Muon Radiative Backgrounds in FASER

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Introduction

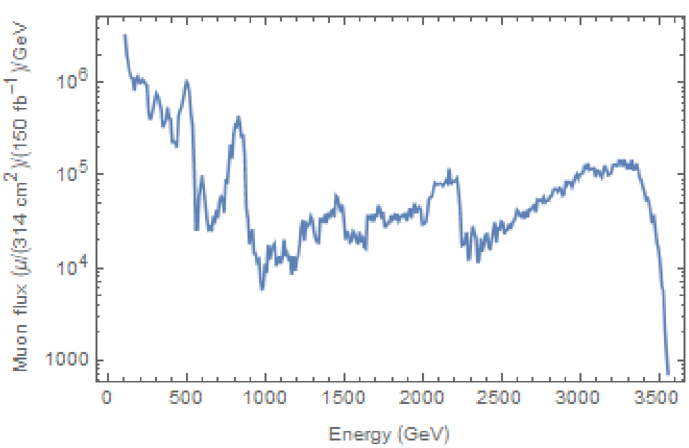
FASER-1 will search for new physics in the form of rare dark photon decays, $A' \to e^+e^-$. In the region of parameter space of interest, the energies of both daughters are over $100\,\mathrm{GeV}$. With FASER's complementary tracking and calorimetry, particles much below this analysis threshold cannot easily mimic the signal.

FLUKA simulation studies[1] indicate that debris from pp collisions at the Point 1 interaction point is the only significant source of particles with these energies. The only Standard Model particles capable of transporting $100\,\mathrm{GeV}$+ over the $485\,\mathrm{m}$ (including well over $100\,\mathrm{m}$ of rock and concrete) to FASER are muons and neutrinos. This note focuses on muon-associated backgrounds:

- unconverted photons from bremsstrahlung,
- electromagnetic showers from converted bremsstrahlung and direct pair production,
- hadronic cascades from photo-nuclear interactions
- delta-ray production
- muon decay in flight

Muon Flux at FASER

The muon flux predicted by the FLUKA form the starting point for this study. As machine-readable data is not yet available, the total muon spectrum ($\mu^+ + \mu^-$) was digitized "by hand" from the FLUKA report. While somewhat imprecise, these approximate values are adequate for present purposes. The spectrum is normalized to an area of $314\,\mathrm{cm}^2$ (i.e. a $10\,\mathrm{cm}$ radius disk) and an integrated luminosity of $150\,\mathrm{fb}^{-1}$ at $14\,\mathrm{TeV}$ center of mass energy. The lowest muon energy in the FLUKA study is $100\,\mathrm{GeV}$.



The FLUKA report estimated an integrated muon flux of $1.9 \times 10^{-8}/\mathrm{cm}^2/\mathrm{bx}$ above $100\,\mathrm{GeV}$, assuming 140 collisions per bunch crossing ("bx"). With a pp cross section of $85\,\mathrm{mb}$, this corresponds to $1.7 \times 10^6/\mathrm{cm}^2/(150\,\mathrm{fb}^{-1})$ or $5.4 \times 10^8/(314\,\mathrm{cm}^2)/(150\,\mathrm{fb}^{-1})$. The corresponding integrated rate computed from the hand-digitized spectrum is 5.2×10^8, which agrees within better than 5%.

Unconverted Photons from Bremsstrahlung

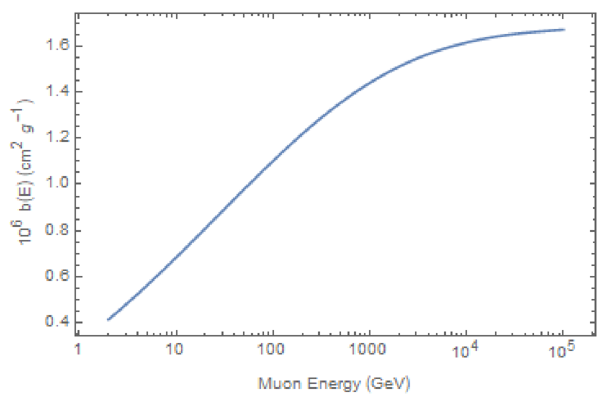
Cross section

The FLUKA report gave only order of magnitude estimates for the rate of energetic photons entering FASER, and no energy spectrum. It is impractical to fully simulate muons entering FASER, since the catastrophic loss processes leading to photons with energies above 100 GeV are very rare. An alternative approach is to work from the simulated muon spectrum to calculate the rate of bremsstrahlung near the detector, and then the flux and spectrum of unconverted photons that enter.

Muon radiative energy losses, including bremsstrahlung, are well-described by[2]

$$-\frac{1}{E}\frac{dE}{dx} = b(E)$$

where $b(E)$ depends on the process, and the material, but is only a slowly-varying function of energy. Tables of $b(E)$ for bremsstrahlung and other processes are provided for many materials by the Particle Data Group.[3]



The energy loss is also related to the cross section:[4]

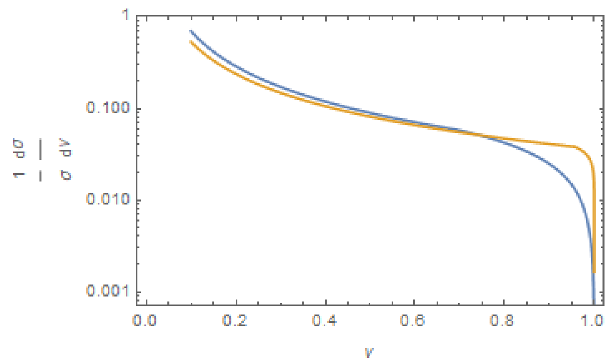
$$-\frac{1}{E}\frac{dE}{dx}\Big|_{\text{brem}} = b = \frac{N_A}{A}\int_0^1 v\frac{d\sigma}{dv}dv = \frac{N_A}{A}\sigma\int_0^1 v\frac{1}{\sigma}\frac{d\sigma}{dv}dv = \frac{N_A}{A}\sigma\cdot\bar{v}$$

Here v is the fractional energy loss in a given process, $\frac{d\sigma}{dv}$ is the differential cross section as a function of v, $\bar{v}$ is the average fractional energy loss, and σ is the total cross section per target atom. Note that $\frac{d\sigma}{dv}$, σ and $\bar{v}$ all depend on the incident muon energy. The bremsstrahlung cross section per atom is then:

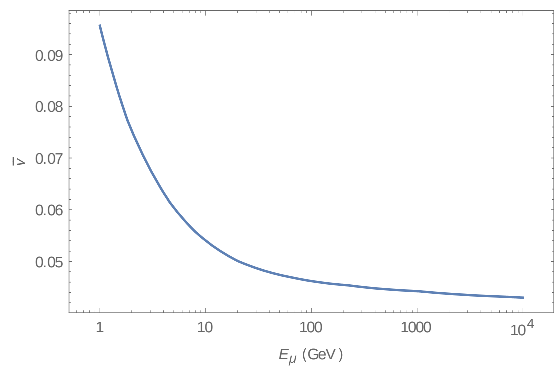
$$\sigma = \frac{A}{N_A}\frac{b}{\bar{v}}$$

Energy loss distribution and $\bar{v}$

The quantity $\bar{v}$ can only be determined by averaging the normalized differential cross section $\frac{1}{\sigma}\frac{d\sigma}{dv}$. which depends on the detailed dynamics of the process. Fortunately, these distributions have been parameterized down to $v = \frac{1\text{ MeV}}{E}$ in a form which is relatively easy to use.[5] The curves for bremsstrahlung at $E_\mu = 100\text{ GeV}$ and $E_\mu = 10\text{ TeV}$ are shown in blue and gold, respectively, and agree with the corresponding ones in Figure 1b of the Van Ginneken paper.



As it turns out, $\bar{\nu}$ is nearly constant ($\sim 4.5\%$) over the energy range of interest ($E_\mu > 100$ GeV).



Mean free path

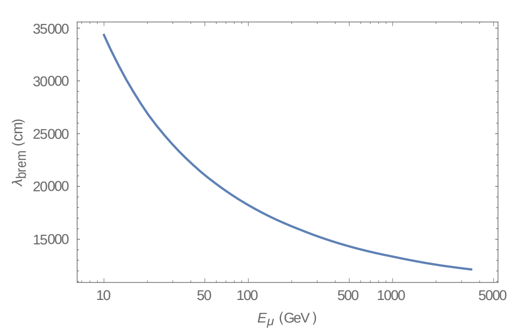
The mean free path λ for a process is related to the cross section per target atom, σ:

$$\lambda^{-1} = \rho \frac{N_A}{A} \sigma = \frac{\rho b}{\bar{\nu}}$$

where ρ is the density of the material, and the previous expression for σ in terms of b and $\bar{\nu}$ has been used. The mean free path is naturally energy-dependent, due to the energy dependence of b and $\bar{\nu}$.

Shielding concrete is used for the local material. As explained below, the results are not very sensitive to this choice. The interpolated Particle Data Group values for $b(E)$ (in units of $10^6\ \mathrm{cm}^2\ \mathrm{g}^{-1}$) are plotted below. The assumed density is $\rho = 2.30\ \mathrm{g\ cm}^{-3}$, with radiation length $X_0 = 11.55$ cm.

The calculated mean free paths for bremsstrahlung are only mildly energy dependent and range from around $\lambda_{\mathrm{brem}} = 180$ m at 100 GeV to $\lambda_{\mathrm{brem}} = 120$ m at 3.5 TeV. The quantity $\lambda_{\mathrm{brem}}^{-1}$ is the average number of interactions per unit length.



Unconverted photon production per muon

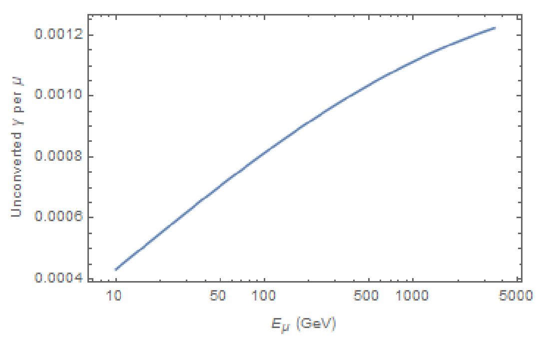
Muons radiate photons along their entire trajectory, but only those created near the boundary of the detector can enter unconverted. A photon which converts to an e^+e^- pair will initiate an electromagnetic shower with charged particles that enter the detector. The energies of these secondaries will rapidly drop below the analysis threshold as the shower develops, and the charged particles in the cascade will be easily detected in the forward veto. Thus, only photons which do not convert before entering the detector cavity can constitute background. The mean free path for photon pair production is approximately

$$\lambda_{pair} = \frac{9}{7} X_0$$

and the expected number of unconverted photons reaching the detector per incident muon is

$$\epsilon = \int_0^\infty \lambda_{\text{brem}}^{-1}\, e^{-\frac{x}{\lambda_{\text{pair}}}}\, dx = \frac{9}{7}\frac{X_0}{\lambda_{\text{brem}}} = \frac{9}{7}\frac{\rho b X_0}{\bar{\nu}}$$

This quantity, like several others, depends only mildly on energy, and is approximately 0.1% between $100\ \text{GeV}$ and $3.5\ \text{TeV}$. Note that this includes all unconverted photons from bremsstrahlung, regardless of their energy. Roughly 1/1000 muons will be accompanied by an unconverted photon.



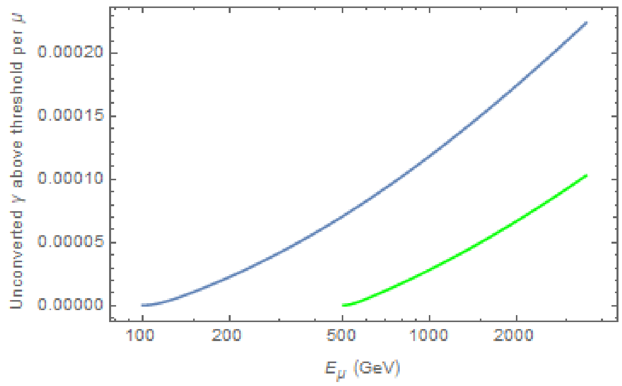
Although $\bar{\nu}$ is virtually independent of material, the numerator of the above expression is the product of three terms that are dependent on the material. Lower values of ϵ will result in fewer unconverted photons from bremsstrahlung, so it is interesting to compare some alternative shielding materials.

Material	Density (g cm^{-3})	$b(1\ \text{TeV})$ ($10^6\ \text{cm}^2\text{g}^{-1}$)	Radiation Length (cm)	Product
Concrete	2.30	1.44	11.55	38.3
Lead	11.35	6.85	0.5612	43.6
Tungsten	19.30	6.35	0.3504	43.0
Air	1.205×10^{-3}	1.025	3.039×10^4	37.5

All four materials have “figures of merit” within 15% of each other; although lead and tungsten are much more effective at converting photons, they are also much more effective at *generating* them. The fact that such different materials all lead to roughly the same rate of unconverted photons from bremsstrahlung suggests that the result is not very sensitive to precise characterization of the surrounding material.

The number of unconverted bremsstrahlung photons per muon that pass a specified photon energy threshold is

$$\epsilon(E_\mu, E_{\rm thr}) = \epsilon(E_\mu) \int_{\frac{E_{\rm thr}}{E_\mu}}^{1} \frac{1}{\sigma}\frac{d\sigma}{d\nu}\, d\nu$$



For a 100 GeV photon energy threshold (in blue), the number of unconverted photons per muon rises from 0 at $E_\mu = 100$ GeV to about 0.00025 at $E_\mu = 3.5$ TeV. The corresponding curve for a 500 GeV threshold is plotted in green.

Unconverted photon flux and spectrum

The total flux of unconverted photons can now be easily computed from the tabulated muon energy spectrum and the efficiency function ϵ.

$$N_\gamma = \int_{100\ \rm GeV}^{3.5\ \rm TeV} dE_\mu \frac{dN_\mu}{dE_\mu}\, \epsilon(E_\mu) = (10\ \rm GeV) \sum_i N_{\mu,i}\, \epsilon(E_i)$$

where 10 GeV is the bin width of the FLUKA spectrum.

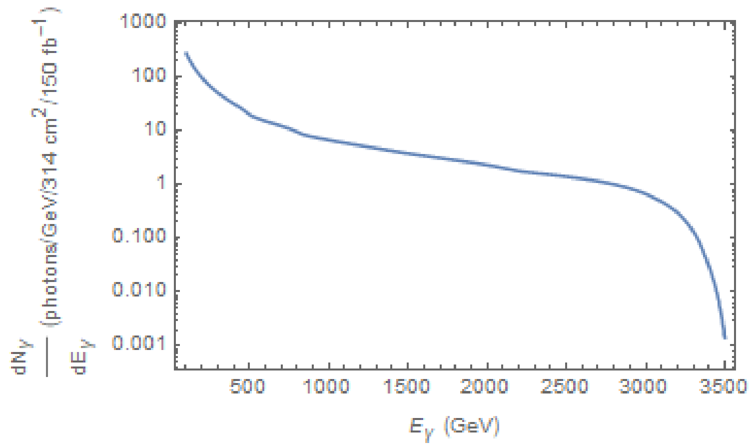
Of more interest is the flux of unconverted photons *above some analysis threshold,* or alternatively, the differential photon flux as a function of energy. Computing either requires folding the muon energy spectrum, the unconverted photon efficiency per muon (ϵ) and the normalized differential cross section $\frac{1}{\sigma}\frac{d\sigma}{d\nu}$:

$$\frac{dN_\gamma}{dE_\gamma} = \int_{100\ \rm GeV}^{3.5\ \rm TeV} dE_\mu \frac{dN_\mu}{dE_\mu}\, \epsilon(E_\mu) \left(\frac{1}{E_\mu}\frac{1}{\sigma}\frac{d\sigma}{d\nu}\right)_{\nu=\frac{E_\gamma}{E_\mu}} = (10\ \rm GeV) \sum_i N_{\mu,i}\, \epsilon(E_i) \left(\frac{1}{E_i}\frac{1}{\sigma}\frac{d\sigma}{d\nu}\right)_{\nu=\frac{E_\gamma}{E_i}}$$

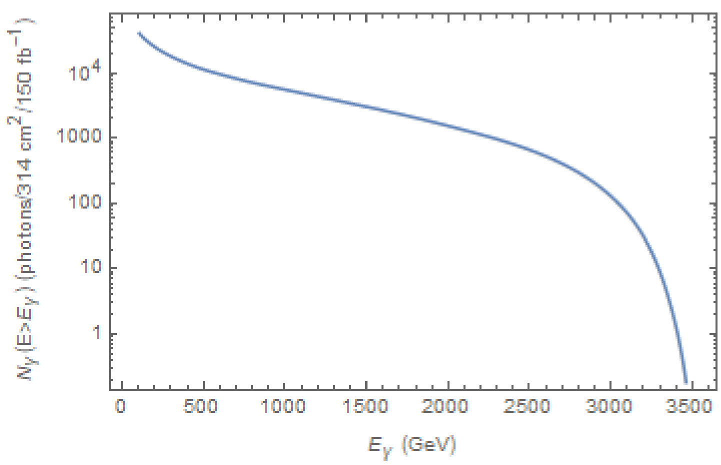
where $\frac{1}{E_\mu}\frac{d\sigma}{d\nu} = \frac{d\sigma}{dE_\gamma}$.

The estimated total number of unconverted bremsstrahlung photons with energy greater than 100 GeV entering 314 $\rm cm^2$ for an integrated luminosity of 150 $\rm fb^{-1}$ is 41,000. Converting this back to the units used in the original FLUKA report, it corresponds to $1.4 \times 10^{-12}\ \rm cm^{-2}\ bx^{-1}$. The FLUKA report quotes the photon flux above 100 GeV as "$\sim 10^{-12}\ \rm cm^{-2}\ bx^{-1}$", which is in surprisingly (and probably accidentally…) good agreement.

Only a handful (<10) of muons should be accompanied by *two* unconverted photons.



The differential photon spectrum is shown in the figure above, and the cumulative distribution (total number with energy greater than threshold E_γ) is plotted below:



Photon Conversion in FASER

Each tracker plane is approximately 0.025 radiation lengths thick; the eight planes together thus comprise 0.2 radiation lengths. The 2 cm plastic scintillator trigger plane contributes 0.05 radiation lengths, and the air within the magnetized region is 0.01 radiation lengths. The total thickness of detector up to the last tracker plane is 0.26 radiation lengths. The photon mean free path is $\frac{9}{7}X_0$, so the probably of conversion in the decay volume or tracker is

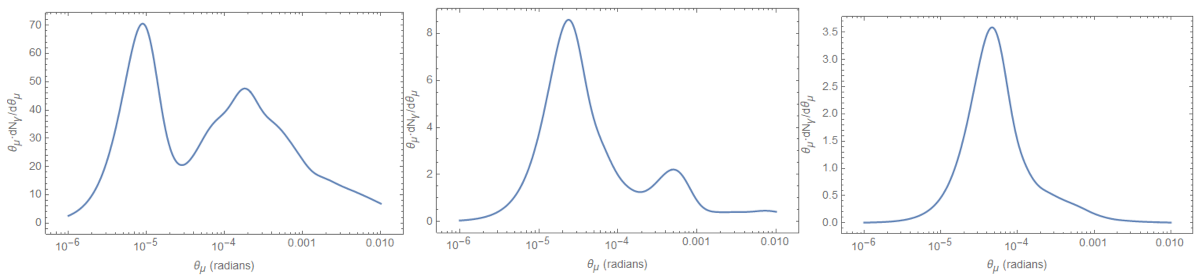
$$P_{\mathrm{convert}} = 1 - e^{-\frac{7}{9}\times 0.26} = 18\%$$

This implies that about 7400 (18% of 41000) unconverted bremsstrahlung products entering the detector will convert inside, potentially leaving some evidence of their passage in the tracker. Additionally, any γ that reaches the calorimeter will certainly convert, but having left no signal in the tracker, it cannot, by itself, be confused with the signal.

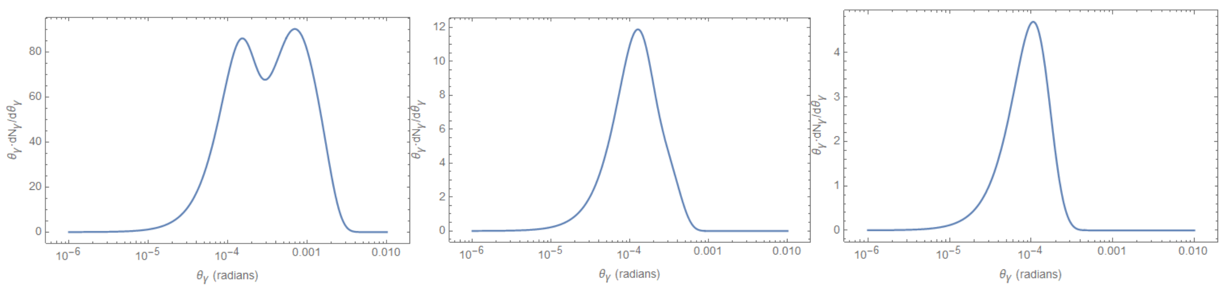
Collimation

Van Ginneken also parameterizes the scattering and emission angle distributions for the incident muon and radiated photon, respectively.

The muon scattering angle distribution for unconverted $100\ \text{GeV}$, $500\ \text{GeV}$ and $1\ \text{TeV}$ photon production are plotted below. The dual-peaked structure reflects small$-\nu$ and large$-\nu$ contributions from different energies in the incident muon spectra. (In these and other log-linear angular distribution graphs, the quantity $(\theta\ dN/d\theta)$ is plotted, so that area on the plot corresponds to event rate.)



The corresponding emission angle distributions (with respect to the incident particle direction) for unconverted $100\ \text{GeV}$, $500\ \text{GeV}$ and $1\ \text{TeV}$ photons are shown in the next three figures.



For the lowest energy photons (100 GeV), the scattering and emission angles can be as large as a few milliradians, but in general the scattered muon and radiated photon are both highly collimated with the incident muon direction (and with each other). Geometrically, it is essentially impossible for one of the outgoing particles to enter the detector without being accompanied by the other.

Electromagnetic Showers

The previous sections computed the flux of *unconverted* photons from bremsstrahlung. Converted photons, as well as directly produced e^+e^- pairs, will produce electromagnetic showers (including charged particles) which also enter the detector.

Converted photon flux and spectrum

For a semi-infinite line source (the muon), the number of photon conversions per unit length along the track will equal the rate of production, $1/\lambda_{\text{brem}}$. A converted photon will initiate an electromagnetic cascade, and the total energy of the cascade will fall exponentially over a distance characterized by the radiation length X_0:

$$E \approx E_0 e^{-x/X_0}$$

here E_0 is the initial energy of the converting photon and x is the distance to the detector cavity.

For unconverted photons, the entering energy is identical to the original energy, but showers entering with a given energy can arise from different initial energies (at different locations) due to energy loss. The differential rate of electromagnetic showers, dN/dE is

$$\frac{dN}{dE} = \int_{100\text{ GeV}}^{1\text{ TeV}} dE_\mu \frac{dN_\mu}{dE_\mu} \int_0^{x_{\max}} \frac{dx}{\lambda_{\text{brem}}} \left(\frac{1}{E_\mu} \frac{1}{\sigma} \frac{d\sigma}{dv} \right)_{v = \left(\frac{E}{E_\mu}\right) e^{\frac{x}{X_0}}}$$

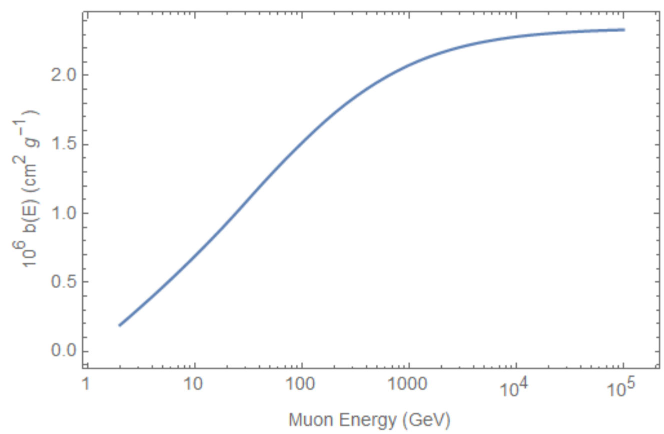
where $x_{\max} = -X_0 \log \frac{E}{E_\mu}$ is the distance from which a shower produced with $v = 1$ will arrive with final energy E.

Direct pair production

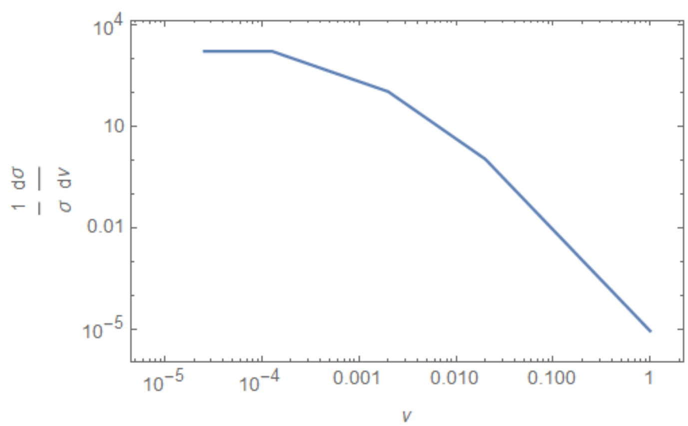
Direct e^+e^- pair production is another muon energy loss process, competing with bremsstrahlung. Direct pair production has a larger cross section but is softer. Entering electromagnetic cascades from direct pair production are experimentally indistinguishable from those produced by converted bremsstrahlung, so the rates can simply be added together.

Pair production cross section, energy loss distribution and mean free path

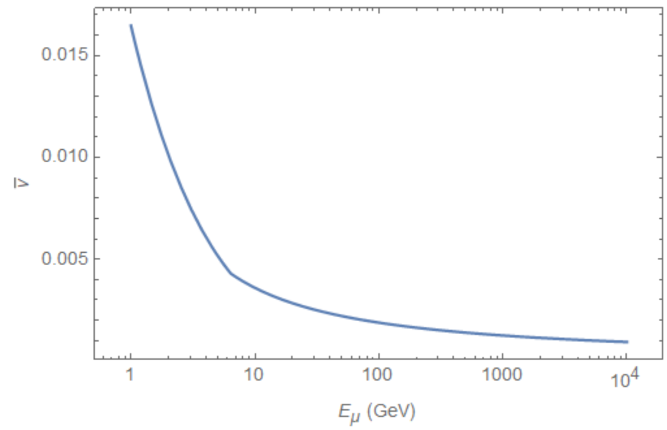
The calculation for pair production is identical to that for converted bremsstrahlung. The cross section is determined from the tabulated radiative energy loss function $b(E)$ for pair production, shown below:



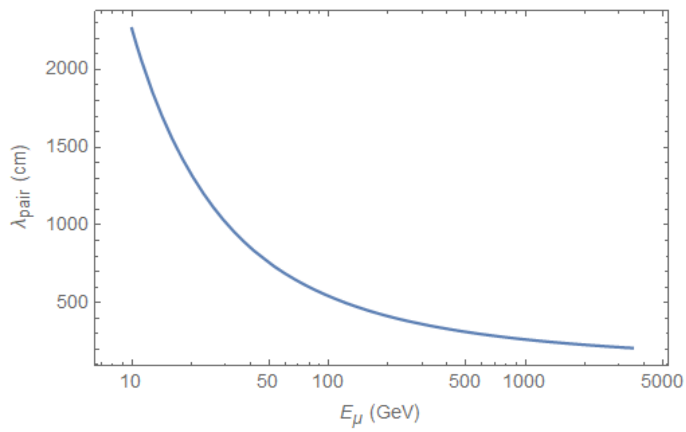
Van Ginneken parameterizes the fractional energy loss distribution, $\frac{1}{\sigma}\left(\frac{d\sigma}{dv}\right)$, plotted for $E_\mu = 100$ GeV:



The average energy loss per scattering, $\bar{\nu}$, is about 30 times smaller than bremsstrahlung:

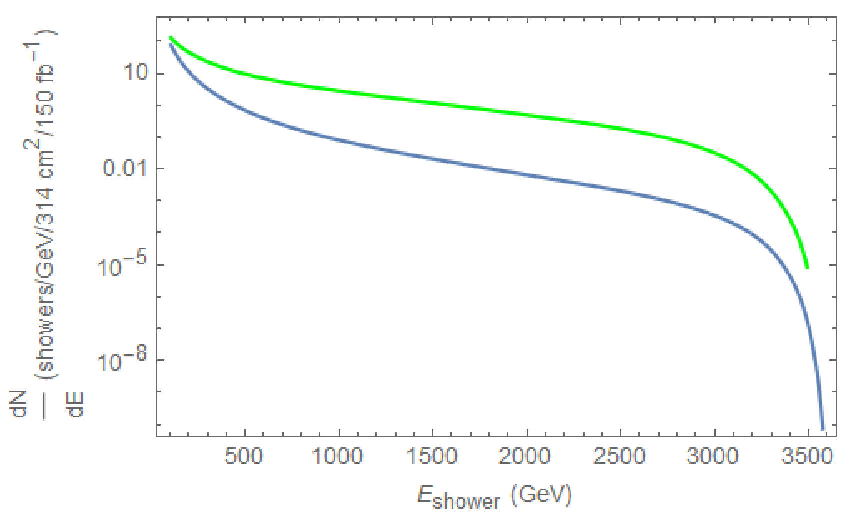


The mean free path, however, is only a few meters:

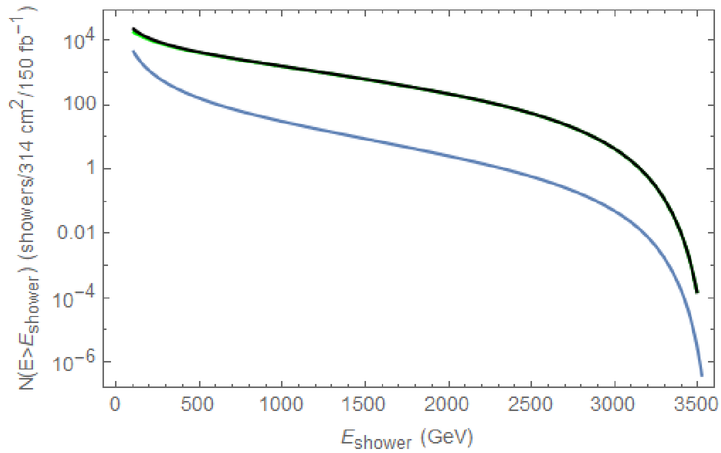


Electromagnetic cascade flux and spectrum

The differential spectrum of entering electromagnetic showers is shown below. The blue and green curves are direct pair production and bremsstrahlung conversion, respectively:



Due to the softness of direct pair production, bremsstrahlung conversions dominate at all but the lowest energies considered. The integral rate (total number above a given shower energy E) is plotted below:

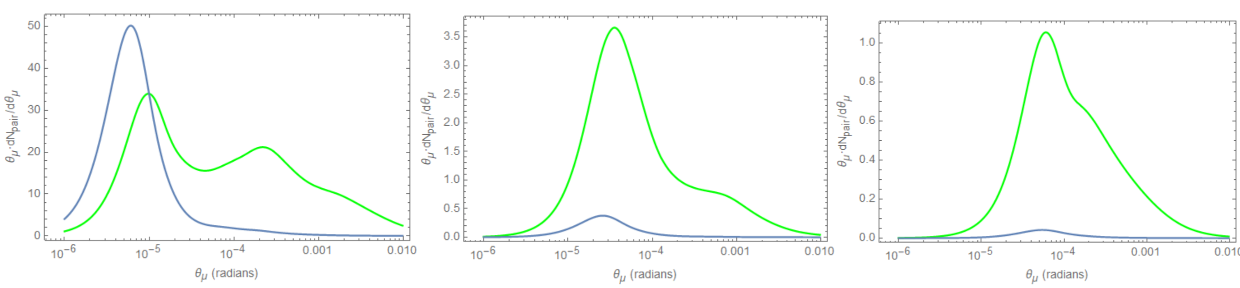


The black curve, which is the sum of both processes, is barely distinguishable from the green.

To total predicted rates of entering showers above $100\ \mathrm{GeV}$ from bremsstrahlung and direct pair production are 18000 and 4400, respectively, for a $314\ \mathrm{cm}^2$ detector and $150\ \mathrm{fb}^{-1}$ integrated luminosity.

Collimation

As for bremsstrahlung, the scattered muons are very tightly collimated. The muon angular distributions for $100\ \mathrm{GeV}$, $500\ \mathrm{GeV}$ and $1\ \mathrm{TeV}$ are plotted below. As before, the blue curves are direct pair production, green are bremsstrahlung photon conversions, and the curves are normalized so that area corresponds to rate:



The transverse development of the electromagnetic cascades is characterized by the Moliere radius, which is $4.9\ \mathrm{cm}$ for shielding concrete. The veto planes should extend beyond nominal aperture of the detector by at least this amount.

Photo-Nuclear Interactions

Muons interacting electromagnetically with atomic nuclei can produce hadrons. Typically, the hadronic energy is shared by multiple particles; as these pass through the target material, a cascade will develop, further decreasing the average energy per particle and eventually absorbing the hadronic energy (apart from escaping neutrinos) completely.

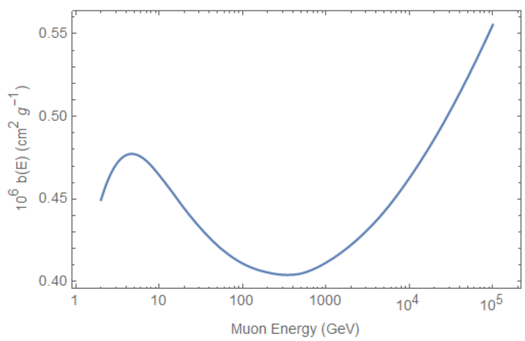
The development of a hadronic shower is complex, but the amount of hadronic energy entering the detector cavity can be roughly characterized by the nuclear interaction length ($\lambda_{\text{int}} = 42.4\ \text{cm}$ for shielding concrete):

$$E \approx E_0 e^{-\frac{x}{\lambda_{\text{int}}}}$$

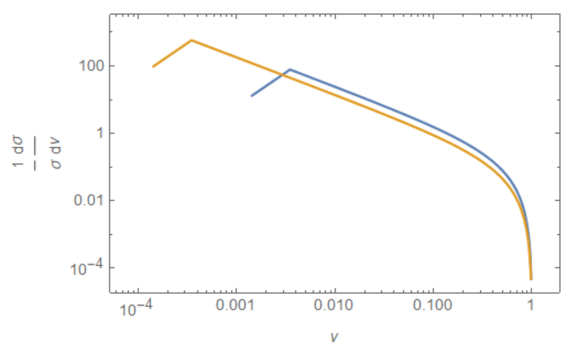
here E_0 is the initial energy of the hadronic and x is the distance from the point of production to the detector cavity.

Photo-nuclear cross section, energy loss distribution and mean free path

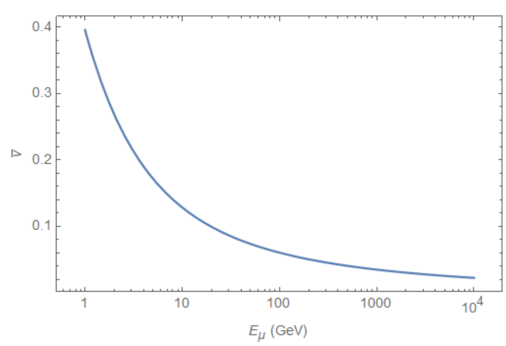
The calculation for photo-nuclear interactions proceeds as above, substituting using the interaction length in place of the radiation length to describe energy loss. The tabulated radiative energy loss function $b(E)$ has some structure, but only very mild energy dependence:



The fractional energy loss distribution parameterized by Van Ginneken (plotted for $100\ \text{GeV}$ and $1\ \text{TeV}$ in blue and gold, respectively) is fairly soft:



The average fractional energy loss, $\bar{v}$, is a few percent at FASER energies:



And the mean free path is several hundred meters:

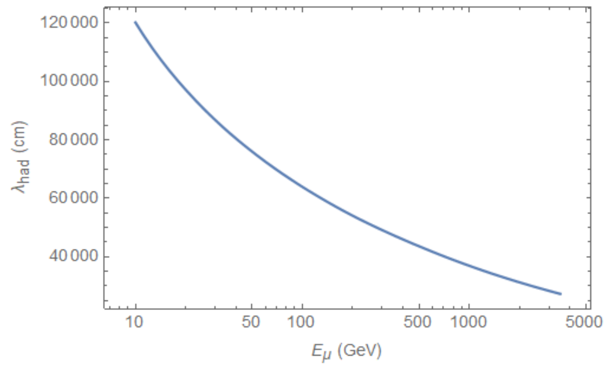
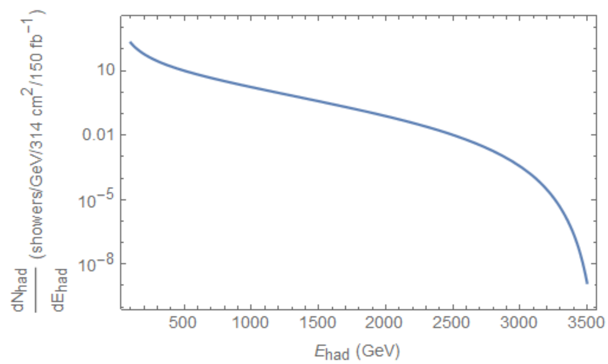
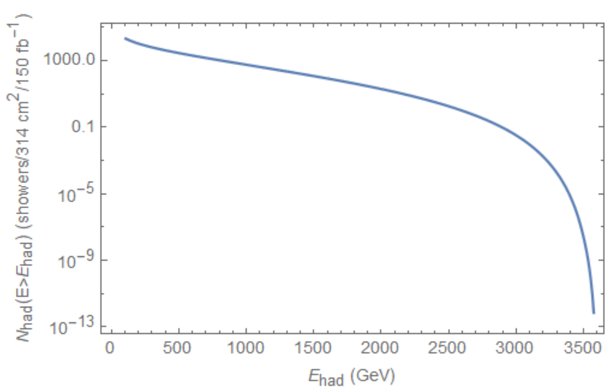


Photo-nuclear flux and spectrum

The differential hadronic energy spectrum falls very rapidly (note that each tick on the vertical axis is a *decade* in rate):



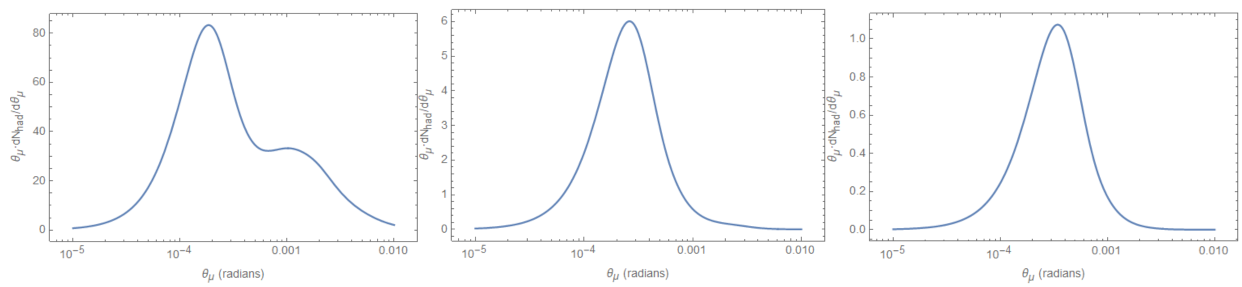
and the cumulative distribution (number of hadronic showers reaching FASER with $E > E_{\mathrm{had}}$) is:



The calculation estimates 21000 hadronic showers reaching FASER with a nominal energy of $100\ \mathrm{GeV}$ or more, but only 550 with nominal energy above $1\ \mathrm{TeV}$. It is important to remember, that the energy *per particle* will be even smaller.

Collimation

As for bremsstrahlung and pair production, muons producing hadronic showers have very small scattering angles. The curves for $E_{\mathrm{had}} = 100\ \mathrm{GeV}, 500\ \mathrm{GeV}$ and $1\ \mathrm{TeV}$ are plotted below.



Delta ray Production

Delta ray production is not a significant source of electrons with $E > 100$ GeV. In the formalism used for other radiative processes, $-\frac{1}{E}\frac{dE}{dx} \equiv b_\delta(E) \sim \frac{1}{E_\mu}$, and $\frac{1}{\sigma}\frac{d\sigma}{dv} \sim \frac{1-v}{v^2}$ strongly suppresses large fractional energy transfers. In addition, high-energy delta rays are essentially collinear with the muon that produces them.

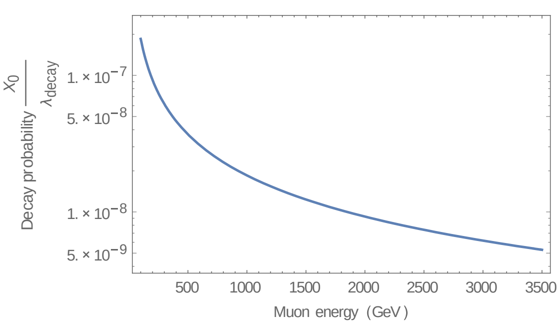
Muon Decay in Flight

All the radiative processes considered can be vetoed based on the collinear high-energy muon that accompanies them. One rare signature that might appear to evade this constraint would be a radiative process like bremsstrahlung, followed immediately by muon decay in flight (before the muon reaches the veto). The effective target volume in which bremsstrahlung followed by decay in flight could result in an unconverted and unvetoed (but see below) photon entering FASER is one radiation length thick.

The lab decay length for a muon of energy E is

$$\lambda_{\text{decay}} = \frac{c\tau_\mu E}{m_\mu} = \left(6.24 \times 10^5 \frac{\text{cm}}{\text{GeV}}\right) E$$

The decay probability per radiation length, $X_0/\lambda_{\text{decay}}$, plotted below, is extremely small:



Folding with the muon energy spectrum, only 30 muon decays in flight within one radiation length are expected in a $150\,\text{fb}^{-1}$ exposure; the expected number of decays in flight accompanied by an unconverted photon is 0.03.

Even these estimates are conservative, since the muon's boosted daughter electron will follow its direction, and produce a sufficiently energetic electromagnetic shower to "punch through" into the veto counter.

Summary

The table summarizes the estimated rates for a $314\ \mathrm{cm}^2$ detector, $150\ \mathrm{fb}^{-1}$ exposure and $100\ \mathrm{GeV}$ analysis threshold:

Process ($E > 100$ GeV in all cases)	Expected events ($150\ \mathrm{fb}^{-1}$)	Muons per event	Bunch crossings per muon
All muons	540M	1	170K
Unconverted entering photon	41K	13K	
…. subset of these converting in tracker	7.4K	73K	
Electromagnetic shower	22K	25K	
Hadronic shower	21K	26K	
Decay + bremsstrahlung	0.03	20B	

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