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BS 5-10) Prove that the state estimation errors are not white

$$E[\tilde{\underline{x}}(k+1|k+1)\tilde{\underline{x}}^T(k|k)] = [I - W(k+1)H(k+1)]F(k)P(k|k)$$

$$\tilde{\underline{x}}(k+1|k+1) = \underline{x}(k+1) - \hat{\underline{x}}(k+1|k+1)$$

$$\tilde{\underline{x}}(k|k) = \underline{x}(k) - \hat{\underline{x}}(k|k)$$

$$\tilde{\underline{x}}(k+1|k+1) = F(k)\underline{x}(k) + G(k)\underline{u}(k) + \Gamma(k)\underline{v}(k) - \ldots$$
$$[\hat{\underline{x}}(k+1|k) + W(k+1)\underline{\nu}(k+1)]$$

$$\hat{\underline{x}}(k+1|k) = F(k)\hat{\underline{x}}(k|k) + G(k)\underline{u}(k)$$

$$\tilde{\underline{x}}(k+1|k+1) = F(k)[\underline{x}(k) - \hat{\underline{x}}(k|k)] - W(k+1)\underline{\nu}(k+1) + \Gamma(k)\underline{v}(k)$$

$$\tilde{\underline{x}}(k+1|k+1) = F(k)\tilde{\underline{x}}(k|k) + \Gamma(k)\underline{v}(k) - \ldots$$
$$W(k+1)[H(k+1)\underline{x}(k+1) + \underline{w}(k+1) - H(k+1)\hat{\underline{x}}(k+1|k)]$$

$$= F(k)\tilde{\underline{x}}(k|k) + \Gamma(k)\underline{v}(k) - \ldots$$
$$W(k+1)[H(k+1)(F(k)(\underline{x}(k) - \hat{\underline{x}}(k|k)) + \Gamma(k)\underline{v}(k)) + \underline{w}(k+1)]$$

$$= [I - W(k+1)H(k+1)]F(k)\tilde{\underline{x}}(k|k) + [I - W(k+1)H(k+1)]\Gamma(k)\underline{v}(k) + \ldots$$
$$W(k+1)\underline{w}(k+1)$$

$$= [I - W(k+1)H(k+1)][F(k)\tilde{\underline{x}}(k|k) + \Gamma(k)\underline{v}(k)] - \underline{w}(k+1)$$

$$E[\tilde{\underline{x}}(k+1|k+1)\,\hat{\tilde{\underline{x}}}^T(k|k)] = \ldots$$

$$[I - W(k+1)H(k+1)][F(k)\,E[\tilde{\underline{x}}(k|k)\,\hat{\tilde{\underline{x}}}^T(k|k)] +$$

expectation is linear $\leftarrow$

$$\Gamma(k)\,E[\underline{v}(k)\,\hat{\tilde{\underline{x}}}^T(k|k)]] +$$

$$W(k+1)\,E[\underline{w}(k+1)\,\tilde{\underline{x}}(k|k)]$$

$$E[\tilde{\underline{x}}(k+1|k+1)\,\hat{\tilde{\underline{x}}}(k|k)] = \ldots$$

$$[I - W(k+1)H(k+1)]\,F(k)\,P(k|k) + 0 + 0 \quad \checkmark$$

→ process noise is <u>white</u>, therefore uncorrelated to current state

→ measurement noise is uncorrelated with state