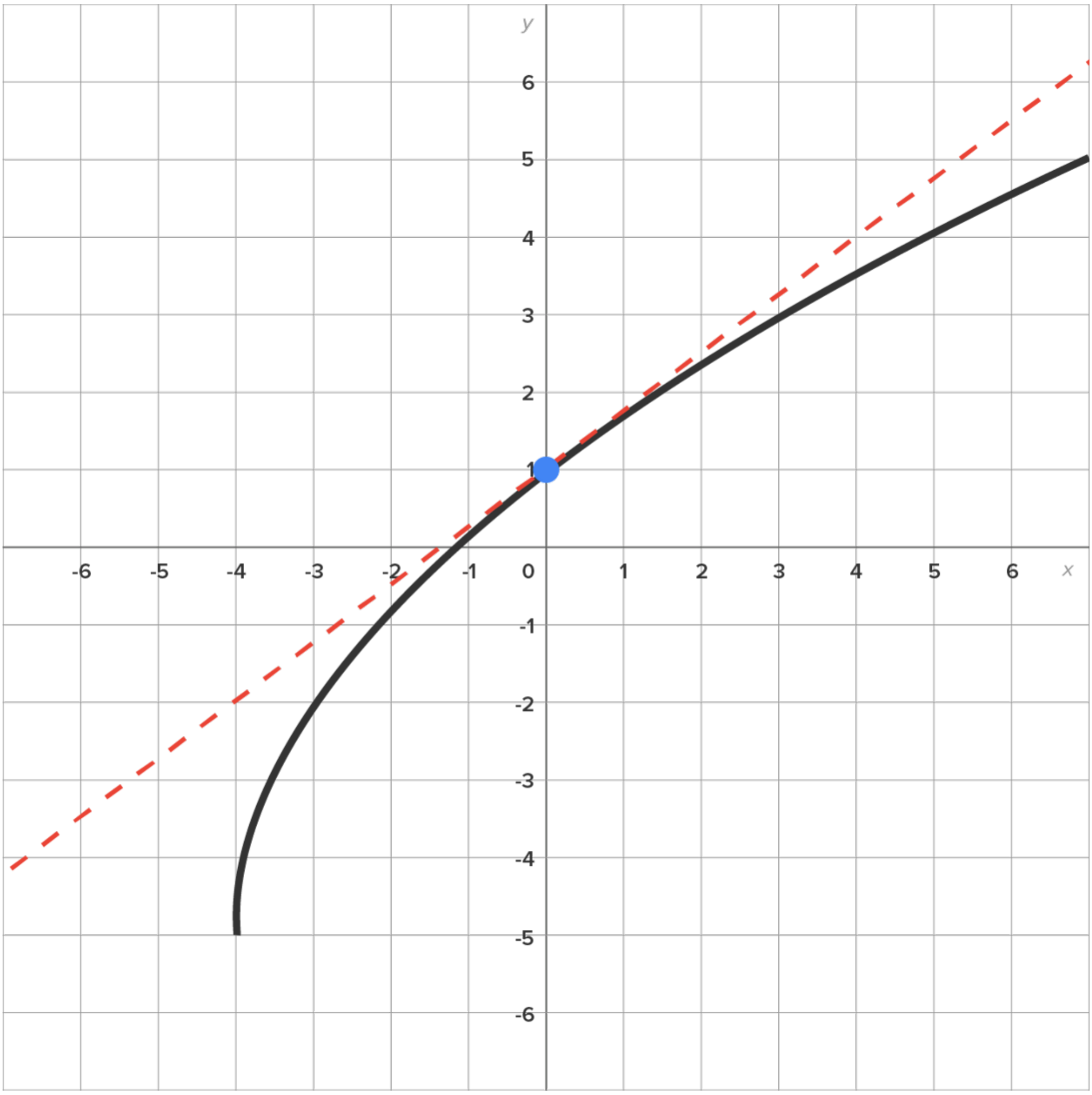
Practice Milestone

Calculus I — Practice Milestone 2

Taking this practice test is a stress-free way to find out if you are ready for the Milestone 2 assessment. You can print it out and test yourself to discover your strengths and weaknesses. The answer key is at the end of this Practice Milestone.

1.



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Use the graph to estimate the slope of the line tangent to the given graph at $(0,1)$.

○ a.) $-\frac{4}{3}$

○ b.) $-\frac{3}{4}$

○ c.) $\frac{3}{4}$

○ d.) $\frac{4}{3}$

2.

Calculate the average rate of change of $f(x) = -3x^4 + 20$ on the interval $[-1, 3]$.

○ a.) 120

○ b.) 60

○ c.) -120

○ d.) -60

3.

Find the instantaneous rate of change of $f(x) = 7x^2 - 5x + 3$ when $x = 2$.

○ a.) -5

○ b.) 9

○ c.) 23

○ d.) 5

4.

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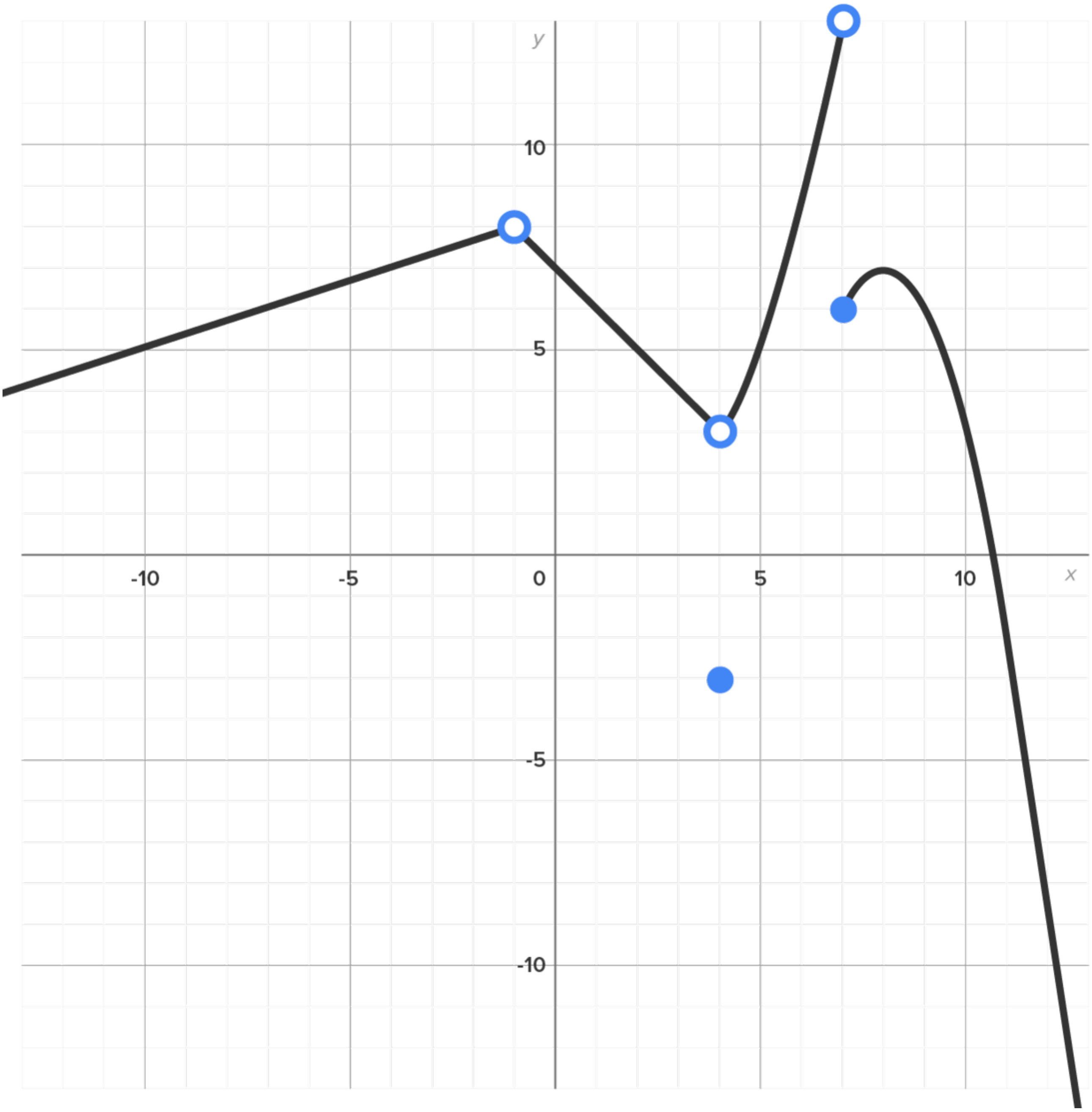
Consider the graph of some function $y = f(x)$.



Evaluate $\lim_{x \to -1} f(x)$, $\lim_{x \to 4} f(x)$, **and** $\lim_{x \to 7} f(x)$.

○ **a.)** $\lim_{x \to -1} f(x) = 8$

$\lim_{x \to 4} f(x) = 3$

$\lim_{x \to 7} f(x)$ *does not exist*

○ **b.)** $\lim_{x \to -1} f(x) = 8$

$\lim_{x \to 4} f(x) = -3$

$\lim_{x \to 7} f(x)$ *does not exist*

○

○ **c.)** $\lim_{x \to -1} f(x) = 8$

$\lim_{x \to 4} f(x) = 3$

$\lim_{x \to 7} f(x) = 6$

○ **d.)** $\lim_{x \to -1} f(x) = 8$

$\lim_{x \to 4} f(x)$ *does not exist*

$\lim_{x \to 7} f(x)$ *does not exist*

5.

Use a table of values to evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 7x + 12}$.

○ **a.)** $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 7x + 12}$ *does not exist*

○ **b.)** $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 7x + 12} = -6$

○ **c.)** $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 7x + 12} = 6$

○ **d.)** $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 7x + 12} = 0$

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6.

Evaluate $\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^3 + 8}$ **algebraically by simplifying if possible, then using direct substitution.**

○ **a.)** $\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^3 + 8} = -\frac{1}{4}$

○ **b.)** $\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^3 + 8} = -\frac{7}{12}$

○ **c.)** $\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^3 + 8}$ *does not exist*

○ **d.)** $\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^3 + 8} = 0$

7.

Consider the function $f(x) = \begin{cases} 6 - 3x^2 & \text{if } x < -3 \\ 8 + 5x & \text{if } x > -3 \end{cases}$.

Evaluate $\lim_{x \to -3^-} f(x)$ **and** $\lim_{x \to -3^+} f(x)$.

○ **a.)** $\lim_{x \to -3^-} f(x) = -42$

$\lim_{x \to -3^+} f(x) = -2$

○ **b.)** $\lim_{x \to -3^-} f(x) = 33$

$\lim_{x \to -3^+} f(x) = 7$

○ **c.)** $\lim_{x \to -3^-} f(x) = -21$

$\lim_{x \to -3^+} f(x) = -7$

○ **d.)** $\lim_{x \to -3^-} f(x) = -7$

$\lim_{x \to -3^+} f(x) = -21$

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8.

Given $\lim_{x \to -9} f(x) = 12$ **and** $\lim_{x \to -9} g(x) = -11$**, evaluate** $\lim_{x \to -9} [3f(x) + 7g(x)]$**.**

○ **a.)** $\lim_{x \to -9} [3f(x) + 7g(x)] = 113$

○ **b.)** $\lim_{x \to -9} [3f(x) + 7g(x)] = -41$

○ **c.)** $\lim_{x \to -9} [3f(x) + 7g(x)] = -113$

○ **d.)** $\lim_{x \to -9} [3f(x) + 7g(x)] = 41$

9.

Suppose $-2x + 1 \le f(x) \le \frac{1}{2}x^2 - 4x + 3$ for all x values near 2, except possibly at 2.

Evaluate $\lim_{x \to 2} f(x)$**.**

○ **a.)** $\lim_{x \to 2} f(x) = -3$

○ **b.)** $\lim_{x \to 2} f(x)$ *does not exist*

○ **c.)** $\lim_{x \to 2} f(x) = -4$

○ **d.)** $\lim_{x \to 2} f(x) = 3$

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10.

Consider the following function:

$$f(x) = \begin{cases} (x+3)^2 - 2 & \text{if } x < -4 \\ 5x + 12 & \text{if } x \geq -4 \end{cases}$$

Determine if $f(x)$ is continuous at $x = -4$. If not, select the option with the correct reasoning as to why not.

○ **a.)** Not continuous at $x = -4$ because $\lim_{x \to -4} f(x)$ does not exist

○ **b.)** Not continuous at $x = -4$ because $f(-4)$ is undefined

○ **c.)** Continuous at $x = -4$

○ **d.)** Not continuous at $x = -4$ because $\lim_{x \to -4} f(x) \neq f(-4)$

11.

Using the properties of combinations of continuous functions, determine the interval(s) over which the function $f(x) = \frac{x^2 + 7x - 30}{x+8}$ is continuous.

○ **a.)** $(-\infty, 8) \cup (8, \infty)$

○ **b.)** $(-\infty, -10) \cup (-10, 3) \cup (3, \infty)$

○ **c.)** $(-\infty, -8) \cup (-8, \infty)$

○ **d.)** $(-\infty, \infty)$

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12.

Let $f(x)=x^2+9x$ on $[-9, -4]$.

Use the IVT to determine if there is a solution to $f(x)=-14$ in the interval between -9 and -4. If so, find the value of *c* in the interval such that $f(c)=-14$.

○ a.)
- $f(x)$ is continuous on [-9, -4]
- -14 is between $f(-9)=0$ and $f(-4)=-20$
- $c=-2$, $c=-7$

○ b.)
- $f(x)$ is continuous on [-9, -4]
- -14 is between $f(-9)=0$ and $f(-4)=-20$
- $c=-7$

○ c.)
- $f(x)$ is continuous on [-9, -4]
- -14 is between $f(-9)=0$ and $f(-4)=-20$
- $c=-2$

○ d.)
- $f(x)$ is continuous on [-9, -4]
- -14 is not in the interval [-9, -4] so the IVT does not apply

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13.

Consider the following limit statement:

$$\lim_{x\to 3}(4x-7)=5$$

Find the corresponding value of δ when $\varepsilon=0.005$.

○ a.) $\delta=3.00125$

○ b.) $\delta=-0.00125$

○ c.) $\delta=0.00125$

○ d.) $\delta=0.005$

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14.

For $\varepsilon > 0$, find $\delta > 0$ necessary to prove $\lim_{x \to 4} (8x - 17) = 15$.

○ a.) $\delta = \frac{\epsilon}{8}$

○ b.) $\delta = \frac{\epsilon}{8} + 28$

○ c.) $\delta = -\frac{\epsilon}{8}$

○ d.) $\delta = \frac{\epsilon}{8} + 4$

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Answer Key

Question	Answer	
1	c	**Concept:** Slope of a Tangent Line Visually **Rationale:** Looking at the graph we see that the line is tangent to the curve at the point $(0,1)$. In order to estimate the slope of a line, two points are needed. Thus, we need another point on the line besides $(0,1)$ to estimate the slope of this line. Inspecting closely, it looks like the point $(4,4)$ is also contained on the line. Thus, the slope of the tangent line is approximately: $m=\frac{4-1}{4-0}=\frac{3}{4}$.
2	d	**Concept:** Average Rate of Change **Rationale:** Evaluate $f(-1)$ and $f(3)$: $f(-1)=-3(-1)^4+20=17$ $f(3)=-3(3)^4+20=-223$ Use this information to find the average rate of change: $\mathit{average\ rate\ of\ change}=\frac{f(3)-f(-1)}{3-(-1)}=\frac{-223-17}{3+1}=\frac{-240}{4}=-60$ This means that on average, the curve falls 60 units for every 1 unit of horizontal increase.
3	c	**Concept:** Instantaneous Rate of Change **Rationale:** The instantaneous rate of change is found by first finding the average rate of change from 2 to 2 + *h* and then setting *h* = 0. $f(2+h)=7(2+h)^2-5(2+h)+3$ $=7(4+4h+h^2)-10-5h+3$ $=28+28h+7h^2-10-5h+3$ $=7h^2+23h+21$ $f(2)=7(2)^2-5(2)+3$ $=28-10+3$ $=21$ Then use this information to find the average rate of change: $\mathit{average\ rate\ of\ change}=\frac{f(2+h)-f(2)}{h}=\frac{7h^2+23h+21-21}{h}=\frac{h(7h+23)}{h}=7h+23$ The instantaneous rate of change is found when *h* = 0: $\mathit{instantaneous\ rate\ of\ change}=7(0)+23=23$
		Concept: The Graph Method **Rationale:** As *x* gets closer to -1, $f(x)$ gets closer to 8.

<table>
<tr>
<td>4</td>
<td>a</td>
<td>As <i>x</i> gets closer to 4, $f(x)$ gets closer to 3. Note that the actual value of $f(4)$ is -3 (closed dot at $x=4$), but the limit tells us what is happening as we get closer and closer to 4, not what is happening right at 4.

As <i>x</i> gets closer to 7 from the left (values smaller than 7), $f(x)$ gets closer to 13.5. However, as <i>x</i> gets closer to 7 from the right (values larger than 7), $f(x)$ gets closer to 6.

Since $f(x)$ approaches two different values as <i>x</i> approaches 7, we say the limit does not exist.</td>
</tr>
<tr>
<td>5</td>
<td>b</td>
<td>Concept: The Table Method
Rationale:
<table>
<tr><td>x</td><td>$f(x)=\frac{x^2-9}{x^2-7x+12}$</td></tr>
<tr><td>2.9</td><td>-5.364</td></tr>
<tr><td>2.99</td><td>-5.931</td></tr>
<tr><td>2.999</td><td>-5.993</td></tr>
<tr><td>3</td><td>---</td></tr>
<tr><td>3.001</td><td>-6.007</td></tr>
<tr><td>3.01</td><td>-6.071</td></tr>
<tr><td>3.1</td><td>-6.778</td></tr>
</table>
Notice that the values of $f(x)$ approach -6 as <i>x</i> approaches 3 from the left and the from the right.</td>
</tr>
<tr>
<td>6</td>
<td>b</td>
<td>Concept: The Algebra Method
Rationale:
Attempting direct substitution, notice that the numerator and denominator are both 0. This means we should try to simplify.

Factor the numerator and the denominator:
$\lim_{x\to -2}\frac{x^2-3x-10}{x^3+8}=\lim_{x\to -2}\frac{(x+2)(x-5)}{(x+2)(x^2-2x+4)}$
Remove the common factor:
$=\lim_{x\to -2}\frac{x-5}{x^2-2x+4}$
Substitute $x=-2$ and simplify:
$=\frac{(-2)-5}{(-2)^2-2(-2)+4}$
$=\frac{-7}{12}$
$=-\frac{7}{12}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Concept: One-Sided Limits
Rationale:</td>
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7	c	Since $f(x)$ changes definitions at $x=-3$, evaluating $\lim\limits_{x\to -3^-} f(x)$ and $\lim\limits_{x\to -3^+} f(x)$ takes some extra care. When evaluating $\lim\limits_{x\to -3^-} f(x)$, we can replace $f(x)$ with $6-3x^2$ and evaluate the limit: $\lim\limits_{x\to -3^-} f(x)=\lim\limits_{x\to -3^-}(6-3x^2)=-21$ When evaluating $\lim\limits_{x\to -3^+} f(x)$, we can replace $f(x)$ with $8+5x$ and evaluate the limit: $\lim\limits_{x\to -3^+} f(x)=\lim\limits_{x\to -3^+}(8+5x)=-7$
8	b	**Concept:** Using Properties of Limits **Rationale:** Apply the sum/difference property: $\lim\limits_{x\to -9}[3f(x)+7g(x)]=\lim\limits_{x\to -9}3f(x)+\lim\limits_{x\to -9}7g(x)$ Apply the constant multiple property: $=3\cdot\lim\limits_{x\to -9}f(x)+7\cdot\lim\limits_{x\to -9}g(x)$ Substitute $\lim\limits_{x\to -9}f(x)=12$ and $\lim\limits_{x\to -9}g(x)=-11$: $=3(12)+7(-11)$ $=36-77$ $=-41$
9	a	**Concept:** Comparing Limits of Functions: Squeeze Theorem **Rationale:** Evaluate: $\lim\limits_{x\to 2}(-2x+1)=-2(2)+1=-3$ $\lim\limits_{x\to 2}(\frac{1}{2}x^2-4x+3)=\frac{1}{2}(2)^2-4(2)+3=-3$ Since $\lim\limits_{x\to 2}(-2x+1)=-3$ and $\lim\limits_{x\to 2}(\frac{1}{2}x^2-4x+3)=-3$, it follows by the Squeeze Theorem that $\lim\limits_{x\to 2}f(x)=-3$.
10	a	**Concept:** Continuous Functions **Rationale:** To determine if $f(x)$ is continuous when $x=-4$, we must first see if the $\lim\limits_{x\to -4}f(x)$ exists. First, evaluate $\lim\limits_{x\to -4}f(x)$. Since $f(x)$ changes definition when $x=-4$, we need to consider the one-sided limits: Left-sided limit (*x* values less than -4): $\lim\limits_{x\to -4^-}f(x)=\lim\limits_{x\to -4^-}(x+3)^2-2=(-4+3)^2-2=(-1)^2-2=1-2=-1$ Right-sided limit (*x* values greater than -4): $\lim\limits_{x\to -4^+}f(x)=\lim\limits_{x\to -4^+}(5x+12)=5(-4)+12=-20+12=-8$ Since the left-sided limit and the right-sided limit do not have the same value, $\lim\limits_{x\to -4}f(x)$ does not exist. The limit as *x* approaches -4 not existing is one of the situations for which the function is

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		not continuous at an -4 value. Therefore, we do not need to find $f(-4)$. We already know the function is not continuous at $x=-4$.
11	c	**Concept:** Which Functions are Continuous? **Rationale:** The function $f(x)=\dfrac{x^2+7x-30}{x+8}$ is a quotient of two continuous functions, $g(x)=x^2+7x-30$ and $h(x)=x+8$. The quotient is continuous for every real number except where the denominator is 0. That is where $x+8=0$, which means $x=-8$. The intervals over which $f(x)$ is continuous are $(-\infty,\ -8)(-8,\ \infty)$.
12	b	**Concept:** Intermediate Value Theorem **Rationale:** The function $f(x)=x^2+9x$ is a polynomial and therefore continuous on $[-9,\ -4]$. First, find $f(-9)$ and $f(-4)$: $f(-9)=(-9)^2+9(-9)=0$ $f(-4)=(-4)^2+9(-4)=-20$ The value -14 is between 0 and -20, so by the IVT, this means that there is at least one value of *c* between -9 and -4 such that $f(c)=-14$. Let's find this value. Since we want $f(c)=-14$, this means $c^2+9c=-14$. Solve this quadratic equation by factoring: $c^2+9c+14=0$ $(c+7)(c+2)=0$ $c=-7\ or\ c=-2$ Since -7 is between -9 and -4, this illustrates the existence of the value of "*c*" in the theorem. Note that -2 is not in the interval $[-9,\ -4]$, so this value is not considered.
13	c	**Concept:** The Intuitive Approach **Rationale:** If $\varepsilon=0.005$, this means we want $\lvert 4x-7-5\rvert<0.005$. Simplify: $\lvert 4x-12\rvert<0.005$ $\lvert x\rvert<a$ means $-a<x<a$, so we can say: $-0.005<4x-12<0.005$ Add 12 to all three parts: $11.995<4x<12.005$

		Divide all three parts by 4: $2.99875 < x < 3.00125$ Subtract 3 from all three parts to get $x - 3$ in the middle: $-0.00125 < x - 3 < 0.00125$ Thus, $\delta = 0.00125$.
14	a	**Concept:** The Formal Definition of a Limit **Rationale:** To find $\delta > 0$ necessary to prove limit $\lim\limits_{x \to 4} (8x - 17) = 15$, start with $\|8x - 17 - 15\| < \in$. Then convert to an inequality with $x - 4$ in the middle. Simplify: $\|8x - 32\| < \in$ $\|x\| < a$ means $-a < x < a$, so we can say: $-\in < 8x - 32 < \in$ Add 32 to all three parts: $-\in + 32 < 8x < \in + 32$ Divide all parts by 8: $-\frac{\in}{8} + 4 < x < \frac{\in}{8} + 4$ Subtract 4 from all three parts to get $x - 4$ in the middle: $-\frac{\in}{8} < x - 4 < \frac{\in}{8}$ Thus, $\delta = \frac{\in}{8}$. Note also that the last inequality can be written $\|x - 4\| < \frac{\in}{8}$. Now, to prove the limit, we have that this works because $0 < \|x - 4\| < \delta$. Since $\delta = \frac{\in}{8}$, this gives $0 < \|x - 4\| < \frac{\in}{8}$. This implies that $\|8x - 17 - 15\| = \|8x - 32\| = 8\|x - 4\| < 8\left(\frac{\in}{8}\right) < \in$.

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