HW4 | Spencer Freeman | AOE 5784 | 11/20/24

BS 4-3) $V_{xx}(k,j) \triangleq E[(\underline{x}(k) - \bar{\underline{x}}(k))(\underline{x}(j) - \bar{\underline{x}}(j))^T]$

in terms of $P_{xx}(j)$ for 4.3.1-14, $k>j$

$$\underline{x}(k+1) = F(k)x(k) + G(k)\underline{u}(k) + \Gamma(k)\underline{v}(k) \quad (4.3.1\text{-}14)$$

① $P_{xx}(k) = E[(\underline{x}(k) - \bar{\underline{x}}(k))(\underline{x}(k) - \bar{\underline{x}}(k))^T]$

suppose $k = j+1$

$$\underline{x}(k) = F(j)\underline{x}(j) + G(j)\underline{u}(j) + \Gamma(j)\underline{v}(j)$$

$$\bar{\underline{x}}(k) = F(j)\bar{\underline{x}}(j) + G(j)\underline{u}(j) + \Gamma(j)\bar{\underline{v}}(j)$$

$$\underline{x}(k) - \bar{\underline{x}}(k) = F(j)[\underline{x}(j) - \bar{\underline{x}}(j)] + \Gamma(j)[\underline{v}(j) - \bar{\underline{v}}(j)]$$

$$V_{xx}(k,j) = E[F(j)(\underline{x}(j) - \bar{\underline{x}}(j))(\underline{x}(j) - \bar{\underline{x}}(j))^T + \ldots$$
$$\Gamma(j)(\underline{v}(j) - \bar{\underline{v}}(j))(\underline{x}(j) - \bar{\underline{x}}(j))^T]$$

Expectation is linear ←

$$V_{xx}(k,j) = F(j)E[(\underline{x}(j) - \bar{\underline{x}}(j))(\underline{x}(j) - \bar{\underline{x}}(j))^T] + \ldots$$
$$\Gamma(j)E[\underline{v}(j) - \bar{\underline{v}}(j))(\underline{x}(j) - \bar{\underline{x}}(j))^T]$$

$$\boxed{V_{xx}(k,j) = F(j)P_{xx}(j) + 0}$$ → process noise is white thus unconelated with current state

② suppose $k >> j$,

Bar Shalom (4.3.2-2)

$$\underline{x}(k) = \left[\prod_{j=0}^{k-1} F(k-1-j)\right]\underline{x}(j) + \ldots$$

$$\sum_{i=0}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right]\left[G(i)\underline{w}(i) + \underline{v}(i)\right]$$

$$\bar{\underline{x}}(k) = \left[\prod_{j=0}^{k-1} F(k-1-j)\right]\bar{\underline{x}}(j) + \ldots$$

$$\sum_{i=0}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right]\left[G(i)\underline{w}(i) + \bar{\underline{v}}(i)\right]$$

$$\underline{x}(k) - \bar{\underline{x}}(k) = \left[\prod_{j=0}^{k-1} F(k-1-j)\right]\left[\underline{x}(j) - \bar{\underline{x}}(j)\right] + \ldots$$

$$\sum_{i=0}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right]\left[\underline{v}(i) - \bar{\underline{v}}(i)\right]$$

$$V_{xx}(k,j) = E\left[\left[\prod_{j=0}^{k-1} F(k-1-j)\right]\left[\underline{x}(j) - \bar{\underline{x}}(j)\right]\left[\underline{x}(j) - \bar{\underline{x}}(j)\right]^T + \ldots\right.$$

$$\left.\sum_{i=0}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right]\left[\underline{v}(i) - \bar{\underline{v}}(i)\right]\left[\underline{x}(j) - \bar{\underline{x}}(j)\right]^T\right]$$

$$V_{xx}(k,j) = \left[\prod_{j=0}^{k-1} F(k-1-j)\right] E\left[(\underline{x}(j) - \bar{\underline{x}}(j))(\underline{x}(j) - \bar{\underline{x}}(j))^T\right] + \ldots$$

$$\sum_{i=0}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right] E\left[(\underline{v}(i) - \bar{\underline{v}}(i))(\underline{x}(j) - \bar{\underline{x}}(j))^T\right]$$

$$\boxed{V_{xx}(k,j) = \left[\prod_{j=0}^{k-1} F(k-1-j)\right] P_{xx}(j) + 0}$$

→ process noise is white $i < k$; all terms uncorrelated