Posting Date: Monday Oct. 21st.

You need not hand in anything. Instead, be prepared to answer any of these problems on an upcoming take-home prelim exam. You may discuss your solutions with classmates up until the time that the prelim becomes available.

Do the following problems from Bar Shalom:

4-1.3 (prove by differentiating eq. 4.2.4-6), 4-3, 4-4, 4-6, 4-7 (include a time-invariant Γ matrix that is different from the identity matrix in your Part 1 derivation in order to make the two parts of this problem consistent.)

Instructions for 4-4.4: The N_e is for a fixed averaging lag window where $y_{fw}(k) = \frac{1}{N_e}\left[\sum_{i=k-N_e+1}^{k} v(i)\right]$. Both $y_{fw}(k)$ and $y(k)$ from the book's problem definition have the advantage of representing, in effect, an average of a number of recent $v(k)$ values. This averaging has the advantage that the steady-state variance of the random errors in $y_{fw}(k)$ and $y(k)$ will be less than the variance of the random errors in the data $v(k)$.

One way to determine the equivalent α for a given N_e is to solve the steady-state Lyapunov equation for σ_{yss}^2 as a function of α and σ_v^2 under the assumption that any random errors in the $v(k)$ data are discrete-time white-noise with variance σ_v^2. One can derive an equivalent σ_{yfw}^2 by analyzing the effects of $v(k)$ random noise components on errors in $y_{fw}(k)$. One can then choose α so that $\sigma_{yfw}^2 = \sigma_{yss}^2$.

An alternate way of choosing α is to match the steady-state errors in $y_{fw}(k)$ and $y(k)$ under the assumption that both moving averages are supposed to track a ramping $v(k)$ time history that has no random errors. In this case, one would want $y_{fw}(k)$ and $y(k)$ to track $v(k)$ exactly, but their averaging processes cause them to lag behind the ramping $v(k)$ time history in steady state. If α is chosen properly, then the steady-state errors $v(k)$- $y(k)$ for very large k and $v(k)$-$y_{fw}(k)$ for very large k will be equivalent. One can find the equivalent α by first determining these steady-state errors under the assumption that $v(k) = k\Delta v$ for a fixed increment Δv. The equivalent α causes the steady-state value of $v(k)$- $y(k)$ to equal the steady-state value of $v(k)$-$y_{fw}(k)$.

Complete part 4-4.4 by determining equivalent α values using both methods.