HW3 P7 Midterm

Test statistic and it's PDF:

$$\underline{P_e} = P^{-1}\underline{e}$$

$$\beta = \underline{z}^T \underline{P_e}$$

$$P.D.F: \quad y = \frac{1}{\sqrt{2\pi}\underline{P_e}^T P \underline{P_e}} e^{-\frac{1}{2}\left(\frac{\beta - \theta \underline{e}^T \underline{P_e}}{\underline{P_e}^T P \underline{P_e}}\right)^2}$$

Code:

```
%% Optimal Neyman-Pearson 2-sided hypothesis test
% Spencer Freeman, 10/21/2024
% AOE 5784, Estimation and Filtering
%
% This script solves number 7 of problem set 3 which is highy related to
% number 1-9 (Bar Shalom) of problem set 1.
% ------------------------------------------------------------------------
clear;clc;close all

disp('HW3-P7_midterm')


%% a

alpha = .01;
P = [1 .5; ...
  .5 2];

Pinv = inv(P);

e = [1; 1];

Pe = Pinv * e;

sig_beta = sqrt(Pe' * P * Pe); % variance of beta
mu_beta  = 0;            % mean of beta

beta0 = -norminv(alpha/2, mu_beta, sig_beta); % threshold value


% create sample measurements and assess the test
thetas =  -10:.01:10;
for i = 1:length(thetas)
  theta = thetas(i);      % signal
  m = 100;%100e3;          % number of samples
  w = mvnrnd([0; 0], P, m)'; % random draw noise terms
  z = theta * e + w;      % noisy samples
  b = z' * Pinv * e;      % test statistic for each sample

  accept_H1 = abs(b) >= beta0; % test hypothesis

  pw_beta(i) = sum(accept_H1) / m; % detection rate (power)
  Power_beta(i) = ...
    normcdf(-beta0,  (theta * e)' * Pinv * e, sig_beta) + ...
    1-normcdf( beta0, (theta * e)' * Pinv * e, sig_beta);

end



%% b

bs = linspace(-5, 10, 500); % beta's to evaluate

sig_beta = sqrt(Pe' * P * Pe); % variance of beta
```

```
mu_beta  = 0;              % mean of beta

y0 = normpdf(bs, mu_beta, sig_beta);

theta1 = 4;
mu_beta  = theta1*e'*Pe;              % mean of beta

y1 = normpdf(bs, mu_beta, sig_beta);



%% plotting
close all

% Be sure to hand in your acquisition test statistic's formula,
% its threshold value, and its probability density functions,
% all with numerical values included where appropriate.

% CDF's of beta and eta
h = figure;
h.WindowStyle = 'Docked';
plot(thetas, pw_beta, 'o', 'Color', "#0072BD"); hold on
plot(thetas, Power_beta, 'LineWidth', 1.5, 'Color', "#D95319")
grid on
title('Part a')
ylabel('Power')
xlabel('Theta')
legend('Observed-Beta', 'Theory-Beta')

% PDF's for beta
h = figure;
h.WindowStyle = 'Docked';
plot(bs, y0, bs, y1)
grid on
title('Part b')

xline(beta0)
ylabel('Probability Density')
xlabel('\beta')
legend('Theta = 0', 'Theta = 4', 'Threshold \beta')

fprintf('\n\tThreshold Beta0: %f\n\t1-Sigma Beta: %f\n', beta0, sig_beta)
```

Output:

HW3-P7_midterm

Threshold Beta0: 2.753677
1-Sigma Beta: 1.069045