P9) $$p(\underline{x}) = \frac{1}{(2\pi)^{n/2}|P|^{1/2}} \exp\left\{-\frac{1}{2}(\underline{x}-\bar{\underline{x}})^T P^{-1}(\underline{x}-\bar{\underline{x}})\right\}$$

$$E[\underline{x}] = \bar{\underline{x}}, \quad E[(\underline{x}-\bar{\underline{x}})(\underline{x}-\bar{\underline{x}})^T] = P, \quad \int p(x)\,dx = 1$$

$$\underline{z} = V^T\underline{x}, \quad V^{-1} = V^T, \quad P = V\,\mathrm{diag}\{\sigma_{z_1}^2 \cdots \sigma_{z_n}^2\}V^T$$

$$\underline{x} = V\underline{z} \qquad P^{-1} = V\,\mathrm{diag}\left\{\frac{1}{\sigma_{z_1}^2}, \cdots \frac{1}{\sigma_{z_n}^2}\right\}V^T$$

$$\bar{\underline{x}} = V\bar{\underline{z}}$$

$$P = VP_zV^T, \quad P^{-1} = VP_z^{-1}V^T$$

$$p(V\underline{z}) = \frac{1}{(2\pi)^{n/2}|VP_zV^T|^{1/2}} \exp\left\{-\frac{1}{2}(V(\underline{z}-\bar{\underline{z}}))^T VP_z^{-1}V^T(V(\underline{z}-\bar{\underline{z}}))\right\}$$

$$p(V\underline{z}) = \frac{1}{(2\pi)^{n/2}|P_z|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\underline{z}-\bar{\underline{z}})^T P_z^{-1}(\underline{z}-\bar{\underline{z}})\right\}$$

$$p(V\underline{z}) = \prod_{j=1}^{n} \frac{1}{(2\pi)^{1/2}\sigma_{z_j}} \exp\left\{-\frac{1}{2\sigma_{z_j}}(z_j-\bar{z}_j)^2\right\}$$

$$E[\underline{x}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \tilde{\underline{x}}\,p(\tilde{\underline{x}})\,d\tilde{x}_n \ldots d\tilde{x}_2\,d\tilde{x}_1 \qquad \underline{z} = V^T\underline{x}$$

$$E[\underline{x}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \frac{V\tilde{\underline{z}}\,p(V\tilde{\underline{z}})}{|\det(V^T)|}\,d\tilde{z} \ldots d\tilde{z}_2\,d\tilde{z}_1$$

$$\boxed{E[\underline{x}] = V\int_{\tilde{\underline{z}}} \tilde{\underline{z}}\,p(V\tilde{\underline{z}})\,d\tilde{\underline{z}} = V\bar{\underline{z}} = \bar{\underline{x}}}$$

$$\mathrm{cov}(\underline{x}) = \int_{\tilde{\underline{x}}} (\tilde{\underline{x}}-\bar{\underline{x}})(\tilde{\underline{x}}-\bar{\underline{x}})^T p(\tilde{\underline{x}})\,d\tilde{\underline{x}}$$

$* \; P_z = \mathrm{diag}\{\sigma_{z_1}^2 \cdots \sigma_{z_n}^2\}$

$$\boxed{\mathrm{cov}(\underline{x}) = \int_{\tilde{\underline{x}}} \frac{V(\tilde{\underline{z}}-\bar{\underline{z}})(\tilde{\underline{z}}-\bar{\underline{z}})^T V^T p(\tilde{\underline{z}})\,d\tilde{\underline{z}}}{|\det(V^T)|} = VP_zV^T = P}$$

$$\int_{\tilde{x}} p(\tilde{x})\,d\tilde{x} = \int_{\tilde{z}} \frac{p(V\tilde{z})\,d\tilde{z}}{|\det(V^T)|} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (1)\,dz_2\,dz_1 \cdots = 1$$