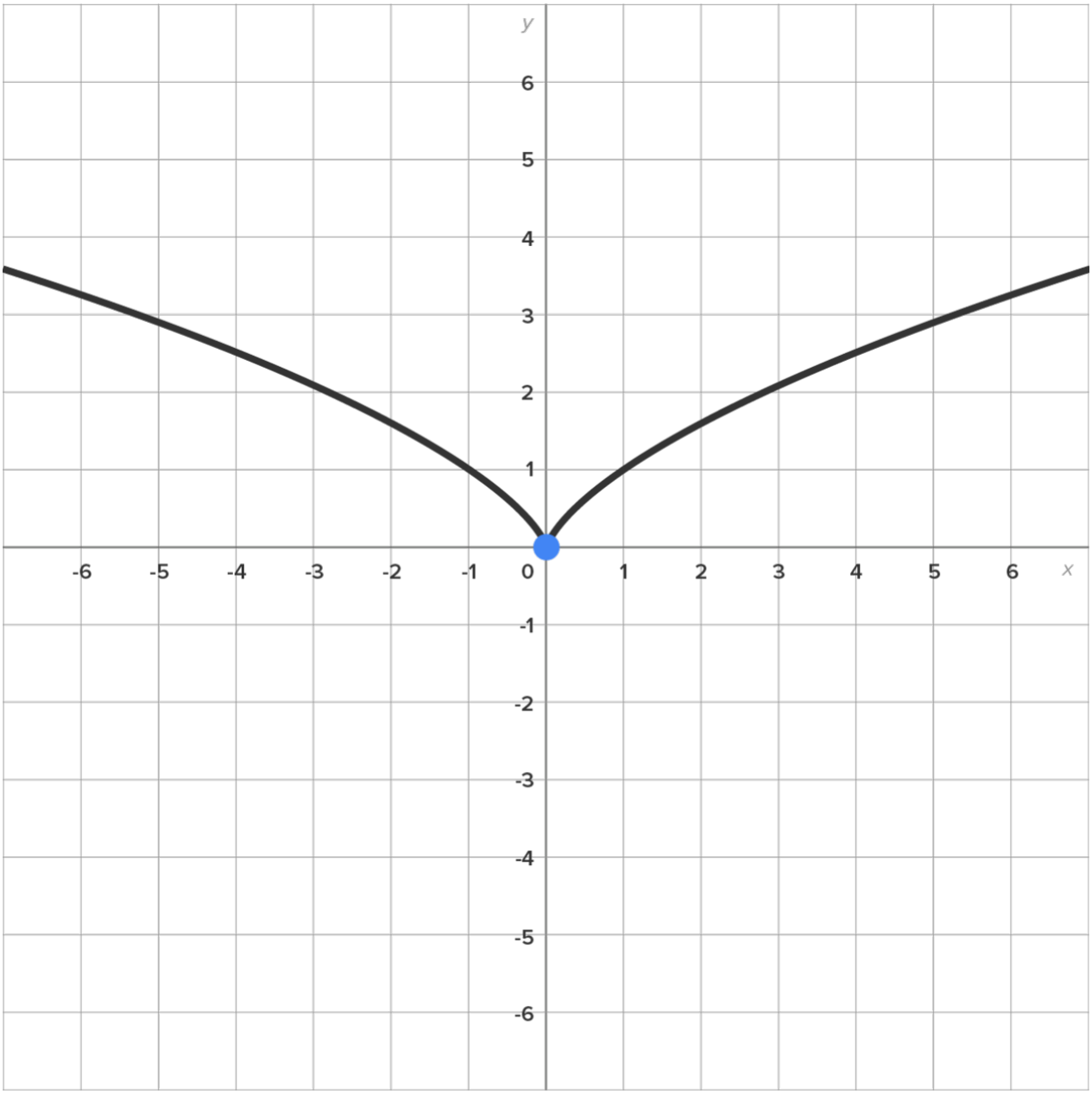
Evaluate the difference quotient for $f(x) = 7x^2 - 4x + 6$.

- a.) $7h - 4$
- b.) $\frac{2xh + h^2 + h - 8x + 12}{h}$
- c.) $7x + 7h - 4$
- d.) $14x + 7h - 4$

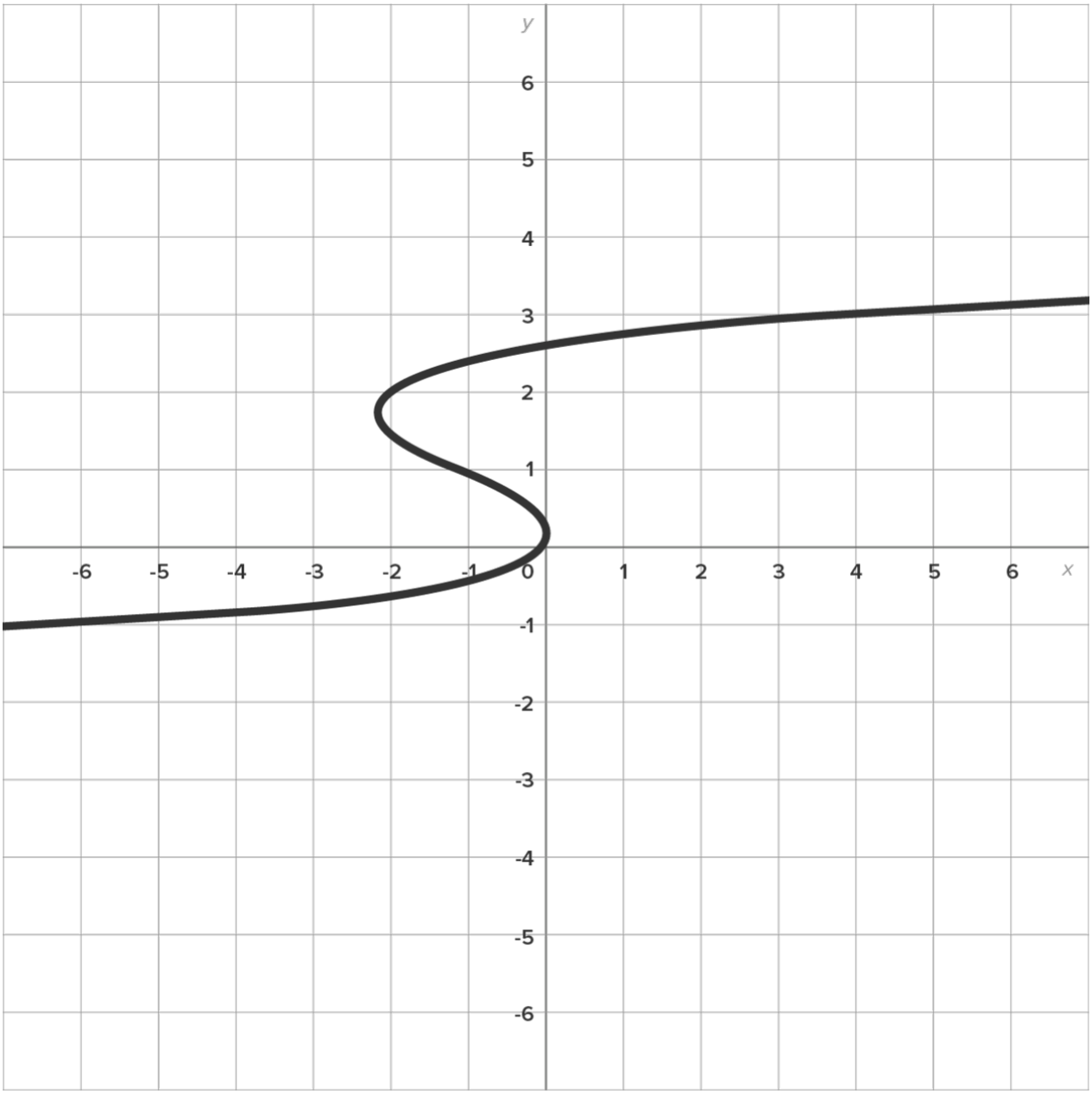
8.

Determine which graph defines *y* to be a function of *x.*

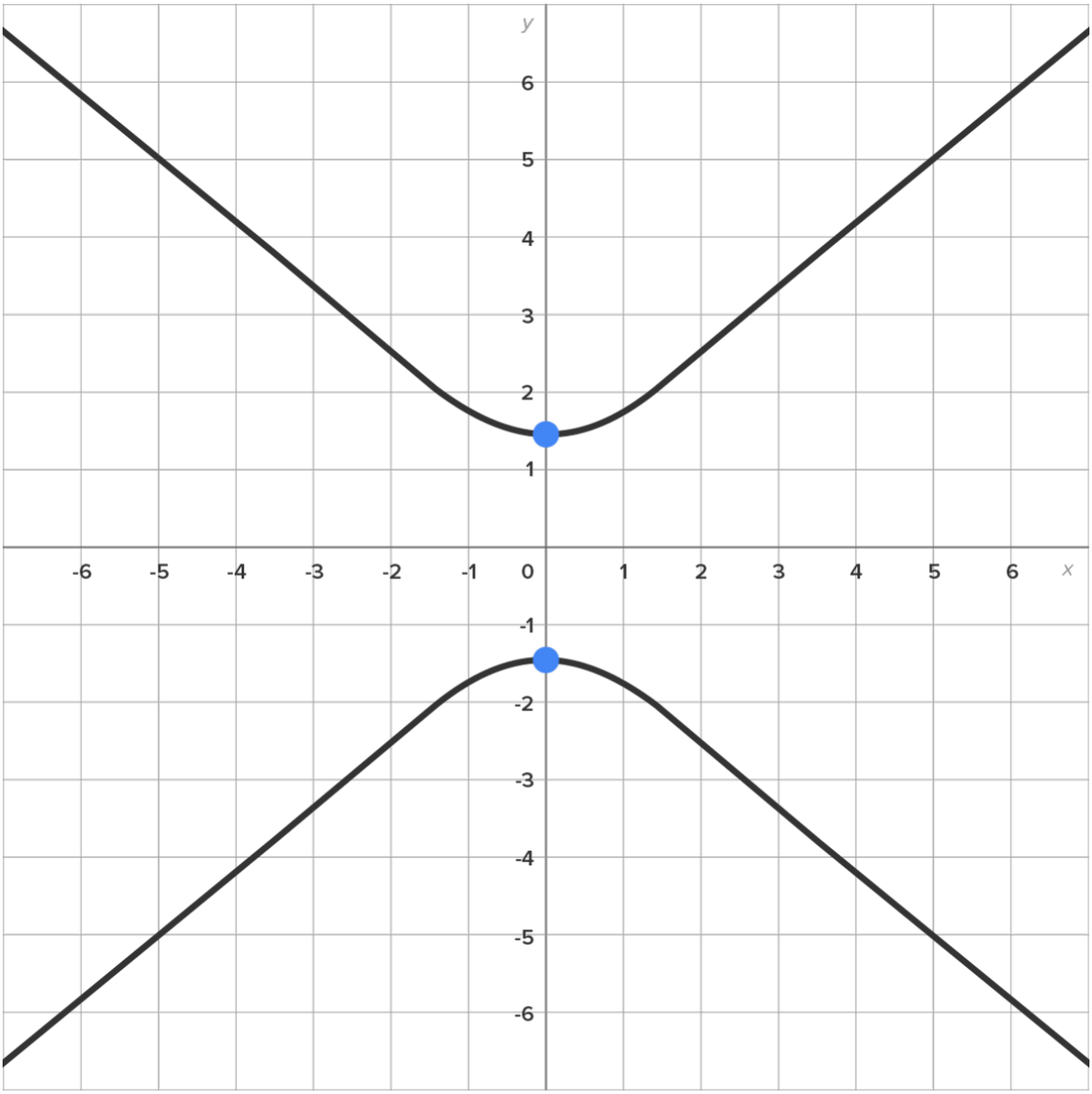
- a.)



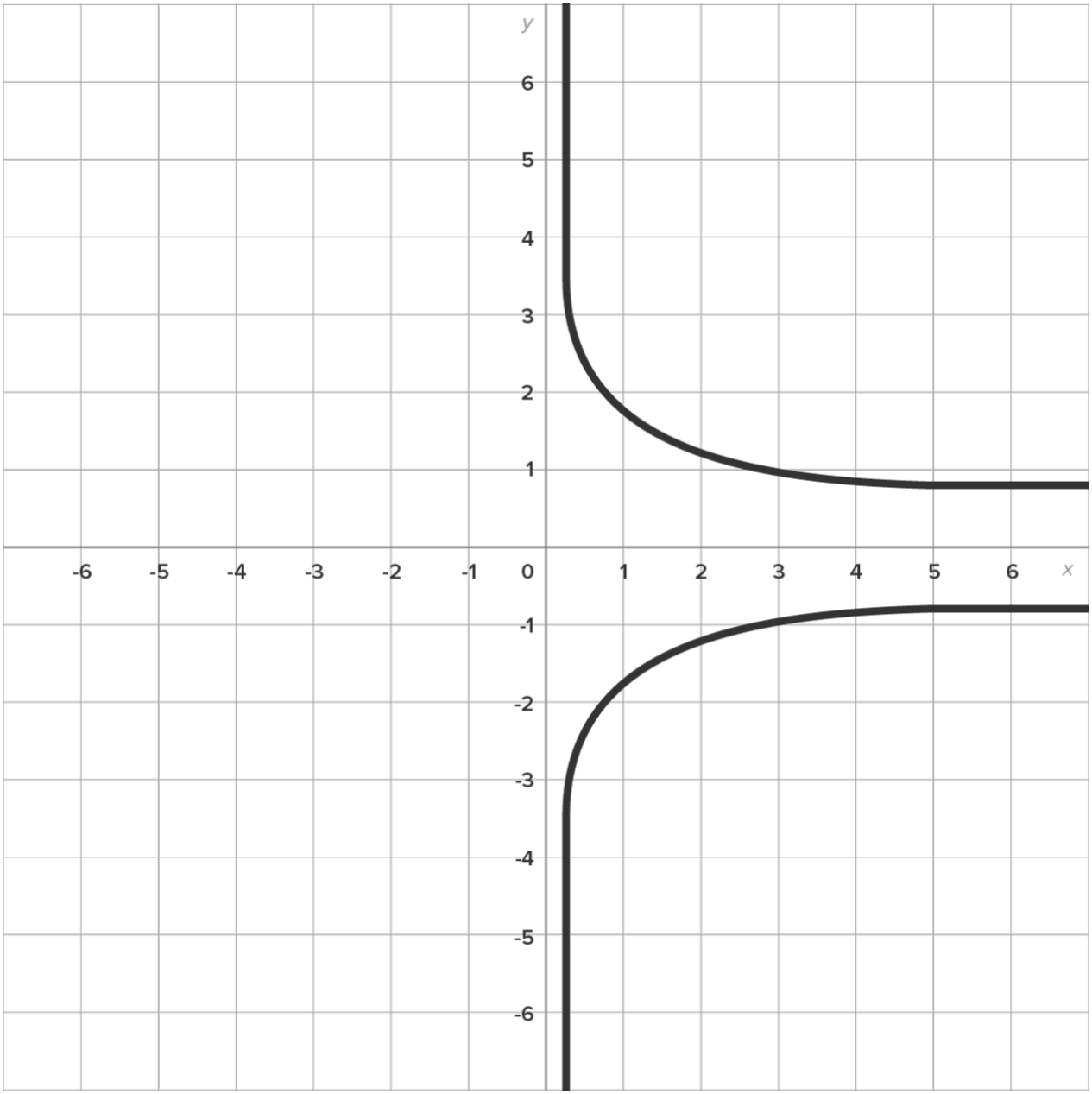
○ b.)



○ **c.)**

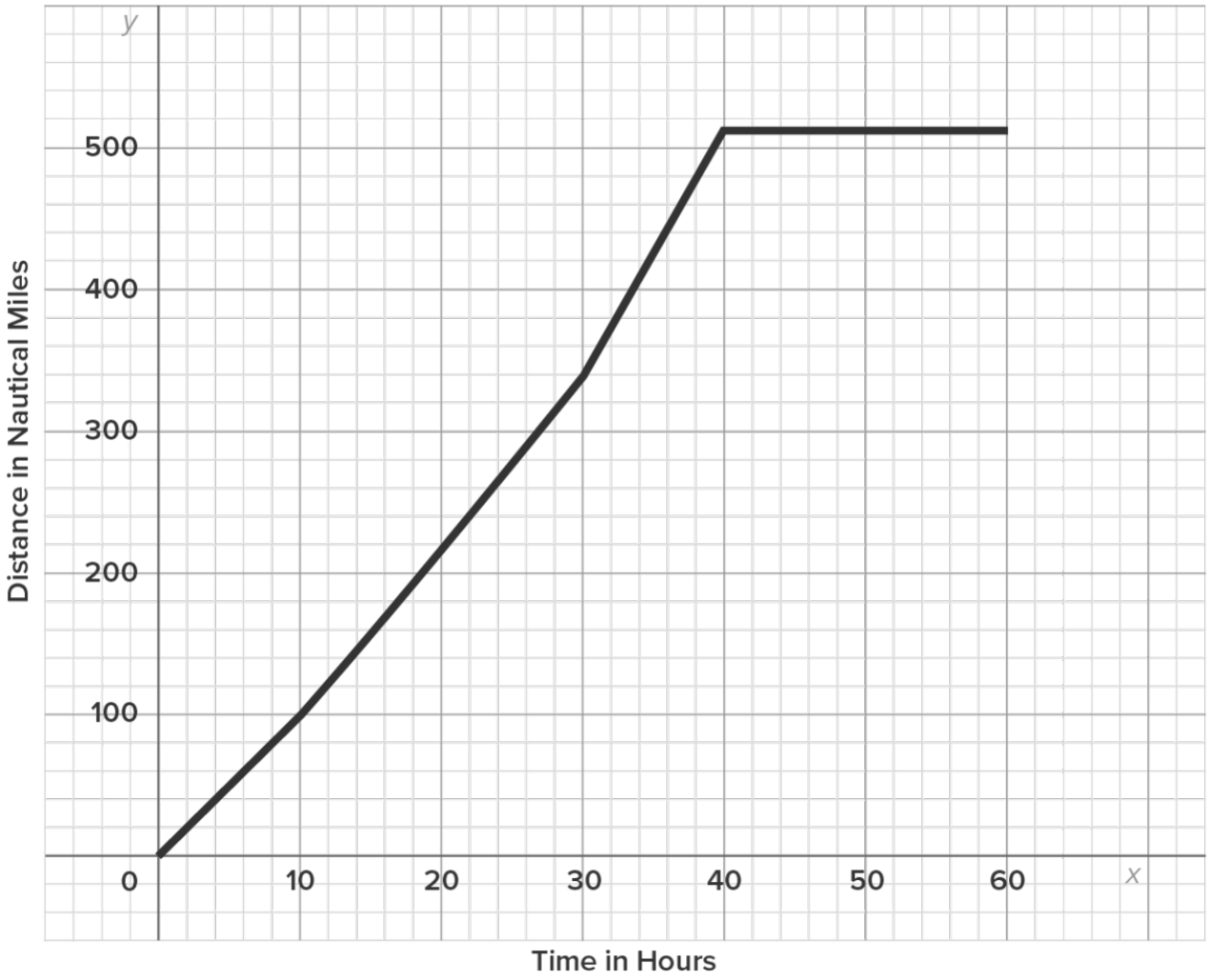


○ **d.)**



9.

Consider the graph below, which shows the number of hours since a ship has left port on the horizontal axis and the distance in nautical miles the ship is from port on the vertical axis.



Determine the distance the ship is from port 30 hours after it departed.

- **a.)** 4 nautical miles
- **b.)** 340 nautical miles
- **c.)** 300 nautical miles
- **d.)** 320 nautical miles

10.

Consider the following piecewise function:

$$f(x)=\begin{cases} 3x & x<3 \\ -2x^3+5x-4 & x\geq 3 \end{cases}$$

Evaluate *f*(-5), *f*(3), and *f*(5).

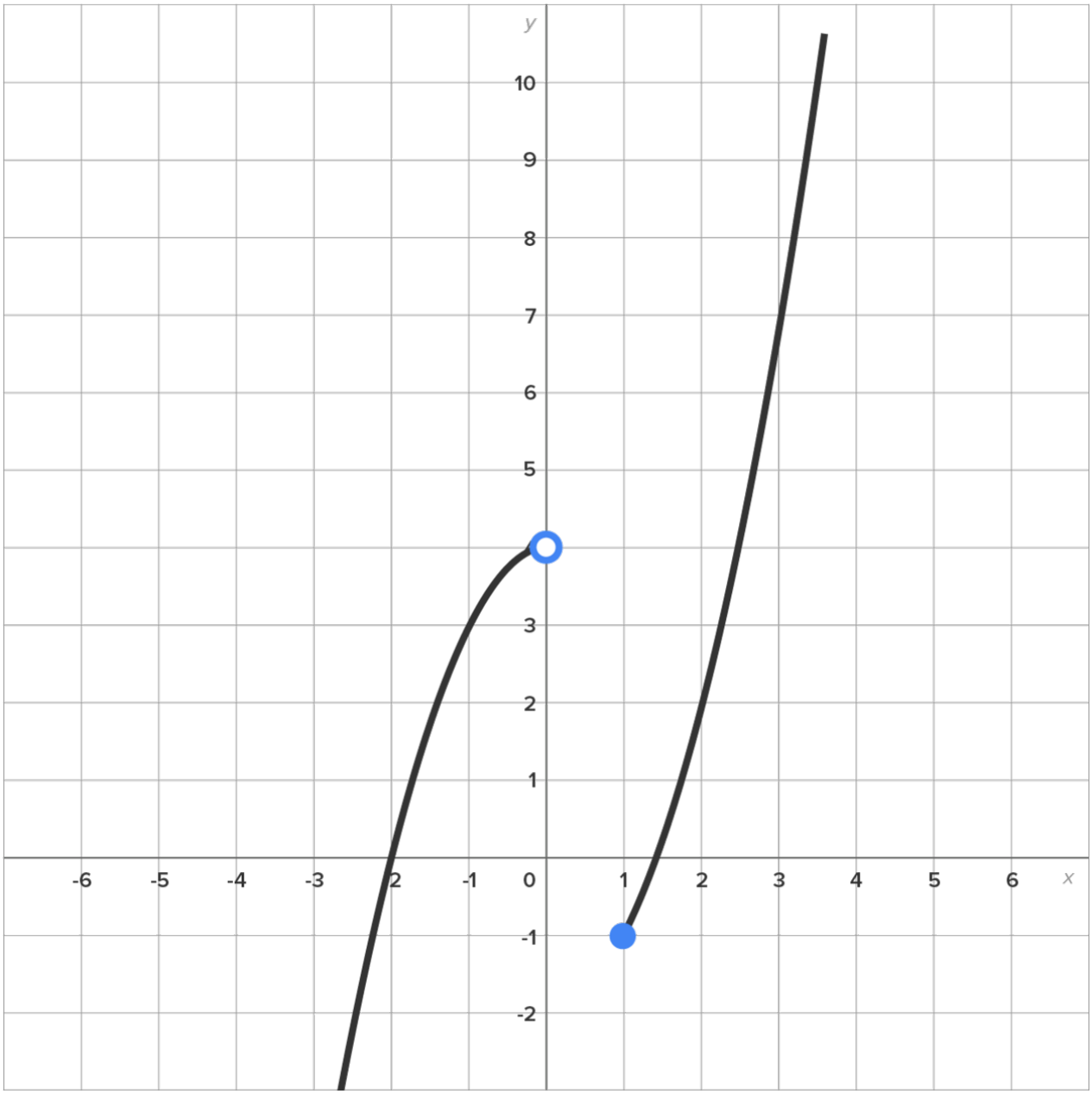
- a.) $f(-5)=-15$, $f(3)=-43$, $f(5)=-229$
- b.) $f(-5)=-15$, $f(3)=-1$, $f(5)=-9$
- c.) $f(-5)=-15$, $f(3)=9$, $f(5)=-9$
- d.) $f(-5)=-15$, $f(3)=9$, $f(5)=-229$

11.

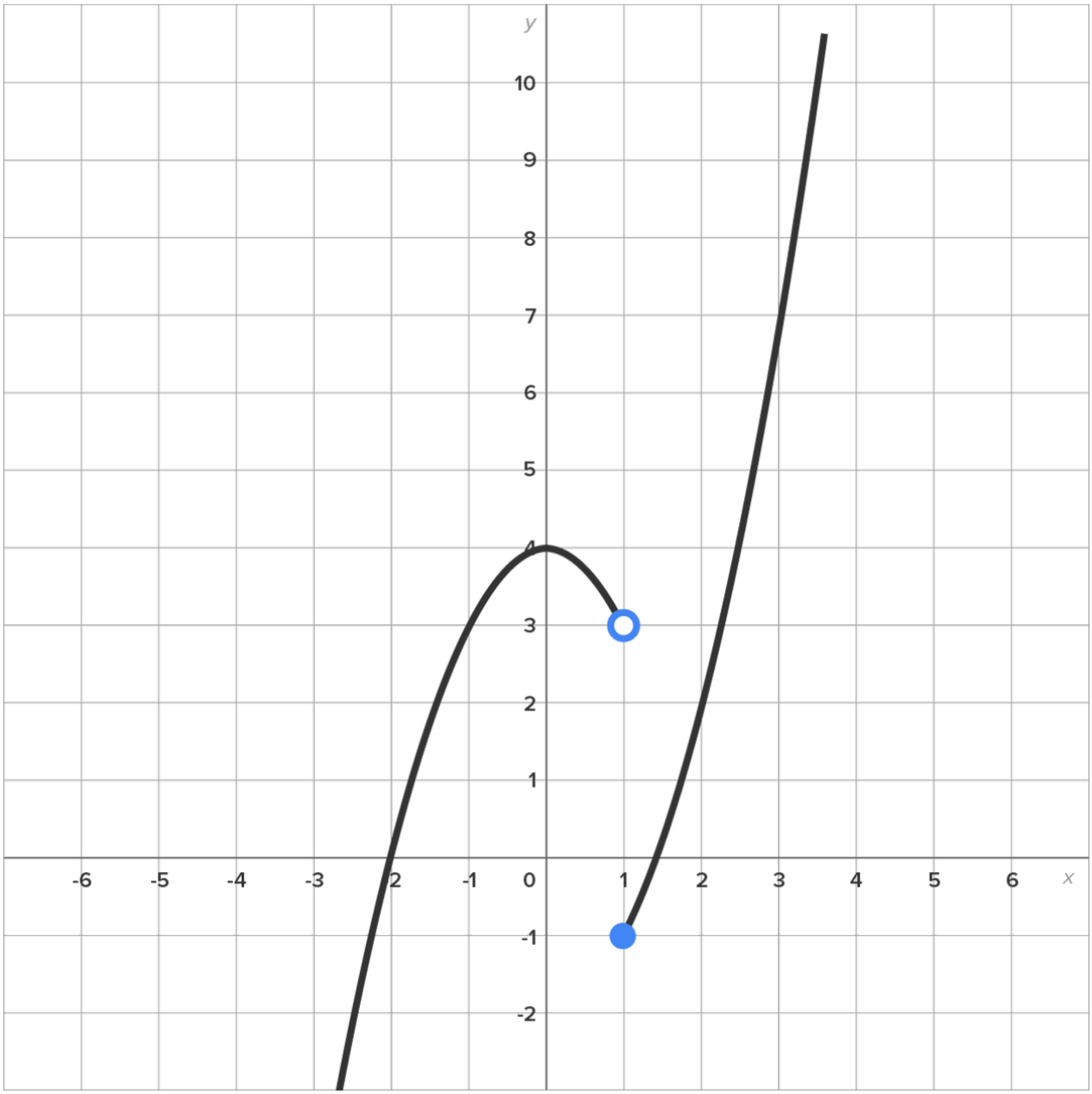
Graph the following function:

$$f(x)=\begin{cases} -x^2+4 & x<1 \\ x^2-2 & x\geq 1 \end{cases}$$

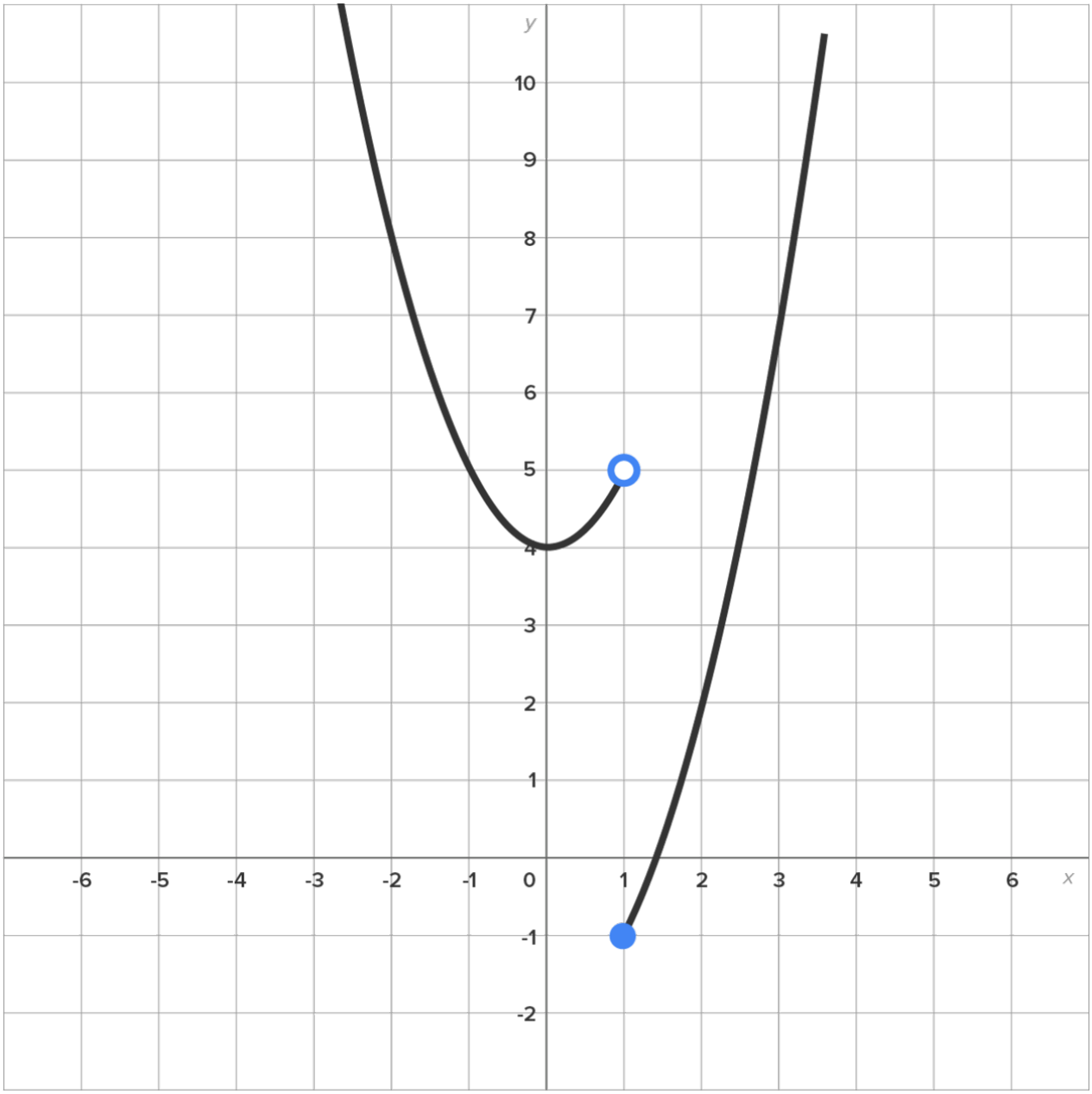
- a.)



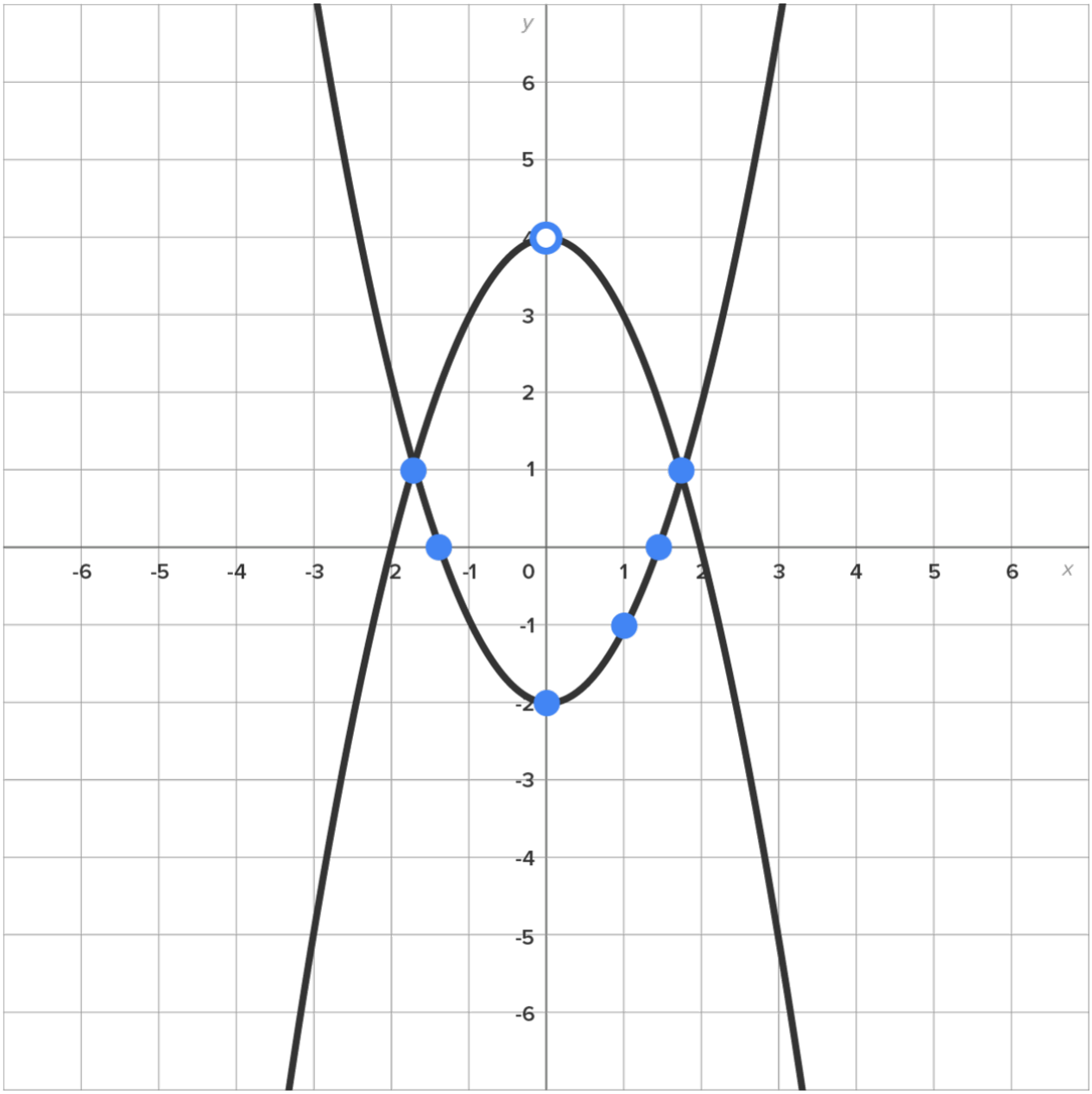
○ b.)



○ c.)



○ **d.)**



12.

Let $f(x) = 3x - 2$ and $g(x) = -5x^2 + 4$.

Find and simplify $(g \circ f)(x)$.

- **a.)** $(g \circ f)(x) = -45x^2 + 60x - 16$
- **b.)** $(g \circ f)(x) = -15x^2 + 10$
- **c.)** $(g \circ f)(x) = -45x^2 - 16$
- **d.)** $(g \circ f)(x) = -45x^2 - 60x + 24$

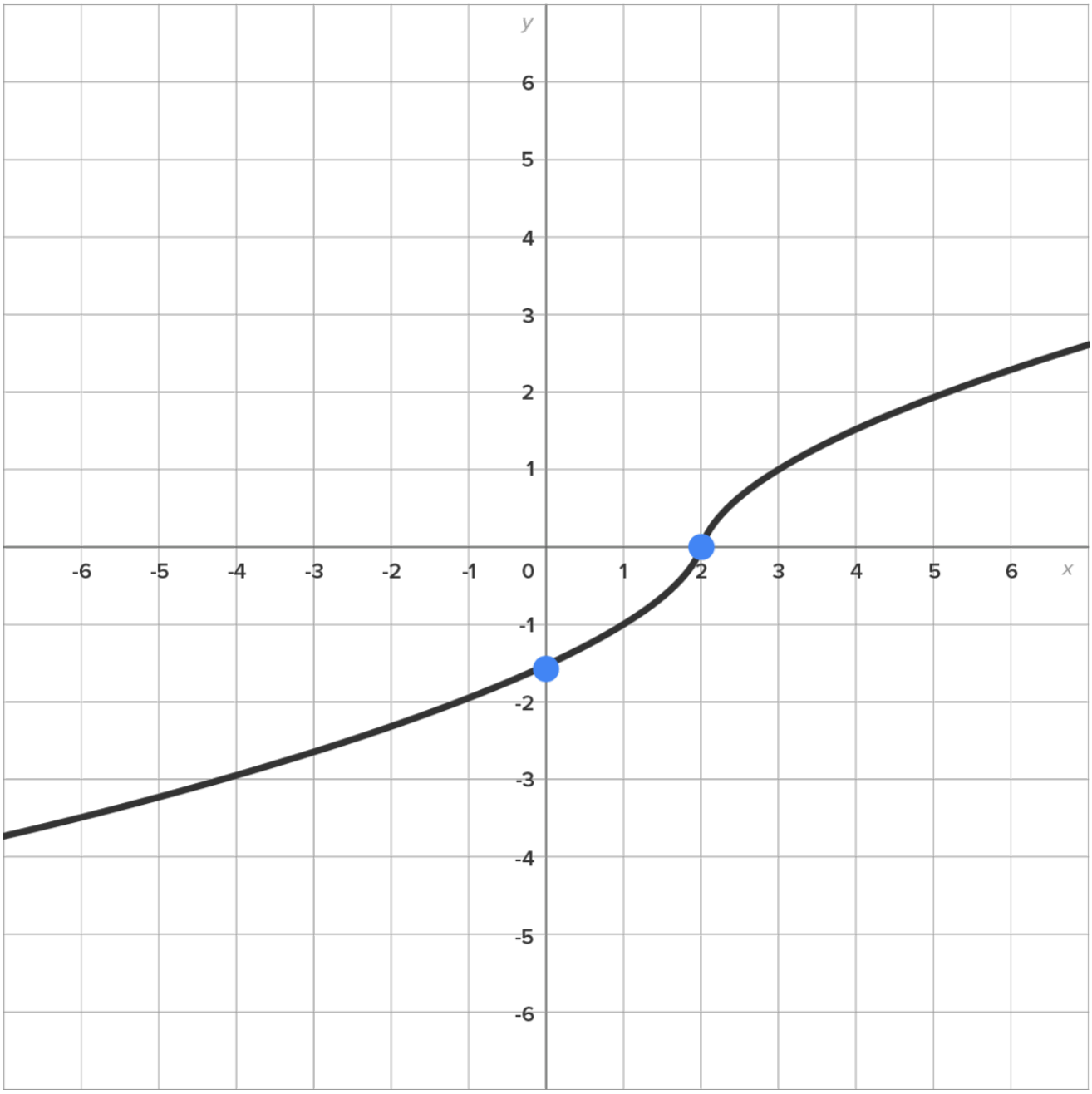
13.

Describe the sequence of transformations that are required to graph $g(x) = -5\sqrt{x-11}+12$ **based on** $f(x) = \sqrt{x}$.

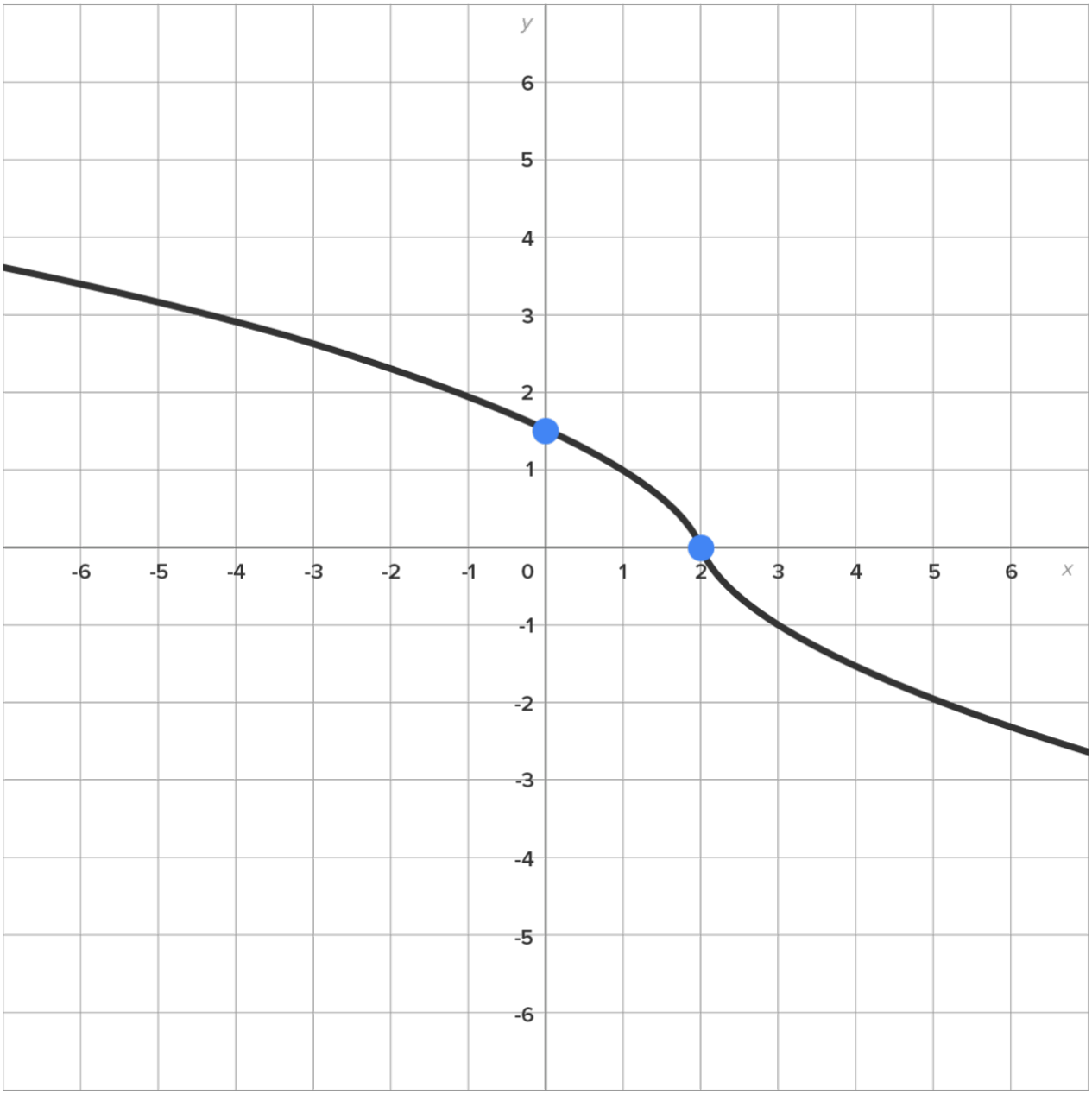
- **a.)** The graph of $f(x) = \sqrt{x}$ is:
  - Shifted right 12 units
  - Stretched vertically by a factor of 5
  - Reflected around the *x*-axis
  - Shifted down 11 units
- **b.)** The graph of $f(x) = \sqrt{x}$ is:
  - Shifted right 11 units
  - Stretched vertically by a factor of 5
  - Reflected around the *x*-axis
  - Shifted up 12 units
- **c.)** The graph of $f(x) = \sqrt{x}$ is:
  - Shifted left 11 units
  - Stretched vertically by a factor of 5
  - Reflected around the *x*-axis
  - Shifted up 12 units
- **d.)** The graph of $f(x) = \sqrt{x}$ is:
  - Shifted right 11 units
  - Stretched vertically by a factor of 5
  - Shifted up 12 units

14.

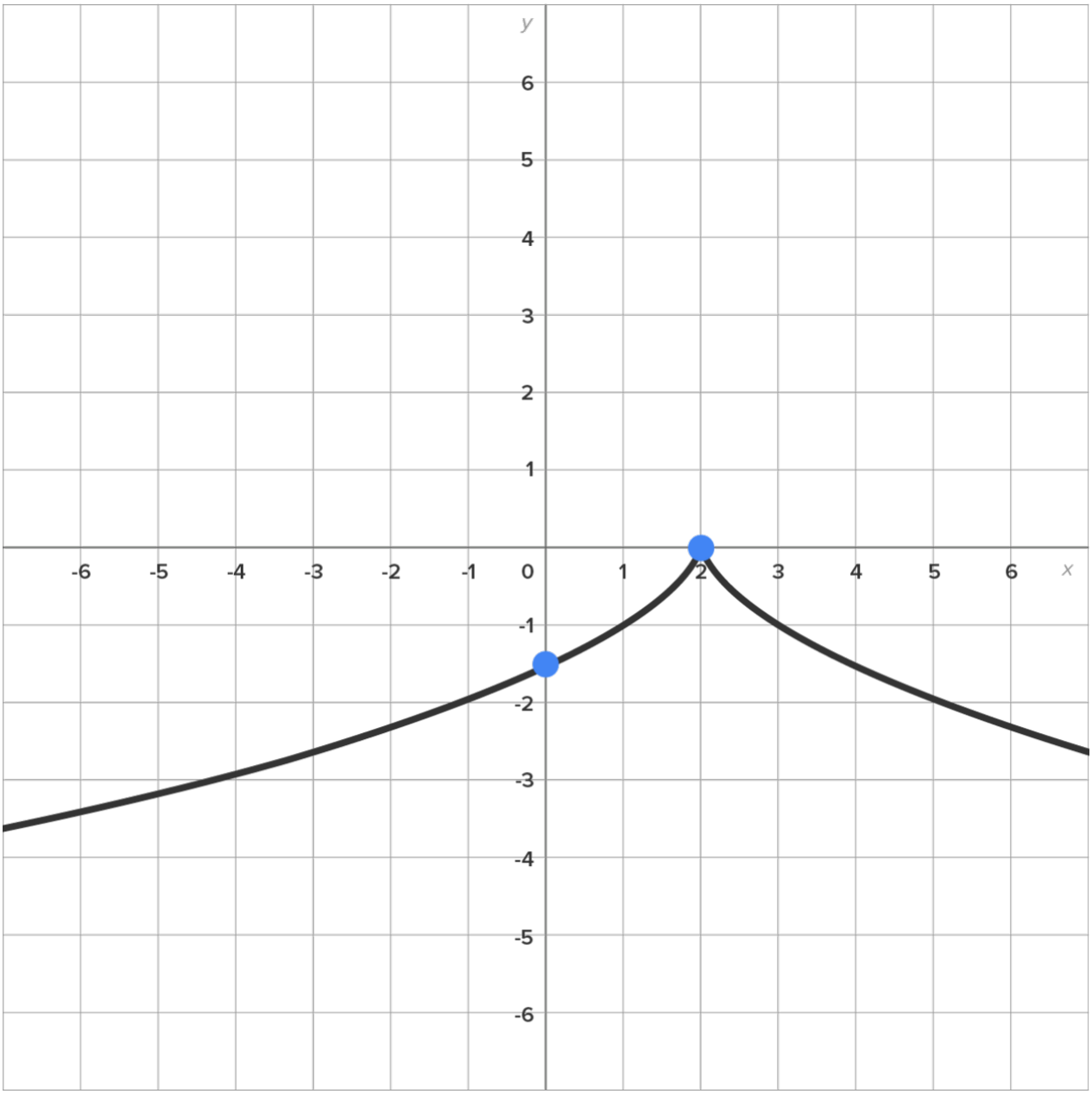
Given the graph of $y = f(x)$ **as shown below, sketch the graph of** $y = |f(x)|$.



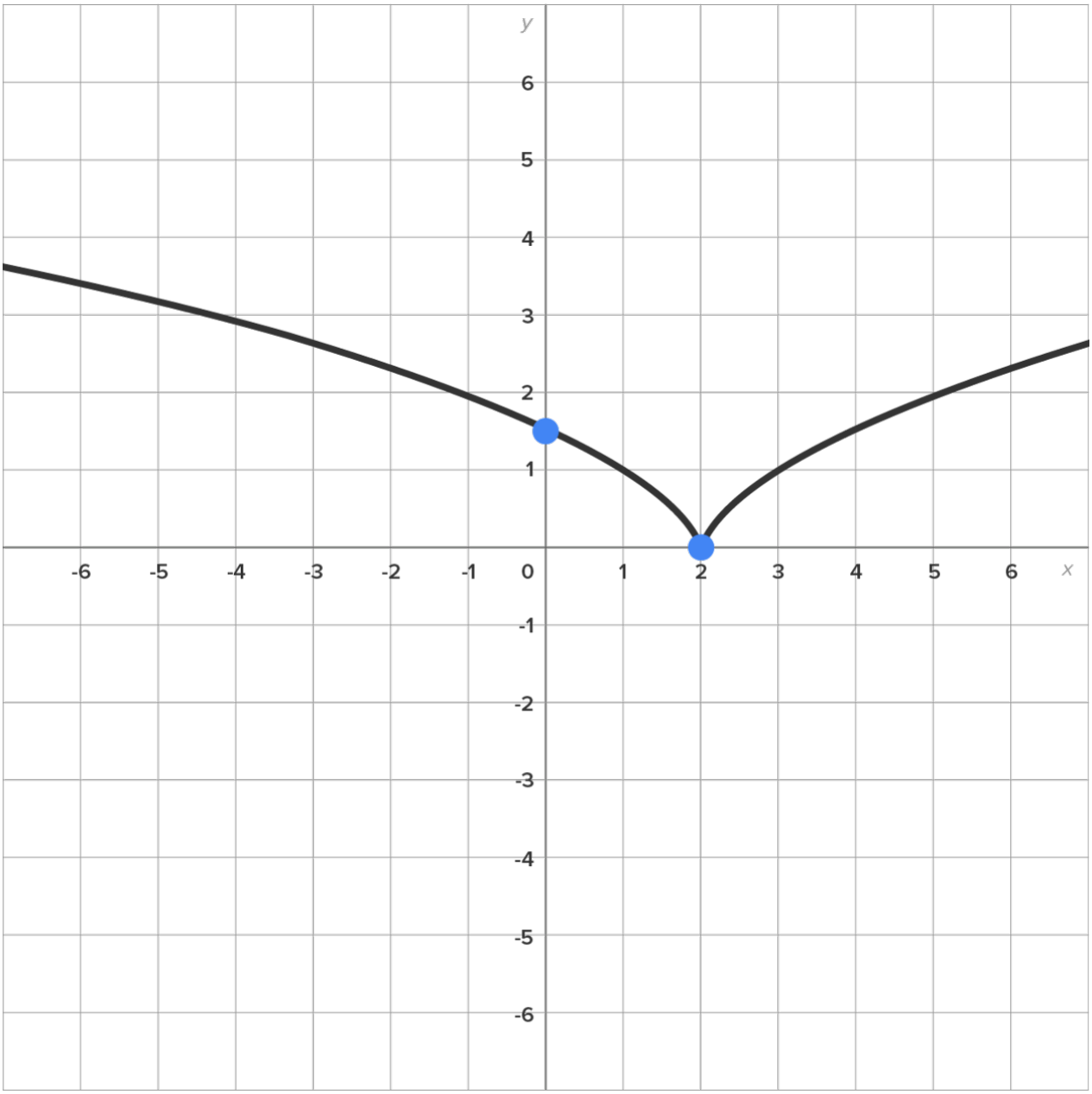
○ a.)



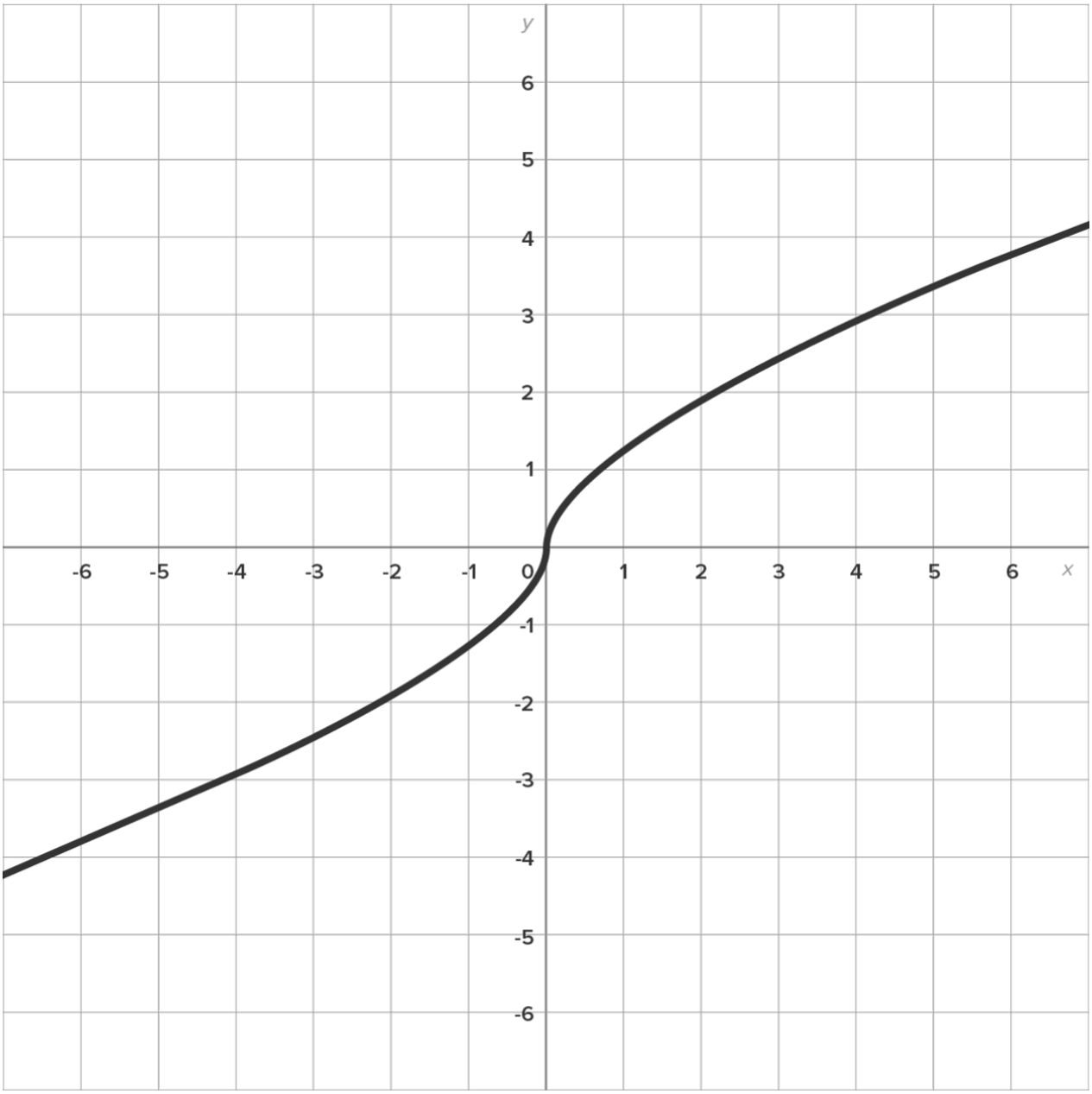
○ b.)



○ c.)



○ **d.)**



15.

For $f(x) = \lfloor x \rfloor$, evaluate $f(12.959)$.

- a.) 13
- b.) 12
- c.) 12.96
- d.) 12.9

16.

Find the exact value of $\sin\left(\frac{\pi}{4}\right)$ by using the unit circle.

- a.) $\frac{\sqrt{3}}{2}$
- b.) $\frac{1}{2}$
- c.) $\frac{\sqrt{2}}{2}$
- d.) $-\frac{\sqrt{2}}{2}$

17.

Use logarithm properties to expand the expression $\log_4\left(\frac{x^8z^5}{y^{12}}\right)$.

- a.) $\frac{10\log_4(xz)}{3\log_4 y}$
- b.) $8\log_4 x + 5\log_4 z - 12\log_4 y$
- c.) $\log_4\left(\frac{10xz}{3y}\right)$
- d.) $\log_4(8x + 5z - 12y)$

Answer Key

Question	Answer	
1	d	**Concept:** Distance Between Two Points **Rationale:** To find the distance between two points, use the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$: $x_1=8, y_1=-3, x_2=12, y_2=2$ $d=\sqrt{(12-8)^2+(2-(-3))^2}$ $d=\sqrt{(4)^2+(5)^2}$ $d=\sqrt{16+25}$ $d=\sqrt{41}$
2	b	**Concept:** Equations of Circles **Rationale:** The center is (h, k) = (3, -7) and its radius is r = 9. The standard form of a circle is $(x-h)^2+(y-k)^2=r^2$. Substituting the center and radius into the standard form, we have: $(x-3)^2+(y-(-7))^2=9^2$ $(x-3)^2+(y+7)^2=81$
3	c	**Concept:** Slope of a Line Between Two Points **Rationale:** To find the slope of the line that passes through two points, use the slope formula. $m=\frac{y_2-y_1}{x_2-x_1}$ $x_1=13, y_1=-8, x_2=2, y_2=8$ $m=\frac{8-(-8)}{2-13}$ $m=\frac{8+8}{2-13}$ $m=\frac{16}{-11}$ $m=-\frac{16}{11}$
4	a	**Concept:** Equations of Lines **Rationale:** Slope-Intercept Form: $y=mx+b$ First, identify the values for m , x_1 , and y_1 . The given information is: • A point the line contains, (-6, 2), gives $x_1=-6, y_1=2$. • Slope $\frac{5}{3}$ gives $m=\frac{5}{3}$. You can then use point-slope form, along with the given point and slope. Point-Slope Form: $y-y_1=m(x-x_1)$ Substitute the values for m and the known point for x_1 and y_1:

		$y-y_1=m(x-x_1)$ $y-2=\frac{5}{3}(x-(-6))$ Simplify the subtraction on the right-hand side: $y-2=\frac{5}{3}x+10$ Add 2 to both sides: $y=\frac{5}{3}x+12$
5	a	**Concept:** What Is a Function? **Rationale:** To determine which equation is a function, solve for *y*. The equation is a function if for each *x*, there is only one *y*. Subtract 6*x* from both sides to isolate the term containing *y*: $-3y=-6x+21$ Divide both sides by -3: $y=2x-7$ The equation $6x-3y=21$ is a function since for each *x* input there is only one *y* output.
6	b	**Concept:** Evaluate Functions **Rationale:** $f(x)=-2x^2+7x+5$ For *f*(3), substitute 3 for *x*: $f(3)=-2(3)^2+7(3)+5$ Follow the order of operations: $f(3)=-2(9)+7(3)+5$ $f(3)=-18+21+5$ $f(3)=8$ For *f*(-4), substitute -4 for *x*: $f(-4)=-2(-4)^2+7(-4)+5$ Follow the order of operations: $f(-4)=-2(16)+7(-4)+5$ $f(-4)=-32-28+5$ $f(-4)=-55$ For *f*(a – 6), substitute (a – 6) for *x*: $f(a-6)=-2(a-6)^2+7(a-6)+5$ Multiply out the binomial square: $f(a-6)=-2(a^2-12a+36)+7(a-6)+5$ Distribute the -2 and the 7: $f(a-6)=-2a^2+24a-72+7a-42+5$ $f(a-6)=-2a^2+31a-109$
		Concept: Set Up and Simplify a Difference Quotient **Rationale:** For *f*(*x* + *h*), substitute (*x* + *h*) for *x*:

7	d	$\frac{[7(x+h)^2-4(x+h)+6]-[7x^2-4x+6]}{h}$ Simplify the first quantity in brackets: $\frac{[7x^2+14xh+7h^2-4x-4h+6]-[7x^2-4x+6]}{h}$ Distribute the subtraction sign: $\frac{7x^2+14xh+7h^2-4x-4h+6-7x^2+4x-6}{h}$ Simplify the numerator: $\frac{14xh+7h^2-4h}{h}$ Factor the numerator: $\frac{h(14x+7h-4)}{h}$ Divide out the common factor: $14x+7h-4$
8	a	**Concept:** Functions Defined by Graphs and Tables of Values **Rationale:** The correct answer is a function. Any vertical line will pass through at most one point on this graph.
9	b	**Concept:** Reading Graphs (Carefully) **Rationale:** Locate 30 on the horizontal axis. Move up vertically to the graph and then horizontally to the vertical axis to reference the distance from the port. Look carefully at the increments. The minor grid lines are in increments of 20 nautical miles. This gives 300+40 which is 340 nautical miles.
10	a	**Concept:** Evaluate Piecewise Functions **Rationale:** For $f(-5)$, use the first rule since $-5 < 3$: $f(-5)=3(-5)$ $f(-5)=-15$ For $f(3)$, use the second rule since $3 \geq 3$: $f(3)=-2(3)^3+5(3)-4$ $f(3)=-2(27)+5(3)-4$ $f(3)=-54+15-4$ $f(3)=-43$ For f(5), use the second rule since $5 \geq 3$: $f(5)=-2(5)^3+5(5)-4$ $f(5)=-2(125)+5(5)-4$ $f(5)=-250+25-4$ $f(5)=-229$
		Concept: Graph Piecewise Functions **Rationale:** The function tells us to use "$-x^2+4$" but only if the input is less than 1 and to use "x^2-2" if the input is greater than or equal to 1. An open circle at the point (1, 3)

11	b	indicates that the graph gets infinitely close to that dot from the left but doesn't include it. A filled dot at (1, -1) indicates that the graph includes that point. Since (1, -1) has an x-value equal to 1, it will be included with the portion of the graph $y = x^2 - 2$. This means that the graph of the function will be "part of" the graph of $y = -x^2 + 4$ along with "part of" the graph of $y = x^2 - 2$.
12	a	**Concept:** Composition of Functions **Rationale:** Rewrite using the definition of composition: $(g \circ f)(x) = g(f(x))$ Substitute $3x - 2$ for $f(x)$: $g(f(x)) = g(3x - 2)$ Evaluate the function and simplify: $g(f(x)) = -5(3x - 2)^2 + 4$ $g(f(x)) = -5(3x - 2)(3x - 2) + 4$ $g(f(x)) = -5(9x^2 - 12x + 4) + 4$ $g(f(x)) = -45x^2 + 60x - 20 + 4$ $g(f(x)) = -45x^2 + 60x - 16$
13	b	**Concept:** Shifting and Stretching Graphs **Rationale:** The "x – 11" tells us that the graph is shifted to the right by 11 units. The "5" multiplied to the square root term tells us that the graph is stretched vertically (since $\lvert -5 \rvert > 1$). The negative coefficient in front of the square term tells us that the graph is reflected around the *x*-axis. The "+ 12" tells us that the graph is then shifted up 12 units.
14	c	**Concept:** Absolute Value Functions **Rationale:** To graph $g(x) = \lvert f(x) \rvert$, notice that the graph of *f(x)* is below or touching the x-axis for x < 2. This part reflects over the x-axis, while the rest of the graph remains the same.
15	b	**Concept:** Greatest Integer Functions **Rationale:** Since 12.959 is not an integer, the function returns the greatest integer below 12.959, which is 12.
16	c	**Concept:** Trigonometric Functions **Rationale:** $\sin\left(\frac{\pi}{4}\right) = \sin(45°) = \frac{\sqrt{2}}{2}$
		Concept: Exponential and Logarithmic Functions **Rationale:** $\frac{x^8 z^5}{y^{12}}$ is a quotient; apply the quotient property:

17	b	$\log_4(x^8z^5)-\log_4(y^{12})$ x^8z^5 is a product; apply the product property: $\log_4 x^8+\log_4 z^5-\log_4 y^{12}$ Apply the power property: $8\log_4 x+5\log_4 z-12\log_4 y$