Spencer Adams HW 7 Final answers highlighted in blue!

Codes for problem 1 and 2 are provided in the zip file submission for this assignment.

Course: MAE 5420 - Compressible Fluid Flow Term: Fall 2023

Instructor: Tyson Smith **Due Date: $13^{rd}$ November, 2023**

**Problem 1**

Consider the geometry represented by Figure 1. Compute the following after each corner:

- Entry and exit Mach wave angles or shock angles
- Mach number
- Static and stagnation pressure
- Static temperature

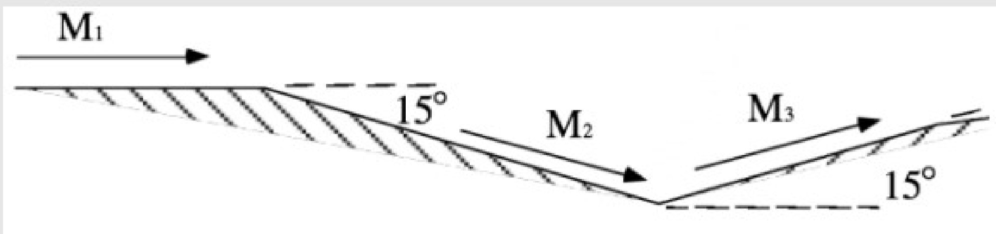



Figure 1

*Given Values:*

$M_1 = 4.0$ $\gamma = 1.25$

$p_1 = 0.01$ atm $\theta_1 = 15^\circ$

$T_1 = 217$ K $\theta_2 = 15^\circ$

Note: The Prandtl-Meyer Function is

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - tan^{-1}(\sqrt{M^2-1}) \quad (1)$$

Given values:

$M_1 := 4.0$ $P_1 := 0.01 \cdot atm = 1.01325\ kPa$

$T_1 := 217 \cdot K$ $\gamma := 1.25$ $theta_1 := 15\ deg = 0.261799\ rad$

$theta_2 := 15 \cdot deg = 0.262\ rad$

first find the entry wave angle

$$angle_{enter} := \operatorname{asin}\left(\frac{1}{M_1}\right) = 14.478\ deg$$

now calc v(M1) in order to plug it into an iterative solver. the equation for v(M2) is the same (the Prandtl-Meyer equation) for v(M2)

$$\nu(M_1)=\sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_1^2-1\right)}\right\}-\tan^{-1}\sqrt{M_1^2-1}$$

see code for vM1 calculation

$vM_1 := 81.19378046482483 \cdot deg$

- Use Iterative Solver to Compute $M_2$

$$M_{2(j+1)}=\frac{\left[\theta+\nu(M_1)-\nu(M_{2(j)})\right]}{\left(\frac{\partial\nu}{\partial M}\right)_{(j)}}+M_{2(j)}$$

see code to see how I made an M2 newton solver based on the equation to the left.

$M_2 := 4.934742565$

The exit angle is calculated with $angle_{exit} = \mu_2 - theta_1$

where

$$\mu_2 := \operatorname{asin}\left(\frac{1}{M_2}\right) = 11.692\ deg$$

$$theta_1 = 15\ deg$$

$$angle_{exit} := \mu_2 - theta_1 = -3.308\ deg$$

To find static and stagnation pressure... The numerator below multiplied by P1 is the stagnation pressure, Po1. (We can use this equation because the flow across an expansion fan is isentropic). See code for actual calculation.

$$p_2 = p_1\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma}{\gamma-1}}$$

$P_{o1} := 246219.75 \cdot Pa = 246.22\ kPa$

$P_{o2} := P_{o1} = 246.22\ kPa\ (isentropic)$

$P_2 := 227.66 \cdot Pa = 0.228\ kPa$

To find static and stagnation temperature... The numerator below multiplied by T1 is the stagnation pressure, To1. (We can use this equation because the flow across an expansion fan is isentropic). See code for actual calculations of To1, To2, and T2.

$$T_2 = T_1\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]$$

$T_{o1} := 651.0 \cdot K$

$T_{o2} := T_{o1} = 651\ K \quad (isentropic)$

$T_2 := 160.980799 \cdot K$

Now, to find M3 and the temperature and pressure terms after the oblique shock at the second turn, we first need to find the shock angle, beta. I chose to solve for beta explicitly, because the equation for beta is a cubic function that could converge to three numbers depending on starting condition. I instead assumed that we had a weak shock (delta = 1) and used the following explicit functions.

See code for actual calculation of lambda, chi, and beta.

$theta_{twototal} := theta_1 + theta_2 = 30\ deg$

$$\lambda = \sqrt{\left(M_1^2 - 1\right)^2 - 3\left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[1 + \frac{\gamma + 1}{2} M_1^2\right]\tan^2(\theta)}$$

$$\chi = \frac{\left(M_1^2 - 1\right)^3 - 9\left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[1 + \frac{\gamma - 1}{2} M_1^2 + \frac{\gamma + 1}{4} M_1^4\right]\tan^2(\theta)}{\lambda^3}$$

sub M2 in for M1 in the equations to the left

$\lambda := 20.7477$

$\chi := 0.967145$

where

$$\tan(\beta) = \frac{\left(M_1^2 - 1\right) + 2\lambda \cos\left(\frac{4\pi\delta + \cos^{-1}(\chi)}{3}\right)}{3\left[1 + \frac{\gamma - 1}{2} M_1^2\right]\tan(\theta)}$$

$\beta := 39.4092396 \cdot deg = 0.688\ rad$

$\beta$ is the shock angle!

Now, to solve the pressures and temperatures at 2 and 3, we can use the following equations. See code for actual calculations.

$M_{n2} = M_2 \cdot sin(\beta) \longrightarrow M_{n2} := 3.1328465$

$$M_{n3} = \sqrt[2]{\frac{\left(1 + \frac{(\gamma - 1)}{2} \cdot \left(M_2 \cdot sin(\beta)\right)^2\right)}{\gamma \cdot \left(M_2 \cdot sin(\beta)\right)^2 - \frac{(\gamma - 1)}{2}}} \longrightarrow M_{n3} := 0.428227$$

$$M_3 := \frac{M_{n3}}{\sin\left(\beta - theta_{twototal}\right)} = 2.619$$

using $theta_{twototal}$ because the turn angle at two does not come in from the free stream direction.

Now, to find the pressure and temperature values after the shock at $\beta$, we can use the following equations. (for our calculation switch from P1, T1, to P2, T2, and from P2, T2 to P3, T3. I just pasted these from the slides into here. You can see I did that in the code for the problem.

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{(\gamma+1)}\left(Mn_1^{\,2} - 1\right) \longrightarrow$$

$P_3 := 2457.4034 \cdot Pa = 2.457\ kPa$

$P_{o3} := 54359.85 \cdot Pa = 54.36\ kPa$

$$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{(\gamma+1)}\left(Mn_1^{\,2} - 1\right)\right]\left[\frac{\left(2 + (\gamma-1)Mn_1^{\,2}\right)}{(\gamma+1)Mn_1^{\,2}}\right] \longrightarrow$$

$T_3 := 350.4456 \cdot K$

$T_{o3} := 651 \cdot K \quad (adiabatic)$

**Problem 2**

A ramjet operates at an altitude of 10,000 meters at a Mach number of 1.7. The external diffusion is based on an oblique shock and normal shock, as shown in Figure 2. Calculate the following:

(a) Stagnation pressure recovery, $P_{02}/P_{0\infty}$

(b) At what Mach number does the oblique shock become detached?

(c) What is the distance x (from cone tip to the outer inlet lip) for the condition described in Figure 2?

(d) What is the best turning angle ($\theta$) in terms of highest pressure ratio, $P_{02}/P_{0\infty}$?

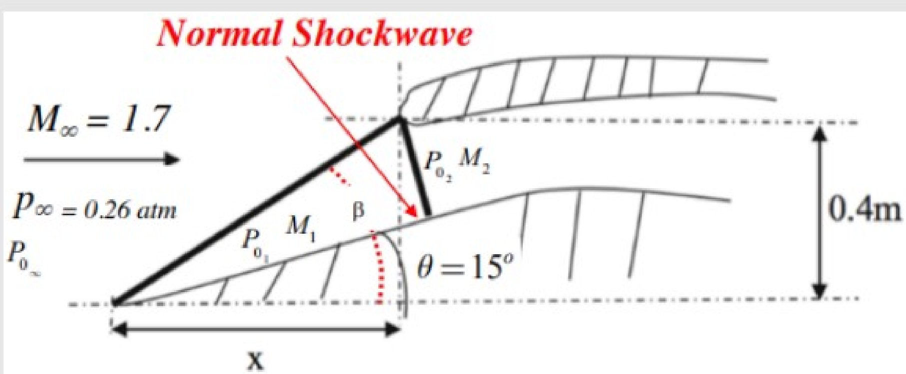



Figure 2: Ramjet Supersonic Diffuser

*Given Values:*

$M_\infty = 1.7$

$p_\infty = 0.26$ atm

$T_\infty = 223$ K

$\gamma = 1.4$

$\theta = 15°$

r = 0.4 meters (radius to outer inlet tip)

Given values:

$M_{\infty} := 1.7$

$P_{\infty} := 0.26 \cdot atm = 26.345\ kPa$

$\gamma := 1.4$

$theta := 15 \cdot deg = 0.262$

$r := 4 \cdot m$

$T_{\infty} := 223 \cdot K$

(a) Stagnation pressure recovery, $P_{02}/P_{0\infty}$

$$P_{o\infty} := P_{\infty} \cdot \left(1 + \frac{(\gamma - 1)}{2} \cdot (M_{\infty})^2\right)^{\frac{\gamma}{(\gamma-1)}} = 130.036\ kPa$$

$$T_{o\infty} := T_{\infty} \cdot \left(1 + \frac{(\gamma - 1)}{2} \cdot (M_{\infty})^2\right) = 351.894\ K$$

$$\lambda = \sqrt{\left(M_1^2 - 1\right)^2 - 3\left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[1 + \frac{\gamma + 1}{2} M_1^2\right]\tan^2(\theta)}$$

For

$$\chi = \frac{\left(M_1^2 - 1\right)^3 - 9\left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[1 + \frac{\gamma - 1}{2} M_1^2 + \frac{\gamma + 1}{4} M_1^4\right]\tan^2(\theta)}{\lambda^3}$$

$$\lambda := \sqrt[2]{\left(M_{\infty}^2 - 1\right)^2 - 3 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M_{\infty}^2\right) \cdot \left(1 + \frac{\gamma + 1}{2} \cdot M_{\infty}^2\right) \cdot (\tan(theta))^2} = 1.433$$

$$\chi := \frac{\left(M_{\infty}^2 - 1\right)^3 - 9 \cdot \left(1 + \frac{\gamma - 1}{2} M_{\infty}^2\right) \cdot \left(1 + \frac{\gamma - 1}{2} M_{\infty}^2 + \frac{\gamma + 1}{4} \cdot M_{\infty}^4\right) \cdot (\tan(theta))^2}{\lambda^3} = 0.011$$

$\delta := 1$

$$\beta := \operatorname{atan}\left(\frac{\left(M_{\infty}^2 - 1\right) + 2 \cdot \lambda \cdot \cos\left(\frac{4 \cdot \pi \cdot \delta + \operatorname{acos}(\chi)}{3}\right)}{3 \cdot \left(1 + \frac{\gamma - 1}{2} M_{\infty}^2\right) \cdot \tan(theta)}\right) = 55.984\ deg$$

Now that we have the oblique shock angle, we can find how the components change as they travel normally through it. (So we can find M1, T1, P1, etc.)

$$M_{n\infty} = M_2 \cdot sin(\beta) = 1.409098$$

$$M_{n1} = \sqrt[2]{\frac{\left(1 + \frac{(\gamma - 1)}{2} \cdot (M_\infty \cdot sin(\beta))^2\right)}{\gamma \cdot (M_\infty \cdot sin(\beta))^2 - \frac{(\gamma - 1)}{2}}} = 0.7359$$

$$M_1 = \frac{M_{n1}}{\sin(\beta - theta)} = 1.122071338$$

$$P_1 = P_\infty \cdot \left(1 + \frac{2 \cdot \gamma}{(\gamma + 1)} \cdot \left({M_{n\infty}}^2 - 1\right)\right) = 56635.9068 \text{ Pa}$$

$$T_1 = T_\infty \cdot \left(1 + \frac{2 \cdot \gamma}{(\gamma + 1)} \cdot \left({M_{n\infty}}^2 - 1\right)\right) \cdot \left(\frac{2 + (\gamma - 1) \cdot {M_{n\infty}}^2}{(\gamma + 1)\ {M_{n\infty}}^2}\right) = 281.108 \text{ K}$$

$$P_{o1} = P_1 \cdot \left(1 + \frac{(\gamma - 1)}{2} \cdot (M_1)^2\right)^{\frac{\gamma}{(\gamma - 1)}} = 124301.0734 \text{ Pa}$$

$$T_{o1} = T_1 \cdot \left(1 + \frac{(\gamma - 1)}{2} \cdot (M_1)^2\right) = 351.894 \text{ K}$$

Because the next shock is a normal shock we plug in 90 degrees for beta and repeat the process. Also theta is zero going from station 1 to 2.

$$M_{n2} = \sqrt[2]{\frac{\left(1 + \frac{(\gamma - 1)}{2} \cdot \left(M_1 \cdot sin\left(\frac{\pi}{2}\right)\right)^2\right)}{\gamma \cdot \left(M_1 \cdot sin\left(\frac{\pi}{2}\right)\right)^2 - \frac{(\gamma - 1)}{2}}} = 0.8950277$$

$$M_2 = \frac{M_{n2}}{sin\left(\frac{\pi}{2} - 0.0\right)} = 0.89502778$$

$M_1$ is parallel to the direction normal to the normal shock, so use that to find P2 and T2

$$P_2 = P_1 \cdot \left(1 + \frac{2 \cdot \gamma}{(\gamma+1)} \cdot \left(M_1{}^2 - 1\right)\right) = 73752.302 \text{ Pa}$$

$$T_2 = T_1 \cdot \left(1 + \frac{2 \cdot \gamma}{(\gamma+1)} \cdot \left(M_1{}^2 - 1\right)\right) \cdot \left(\frac{2 + (\gamma-1) \cdot M_1{}^2}{(\gamma+1)\, M_1{}^2}\right) = 303.30069 \text{ K}$$

$$P_{o2} = P_2 \cdot \left(1 + \frac{(\gamma-1)}{2} \cdot (M_2)^2\right)^{\frac{\gamma}{(\gamma-1)}} = 124068.114 \text{ Pa}$$

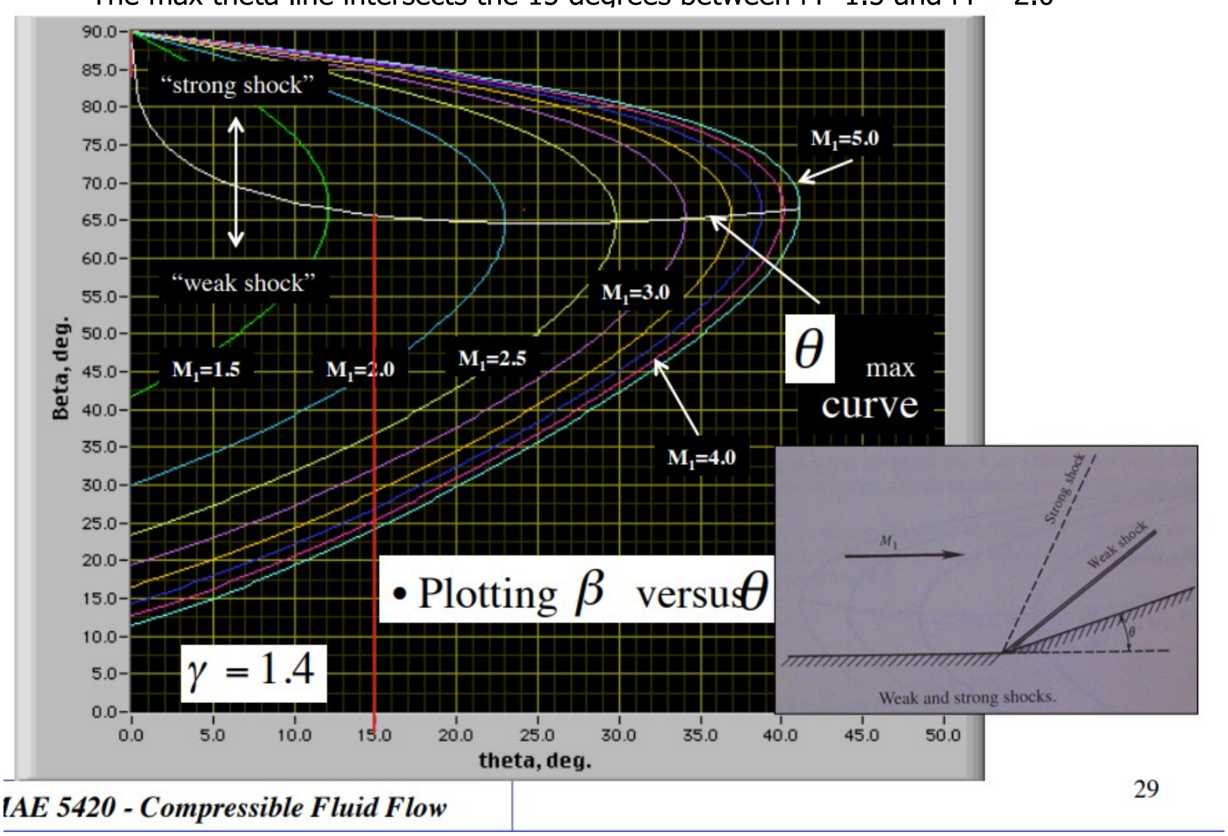
$$T_{o2} = T_2 \cdot \left(1 + \frac{(\gamma-1)}{2} \cdot (M_2)^2\right) = 351.8939 \text{ K}$$

Now we can finally calculate the stagnation pressure recovery.

$P_{o2}/P_{o\infty}$ = 0.954104

(b) At what Mach number does the oblique shock become detached?

To answer this question, draw a line on the curve at the condition theta. The max theta line intersects the 15 degrees between M=1.5 and M = 2.0



based on the theta, beta, mach chart, we get a max mach of around

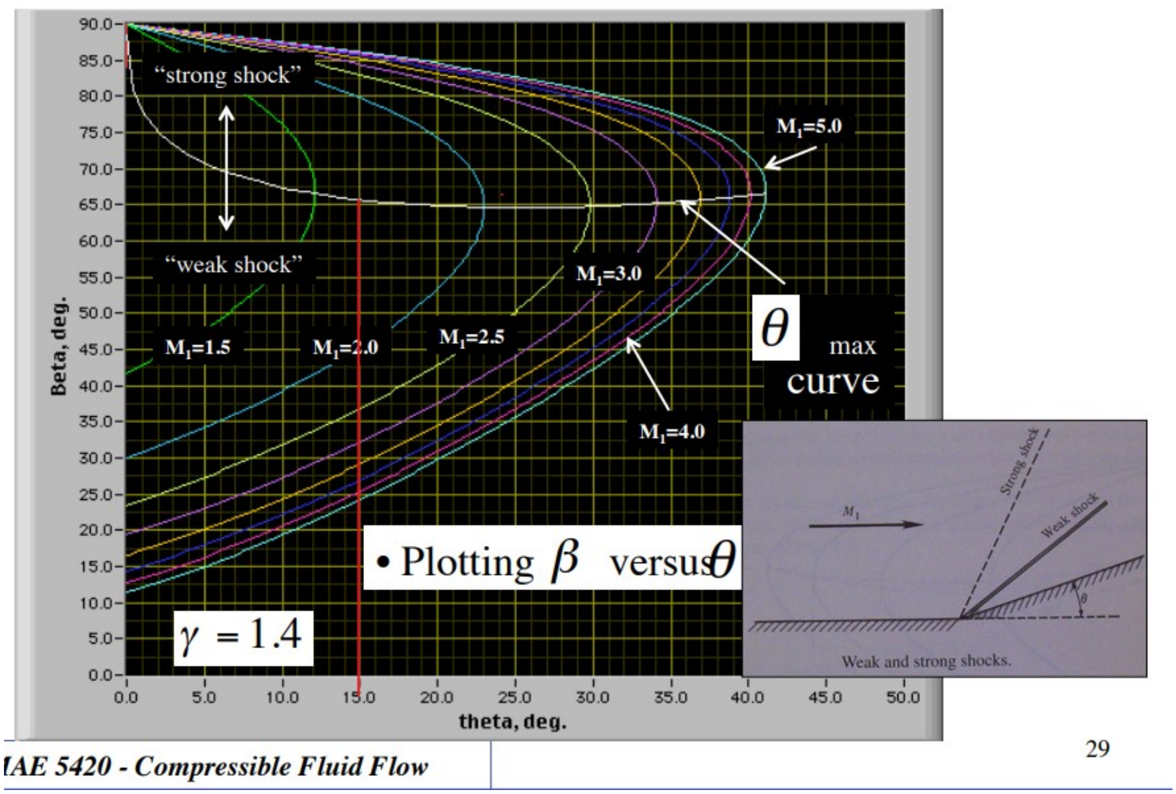
$M_{detached} := 1.62$

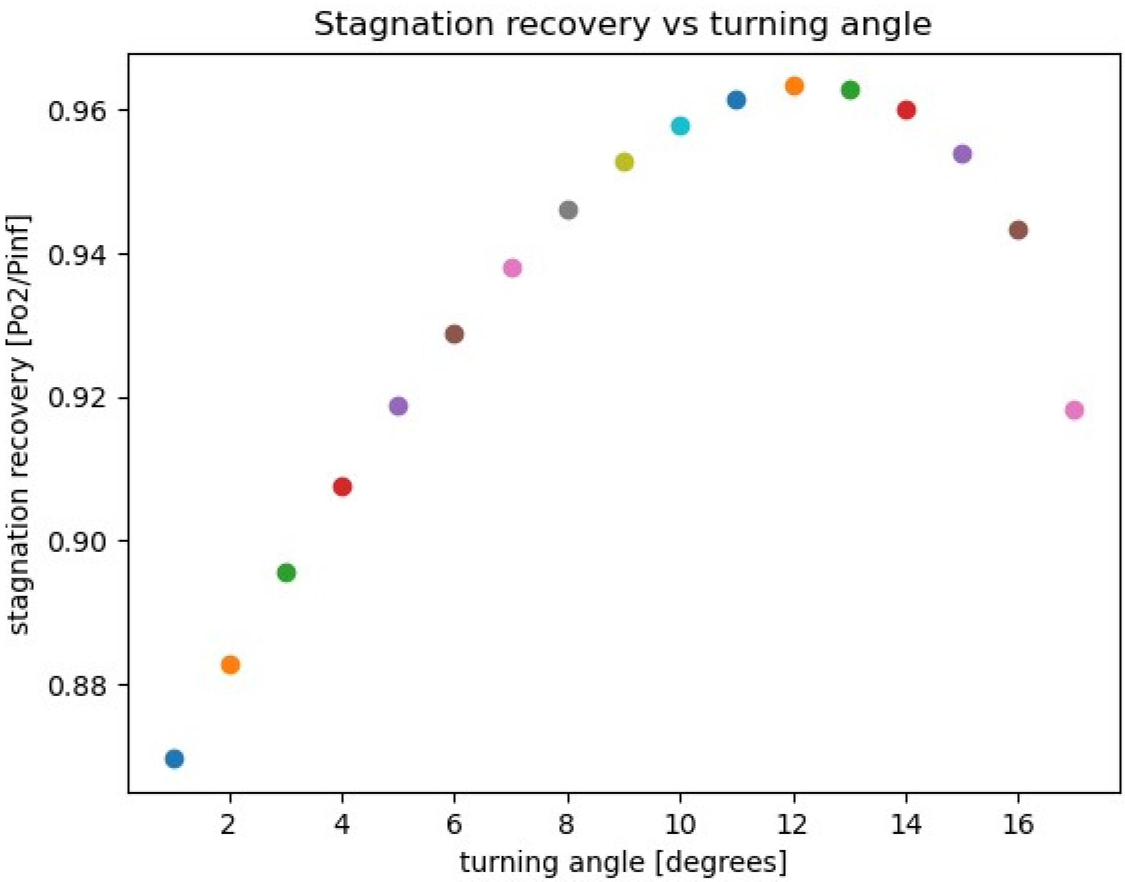
(c) What is the distance x (from cone tip to the outer inlet lip) for the condition described in Figure 2?

solve for x using beta and r

$$x := \frac{r}{\tan(\beta)} = 2.7\ m$$

(d) What is the best turning angle ($\theta$) in terms of highest pressure ratio, $P_{02}/P_{0\infty}$?






I ran my code over a range of zero to 100. This is the area shows where the stagnation recovery maxes out. This is when theta is around 12.15 degrees.

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