APL

C++

Meetup

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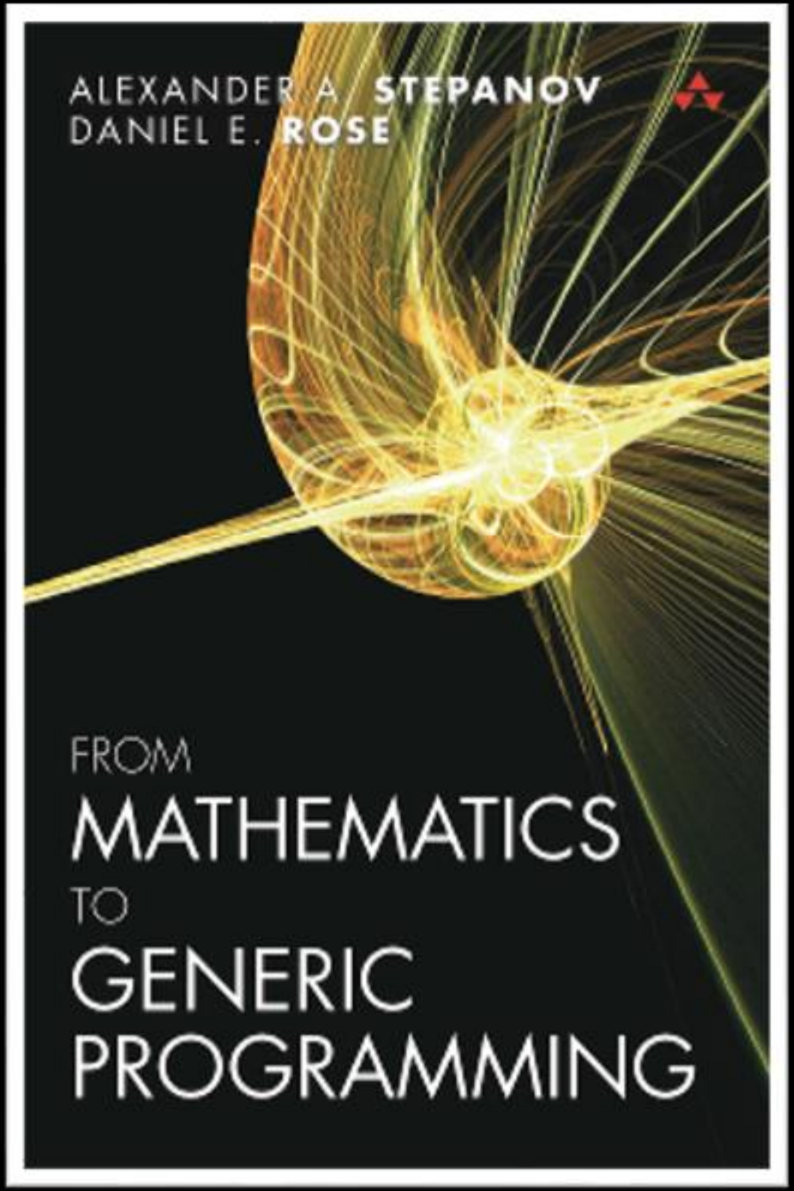

From Mathematics to Generic Programming

Chapter 2/9

1. What This Book Is About
2. The First Algorithm
3. Ancient Greek Number Theory
4. Euclid's Algorithm
5. The Emergence of Modern Number Theory
6. Abstraction in Mathematics
7. Deriving a Generic Algorithm
8. More Algebraic Structures
9. Organizing Mathematical Knowledge
10. Fundamental Programming Concepts
11. Permutation Algorithms
12. Extensions of GCD
13. A Real-World Application

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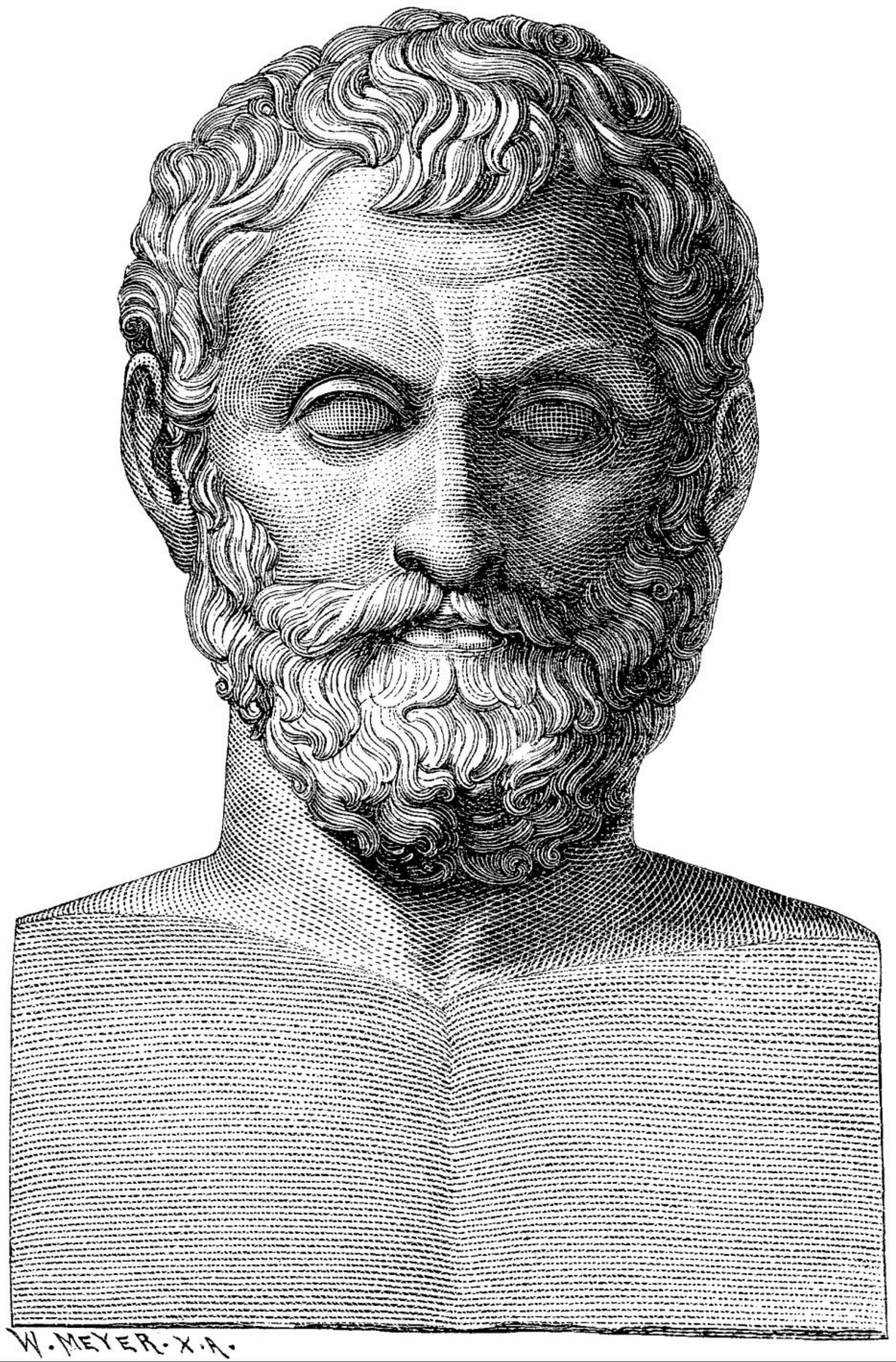
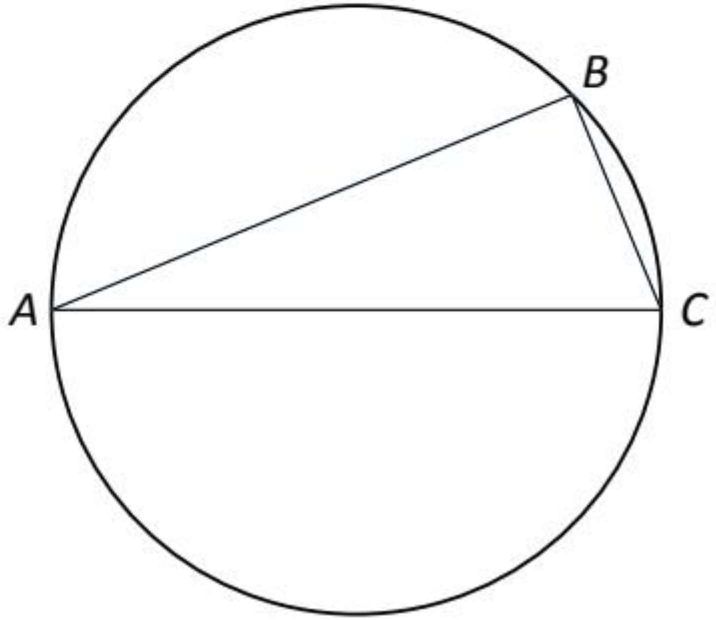
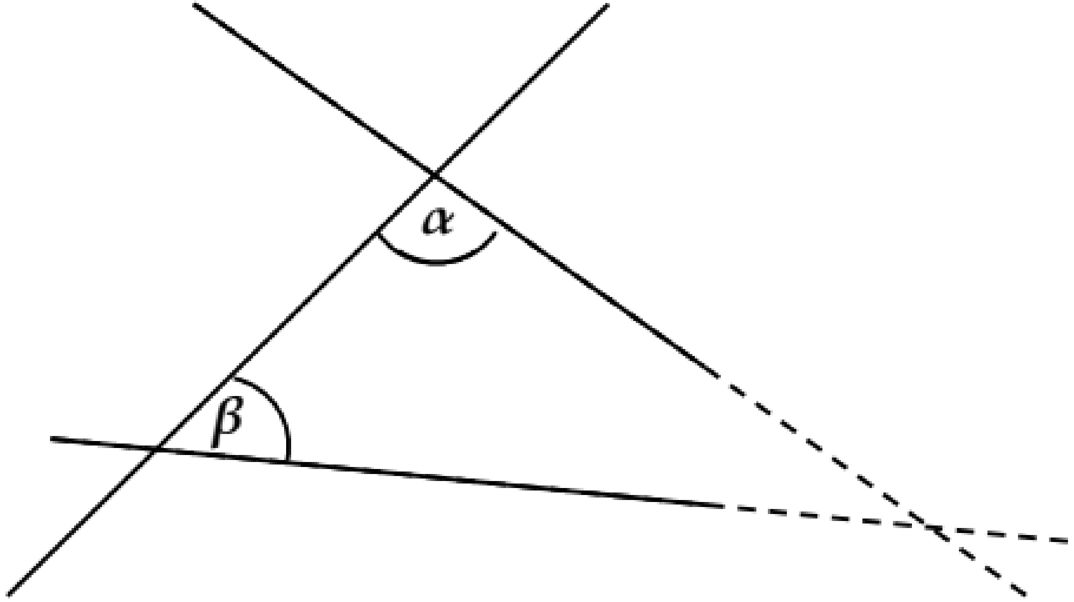

W.MEYER·X.A.

Theorem 9.1 (Thales' Theorem): *For any triangle ABC formed by connecting the two ends of a circle's diameter (AC) with any other point B on the circle,* $\angle ABC = 90°$.





$$\alpha + \beta < 180°$$

```cpp
auto multiply0(int n, int a) -> int {
  if (n == 1) return a;
  return multiply0(n - 1, a) + a;
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

// the Egyptian multiplication algorithm in C++:
int multiply1(int n, int a) {
  if (n == 1) return a;
  int result = multiply1(half(n), a + a);
  if (odd(n)) result = result + a;
  return result;
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }


// multiply-accumulate function:
int mult_acc0(int r, int n, int a) {
  if (n == 1) return r + a;
  if (odd(n)) {
    return mult_acc0(r + a, half(n), a + a);
  } else {
    return mult_acc0(r, half(n), a + a);
  }
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

int mult_acc1(int r, int n, int a) {
  if (n == 1) return r + a;
  if (odd(n)) r = r + a;
  return mult_acc1(r, half(n), a + a);
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

// So we can reduce the number of times we have to compare
// w/ 1 by a factor of 2 simply by checking for oddness 1st:
int mult_acc2(int r, int n, int a) {
  if (odd(n)) {
    r = r + a;
    if (n == 1) return r;
  }
  return mult_acc2(r, half(n), a + a);
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

// getting back tail recursion
int mult_acc3(int r, int n, int a) {
  if (odd(n)) {
    r = r + a;
    if (n == 1) return r;
  }
  n = half(n);
  a = a + a;
  return mult_acc3(r, n, a);
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

// change to iterative
int mult_acc4(int r, int n, int a) {
  while (true) {
    if (odd(n)) {
      r = r + a;
      if (n == 1) return r;
    }
    n = half(n);
    a = a + a;
  }
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

int mult_acc4(int r, int n, int a) {
  while (true) {
    if (odd(n)) {
      r = r + a;
      if (n == 1) return r;
    }
    n = half(n);
    a = a + a;
  }
}

int multiply2(int n, int a) {
  if (n == 1) return a;
  return mult_acc4(a, n - 1, a);
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

int mult_acc4(int r, int n, int a) {
  while (true) {
    if (odd(n)) {
      r = r + a;
      if (n == 1) return r;
    }
    n = half(n);
    a = a + a;
  }
}
```

```cpp
// This is pretty good, except when n is a power of 2. The first thing we do is
// subtract 1, which means that mult_acc4 will be called with a number whose bi-
// nary representation is all 1s, the worst case for our algorithm. So we’ll
// avoid this by doing some of the work in advance when n is even, halving it
// (and doubling a) until n becomes odd:
int multiply3(int n, int a) {
  while (!odd(n)) {
     a = a + a;
    n = half(n);
  }
  if (n == 1) return a;
  return mult_acc4(a, n - 1, a);
}
```

```cpp
auto odd(int n) -> bool { return n & 0x1; }
auto half(int n) -> int { return n >> 1; }

int mult_acc4(int r, int n, int a) {
  while (true) {
    if (odd(n)) {
      r = r + a;
      if (n == 1) return r;
    }
    n = half(n);
    a = a + a;
  }
}


// But now we notice that we're making mult_acc4 do one unnecessary test for
// odd(n), because we're calling it with an even number. So we'll do one halving
// and doubling on the arguments before we call it, giving us our final version:
int multiply4(int n, int a) {
  while (!odd(n)) {
    a = a + a;
    n = half(n);
  }
  if (n == 1) return a;
  // even(n - 1) ==> n - 1 ≠ 1
  return mult_acc4(a, half(n - 1), a + a);
}
```

discussion

APL

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