

## Table of Integration Formulas

In all integrals, assume that  $k$ ,  $n$ ,  $a$ , and  $b$  represent real numbers.

### Part One: Integrals Involving Powers

1.  $\int k dx = kx + C$
2.  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$
3.  $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$
4.  $\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + C, n \neq -1$
5.  $\int (ax+b)^{-1} dx = \int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$
6.  $\int \frac{1}{x(ax+b)} dx = \frac{1}{b} \ln \left| \frac{x}{ax+b} \right| + C$
7.  $\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} [\ln|x+a| - \ln|x+b|] + C = \frac{1}{b-a} \ln \left| \frac{x+a}{x+b} \right| + C$  if  $a \neq b$
8.  $\int \frac{1}{(x+a)(x+a)} dx = \int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a} + C$
9.  $\int x(ax+b)^n dx = \frac{1}{a^2} (ax+b)^{n+1} \left[ \frac{ax+b}{n+2} - \frac{b}{n+1} \right] + C, n \neq -1, -2$
10.  $\int x(ax+b)^{-1} dx = \int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$
11.  $\int x(ax+b)^{-2} dx = \int \frac{x}{(ax+b)^2} dx = \frac{1}{a^2} \left[ \ln|ax+b| + \frac{b}{ax+b} \right] + C$

### Part Two: Integrals Involving Trigonometric Functions

12.  $\int \sin(ax) dx = \frac{-1}{a} \cos(ax) + C$
13.  $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$

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$$14. \quad \int \tan(ax) \, dx = \int \frac{\sin(ax)}{\cos(ax)} \, dx = \frac{-1}{a} \ln |\cos(ax)| + C = \frac{1}{a} \ln |\sec(ax)| + C$$

$$15. \quad \int \cot(ax) \, dx = \int \frac{\cos(ax)}{\sin(ax)} \, dx = \frac{1}{a} \ln |\sin(ax)| + C$$

$$16. \quad \int \sec(ax) \, dx = \frac{1}{a} \ln |\sec(ax) + \tan(ax)| + C$$

$$17. \quad \int \csc(ax) \, dx = \frac{-1}{a} \ln |\csc(ax) + \cot(ax)| + C$$

$$18. \quad \int \sin^2(ax) \, dx = \frac{1}{2}x - \frac{1}{4a} \sin(2ax) + C = \frac{1}{2}x - \frac{1}{2a} \sin(ax) \cos(ax) + C$$

$$19. \quad \int \cos^2(ax) \, dx = \frac{1}{2}x + \frac{1}{4a} \sin(2ax) + C = \frac{1}{2}x + \frac{1}{2a} \sin(ax) \cos(ax) + C$$

$$20. \quad \int \tan^2(ax) \, dx = \frac{1}{a} \tan(ax) - x + C$$

$$21. \quad \int \cot^2(ax) \, dx = \frac{-1}{a} \cot(ax) - x + C$$

$$22. \quad \int \sec^2(ax) \, dx = \frac{1}{a} \tan(ax) + C$$

$$23. \quad \int \csc^2(ax) \, dx = \frac{-1}{a} \cot(ax) + C$$

$$24. \quad \int \sin^3(ax) \, dx = \frac{-1}{3a} \sin^2(ax) \cos(ax) - \frac{2}{3a} \cos(ax) + C$$

$$25. \quad \int \cos^3(ax) \, dx = \frac{1}{3a} \cos^2(ax) \sin(ax) + \frac{2}{3a} \sin(ax) + C$$

$$26. \quad \int \tan^3(ax) \, dx = \frac{1}{2a} \tan^2(ax) + \frac{1}{a} \ln |\cos(ax)| + C$$

$$27. \quad \int \cot^3(ax) \, dx = \frac{-1}{2a} \cot^2(ax) - \frac{1}{a} \ln |\sin(ax)| + C$$

$$28. \quad \int \sec^3(ax) \, dx = \frac{1}{2a} \sec(ax) \tan(ax) + \frac{1}{2a} \ln |\sec(ax) + \tan(ax)| + C$$

$$29. \quad \int \csc^3(ax) \, dx = \frac{-1}{2a} \csc(ax) \cot(ax) - \frac{1}{2a} \ln |\csc(ax) + \cot(ax)| + C$$

$$30. \quad \int \sin(ax) \sin(bx) \, dx = \frac{1}{2(a-b)} \sin((a-b)x) - \frac{1}{2(a+b)} \sin((a+b)x) + C, \quad |a| \neq |b|$$

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$$31. \quad \int \cos(ax) \cos(bx) dx = \frac{1}{2(a-b)} \sin((a-b)x) + \frac{1}{2(a+b)} \sin((a+b)x) + C, \quad |a| \neq |b|$$

$$32. \quad \int \sin(ax) \cos(bx) dx = \frac{-1}{2(a-b)} \cos((a-b)x) - \frac{1}{2(a+b)} \cos((a+b)x) + C, \quad |a| \neq |b|$$

### Part Three: Integrals Involving Exponential & Logarithmic Functions

$$33. \quad \int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$34. \quad \int b^{ax} dx = \frac{1}{a \ln b} b^{ax} + C$$

$$35. \quad \int x e^{ax} dx = \frac{1}{a} x e^{ax} - \frac{1}{a^2} e^{ax} + C$$

$$36. \quad \int x^2 e^{ax} dx = \frac{1}{a} x^2 e^{ax} - \frac{2}{a^2} x e^{ax} + \frac{2}{a^3} e^{ax} + C$$

$$37. \quad \int \ln x dx = x \ln x - x + C$$

$$38. \quad \int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1$$

$$39. \quad \int x^{-1} \ln x dx = \int \ln x \cdot \frac{1}{x} dx = \frac{1}{2} (\ln x)^2 + C$$

$$40. \quad \int \frac{1}{x \ln x} dx = \ln |\ln x| + C$$

$$41. \quad \int e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2 + b^2} [a \sin(bx) - b \cos(bx)] + C$$

$$42. \quad \int e^{ax} \cos(bx) dx = \frac{e^{ax}}{a^2 + b^2} [a \cos(bx) + b \sin(bx)] + C$$

### Part Four: Integrals Involving $a^2 \pm x^2$ and $x^2 \pm a^2$

$$43. \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$44. \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

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$$45. \quad \int \frac{1}{|x|\sqrt{x^2 - a^2}} dx = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C$$

$$46. \quad \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$47. \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$48. \quad \int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$49. \quad \int x^2 \sqrt{x^2 \pm a^2} \, dx = \frac{1}{8} x (2x^2 \pm a^2) \sqrt{x^2 \pm a^2} - \frac{1}{8} a^4 \ln |x + \sqrt{x^2 \pm a^2}| + C$$