Terms to Know

A Power

The result of raising a number to an exponent. For example, $2^5 = 32$, and we say that 32 is the 5th power of 2.

Absolute Value

The distance that a number is from 0 on the number line.

Acute Angle

An angle whose measure is more than 0° and less than 90°.

Composition of Functions

Written $(f \circ g)(x)$, it is a function that is obtained by substituting one function into another function.

Coterminal Angles

Angles that have the same terminal side.

Difference Quotient

An expression that represents the average rate of change between two points on a curve between input values *x* and *x* + *h*.

Distance

The length of a line segment between two points.

Function

A correspondence between a set of inputs (*x*) and a set of outputs (*y*) such that each input corresponds to at most one output.

Greatest Integer Function

Returns the greatest integer that is less than or equal to the input value.

Logarithmic Function

$f(x) = \log_a x$ uses the power as its input and returns the exponent required to produce that power when the base is *a*.

Piecewise Function

Assigns an input to an output, but the rule used to determine the output depends on the value of the input.

Radian

The angle required to produce a circular arc whose length is equal to the radius. One radian is $\frac{180}{\pi}$ degrees.

Restricted Domain

Part of, but not the entire, domain of a function.

Slope

The ratio of the change in *y* to the change in *x*; measure of the steepness of a line.

Trigonometric Equation

An equation in which trigonometric functions are involved and the angle is unknown.

Trigonometric Function

Uses an angle as an input and returns a ratio as the output.

Vertical Compression

A translation that makes all y-values of a graph smaller in magnitude, pulling a graph toward the x-axis. This is represented by $y = a \cdot f(x)$, where $|a| < 1$.

Vertical Stretch

A translation that makes all y-values of a graph larger in magnitude, pulling a graph toward the y-axis. This is represented by $y = a \cdot f(x)$, where $|a| > 1$.

Formulas to Know

Conversions Between Degrees and Radians

$$1\ degree = \frac{\pi}{180}\ radians$$

$$1\ radian = \frac{180}{\pi}\ degrees$$

Definitions of the Sine, Cosine, and Tangent Functions

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

Distance in the xy-Plane

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Distance on a Number Line

$$dist(a,b)=|b-a|$$

Logarithm Definition

$y=\log_a x$ if $a^y=x$ where $a>0$ and $a\neq 1$.

Point-Slope Form

$$y-y_1=m(x-x_1)$$

Power Property

$$\log_a(x^y)=y\cdot\log_a x$$

Product Property

$$\log_a(xy)=\log_a x+\log_a y$$

Quotient Property

$$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$$

Radius of a Circle

$$r=\sqrt{(x-h)^2+(y-k)^2}$$

Where: (h, k) is the center and (x, y) is a point on the circle.

Slope

$$m=\frac{\text{rise}}{\text{run}}=\frac{\text{change in } y}{\text{change in } x}=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$$

Slope-Intercept Form

$$y=mx+b$$

Standard Form Equation of a Circle

$$(x-h)^2+(y-k)^2=r^2$$

Where: (h, k) is the center and r is the radius.

The Piecewise Greatest Integer Function

$$\lfloor x\rfloor=\begin{cases} x & \textit{if } x \textit{ is an integer} \\ \textit{the closest integer less than } x & \textit{if } x \textit{ is NOT an integer}\end{cases}$$

Terms to Know

Average Rate of Change

The net change divided by the length of the interval.

Continuous From the Left

A function is continuous from the left at ***x*** = ***a*** if $\lim_{x \to a^-} f(x) = f(a)$.

Continuous From the Right

A function is continuous from the left at ***x*** = ***a*** if $\lim_{x \to a^+} f(x) = f(a)$.

Continuous Function

A function that has no breaks in the graph. That is, $\lim_{x \to a} f(x) = f(a)$.

Formal Definition of a Limit

$\lim_{x \to a} f(x) = L$ means that for every given $\varepsilon > 0$, there exists $\delta > 0$ so that:

- If x is within δ units of a (and $x \neq a$), then $f(x)$ is within ε units of L.
- This translates to $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

Instantaneous Rate of Change

The rate of change of a function at a specific point.

Intermediate Value Theorem (IVT)

Suppose $f(x)$ is a continuous function on the closed interval $[a, b]$. Let V be a value between $f(a)$ and $f(b)$. Then, there is at least one value of c between a and b such that $f(c) = V$.

Limit

The value that a function $f(x)$ approaches as x gets closer to a specified number.

Rational Function

A function in the form $f(x) = \frac{N(x)}{D(x)}$ where $N(x)$ and $D(x)$ are polynomials. A rational function is continuous at all real numbers except for those where $D(x) = 0$.

Secant Line

A line that contains two points of the same function.

Tangent Line

A line that touches (but does not cross) the graph of a function at a specific point.

Velocity

The speed of some object relative to some starting point. Unlike speed, velocity can be negative.

Formulas to Know

Average Rate of Change on the Interval [a, b]

$$\frac{\text{change in } f}{\text{change in } x} = \frac{f(b)-f(a)}{b-a}$$

Terms to Know

Acceleration

An object's change in velocity with respect to time.

Cusp

A pointed end where two parts of a curve meet at a vertical tangent.

Derivative

The slope of the tangent line to the graph of a function at a point is also known as the derivative of the function at that point.

Differentiable

A function $y=f(x)$ is said to be differentiable at $x = a$ if $f(x)$ is continuous at $x = a$ and $f'(a)$ is defined.

Fixed Cost (or Overhead)

The costs that are incurred before any items are produced. Mathematically, it is the total cost of producing 0 items.

Inverse Trigonometric Functions

A function that receives a real number as its input and returns an angle as its output.

Jerk

An object's change in acceleration with respect to time.

Linear Approximation of f(x) at x = a

The tangent line to the graph of $f(x)$ at $x = a$.

Marginal Cost Function

The derivative (rate of change) of the cost function. Given a production level ***x***, it approximates the cost of the next item.

Velocity

An object's change in distance with respect to time.

Formulas to Know

Average Cost Function

$AC(x) = \frac{C(x)}{x}$

Chain Rule

Suppose $y = f(u)$, a composite function, where ***u*** is a function of ***x***.

Then, $\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$.

Using "prime" notation, we can write $\frac{dy}{dx} = f'(u) \cdot u'$.

Using "D" notation, we can write $\frac{dy}{dx} = f'(u) \cdot D[u]$.

Derivative of Cosecant

$D[\csc x] = -\csc x \cot x$

Derivative of Cosine

$\frac{d}{dx}[\cos x] = -\sin x$

Derivative of Cotangent

$D[\cot x] = -\csc^2 x$

Derivative of Secant

$D[\sec x] = \sec x \tan x$

Derivative of Sine

$\frac{d}{dx}[\sin x] = \cos x$

Derivative of Tangent

$D[\tan x] = \sec^2 x$

Derivative of a Composite Logarithm Function, Base a

$D[\log_a u] = \frac{1}{u \cdot \ln a} \cdot u' = \frac{u'}{u \cdot \ln a}$

Derivative of a Constant Multiple

$D[k \cdot f(x)] = k \cdot D[f(x)]$

Derivative of a Difference

$D[f(x) - g(x)] = D[f(x)] - D[g(x)]$

Derivative of a Logarithm Function, Base a

$$D[\log_a x] = \frac{1}{x \cdot \ln a}$$

Derivative of a Sum

$$D[f(x) + g(x)] = D[f(x)] + D[g(x)]$$

Derivative of lnu, Where u Is a Function of x

$$D[\ln u] = \frac{1}{u} \cdot u'$$

Derivative of the Inverse Cosecant Function

$$\frac{d}{dx}[\csc^{-1} x] = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}[\csc^{-1} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$$

Derivative of the Inverse Cosine Function

$$\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}[\cos^{-1} u] = \frac{-u'}{\sqrt{1 - u^2}}$$

Derivative of the Inverse Cotangent Function

$$\frac{d}{dx}[\cot^{-1} x] = \frac{-1}{1 + x^2}$$

$$\frac{d}{dx}[\cot^{-1} u] = \frac{-u'}{1 + u^2}$$

Derivative of the Inverse Secant Function

$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}[\sec^{-1} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$$

Derivative of the Inverse Sine Function

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}[\sin^{-1} u] = \frac{u'}{\sqrt{1 - u^2}}$$

Derivative of the Inverse Tangent Function

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}[\tan^{-1} u] = \frac{u'}{1 + u^2}$$

Derivative of the Natural Logarithmic Function

$D[\ln x] = \frac{1}{x}$

Differential of f

$df = f'(x)dx$ for any choice of *x* and any real number *dx*.

When $y = f(x)$, we can also write $dy = f'(x)dx$.

Equation of a Tangent Line to y = f(x) at x = a

$y = f(a) + f'(a)(x - a)$

Evaluating Inverse Cosecant

$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

Evaluating Inverse Cotangent

$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$

Evaluating Inverse Secant

$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

General Power Rule for Derivatives of Functions

If $f(x)$ is some function, then $D\left[[f(x)]^n\right] = n \cdot [f(x)]^{n-1} \cdot f'(x)$.

Limit Definition of Derivative

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Linear Approximation Error

$\text{Error} = |f(x) - L(x)|$

Newton's Method

To find the next estimate for an x-intercept, use the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

Power Property

$\ln(a^b) = b \cdot \ln a$

Power Rule

$$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$$

Product Property

$$\ln(ab) = \ln a + \ln b$$

Product Rule for Derivatives

$$D[f(x) \cdot g(x)] = D[f(x)] \cdot g(x) + f(x) \cdot D[g(x)]$$

Using alternate notation: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Quotient Property

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

Quotient Rule for Derivatives

Using "Prime" Notation: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Using "D" Notation: $D\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot D[f(x)] - f(x) \cdot D[g(x)]}{[g(x)]^2}$

"High and Low" Version: $D\left[\frac{high}{low}\right] = \frac{low\ dee\ high - high\ dee\ low}{low\ low}$

Slope of the Line Passing Through the Points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The Derivative of a^x

$$D[a^x] = a^x \cdot \ln a$$

The Derivative of a^u, Where u Is a Function of x

$$D[a^u] = (a^u \cdot \ln a) \cdot u'$$

The Derivative of e^x

$$D[e^x] = e^x$$

The Derivative of e^u, Where u Is a Function of x

$$D[e^u] = e^u \cdot u'$$

Terms to Know

Concave Down

When a graph opens downward on an interval.

Concave Up

When a graph opens upward on an interval.

Concavity

Refers to the direction in which a graph opens. A graph is concave up if it opens upward and concave down if it opens downward.

Critical Number

A value of *c* in the domain of $f(x)$ for which $f'(c) = 0$ or $f'(c)$ is undefined, provided that $f(c)$ is defined.

Extrema

Another word for extreme values.

Extreme Value Theorem

If $f(x)$ is a continuous function on some closed interval $[a, b]$, then $f(x)$ has global maximum and global minimum values on the interval $[a, b]$.

Extreme Values

The minimum or maximum values of a function.

First Derivative Test

Used to identify possible local maximum and minimum points.

Global (or Absolute) Maximum

A function $f(x)$ has a global (or absolute) maximum at $x = a$ if $f(a) \geq f(x)$ for all *x*. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x = a$.

Global (or Absolute) Minimum

A function $f(x)$ has a global (or absolute) minimum at $x = a$ if $f(a) \leq f(x)$ for all *x*. In other words, $f(a)$ is the smallest value of a function $f(x)$, and occurs when $x = a$.

Horizontal Asymptote

A horizontal line in the form $y = c$ for the graph of $f(x)$ if either $\lim_{x \to \infty} f(x) = c$ or $\lim_{x \to -\infty} f(x) = c$.

Indeterminate Form

A form of a limit, such as "$\frac{0}{0}$", that doesn't always yield the same value. Further analysis is needed to determine its value, if it exists.

Inflection Point (Point of Inflection)

A point on a curve at which concavity changes.

Local (or Relative) Maximum

A function $f(x)$ has a local (or relative) maximum at $x = a$ if $f(a) \geq f(x)$ for all *x* close to $x = a$. In other words, $f(a)$ is the largest value of a function $f(x)$ for values near $x = a$.

Local (or Relative) Minimum

A function $f(x)$ has a local (or relative) minimum at $x = a$ if $f(a) \leq f(x)$ for all *x* close to $x = a$. In other words, $f(a)$ is the smallest value of a function $f(x)$ for values near $x = a$.

Mean Value Theorem for Derivatives

Let $f(x)$ be continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b).

Then, there is at least one value of *c* between *a* and *b* for which $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Nonlinear Asymptote

The curve that a graph approaches as $x \to \pm\infty$.

Optimization Problem

A problem in which the maximum or minimum value is sought, whichever is relevant.

Second Derivative Test

Suppose $f'(c) = 0$, which means $f(x)$ has a horizontal tangent at $x = c$.

• If $f''(c) < 0$, this means $f(x)$ is concave down around *c*, which means there is a local maximum at *c*.

• If $f''(c) > 0$, this means $f(x)$ is concave up around *c*, which means there is a local minimum at *c*.

• If $f''(c) = 0$, the test is inconclusive, and the first derivative test needs to be used to determine the behavior at *c*.

Slant (Oblique) Asymptote

The slanted line that a graph approaches as $x \to \pm\infty$.

Vertical Asymptote

A vertical line in the form $x = a$ for the graph of $f(x)$ if either $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.

Terms to Know

Antiderivative

$F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Circumscribed (Rectangles)

A rectangle is circumscribed outside a region if it is the smallest rectangle that encompasses the region.

Differential Equation

An equation that contains derivatives of some function *y*.

General Solution

The general solution of a differential equation is a function of the form $y = F(x) + C$ that satisfies a differential equation regardless of the value of *C*.

Indefinite Integral of f(x)

The collection of functions whose derivatives are equal to $f(x)$. In other words, the indefinite integral of $f(x)$ is the antiderivative of $f(x)$.

Initial Condition

From a differential equation, a point that the solution's graph passes through.

Inscribed (Rectangles)

A rectangle is inscribed inside a region if it is the largest rectangle that stays inside the region.

Integrable

If the value of $\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \cdot \Delta x$ exists and is equal to *A* regardless of the values of c_k used in each subinterval, then we say that $f(x)$ is integrable on the interval $[a, b]$.

Particular Solution

The solution to a differential equation that doesn't contain an arbitrary constant. The particular solution satisfies the differential equation as well as the initial condition.

Partition

A set of x-values that are used to split the interval $[a, b]$ into smaller intervals.

Riemann Sum
The sum obtained from the areas of rectangles that are used to approximate the area between a curve and the x-axis.

Subinterval

A smaller interval that is part of a larger interval.

Summand

The expression being used to determine the numbers that are added in a sum.

Summation

An expression that implies that several numbers are being added together. These are often written using sigma notation.

The First Fundamental Theorem of Calculus

Let $F(x)$ be an antiderivative of $f(x)$, meaning that $F'(x) = f(x)$.

Then, $\int_a^b f(x)dx = F(b) - F(a)$, which means we evaluate the antiderivative at the endpoints, then subtract.

The Mean Value Theorem for Integrals
If $f(x)$ is continuous on $[a, b]$, then at some point *c* in $[a, b]$:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

The Second Fundamental Theorem of Calculus
Let $f(x)$ be a continuous function on the closed interval $[a, b]$ with $a \le x \le b$.

Let $F(x) = \int_a^x f(t)dt$. Then, $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$.

Formulas to Know

Antiderivative of a Constant

$$\int k dx = kx + C$$

$$\int k \cdot f(x)dx = k \cdot \int f(x)dx$$

Antiderivative of a Constant Multiple of a Function

Antiderivative of a Difference of Functions

$$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

Antiderivative of a Sum of Functions

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

Antiderivative of cos x

$$\int \cos x dx = \sin x + C$$

Antiderivative of csc x cot x

$$\int \csc x \cot x dx = -\csc x + C$$

Antiderivative of $\csc^2$ x

$$\int \csc^2 x dx = -\cot x + C$$

Antiderivative of e^{kx}, Where k is a Constant

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + C$$

Antiderivative of sec x tan x

$$\int \sec x \tan x dx = \sec x + C$$

Antiderivative of $\sec^2$ x

$$\int \sec^2 x dx = \tan x + C$$

Antiderivative of sin x

$$\int \sin x dx = -\cos x + C$$

Antiderivatives of Exponential Functions

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Area Between Two Curves, x = h(y) and x = k(y), Assuming h(y) ≥ k(y) on [c, d] (Horizontal Sub-rectangles)

$$Area = \int_c^d [h(y) - k(y)]dy$$

Area Between Two Curves, y = f(x) and y = g(x), Assuming f(x) ≥ g(x) on [a, b]

$$Area = \int_a^b [f(x) - g(x)]dx$$

Average Value of a Function

If $f(x)$ is continuous on the closed interval $[a, b]$, then the average value of $f(x)$ on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.

Definite Integral Over a Partition of an Interval, with a ≤ b ≤ c

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Definite Integral When Lower and Upper Bounds Are Equal

$$\int_a^a f(x)dx = 0$$

Definite Integral When Upper and Lower Bounds Are Interchanged

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

Definite Integral of a Constant Function

$$\int_a^b kdx = k(b-a)$$

Definite Integral of a Constant Multiple of a Function

$$\int_a^b k \cdot f(x)dx = k\int_a^b f(x)dx$$

Definite Integral of a Difference of Two Functions

$$\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

Definite Integral of a Sum of Two Functions

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

Fundamental Theorem of Calculus

Let $F(x)$ be an antiderivative of a continuous function $f(x)$ on the interval $[a, b]$.

Then, $\int_a^b f(x)dx = F(b) - F(a)$.

Natural Logarithm Rule

$$\int \frac{1}{x}dx = \ln|x| + C$$

Power Rule for Antiderivatives

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

Riemann Sum

When approximating the area between a nonnegative function $y = f(x)$ and the x-axis by using *n* rectangles, the summation $\sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$ is called the Riemann Sum, where c_k is a value of *x* in the k^{th} subinterval and Δx_k is the width of the k^{th} subinterval.

Simpson's Rule

The value of $\int_a^b f(x)dx$ is approximated by

$\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + ... + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ where $\Delta x = \frac{b-a}{n}$ and $a = x_0, x_1, x_2, ..., x_{n-1}, x_n = b$ are the endpoints of each equally spaced subinterval. Note: *n* must be even.

Summation of a Constant Multiple

$$\sum_{k=1}^{n} C \cdot a_k = C \cdot \sum_{k=1}^{n} a_k$$

Summation of a Sum or Difference

$$\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

Summations of Powers of Consecutive Numbers

$$\sum_{k=1}^{n} k = 1 + 2 + ... + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = 1^4 + 2^4 + ... + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

The Summation of a Constant

If C is a constant, $\sum_{k=1}^{n} C = C \cdot n$

Trapezoidal Rule

$\int_a^b f(x)dx$ can be approximated by the sum $\frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+\ldots+2f(x_{n-1})+f(x_n)]$,

where $\Delta x = \frac{b-a}{n}$ and $a = x_0, x_1, x_2, \ldots, x_{n-1}, x_n = b$ are the endpoints of each equally spaced subinterval.