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Designing a Low Pass Filter

Introduction

In the realm of engineering, we often encounter many different types of filters, such as low pass, high pass, and band pass filters. These filters are used throughout people's everyday lives, whether it be radio communication or amplification of audio signals for humans to hear. However, how does someone go about actually designing and implementing one of these filters? By the end of this document, the answer to that question will be apparent.

The process that we will investigate is how to engineer different low-pass filters for a PWM (pulse width modulation) based DAC (digital-to-analog converter). The given parameters for the scenario are a PWM frequency of 10kHz, operation on a single-supply +5V source, at least 8-bit accuracy in the analog voltage output, a "K" value of 1 for each filter's transfer function, the absence of inductors, and a corner frequency of 2kHz. In addition, the 4 types of filters that can be used are the Bessel, Butterworth, 1 dB Chebyshev type I, and an RC-Cascade with identical sections.

Step 1: Obtaining the Transfer Function

The most fundamental building block upon which a filter is made is the transfer function, or TF. It's important to not worry about the specifications of the hardware needed (i.e. the capacitors, resistors, and Op-Amps) until the TF is obtained. Once we have the TF for each filter, it will tell us the specific details that we need in order to design each circuit.

There are two different ways to get the TF for each filter. The first deals only with the RC-Cascade, since it is composed of sections that are identical and first order. The TF of the RC-Cascade will have the form of:

$$\frac{K}{(1+\left(\frac{S}{|\mathrm{P}|_n}\right))^n}$$

Where K = is a constant, S = jω, n = the order, and $|\mathrm{P}|_n$= the Pole at the specified order. Starting at a first order and going up, the accuracy of the filter must be checked each time until an accuracy of 8 bits is achieved. To check if the ENOB (effective number of bits) is at least 8, a SPICE netlist representing the TF must be created to take the input signal and simulate how the output signal behaves based on the TF. After generating a ".raw" file, this file can be used by MATLAB/Octave to determine what ENOB value of the filter is. These two programs can be written manually but are fortunately provided for us in this project. Fortunately, as the order increases, the only thing that changes is the pole of the transfer function. To calculate the pole, the following formula can be utilized:

$$\frac{\omega_c}{\sqrt{2^{(1/n)}-1}}$$

Where c = corner frequency (in radians/sec) and n = the order. It is important to note that these two formulas can be used for any corner frequency, K value, and order; it is not only restricted to this scenario. It turns out that to meet the specifications of our project, a 14^{th} order TF is required to achieve an ENOB accuracy of at least 8. By substituting n=14, K=1, c=2π*2000, we obtain our TF of:

$$H(S) = \frac{1}{(1 + (\frac{S}{55778.06}))^{14}}$$

To verify that a minimum of 8 bits was achieved, the TF was simulated in SPICE and the "enob_DAC_OUT" command in Octave was used to check the accuracy, as seen in **Appendix 1c-1g**.

Now that we know how to find the TF for the RC-Cascade, it's time to discover how to obtain the TF for the other 3 filters. The first thing to consider when choosing a circuit design is which topology to use. There are many topologies out there such as Sallen-Key, Multiple Feedback, Biquad, State Variable Biquad, etcetera. The most commonly used of these are the Multiple Feedback and the Sallen-Key, since they only require 1 operational amplifier and are relatively simple. One advantage that the Multiple Feedback has on Sallen-Key is that it has higher quality factor, meaning that it attenuates signals at a much steeper rate than the Sallen-Key does. The reason why Sallen-Key topologies have higher noise gain toward higher frequencies is because the inverting input loops back to the output of the Op-Amp. In multiple feedback, this loopback does not occur, which results in a decrease in noise gain for very high frequencies. However, for our purposes, the Sallen-Key topology will be utilized due to its increased simplicity and its use of slightly less components.

On the bright side, the process for finding the TF for the Chebyshev, Butterworth, and Bessel filters are all the same. All these filters are composed of (S+1) or (S^2+S+1) denominator terms, which are represented by either an RC section or a Sallen-Key section, respectively. The terms for each order can be found by consulting a pole locations sheet, which can be easily found online through a search engine. The most straightforward strategy is to begin by using the lowest order pole locations and checking the ENOB. If the ENOB value is lower than what is desired, then the order must be increased, and the process must be repeated. Once the ENOB value is above what is required, then the TF that was used to obtain that ENOB value can be utilized to design the corresponding circuit. To demonstrate how to find the TF of these kinds of filters, the Butterworth design will be used (see **Appendix 11a** for reference). Starting at 2^{nd} order, the pole locations were used to find the 2^{nd} order denominator term, and the corresponding TF was entered in SPICE and Octave to produce an ENOB value of 3.705. An important side note is that every instance of S <u>must be replaced</u> with ($S/(2\pi*\omega_c)$) when entering the TF in SPICE to ensure that the specified corner frequency is realized and to convert from radians/sec to Hz. If S is not properly entered, then the ENOB from Octave/MATLAB will not be accurate, thus producing a circuit that will not meet the required specifications. The order was increased until an ENOB value of at least 8 was produced, which occurred at a 4^{th} order for the Butterworth design. Utilizing an online tool (such as [Wolfram Alpha]) can help expedite this process since multiplying out 2nd order polynomial terms can be somewhat tedious. By following this process for the Butterworth, Bessel, and Chebyshev designs, the following TFs in standard form were obtained:

$$Butterworth: H(S) = \frac{1}{(S^2 + 0.7654S + 1)(S^2 + 1.8478S + 1)}$$

$$Chebyshev: H(S) = \frac{1}{(1.20596S^2 + 0.54437S + 1)(2.2158S + 1)}$$

$$Bessel: H(S) = \frac{1}{(S^2 + 0.7654S + 1)(S^2 + 1.8478S + 1)}$$

Before proceeding, it is always a good idea to have a means of checking to verify that our results are valid. We can do so by using the "tf" and "bodemag" functions in MATLAB/Octave to graph the Bode Magnitude plots to check that each filter starts at 0dB, has a corner frequency of 2kHz, and attenuates as for high frequencies. Using the TFs shown above, Octave was used to validate the results as shown in **Appendix 1a and 1b.**

An alternative approach that does not require the use of a computer is to sketch the straight-line approximated Bode Magnitude plot for each transfer function. By using the transfer functions listed above and replacing each S with ($S/(2\pi*2000)$), we can use superposition to graph each denominator term individually, and then add them up to produce one straight-line plot. Each term will begin to affect the plot at each pole: for 2nd order denominator terms, the magnitude will decrease at a rate of -40dB/decade, and for 1st order terms it will drop off at a rate of -20dB/decade. The method shown in **Appendix 11d** can be used to obtain the straight-line approximated Bode plots, the values of each pole in Hz, and the approximated corner frequency in Hz, for any low pass filter. Utilizing this method produces the approximated Bode Magnitude plots for all four filters as presented in **10a-10d of the Appendix**.

When looking at these plots, there are noticeable differences from the actual plots made using Octave, especially regarding the corner frequency. Although both plots look very similar for very small ω values and very large ω values, the corner frequencies of the approximations are a significant amount different. For each approximation, the corner frequency was always larger than the real and ideal 2kHz corner frequencies. The filter that deviated the largest was the RC-Cascade, whose corner frequency was 9099.08 Hz. Nonetheless, it is expected that for LPF straight-line approximations that the corner frequency will be larger than ideal since hand drawn Bode Plots always drop off a bit slower than a real Bode plot at first. This happens for two main reasons. The first is because the poles of second order terms only consider the "a" coefficient, whereas a real Bode Plot considers both the "a" and the "b" coefficients of each term. The second reason is because approximated Bode Plots are linear, and don't consider the curvature that a true plot has. The more important takeaway from these approximations is that they indicate how the circuit behaves as whole and gives us an idea if our TF is on the right track. If we were to hand draw the plots and the corner frequency were to be less than ideal (i.e. lower than 2kHz), or outrageously large (such as 15GHz), it would be a red flag that our transfer function was incorrect.

Step 2: Determining Component Values

Now that we have all 4 transfer functions, we can use the coefficients of each S term to determine the appropriate resistor and capacitor values for each section. As mentioned previously, each instance of S needs to be replaced with (S/(2π*2000)) before proceeding. To use the Chebyshev filter as an example, the true TF becomes:

$$H(S) = \frac{1}{((7.6368x10^{-9})S^2 + (4.33195x10^{-5})S + 1)((1.7633x10^{-4})S + 1)}$$

For each second order denominator term, we can use the following design equations (*for Sallen-Key architecture **only***):

- $R_1R_2C_1C_2 = a$
- $(R_1 + R_2)C_2 = b$

Where "a" is the coefficient of the S^2 term and "b" is the coefficient of the S term. For first order RC sections, the design equation is simply:

- $R_1C_1 = a$

Where "a" is the coefficient of the S term. To solve for these equations and ultimately determine the component values we need, one approach (for second order sections) is to start by choosing the value of C_2. Using C_2, treat R_1 and R_2 as a single resistance and solve for it. Once each resistance value is chosen for both resistors, plug all 3 values into the first equation and solve for C_1. The approach for first order sections is very straightforward. Simply choose either R_1 or C_1 and solve for the unknown. It is recommended to choose impedances that can be purchased in real life through a vendor, because the overall goal is to design a circuit that can be built, not just one that works in theory. The values for each resistor and capacitor can be obtained for all 4 filter types by following this pathway (as shown in **Appendix 11b**).

Step 3: Using Ideal Operational Amplifiers to Simulate Design

Since we have our impedance values, the next phase of the process is to simulate each circuit design in SPICE by creating netlists that use ideal Op-Amps. Simulating each filter is a good way to verify that the overall design of each section is performing as expected and that the circuit meets the requirements that are expected of it. Furthermore, SPICE can allow us to troubleshoot any issues and will indicate if any of our values are incorrect before proceeding. Before writing the netlist, it is always good practice to draw out the whole circuit for each filter beforehand and label each node, as shown in **Appendix 7a-7d**.

Upon creating the netlist for each filter, we can graph the Bode Magnitude plot by right clicking the plot pane and adding a trace for "Vout". Automatically, SPICE will plot both the magnitude and the phase. To observe the magnitude only, right click the y-axis on the right side of the plot and click the button that says, "Don't plot phase" when the window appears. Now the Bode Magnitude plots can be used to check that each filter is performing correctly, and that the Magnitude of each design is very close to -3dB at the specified corner frequency. For our design, each Bode Magnitude plot is shown in **Appendix 2a-2d** and the corner frequencies are verified by using SPICE's zoom feature as shown in figures **3a-3d** of the **Appendix**. An important detail to notice is that not all the Ideal Op-Amp simulations achieve exactly -3dB at a 2kHz frequency. However, this is a negligible amount because real world results will vary due to noise and the tolerances of resistors and capacitors. Since we validated that each filter has the required ENOB value early on, we can assume that they will produce acceptable results.

Step 4: Choosing a Real Op-Amp

Before testing every filter with true Op-Amps, it must first be guaranteed that the Op-Amps that are selected will produce the desired outcome and will meet the requirements of the situation. In our case, each Op-Amp must operate on a single-supply voltage of 5V. One such device that fits the bill is the Single-Supply, Rail-to-Rail, Low Power AD820 Op-Amp.

Upon choosing an Op-Amp, we must test each section individually that uses an Op-Amp to verify that the gain bandwidth is high enough for that section. To do so, we can utilize the following relationship:

$$GBW \geq 100 * G * Q * f_c$$

Where GBW is the gain bandwidth out the Op-Amp in question, G is the gain (in our case we will always assume G=1), Q = ($\sqrt{a}$)/b, and f_c = corner frequency, in Hz. The parameters a and b are the same coefficients of a second order term that were mentioned previously. The Chebyshev filter will be used as an example to reveal how this relationship is used to verify the suitability of an Op-Amp (see **Appendix 11c**). After going through the motions, we know that the AD820 is a good choice since $100 * G * Q * f_c$ = 4.346x10^5, which is less than the 1.8MHz GBW of the device.

An additional thing to look out for when checking the GBW of a section is the Q value. If the Q value happens to be greater than 2, we begin to observe a "peaking" phenomenon that takes place with our filters. This "peaking" effect can be observed on the Ideal and Real SPICE simulations for the Chebyshev filter, since its Q value turned out to be 2.0173. For this project, whether peaking occurs or not is irrelevant since it was not mentioned in the requirements and the circuit still meets all mandatory fields. However, these anomalies should not be overlooked, especially when designing a real world circuit. For instance, if a requirement was set in place that the output signal must not be amplified whatsoever and a section of one of the filters had a Q value of 8, this circuit would not be suitable for use and a different design would need to be utilized. Fortunately, after verifying the GBW of every 2nd order section of each filter, the AD820 turned out to be a suitable candidate for the entire project.

Step 5: Using Real Op-Amps to Finalize the Design

Upon determining which Op-Amp is suitable for each section, we can now simulate each filter using all physical circuit components. The designs utilized for our scenario are very similar to the initial design, except that each Ideal Op-Amp will be changed out for an AD820, and each impedance value will be composed of resistors and capacitors that can be purchased in the physical world.

SPICE was used once again to check the functionality of each circuit by changing out each "E" source in the netlist with a real Op-Amp, and by changing each resistor and capacitor to the values of real components. We can connect resistors in series to achieve resistance values that are close to what was chosen for the ideal resistors. Similarly, we can connect capacitors in parallel to obtain capacitance values that are near the ideal capacitors. For instance, the ideal value of R_3 of our Chebyshev filter turned out to be 1.7633 kΩ, so we can use 3 real resistors (1.5kΩ, 220Ω, and 39Ω) in series to get a real R_3 value of 1.759 kΩ. Although this resistance is not exact, it is close enough to where we can still deliver acceptable results from the real Chebyshev filter. Practically speaking, an employer would specify a specific tolerance for which a filter must stay within (such as a +/- 5% corner frequency or a +/1dB ripple). Fortunately for our purposes no such tolerances were given, so we are just seeking a filter that gets reasonably close. For instance, one of the filters that was slightly off when using real Op-Amps was the Bessel filter. At a 2kHz frequency, the magnitude wasn't quite as close to -3dB as the other filters, so the value of C_5 was changed from about 115 nF to 105 nF to correct the issue. In the real world, small adjustments like these should be made to the filter design if it is not fulfilling the standards required for it to function properly.

After replacing all the ideal values with our real values and Op-Amps in SPICE, each filter ended up producing results that were satisfactory. Like the case for our ideal design, SPICE was used to plot the Bode Magnitude and check the corner frequency for each filter to make sure they met each specification, as can be seen in **Appendix 4a-4d** and **5a-5d**. Now that each filter has been verified, we can finalize our design as shown in the schematic diagrams of **Appendix 8a-8d**. At this point, we have four filter designs that could be built in the physical world and used for practical applications.

Analysis

It's always a good idea to take a second look at our work and evaluate the pros and cons of each filter that we have created. One such way to compare each of our designs is by looking at the corner frequency of each filter to see which one got the closest. As previously stated, we want our circuits to be generating a corner frequency that is as close to 2kHz as possible. Using the traced cursor feature in SPICE, we can determine what the exact corner frequency is of each design is, as shown in **Appendix 6a-6d**. As expected, the RC-Cascade achieved a corner frequency that was closest to 2kHz, and the Chebyshev filter was the furthest away from 2kHz. As a general trend, as the order of a circuit increases, the accuracy of that circuit will also increase, if the math behind obtaining the TF was all done correctly. From a practical perspective, there are two main things to consider regarding the accuracy of the corner frequency. If we were to be working a job where accuracy was of the utmost importance, the RC-Cascade would be the optimal design since it was the most accurate filter. However, if overall size and hardware simplicity were a higher priority, the Chebyshev filter would be the superior design.

An additional aspect to consider for each design is the cost of each circuit. The most common way to track the price of a project is by creating a Bill of Materials. The BOM for our four filters can be found in sections **9a-9d of the Appendix**. It comes as no surprise that the 3rd order Chebyshev filter costs only \$7.34 in stark contrast to the \$79.24 price tag of the 14th order

RC-Cascade filter. The cost of a design should always be taken into account, especially in the workforce. There is almost always a budget set for any project, and the smaller the company is the smaller the budget it likely going to be. As engineers, it's our job to determine which design will best suit the situation. For example, if we had a budget of over $80 for our project, the RC-Cascade would be the best choice, whereas the Chebyshev would be the best option if the cost was more important than the filter's precision. We could even use the Bessel or Butterworth filters to find a good in between zone that more evenly balanced price of the project with the accuracy of our results. Overall, it is up to the engineer to possess the ability to make the best decision given the circumstances.

Another thing worth analyzing is the difference between the real and ideal Bode Magnitude plots. For some of the filters, the signal appeared to stop attenuating at a certain point and began to rise. Both the Bessel and Butterworth filters are somewhat unstable because their signals dropped off at first, but began to increase as ω approached infinity, The Chebyshev was unique since it attenuated to a certain point, and then flatlined for very large frequencies. The only filter that attenuated indefinitely was the RC-Cascade filter. These are critical things to make note of, especially in the real world. If we were required to design a LPF that eliminated every conceivable signal at frequencies greater than the specified corner frequency, we would need to watch out for Bode Plots like the Bessel and Butterworth (and perhaps choose a different circuit topology), as they would not be valid.

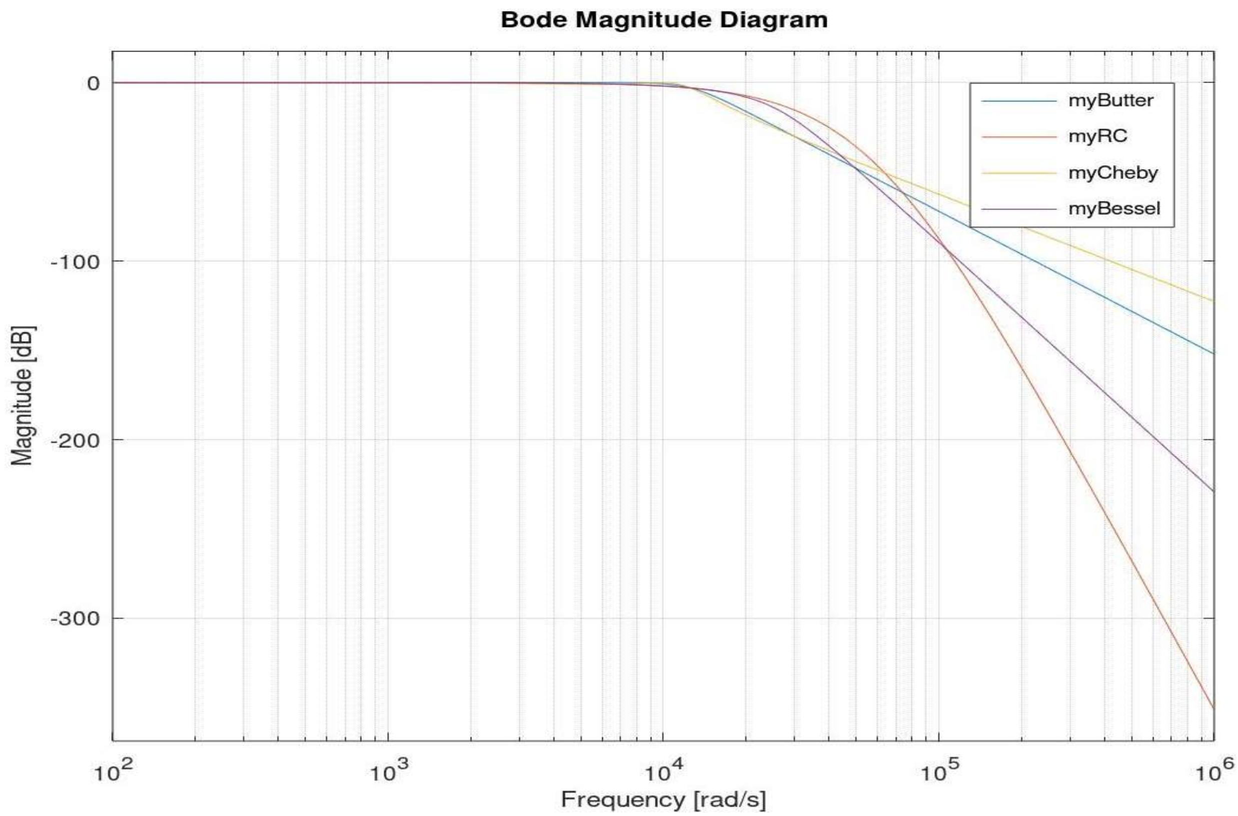
The last thing to consider when finalizing a design and choosing a filter is group delay. Technically speaking, group delay is the negative derivative of the phase with respect to the frequency. The significance of this delay is that it indicates how the signal will change as it passes through a filter. The more constant the group delay is, the less the input signal will be distorted by the filter. The highest group delay peak from about 0 Hz to 2kHz for our filters can be seen in **Appendix 12a-12d**. The Chebyshev filter turned out to have the most turbulent group delay with a peak of 406.2 μs in contrast to the Bessel filter that had the most stable group delay of 230.8 μs. Hence, the Bessel filter would be the superior choice if our main goal was to preserve the input signal as much as possible. If preservation were not as important, then the Chebyshev filter would be a better option since it uses less hardware.

Conclusion

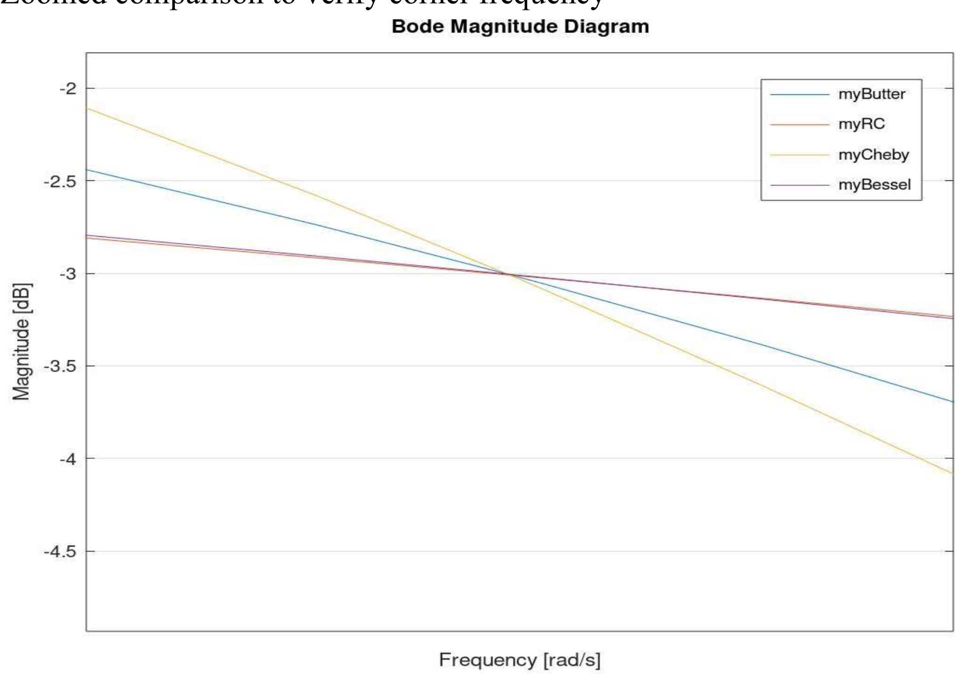
At this point, we have all the tools necessary to design a LPF given a set of requirements. By finding the TF of a filter, we can use MATLAB/Octave and SPICE to ensure that our filter will behave properly. We then know how to choose the correct Op-Amp based on the corner frequency given, and how to test the circuit a second time using real component values. After verifying a filter via simulations, we can then build the circuit in the real world to be used for its specified purpose. In addition, we know that the cost, complexity, size, and group delay of our circuits are all very important aspects to consider before finalizing our design. By following the process outlined in this report, we now have the confidence to design any Low Pass Filter that the world throws our way.

Appendix

1. **Octave References**
   a. Full Comparison of all 4 filters



   b. Zoomed comparison to verify corner frequency



c. LTSPICE example of Cascade TF

```
behavioral_PWM_gen.cir
Rdummy3 compOut 0 1k

*My LPF
*The code below is a 14th order RC LPF with corner frequency = 2kHz
Blpf lpfOut 0 V=V(compOut) Laplace=1/((1+(s/55778.06414))^14)

*The code below is a 4th order Butterworth LPF with corner frequency
*Blpf lpfOut 0 V=V(compOut) Laplace=1/(((s/(2*pi*2000))^2+(0.7654*(s

*The code below is a 7th order Bessel LPF with corner frequency of 2
*Blpf lpfOut 0 V=V(compOut) Laplace=69.5044/(((s/(2*pi*2000))^7+(9.4

*The code below is a 3rd order Chebyshev LPF with corner frequency c
*Blpf lpfOut 0 V=V(compOut) Laplace= 1/(((1.20596*(s/(2*pi*2000))^2)

Rdummy4 lpfOut 0 1k
```

d. Octave verification of 8-bit accuracy for RC-Cascade

```
Command Window
extracting LTspice results ...
finished extracting LTspice results after 7.44473 seconds.
>>

>> enob_DAC_OUT
enob_DAC_OUT =  8.1202
>>
```

e. Octave verification of 8-bit accuracy for Butterworth

```
Command Window
extracting LTspice results ...
finished extracting LTspice results after 7.3839 seconds.
>>

>> enob_DAC_OUT
enob_DAC_OUT =  8.7532
>>
```

f. Octave verification of 8-bit accuracy for Bessel

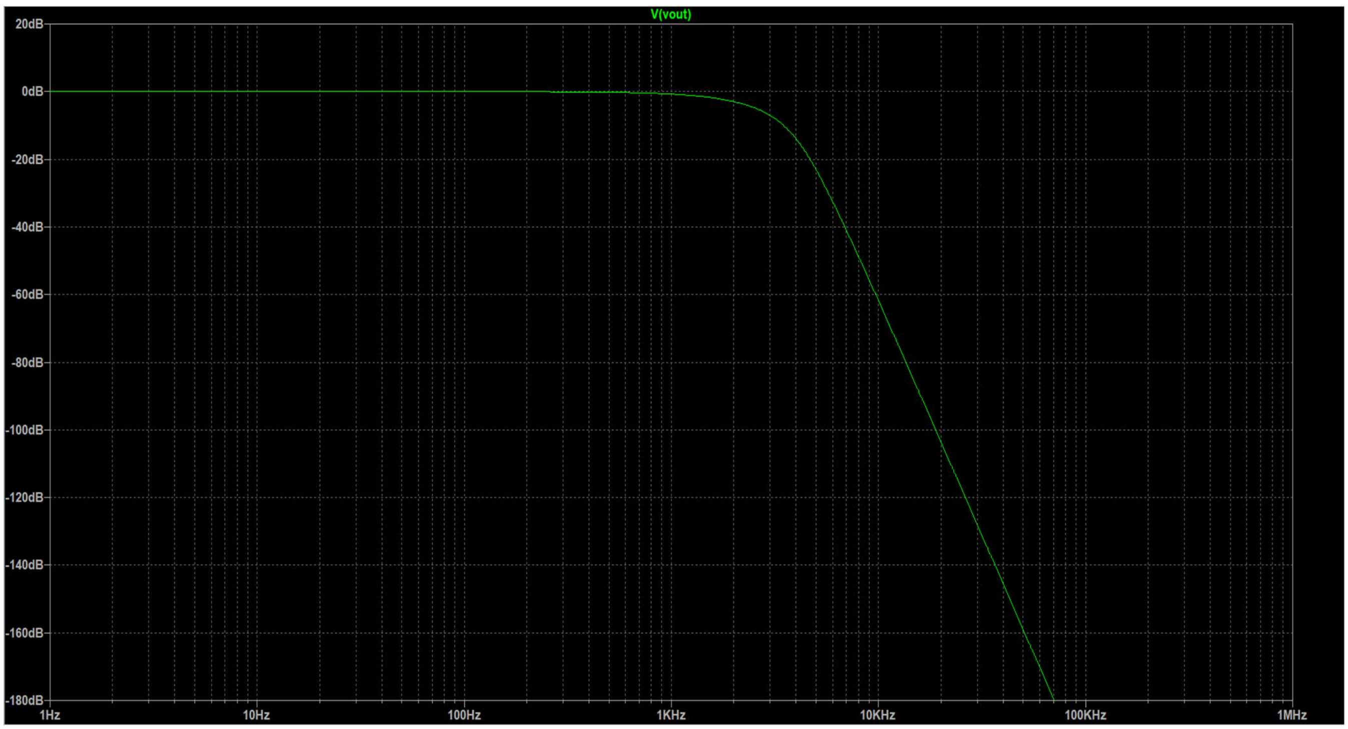
```
Command Window
extracting LTspice results ...
finished extracting LTspice results after 7.02205 seconds.
>>

>> enob_DAC_OUT
enob_DAC_OUT =  8.2141
>>
```

g. Octave verification of 8-bit accuracy for Chebyshev

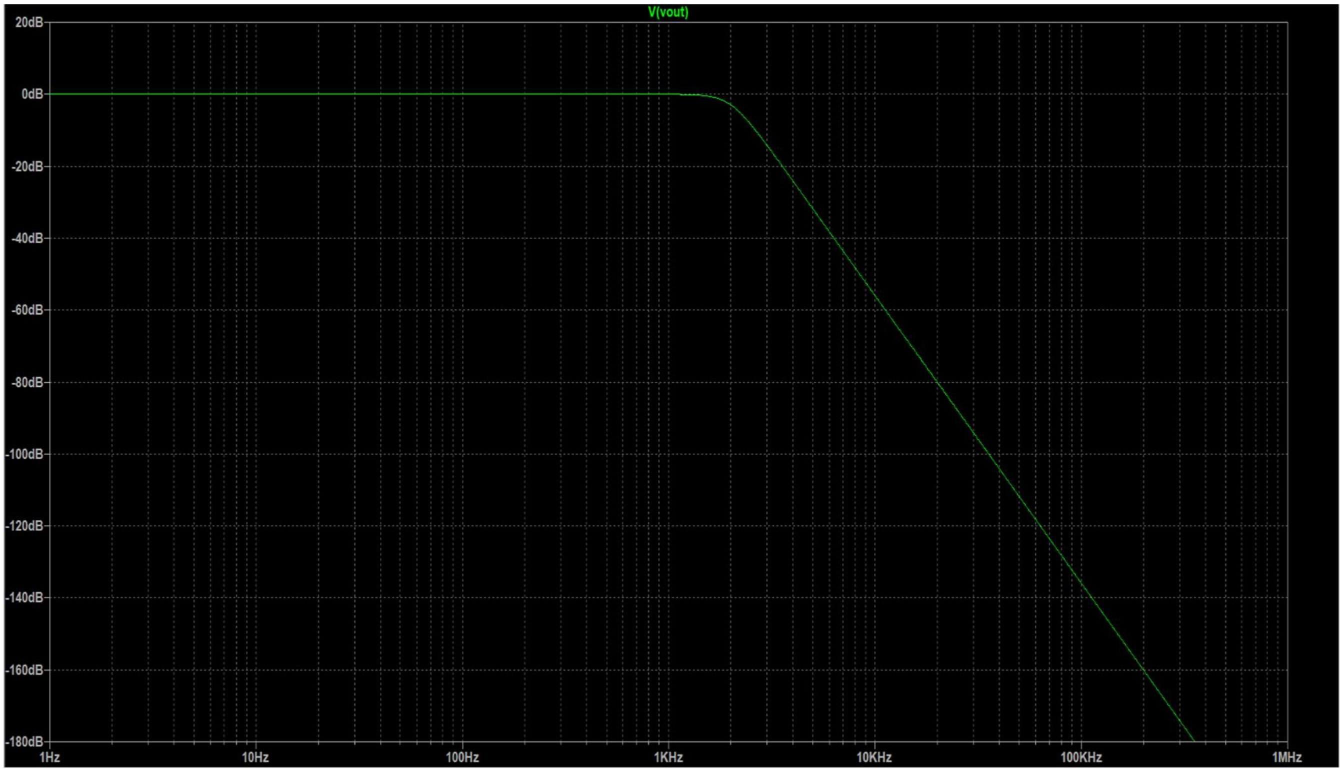
```
Command Window
extracting LTspice results ...
finished extracting LTspice results after 7.0386 seconds.
>>

>> enob_DAC_OUT
enob_DAC_OUT =  9.1384
>> |
```

2. Ideal Op-Amp Bode Plots in LTSPICE

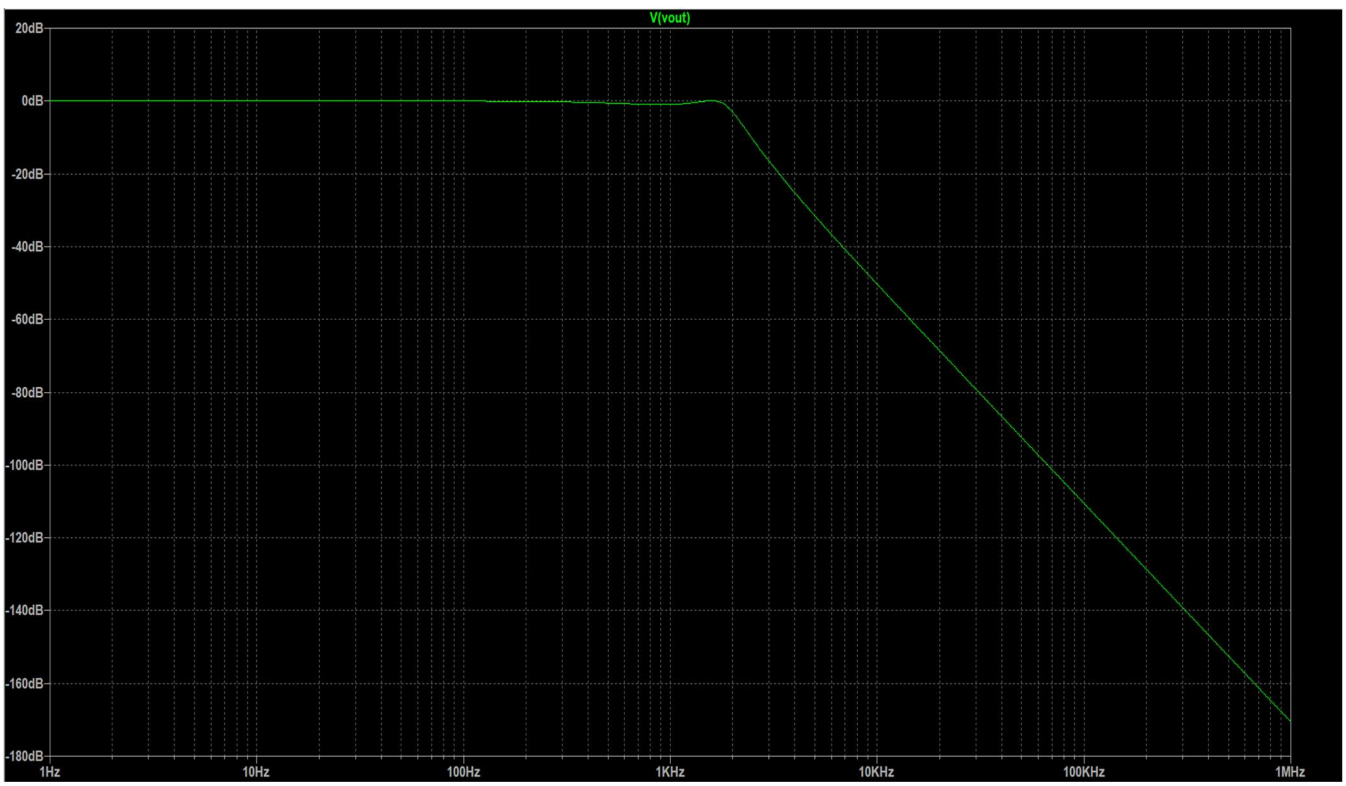
a. Bessel



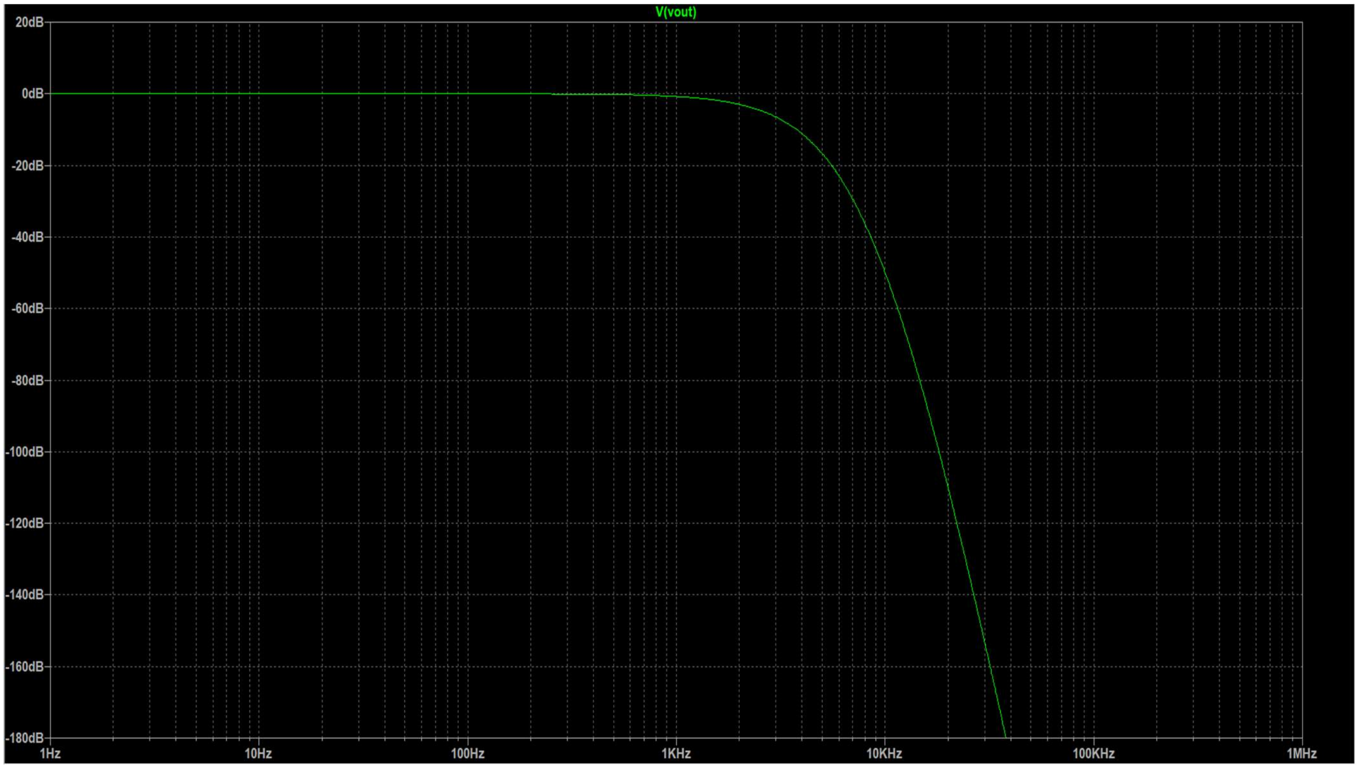
b. Butterworth



c. Chebyshev

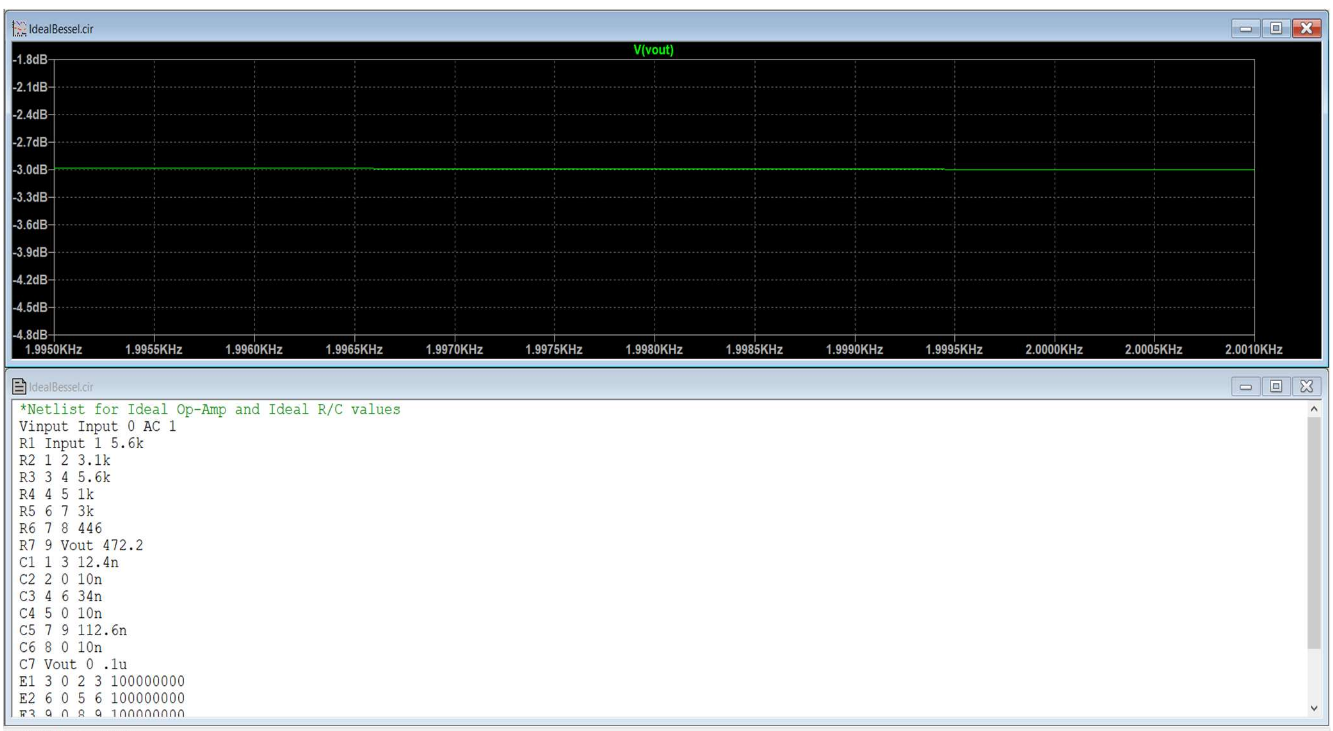


d. RC-Cascade

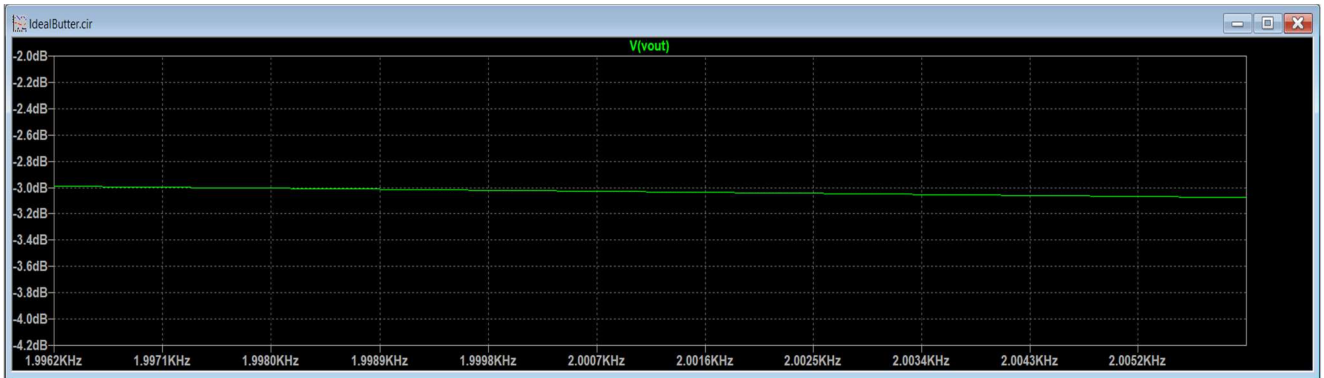


3. Ideal Op-Amp comparison with 2kHz frequency in LTSPICE

a. Bessel



b. Butterworth

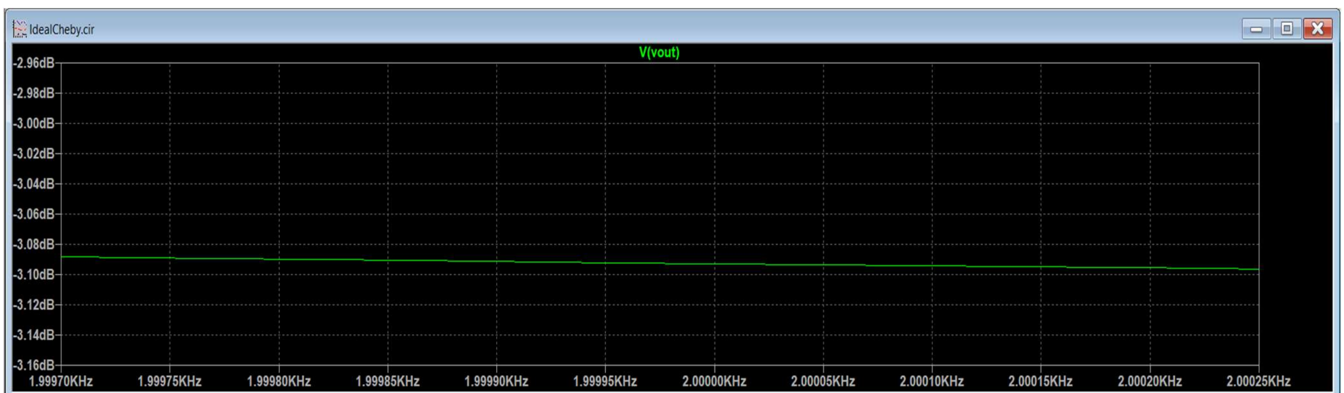


IdealButter.cir

```
*tgfor Ideal Op-Amp and Ideal R/C values
Vinput Input 0 AC 1
R1 Input 1 270
R2 1 2 339.9
R3 3 4 1k
R4 4 5 470.43
C1 1 3 689.2n
C2 2 0 .1u
C3 4 Vout 134.74n
C4 5 0 .1u
E1 3 0 2 3 100000000
E2 Vout 0 5 Vout 100000000
.AC DEC 1000 1 1Meg
.END
```

x = 2.000KHz y = -3.029dB

c. Chebyshev

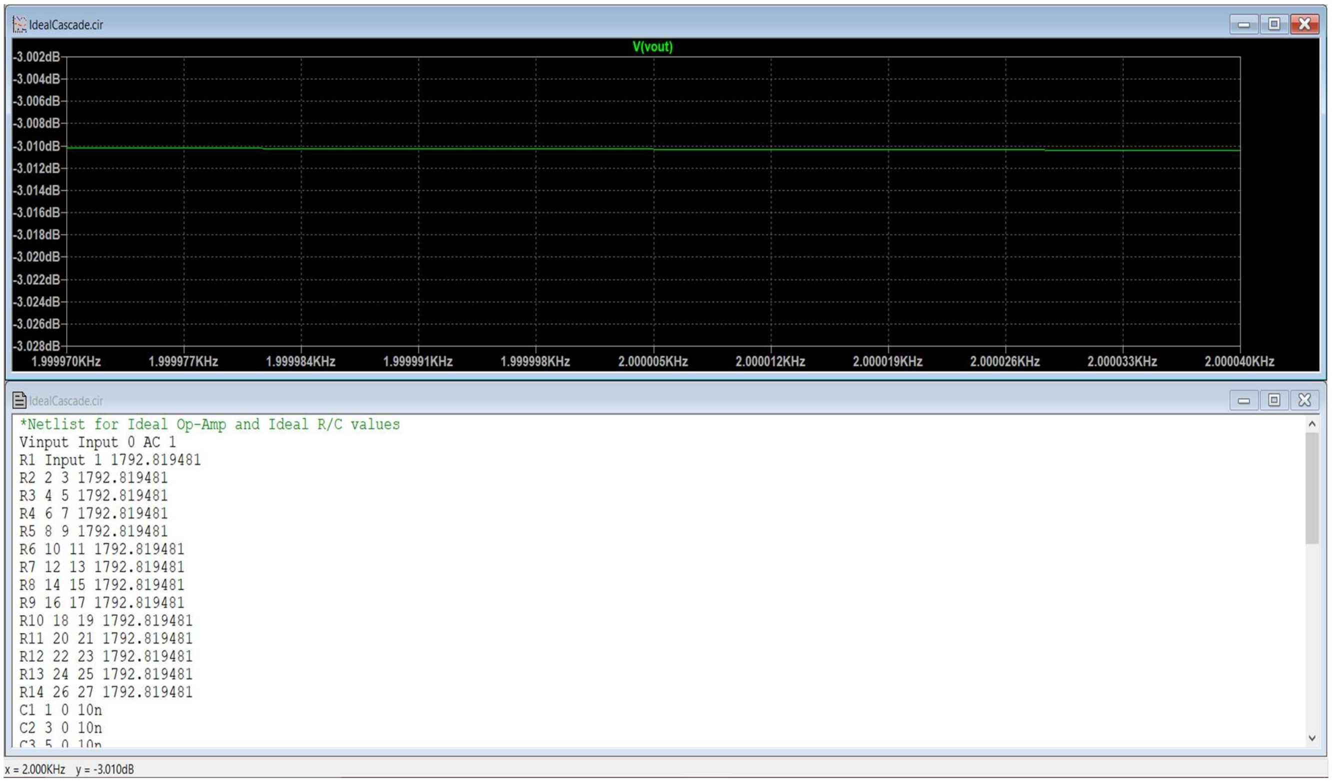


IdealCheby.cir

```
*Netlist for Ideal Op-Amp and Ideal R/C values
Vinput Input 0 AC 1
R1 Input 1 3.9k
R2 1 2 433.95
R3 3 Vout 1.7633k
C1 1 3 456.4n
C2 2 0 10n
C3 Vout 0 100n
E1 3 0 2 3 100000000
.AC DEC 1000 1 1Meg
.END
```

x = 2.000KHz y = -3.093dB

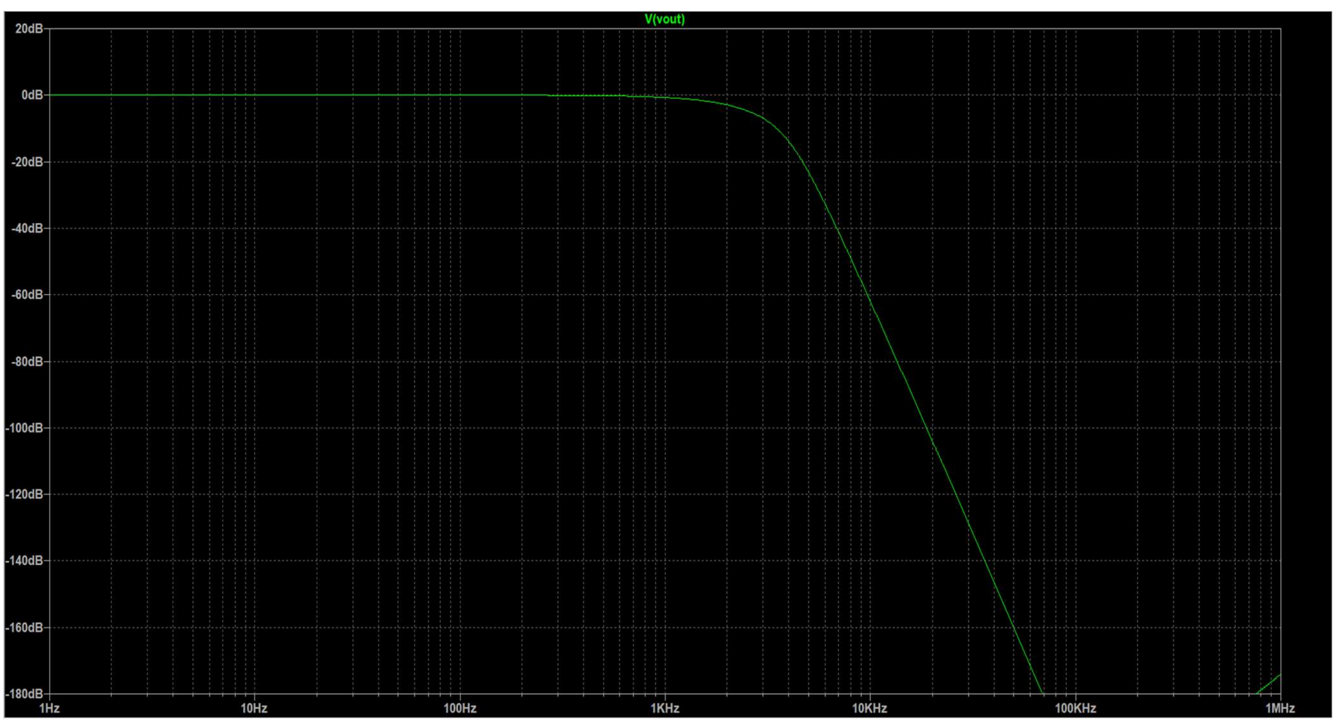
d. RC-Cascade



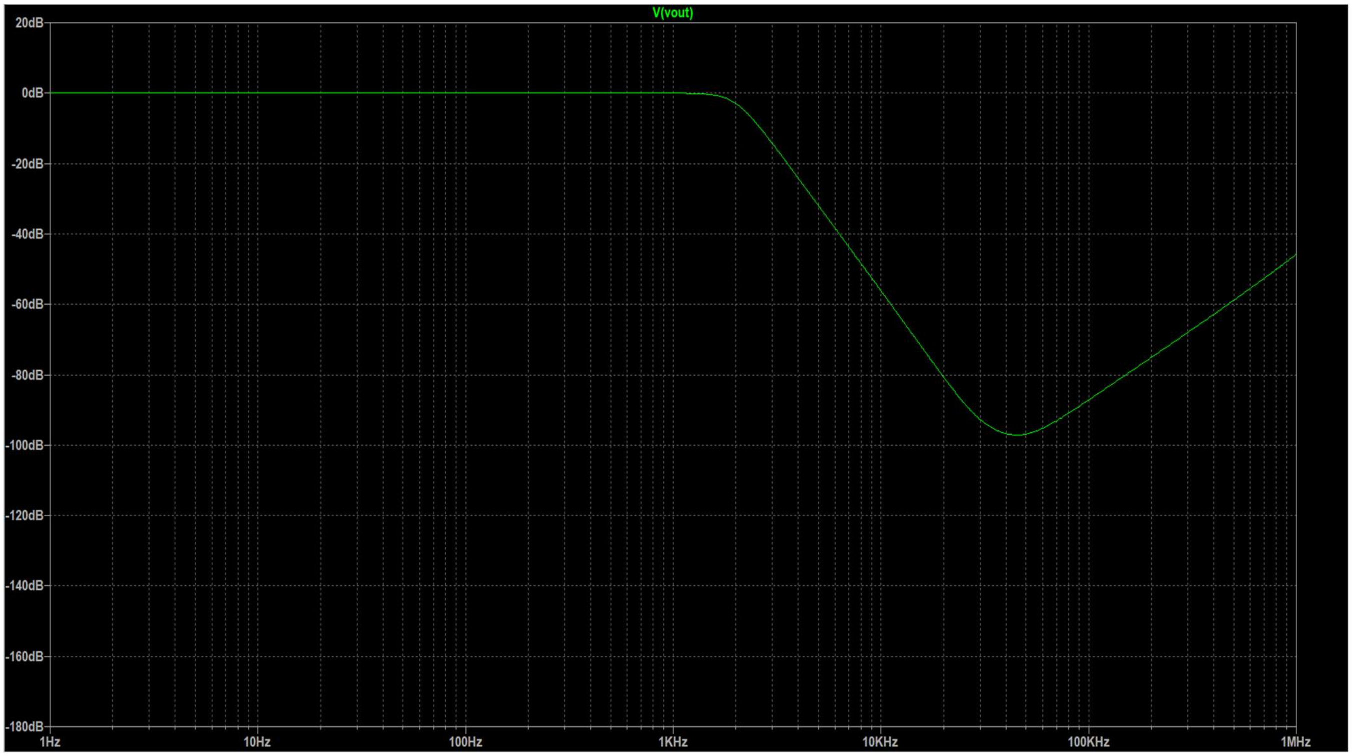

```
*Netlist for Ideal Op-Amp and Ideal R/C values
Vinput Input 0 AC 1
R1 Input 1 1792.819481
R2 2 3 1792.819481
R3 4 5 1792.819481
R4 6 7 1792.819481
R5 8 9 1792.819481
R6 10 11 1792.819481
R7 12 13 1792.819481
R8 14 15 1792.819481
R9 16 17 1792.819481
R10 18 19 1792.819481
R11 20 21 1792.819481
R12 22 23 1792.819481
R13 24 25 1792.819481
R14 26 27 1792.819481
C1 1 0 10n
C2 3 0 10n
```

4. Real Op-Amp Bode Plots in LTSPICE

a. Bessel

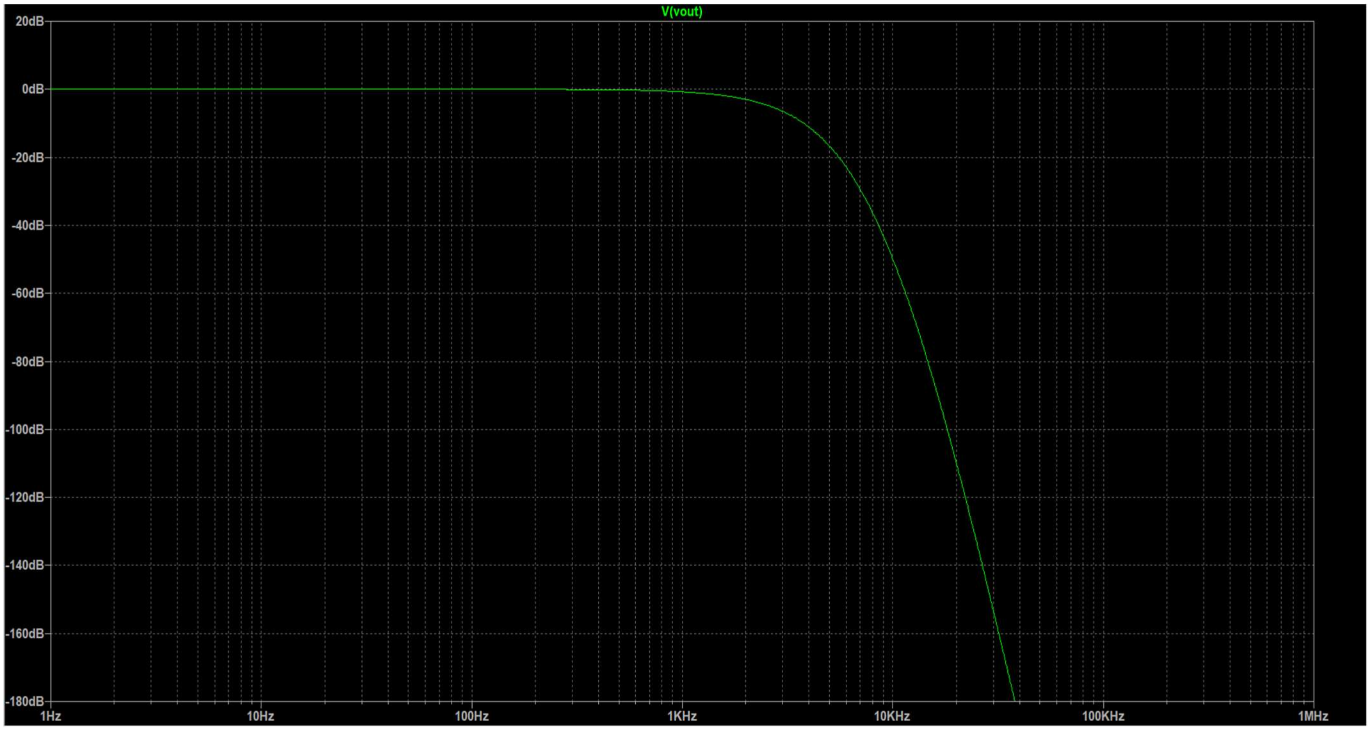


b. Butterworth



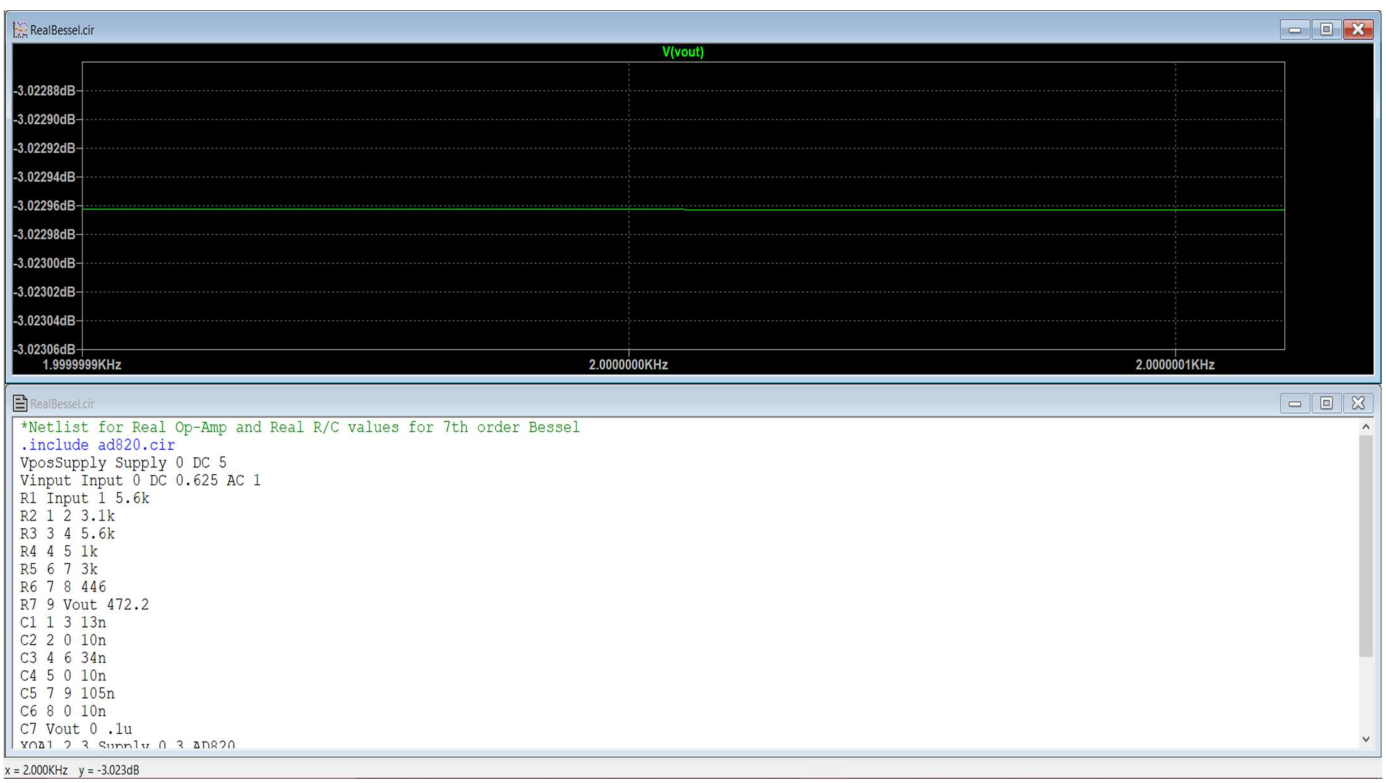
c. Chebyshev

d. RC-Cascade

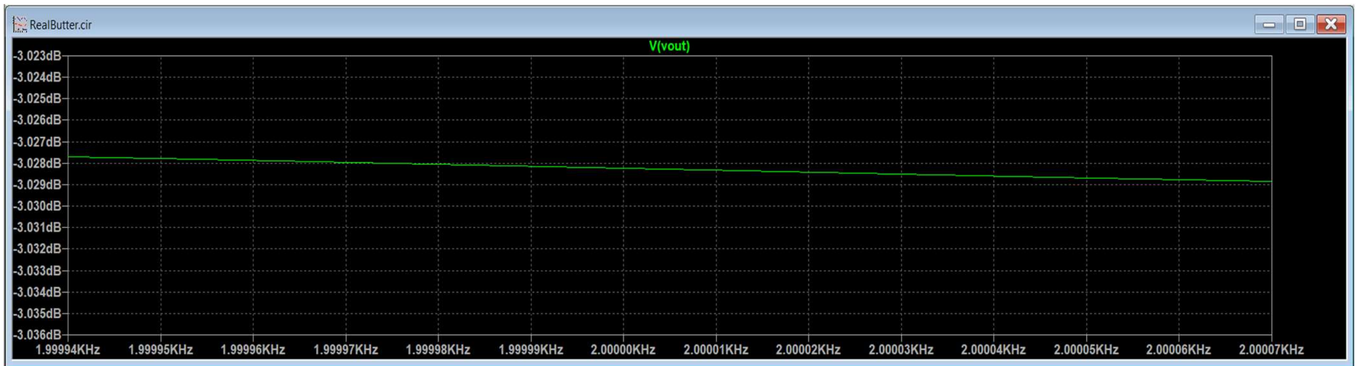


5. Real Op-Amp comparison with 2kHz frequency LTSPICE

a. Bessel



b. Butterworth

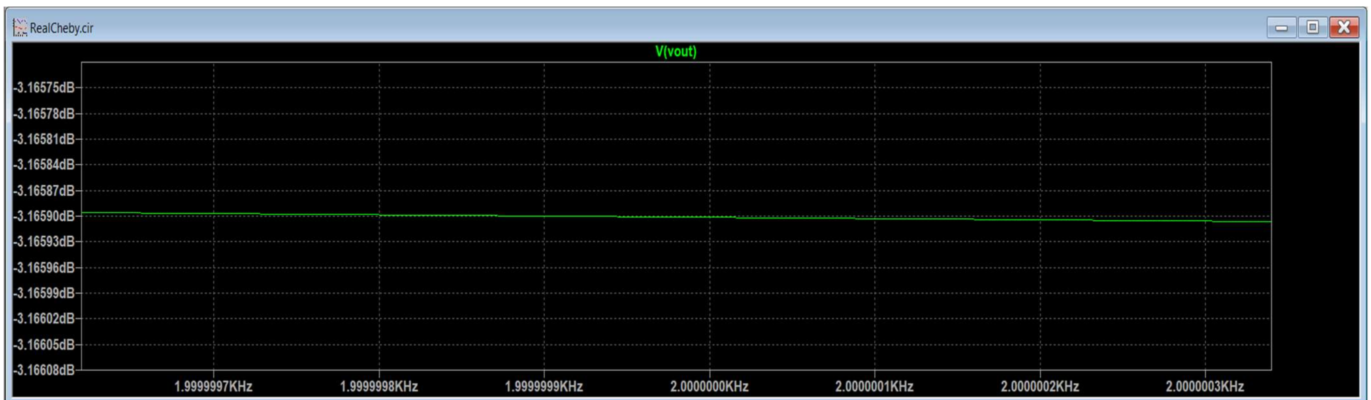


RealButter.cir

```
*Netlist for Real Op-Amp and Real R/C values for 4th Order Butterworth
.include ad820.cir
VposSupply Supply 0 DC 5
Vinput Input 0 DC 0.625 AC 1
R1 Input 1 270
R2 1 2 340
R3 3 4 1k
R4 4 5 470
C1 1 3 700n
C2 2 0 .1u
C3 4 Vout 135n
C4 5 0 .1u
XmyOpAmp1 2 3 Supply 0 3 AD820
XmyOpAmp 5 Vout Supply 0 Vout AD820
.AC DEC 1000 1 1Meg
.END
```

x = 2.000KHz y = -3.028dB

c. Chebyshev

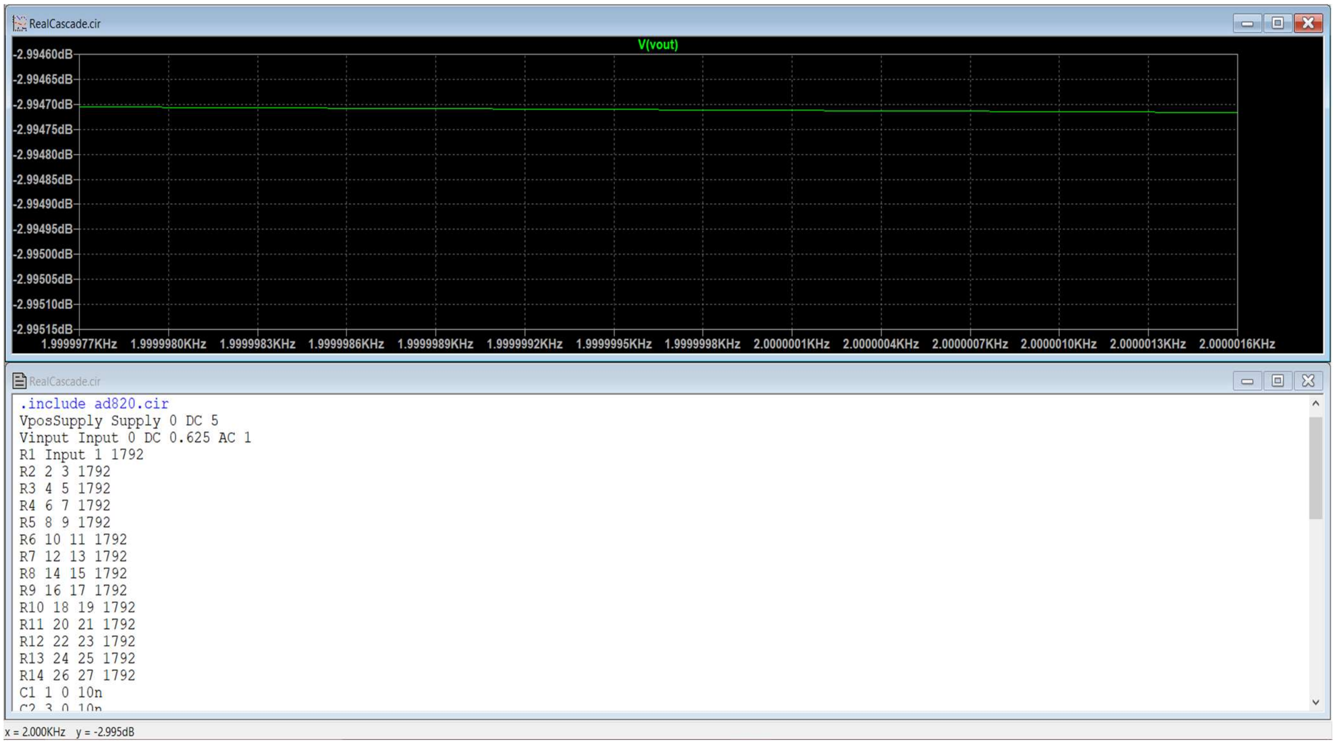


RealCheby.cir

```
*Netlist for Real Op-Amp and Real R/C values for Chebyshev
.include ad820.cir
VposSupply Supply 0 DC 5
Vinput Input 0 DC 0.625 AC 1
R1 Input 1 3.9k
R2 1 2 433.7
R3 3 Vout 1.759k
C1 1 3 456n
C2 2 0 10n
C3 Vout 0 100n
XmyOpAmp 2 3 Supply 0 3 AD820
.AC DEC 1000 1 1Meg
.END
```

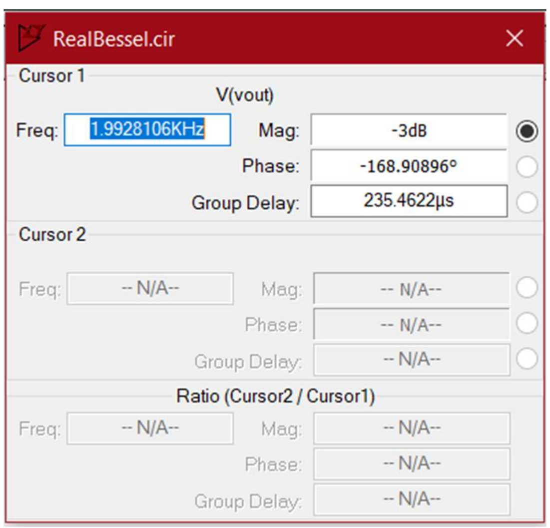
x = 2.000KHz y = -3.166dB

d. RC-Cascade

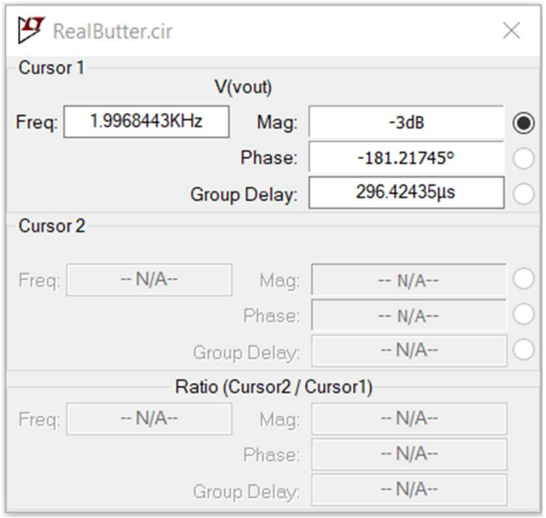


6. Real Op-Amp corner frequencies

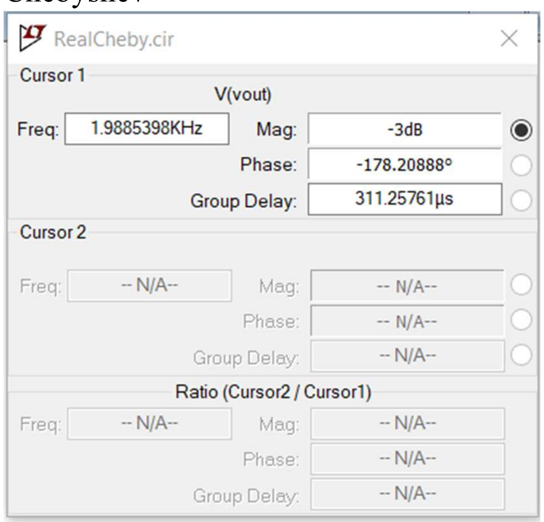
a. Bessel



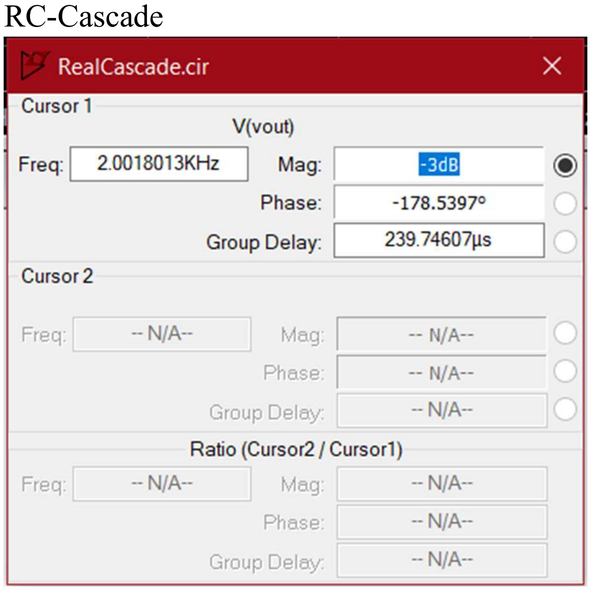
b. Butterworth



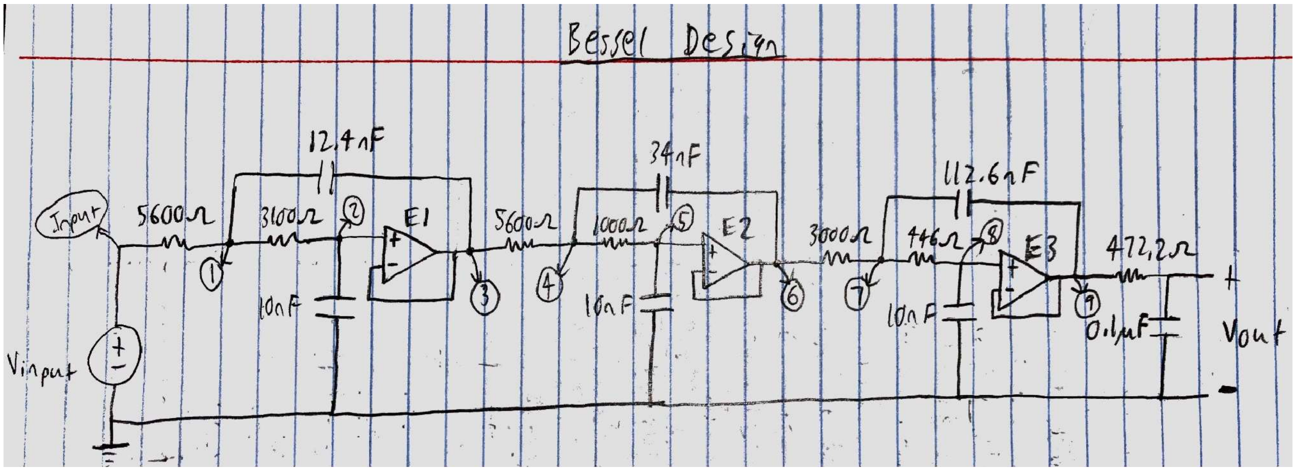
c. Chebyshev



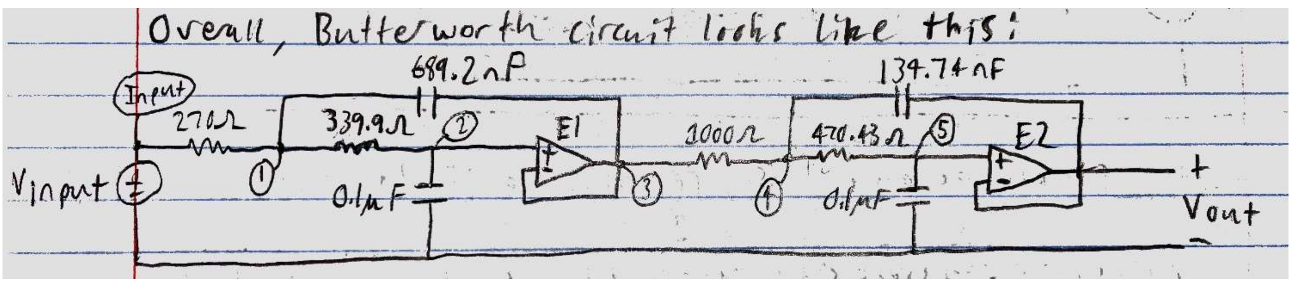
d. RC-Cascade



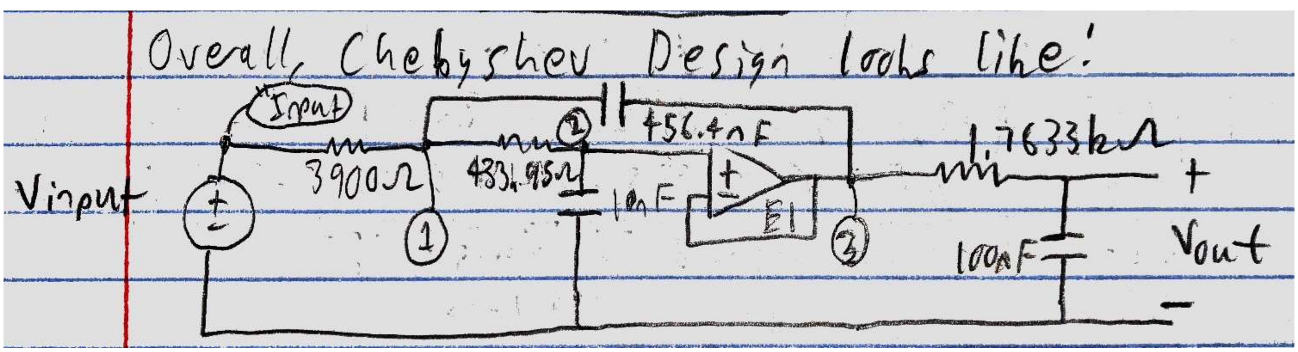
7. **Initial Circuit Design**
   a. Bessel



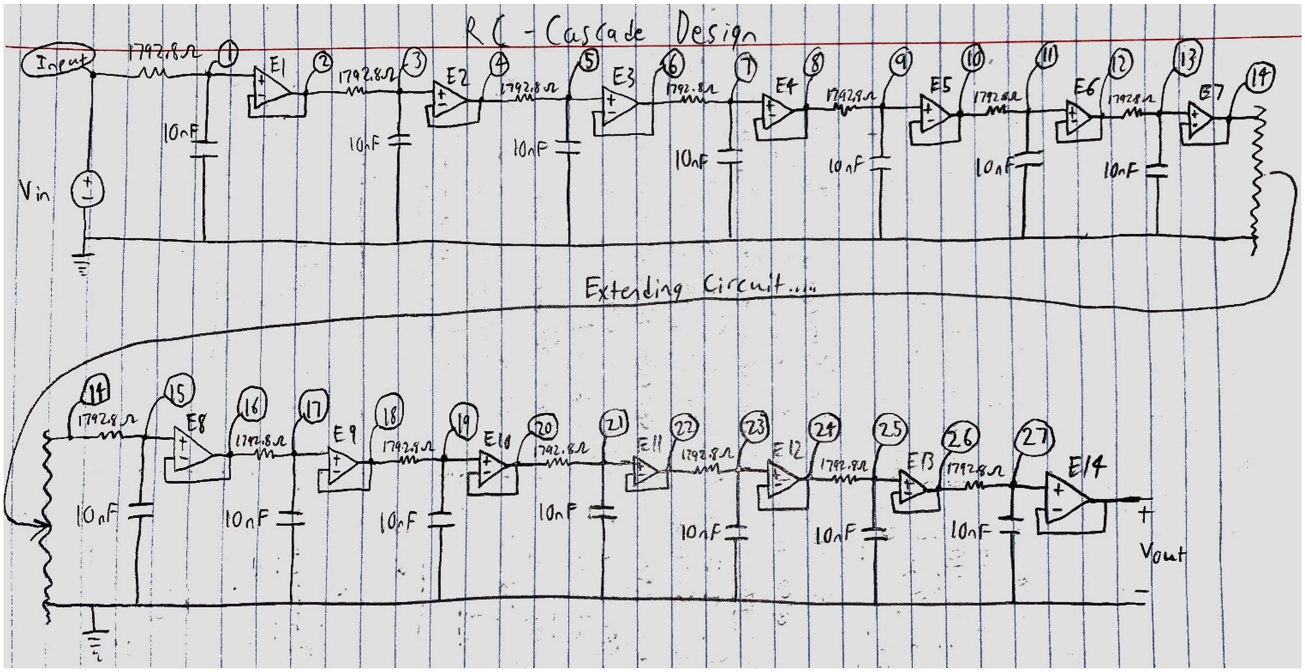
   b. Butterworth



   c. Chebyshev

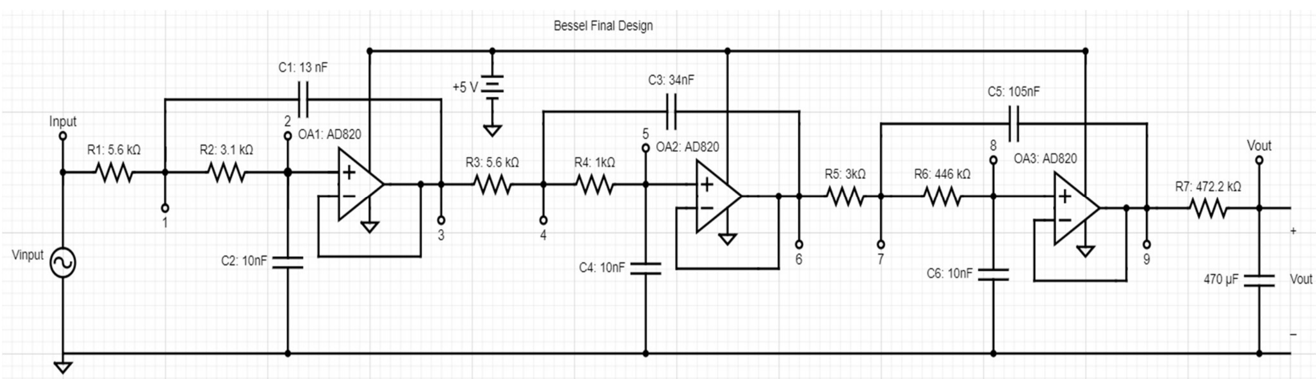


d. RC-Cascade

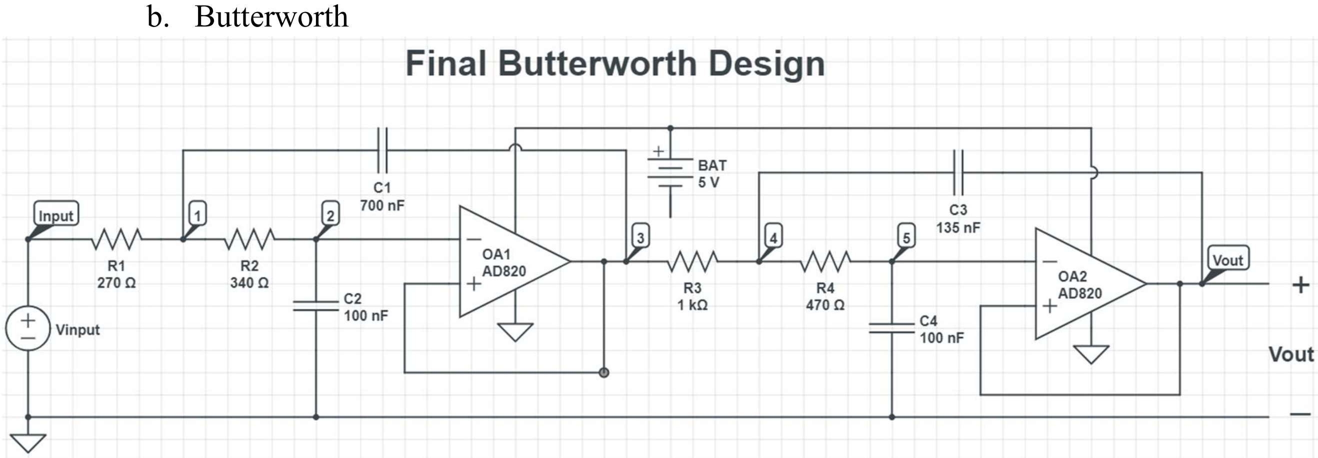


8. Final Circuit Design

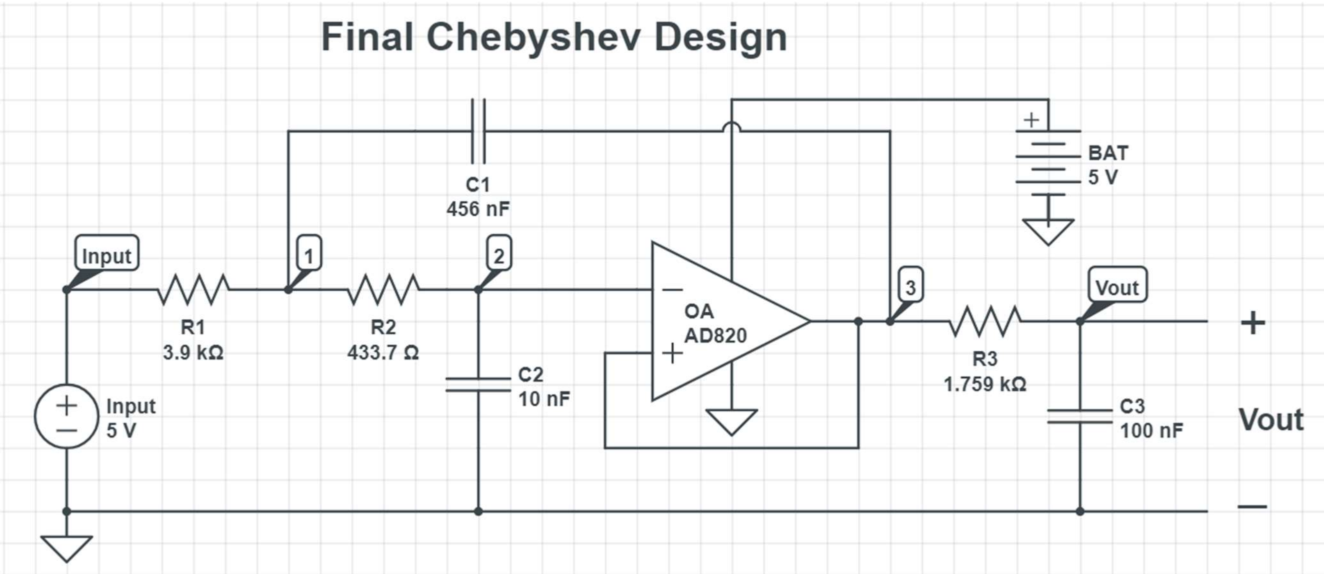
a. Bessel



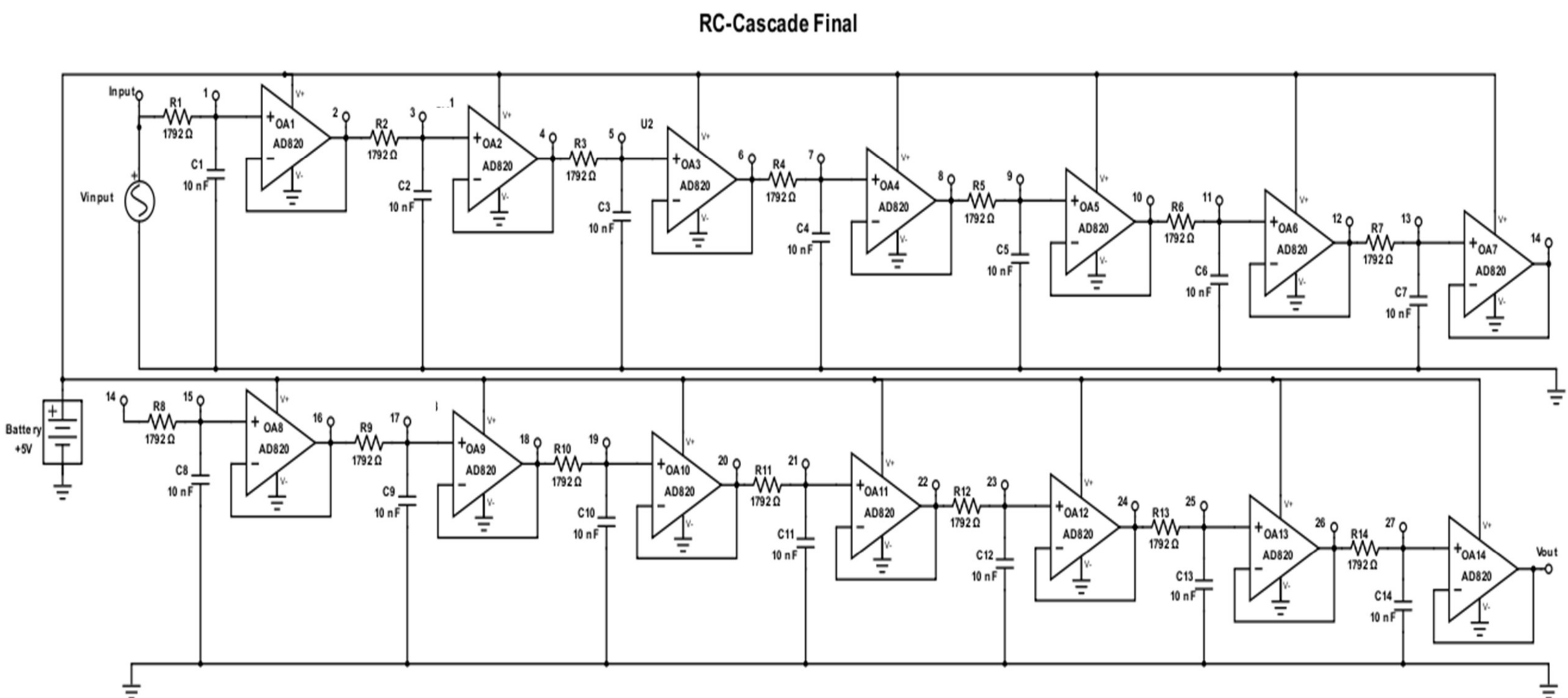
b. Butterworth



c. Chebyshev



d. RC-Cascade



9. Bill of Materials

a. Bessel

Part Description	Manufacturer	Mfg. Part #	Mouser Catalog #	Qty	Unit Cost	Combined Cost	Total
Metal Oxide Resistor 5.6K ohm 5% 1W	KOA Speer	MOS1CT528R562J	660-MOS1CT528R562J	2	$0.10	$0.20	$25.79
Carbon Film Resistor - Through Hole 1/6W 3K Ohm 5%	Yageo	CFR-12JB-52-3K	603-CFR-12JB-52-3K	2	$0.10	$0.20	
Metal Oxide Resistor 1K ohm 5% 1W	KOA Speer	MOS1CT528R102J	660-MOS1CT528R102J	1	$0.10	$0.10	
Metal Oxide Resistor 470 ohm 5% 1W	KOA Speer	MOS1CT528R471J	660-MOS1CT528R471J	1	$0.10	$0.10	
Metal Oxide Resistor 390 ohm 5% 1W	KOA Speer	MOS1CT528R391J	660-MOS1CT528R391J	1	$0.10	$0.10	
Metal Oxide Resistor 100 ohm 5% 1W	KOA Speer	MOS1CT528R101J	660-MOS1CT528R101J	1	$0.10	$0.10	
Metal Film Resistor - Through Hole 0.6W 1% 50ppm 56 ohm	Yageo	MF0207FTE52-56R	603-MF0207FTE52-56R	1	$0.17	$0.17	
Metal Film Resistor - Through Hole 0.6W 2.2 Ohm 1%	Yageo	MF0207FTE52-2R2	603-MF0207FTE52-2R2	1	$0.19	$0.19	
Film Capacitor 250V 0.1uF 5% LS=5mm AEC-Q200	KEMET	R82IC3100SH55J	80-R82IC3100SH55J	2	$0.53	$1.06	
Multilayer Ceramic Capacitor - 100V .01uF X7R 0805 10%	AVX	FS051C103K4Z2A	581-FS051C103K4Z2A	7	$0.32	$2.24	
Multilayer Ceramic Capacitor - 50V 1000pF X7R 0402 5%	KEMET	C0402S102J5RAC7867	80-C0402S102J5R7867	12	$0.58	$6.96	
Precision Amplifiers SGL-Supply RR Lo Pwr FET-Inpt	Analog Devices	AD820ARZ	584-AD820ARZ	3	$4.79	$14.37	

b. Butterworth

Part Description	Manufacturer	Mfg. Part #	Mouser Catalog #	Qty	Unit Cost	Combined Cost	Total
Metal Oxide Resistor 1K ohm 5% 1W	KOA Speer	MOS1CT528R102J	660-MOS1CT528R102J	1	$0.10	$0.10	$18.65
Metal Oxide Resistor 470 ohm 5% 1W	KOA Speer	MOS1CT528R471J	660-MOS1CT528R471J	2	$0.10	$0.20	
l Film Resistor - Through Hole 330 ohm 5% 200 ppm AEC-	KOA Speer	MF1/2LCT528R331J	660-MF1/2LCT528R331J	1	$0.15	$0.15	
Metal Oxide Resistor 10 ohm 5% 1W	KOA Speer	MOS1CT528R100J	660-MOS1CT528R100J	1	$0.10	$0.10	
Film Capacitor 250V 0.1uF 5% LS=5mm AEC-Q200	KEMET	R82IC3100SH55J	80-R82IC3100SH55J	10	$0.53	$5.30	
Multilayer Ceramic Capacitor MLCC - SMD/SMT 100V .03uF X7R 1206 5% Tol	AVX	12061C303JAT2A	581-12061C303JAT2A	1	$0.32	$0.32	
Multilayer Ceramic Capacito - 50V 1000pF X7R 0402 5%	KEMET	C0402S102J5RAC7867	80-C0402S102J5R7867	5	$0.58	$2.90	
Precision Amplifiers SGL-Supply RR Lo Pwr FET-Inpt	Analog Devices	AD820ARZ	584-AD820ARZ	2	$4.79	$9.58	

c. Chebyshev

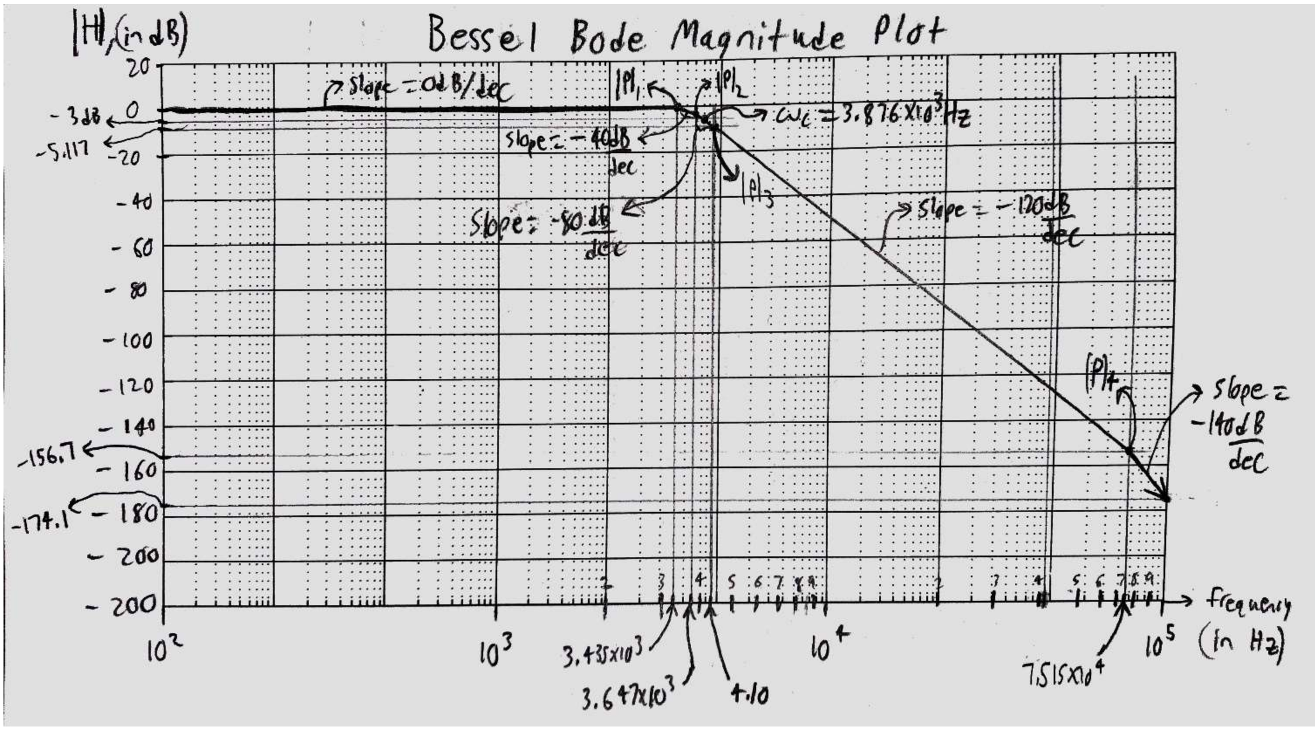
Part Description	Manufacturer	Mfg. Part #	Mouser Catalog #	Qty	Unit Cost	Combined Cost	Total
Wirewound Resistor - 3.9K ohm 5% 7W	IRC / TT Electronics	W22-3K9JI	756-W22-3K9JI	1	$1.92	$1.92	$7.34
Metal Film Resistor - 1.5K ohm 1% 0.5W	W.C. / TT Electronics	MFR4-1K5FI	756-MFR4-1K5FI	1	$0.16	$0.16	
Metal Oxide Resistor 390 ohm 5% 1W	KOA Speer	MOS1CT528R391J	660-MOS1CT528R391J	1	$0.10	$0.10	
Metal Oxide Resistor 220 ohm 5% 1W	KOA Speer	MOS1CT528R221J	660-MOS1CT528R221J	1	$0.10	$0.10	
Metal Oxide Resistor 39 ohm 5% 1W	KOA Speer	MOS1CT528R390J	660-MOS1CT528R390J	2	$0.10	$0.20	
Wirewound Resistor - 1W 4.7 Ohm 5%	Yageo	FKN1WSJR-52-4R7	603-FKN1WSJR-52-4R7	1	$0.29	$0.29	
Film Capacitor 250V 0.1uF 5% LS=5mm	KEMET	R82IC3100SH55J	80-R82IC3100SH55J	5	$0.53	$2.65	
Multilayer Ceramic Capacitor 100V .01uF	AVX	FS051C103K4Z2A	581-FS051C103K4Z2A	6	$0.32	$1.92	
Multilayer Ceramic Capacitor 50V 1000pF	KEMET	C0402S102J5RAC7867	80-C0402S102J5R7867	6	$0.58	$3.48	
Precision Amplifiers SGL-Supply RR Lo Pwr	Analog Devices	AD820ARZ	584-AD820ARZ	1	$4.79	$4.79	

d. RC-Cascade

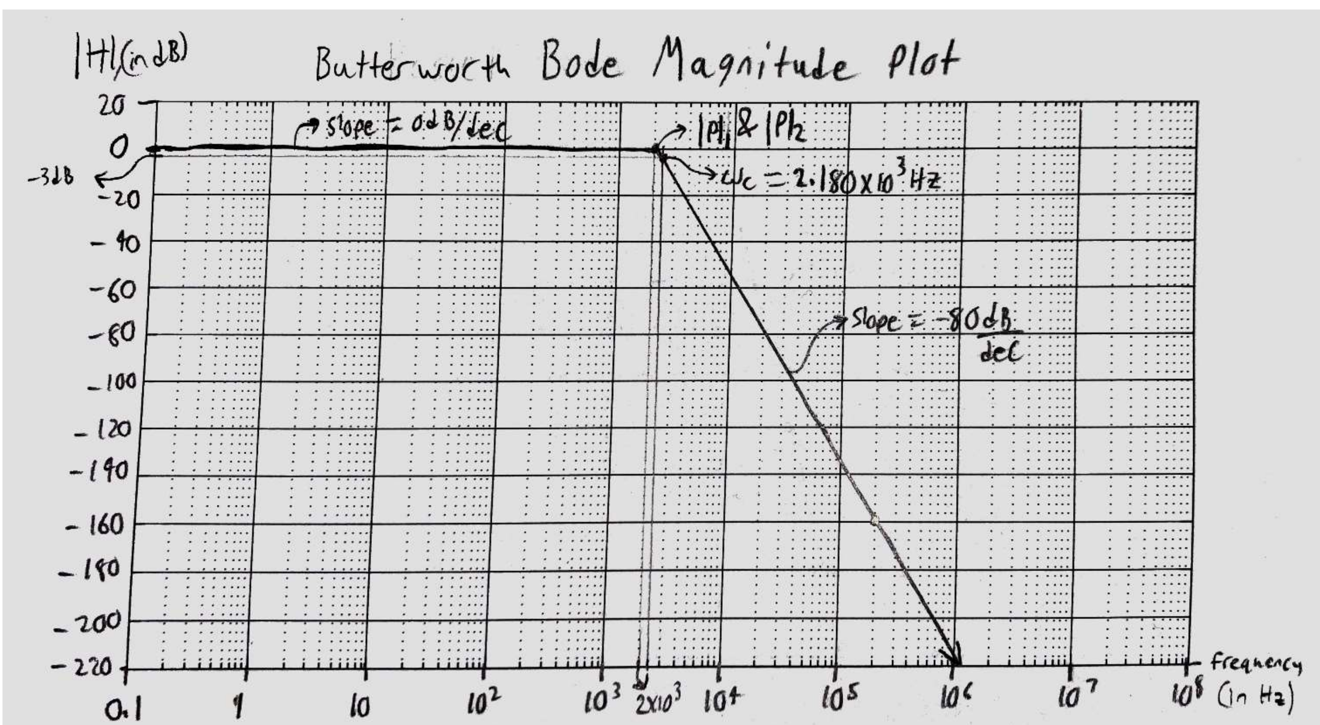
Part Description	Manufacturer	Mfg. Part #	Mouser Catalog #	Qty	Unit Cost	Combined Cost	Total
Metal Film Resistor - 1.5K ohm 1% 0.5W	W.C. / TT Electronics	MFR4-1K5FI	756-MFR4-1K5FI	14	$0.16	$2.24	$79.24
Metal Oxide Resistor 270 ohm 5% 1W	KOA Speer	MOS1CT528R271J	660-MOS1CT528R271J	14	$0.10	$1.40	
Wirewound Resistor - 3W 22 Ohm 5%	Yageo	PNP300JR-73-22R	603-PNP300JR-73-22R	14	$0.61	$8.54	
Precision Amplifiers SGL-Supply RR Lo Pwr	Analog Devices	AD820ARZ	584-AD820ARZ	14	$4.79	$67.06	

10. Hand-sketched straight-line approximated Bode Magnitude plots

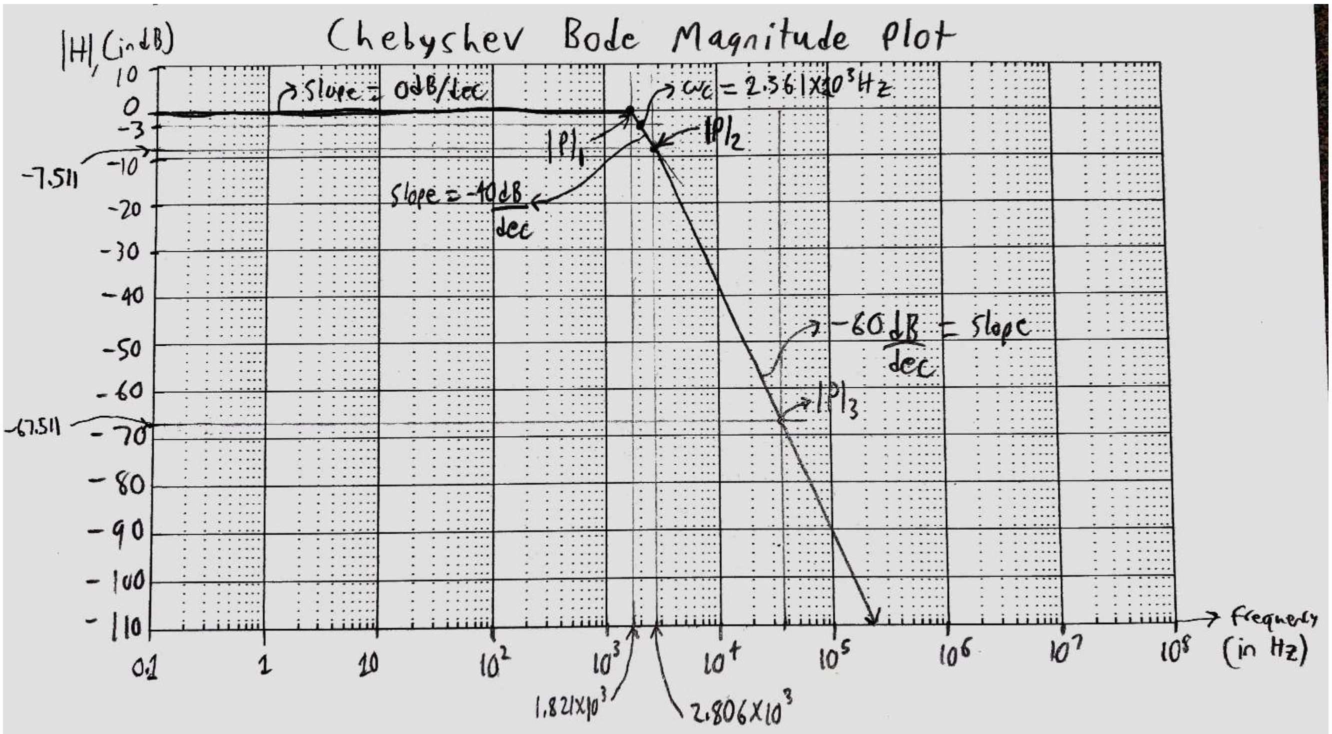
a. Bessel



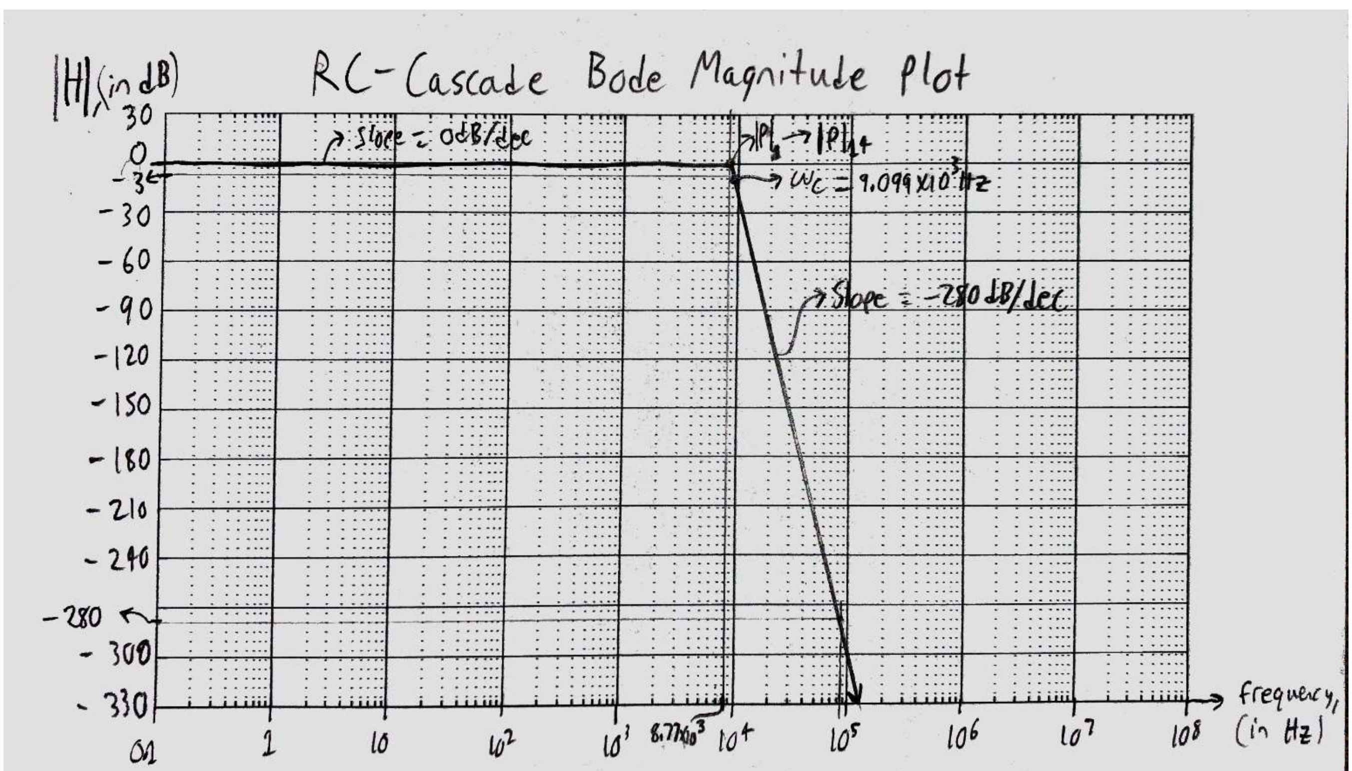
b. Butterworth



c. Chebyshev



d. RC-Cascade



11. Calculations

a. Obtaining the Transfer Function for Butterworth filter

For Butterworth;

First, testing order 2: $-a = 0.7071$; Imag $\pm jb = 0.7071$

$$H = \frac{K}{(s + 0.7071 + j0.7071)(s + 0.7071 - j0.7071)}$$

$$\hookrightarrow D(s) = s^2 + 0.7071s - \cancel{js0.7071} + 0.7071s + 0.5 \cancel{- j0.5}$$
$$+ \cancel{js0.7071} + \cancel{0.5}j + 0.5$$

$D(s) = s^2 + 1.4142s + 1$ → replaced s with $s/2\pi \cdot 2000$

Order 2 yields endb of 3.7049, so increase N

Testing order 3: Real: 0.5, 1.0; Imag: 0.8660

$$H = \frac{K}{(s + 0.5 + j0.866)(s + 0.5 - j0.866)(s + 1)}$$

$$H = \frac{1}{(s^2 + s + 1)(s + 1)} \rightarrow \text{using } K = 1$$

endb yields 7.3907 → trying again

Testing order 4: →

Real	Imag
0.9239	0.3827
0.3827	0.9239

$$H = \frac{1}{(s + 0.9239 + j0.3827)(s + 0.9239 - j0.3827)(s + 0.3827 + j0.9239)(s + 0.3827 - j0.9239)}$$

$$H = \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)}$$

$$H = \frac{1}{\left[\left(\frac{s}{2\pi \cdot 2000}\right)^2 + 0.7654\left(\frac{s}{2\pi \cdot 2000}\right) + 1\right] \cdot \left[\left(\frac{s}{2\pi \cdot 2000}\right)^2 + 1.8478\left(\frac{s}{2\pi \cdot 2000}\right) + 1\right]}$$

yields endb of 8.7532, so we need $\boxed{\text{order 4}}$

b. Finding component values for Chebyshev filter

Chebyshev Design

$$H = \frac{1}{(s^2 + 0.4514s + 0.829217)(s + 0.4513)} \cdot \frac{1}{\left(\frac{1}{0.829217}\right)\left(\frac{1}{0.4513}\right)}$$

After factoring out denominator terms to get "+1" at the end of each term

$$H = \frac{1}{(s^2(1.20596) + s(0.54437) + 1)\cdot(s\cdot(2.2158) + 1)}$$ → yields end of 9.1384

Now we sub in $\frac{s}{2\pi\cdot 2000}$ for each s:

$$H = \frac{1}{\underbrace{(s^2(7.6368\times10^{-9}) + s(4.33195\times10^{-5}) + 1)}_{\text{section 1}}\cdot\underbrace{(s(1.7633\times10^{-4}) + 1)}_{\text{section 2}}}$$

For section 1:

$7.6368\times10^{-9} = R_1R_2C_1C_2$; $4.33195\times10^{-5} = (R_1+R_2)\cdot C_2$

Let's choose 10nF for C_2: $R_1 + R_2 = \frac{4.33195\times10^{-5}}{(10\times10^{-9})} = 4{,}331.95\Omega$

Let's choose $R_1 = 3.9k\Omega$, $R_2 = 433.7\Omega \rightarrow (390 + 39 + 4.7)$

$C_1 = \frac{7.6368\times10^{-9}}{(3900)(429)(10\times10^{-9})} \cong 456.4nF \cong 456nF$

In summary: $\boxed{C_2 = 10nF,\ C_1 = 456.0nF \rightarrow (4\times100nF,\ 5\times10nF,\ 6\times1000pF);\ R_1 = 3.9k\Omega,\ R_2 = 433.7\Omega}$

c. How to Choose the Correct Op-Amp

Choosing an Op-Amp For each Section:

To Start, we will find op-amps for the Chebyshev Filter. First, we will test the AD820, single-supply, rail-to-rail Op-Amp.

- This particular op-amp has a gain bandwidth of $\boxed{1.8\,MHz}$.
- The formula we will use to determine if our Op-Amp is valid to use is:

$$GBW \geq (100)(G)(Q)(\omega_c)$$

→ In this eq., G = Gain of specific section

$$Q = \frac{\sqrt{a}}{b}$$

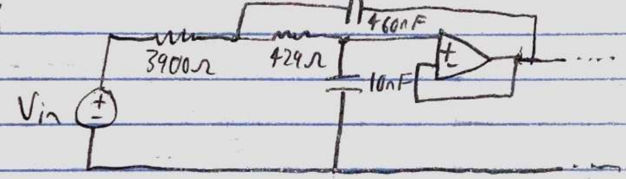
ω_c = Corner frequency, in Hz

→ It is also important to note:

- For this project, we will assume $G = 1$ for each case
- For Q, a = coefficient of S^2 term, and b = coefficient of the S term.
- For each second order section of our circuit, the corresponding equation in standard form is represented by "$aS^2 + bS + 1$"

→ In addition, we don't have to worry about the $(S+1)$ terms since they do not contain any Op-Amps.

For section 1:



Denominator form of Section 1:

$$S^2\underbrace{(7.6368\times10^{-9})}_{a} + S\underbrace{(4.33195\times10^{-5})}_{b} + 1$$

$$Q = \frac{\sqrt{a}}{b} = \frac{\sqrt{7.6368\times10^{-9}}}{4.33195\times10^{-5}} = 2.0173$$

Note: Our Q is > 2, which explains why we notice "peaking" with our Cheby. LPF.

$$GBW \geq (100)(G)(Q)(\omega_c)$$

$$1.8\times10^6\,Hz \geq (100)(1)(2.0173)(2000)$$

$$1.8\times10^6\,Hz \geq 40.346\times10^4 \rightarrow \text{True!}$$

Since our GBW is greater than $100 \cdot G \cdot Q \cdot \omega_c$, we can use the AD 820 Op-Amp for this section.

d. Finding the Corner Frequency for approximated Bode Magnitude Plots

Straight line Approximation Points / Corner Frequency for Chebyshev

$$H = \frac{1}{\underbrace{\left(s^2(7.6368\times10^{-9}) + s(4.33195\times10^{-5}) + 1\right)}_{\text{Section 1, 2nd order}}\underbrace{\left(s(1.7633\times10^{-4}) + 1\right)}_{\text{Section 2, 1st order}}}$$

Note 1: We need to ensure that we convert back to Hz when finding our poles for our plots.

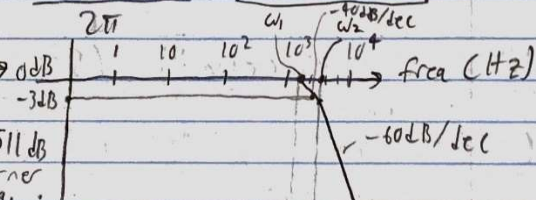
Note 2: For 2nd order straight-line Bode magnitude plots, we use the "a" value of the term to determine what frequency the section begins having an effect.

Converts to Hz

For section 1: $\omega_1 = \left(\frac{1}{\sqrt{a}}\right)\cdot\frac{1}{2\pi} = \frac{1}{\sqrt{7.6368\times10^{-9}}}\cdot\frac{1}{2\pi} \cong \boxed{1.821\times10^3\ Hz}$

For section 2: $\omega_2 = \frac{|P|}{2\pi} = \frac{1.7633\times10^4}{2\pi} \cong \boxed{2.806\times10^3\ Hz}$

K = 1, so has no effect → 0dB



We know ω_c is between 1.8×10^3 and 2.81×10^3 because $-40\log_{10}(\omega_2/\omega_1) \cong -7.511$ dB which is less than -3dB.

slope in dB/dec, corner freq.

$-40\log\left(\frac{\omega_2}{\omega_1}\right) = |H|_c - 40\log_{10}\left(\frac{\omega_c}{\omega_1}\right) \rightarrow |H|_c = -3dB, \rightarrow$ desired dropoff

frequency of 1st section

$-7.511 = -3dB - 40\log\left(\frac{\omega_c}{1.821\times10^3}\right) \rightarrow 10^{(-4.511/-40)} = \frac{\omega_c}{1821}$

$\omega_c = (1821)(1.2965) = \boxed{2360.95\ Hz} \rightarrow$ Straight-line Approx

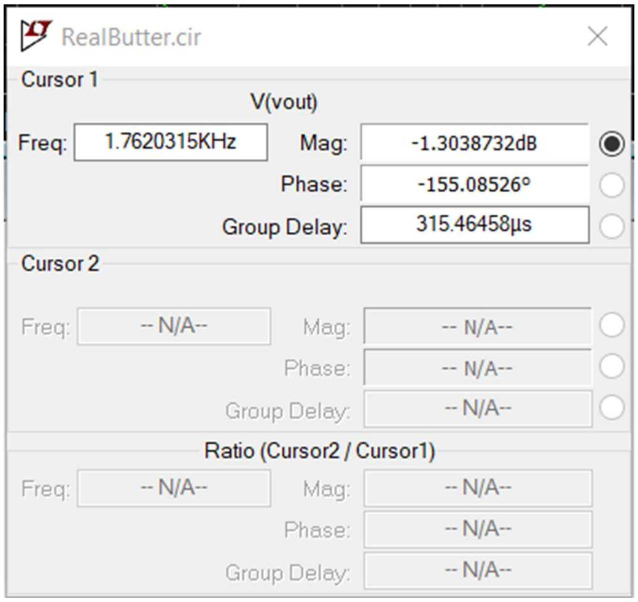
For SPICE, our ω_c is:

Ideal → $\boxed{\omega_c = 2000\ Hz}$

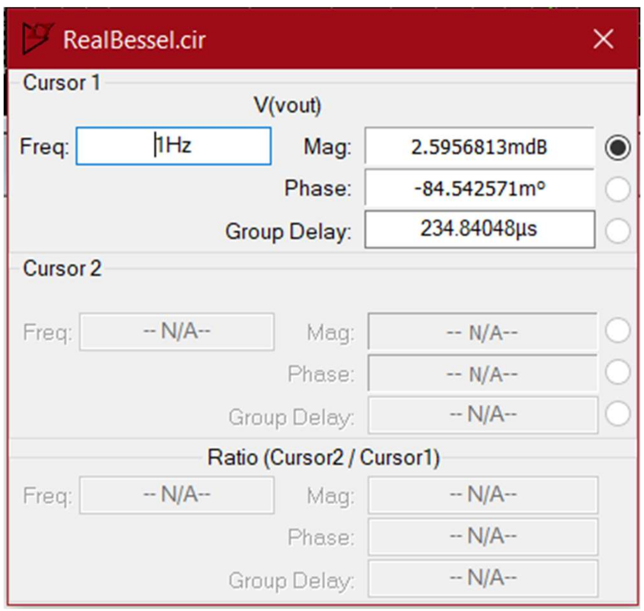
Real → $\boxed{\omega_c = 1988.5\ Hz}$

12. Group Delay

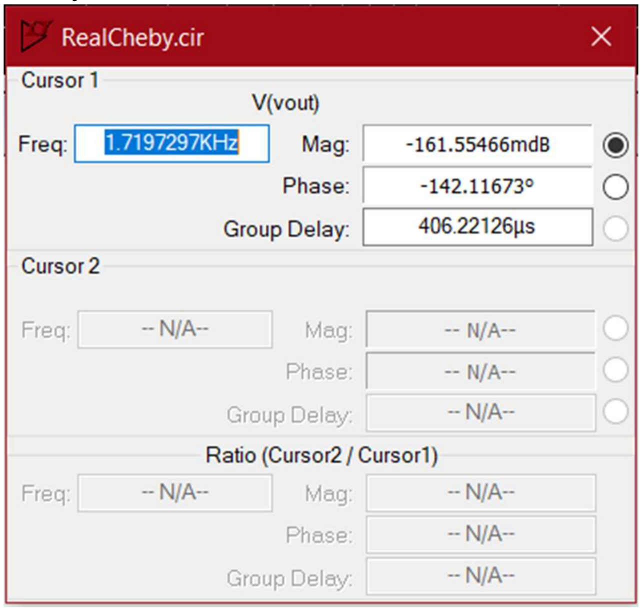
a. Butterworth



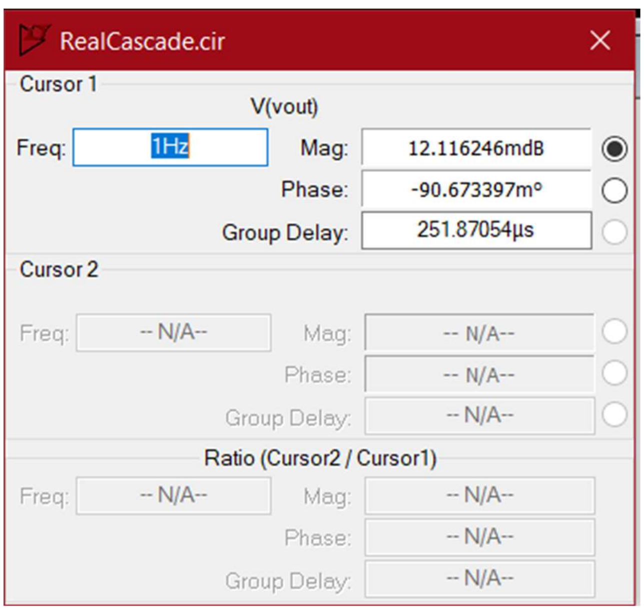
b. Bessel



c. Chebyshev



d. RC-Cascade



13. Sources

a. https://www.maximintegrated.com/en/app-notes/index.mvp/id/1762

b. https://ez.analog.com/amplifiers/f/q-a/15186/filter-wizard-noise-in-sallen-key-vs-multiple-feedback