(3)

EKF:

$$\underline{x}(k+1) = \underline{f}(k, \underline{x}(k), \underline{u}(k), \underline{v}(k)) \approx$$

$$\underbrace{\underline{f}(k, \hat{\underline{x}}(k), \underline{u}(k), 0) + F(k)[\underline{x}(k) - \hat{\underline{x}}(k)] + \Gamma(k)\underline{v}(k)}_{\text{1st order Taylor series approximation}}$$

$$\underline{h}(k+1, \underline{x}(k+1)) \approx \underbrace{\underline{h}(k+1, \bar{\underline{x}}(k+1)) + H(k+1)[\underline{x}(k+1) - \bar{\underline{x}}(k+1)]}_{\text{1st order Taylor series approximation}}$$

SRIF Propagation:

Solve dynamics model equation for $x(k)$ in terms of $\underline{x}(k+1)$, $\underline{w}(k)$, & $\underline{v}(k)$. Substitute result into a posteriori $\underline{x}(k)$ SRI equation

$$\underline{z}_x(k) = \mathcal{R}_{xx}(k)\underline{x}(k) + \underline{w}_x(k) \quad \rightarrow \text{ SRI equation}$$

$$\underline{x}(k) = F^{-1}(k)[\underline{x}(k+1) - \underline{f}(k, \hat{\underline{x}}(k), \underline{u}(k), 0) - \Gamma(k)\underline{v}(k)] + \hat{\underline{x}}(k)$$

$$0 = \mathcal{R}_{vv}(k)\underline{v}(k) + \underline{w}_v(k)$$

$$\begin{bmatrix} 0 \\ z_x + \mathcal{R}_{xx}(k)[F^{-1}(k)\underline{f}(k, \hat{\underline{x}}(k), \underline{u}(k), 0) - \hat{\underline{x}}(k)] \end{bmatrix} = \dots$$

$$\begin{bmatrix} \mathcal{R}_{vv}(k) & 0 \\ -\mathcal{R}_{xx}(k)F^{-1}(k)\Gamma(k) & \mathcal{R}_{xx}(k)F^{-1}(k) \end{bmatrix} \begin{bmatrix} \underline{v}(k) \\ \underline{x}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{w}_v(k) \\ \underline{w}_x(k) \end{bmatrix}$$

QR Factorize:

$$\begin{bmatrix} \bar{R}_{vv}(k) & \bar{R}_{vx}(k+1) \\ 0 & \bar{R}_{xx}(k+1) \end{bmatrix} = T_a(k) \begin{bmatrix} R_{vv}(k) & 0 \\ -R_{xx}(k)F^{-1}(k)\Gamma(k) & R_{xx}(k)F^{-1}(k) \end{bmatrix}$$

Thus:

$$\begin{bmatrix} \bar{\xi}_v(k) \\ \bar{\xi}_x(k+1) \end{bmatrix} = T_a(k) \begin{bmatrix} 0 \\ \xi_x(k) + R_{xx}(k)[F^{-1}(k)\underline{f}(k, \hat{\underline{x}}(k), \underline{v}(k), 0) - \hat{\underline{x}}(k)] \end{bmatrix}$$

and:

$$\begin{bmatrix} \bar{\underline{w}}_v(k) \\ \bar{\underline{w}}_x(k+1) \end{bmatrix} = T_a(k) \begin{bmatrix} \underline{w}_v(k) \\ \underline{w}_x(k) \end{bmatrix}$$

left multiply original eqn by $T_a(k)$

$$\begin{bmatrix} \bar{\xi}_v(k) \\ \bar{\xi}_x(k+1) \end{bmatrix} = \begin{bmatrix} \bar{R}_{vv}(k) & \bar{R}_{vx}(k+1) \\ 0 & \bar{R}_{xx}(k+1) \end{bmatrix} \begin{bmatrix} \underline{v}(k) \\ \underline{x}(k+1) \end{bmatrix} + \begin{bmatrix} \bar{\underline{w}}_v(k) \\ \bar{\underline{w}}_x(k) \end{bmatrix}$$

$$\bar{\xi}_x(k+1) = \bar{R}_{xx}(k+1)\underline{x}(k+1) + \bar{\underline{w}}_x(k)$$

Measurement Update:

$$\underline{z}(k+1) = \underline{h}(k+1, \bar{\underline{x}}(k+1)) + H(k+1)[\underline{x}(k+1) - \bar{\underline{x}}(k+1)] + \underline{w}(k+1)$$

$$\underline{z}_a(k+1) = R_a^{-T}(k+1)\underline{z}(k+1)$$

$$\underline{z}_a(k+1) = R_a^{-T}[\underline{h}(k+1, \bar{\underline{x}}(k+1)) + H(k+1)[\underline{x}(k+1) - \bar{\underline{x}}(k+1)] + \underline{w}(k+1)]$$

$$\begin{bmatrix} \bar{\xi}_x(k+1) \\ \underline{z}_a(k+1) - R_a^{-T}[\underline{h}(k+1, \bar{\underline{x}}(k+1)) + H(k+1)\bar{\underline{x}}(k+1)] \end{bmatrix} = \begin{bmatrix} \bar{R}_{xx}(k+1) \\ R_a^{-T}H(k+1) \end{bmatrix} \underline{x}(k+1) + \dots$$

$$\begin{bmatrix} \bar{\underline{w}}_x(k) \\ R_a^{-T}\underline{w}(k+1) \end{bmatrix}$$

$$\begin{bmatrix} \underline{\xi}_x(k+1) \\ \underline{z}_a(k+1) - \underline{h}_a(k+1, \bar{\underline{x}}(k+1)) - H_a(k+1)\bar{\underline{x}}(k+1) \end{bmatrix} = \dots$$

$$\begin{bmatrix} \bar{R}_{xx}(k+1) \\ H_a(k+1) \end{bmatrix} \underline{x}(k+1) + \begin{bmatrix} \underline{w}_x(k) \\ \underline{w}_a(k+1) \end{bmatrix}$$

$$\underline{h}_a(k+1, \bar{\underline{x}}(k+1)) = R_a^{-T} \underline{h}(k+1, \bar{\underline{x}}(k+1))$$

$$H_a(k+1) = R_a^{-T} H(k+1)$$

$$\underline{w}_a(k+1) = R_a^{-T} \underline{w}(k+1)$$

QR Factorize:

$$\begin{bmatrix} R_{xx}(k+1) \\ 0 \end{bmatrix} = T_b(k+1) \begin{bmatrix} \bar{R}_{xx}(k+1) \\ H_a(k+1) \end{bmatrix}$$

Thus:

$$\begin{bmatrix} \underline{\xi}_x(k+1) \\ \underline{\xi}_r(k+1) \end{bmatrix} = T_b(k+1) \begin{bmatrix} \bar{\underline{\xi}}_x(k+1) \\ \underline{z}_a(k+1) - \underline{h}_a(k+1, \bar{\underline{x}}(k+1)) - H_a(k+1)\bar{\underline{x}}(k+1) \end{bmatrix}$$

And:

$$\begin{bmatrix} \underline{w}_x(k+1) \\ \underline{w}_r(k+1) \end{bmatrix} = T_b(k+1) \begin{bmatrix} \bar{\underline{w}}_x(k+1) \\ \underline{w}_a(k+1) \end{bmatrix}$$

left multiply by $T_b(k+1)$:

$$\begin{bmatrix} \underline{\xi}_x(k+1) \\ \underline{\xi}_r(k+1) \end{bmatrix} = \begin{bmatrix} R_{xx}(k+1) \\ 0 \end{bmatrix} \underline{x}(k+1) + \begin{bmatrix} \underline{w}_x(k+1) \\ \underline{w}_r(k+1) \end{bmatrix}$$

Recapitulating for Filtering:

① Propagate:

$$\begin{bmatrix} \bar{R}_{vv}(k) & \bar{R}_{vx}(k+1) \\ 0 & \bar{R}_{xx}(k+1) \end{bmatrix} = T_a(k) \overbrace{\begin{bmatrix} R_{vv}(k) & 0 \\ -R_{xx}(k)F^{-1}(k)\Gamma(k) & R_{xx}(k)F^{-1}(k) \end{bmatrix}}^{qr()}$$

$$\begin{bmatrix} \xi_v(k) \\ \bar{\xi}_x(k+1) \end{bmatrix} = T_a(k) \begin{bmatrix} 0 \\ \xi_x(k) + R_{xx}(k)\left[F^{-1}(k)\,\underline{f}(k, \hat{\underline{x}}(k), \underline{u}(k), 0) - \hat{\underline{x}}(k)\right] \end{bmatrix}$$

② Measurement Update:

$$\begin{bmatrix} R_{xx}(k+1) \\ 0 \end{bmatrix} = T_b(k+1) \overbrace{\begin{bmatrix} \bar{R}_{xx}(k+1) \\ H_a(k+1) \end{bmatrix}}^{qr()}$$

$$\begin{bmatrix} \xi_x(k+1) \\ \xi_r(k+1) \end{bmatrix} = T_b(k+1) \begin{bmatrix} \bar{\xi}_x(k+1) \\ \underline{z}_a(k+1) - \underline{h}_a(k+1, \bar{\underline{x}}(k+1)) - H_a(k+1)\bar{\underline{x}}(k+1) \end{bmatrix}$$

where:

$$\hat{\underline{x}}(k) = R_{xx}^{-1}(k)\,\underline{\xi}_x(k) \qquad P(k+1) = R_{xx}^{-1}(k+1)R_{xx}^{-T}(k+1)$$

$$\bar{\underline{x}}(k+1) = \bar{R}_{xx}^{-1}(k+1)\,\bar{\underline{\xi}}_x(k+1)$$

$$\hat{\underline{x}}(k+1) = R_{xx}^{-1}(k+1)\,\underline{\xi}_x(k+1)$$

$$F(k) = \frac{d\underline{f}}{d\underline{x}(k)}\bigg|_{[k, \hat{\underline{x}}(k), \underline{u}(k), 0]} \qquad \Gamma(k) = \frac{d\underline{f}}{d\underline{v}(k)}\bigg|_{[k, \hat{\underline{x}}(k), \underline{u}(k), 0]}$$

$$H(k) = \frac{d\underline{h}}{d\underline{x}(k)}\bigg|_{\underline{x}(k)} \qquad H_a(k) = \mathrm{chol}(R(k))^{-T}H(k)$$

$$\underline{h}_a(k) = \mathrm{chol}(R(k))^{-T}\,\underline{h}(k)$$

$$\underline{z}_a(k) = \mathrm{chol}(R(k))^{-T}\,\underline{z}(k)$$