BS 5-5) ① Prove the Joseph form cov. update holds for arbitrary gain @ time k+1

$$P(k+1|k+1) = [I - W(k+1)H(k+1)]P(k+1|k)[I - W(k+1)H(k+1)]^T + \ldots$$
$$W(k+1)R(k+1)W^T(k+1) \qquad (5.2.3-18)$$

$$P(k+1|k+1) = E[\underline{\tilde{x}}(k+1|k+1)\underline{\tilde{x}}(k+1|k+1)^T]$$

$$\underline{\tilde{x}}(k+1|k+1) = \underline{x}(k+1) - \underline{\hat{x}}(k+1|k+1)$$

$$\underline{\tilde{x}}(k+1|k) = \underline{x}(k+1) - \underline{\hat{x}}(k+1|k)$$

$$\underline{\tilde{x}}(k+1|k+1) = \underline{x}(k+1) - (\underline{\hat{x}}(k+1|k) + W(k+1)\underline{\nu}(k+1))$$

$$\underline{\tilde{x}}(k+1|k+1) = \underline{x}(k+1) - \underline{\hat{x}}(k+1|k) - W(k+1)[H(k+1)\underline{x}(k+1) + \ldots$$
$$\underline{w}(k+1) + H(k+1)\underline{\hat{x}}(k+1|k)]$$

$$\underline{\tilde{x}}(k+1|k+1) = [I - W(k+1)H(k+1)][\underline{x}(k+1) - \underline{\hat{x}}(k+1|k)] + W(k+1)\underline{w}(k+1)$$

$$\underline{\tilde{x}}(k+1|k+1) = [I - W(k+1)H(k+1)]\underline{\tilde{x}}(k+1|k) + W(k+1)\underline{w}(k+1)$$

$$P(k+1|k+1) = E[((I - W(k+1)H(k+1))\underline{\tilde{x}}(k+1|k) + W(k+1)\underline{w}(k+1))($$
$$(I - W(k+1)H(k+1))\underline{\tilde{x}}(k+1|k) + W(k+1)\underline{w}(k+1))^T]$$

$$P(k+1|k+1) = E[(I - W(k+1)H(k+1))\underline{\tilde{x}}(k+1|k)\underline{\tilde{x}}^T(k+1|k)(I - W(k+1)H(k+1))^T] + \ldots$$

$$E[W(k+1)\underline{w}(k+1)\underline{w}^T(k+1)W^T(k+1)]$$

$$\rightarrow E[\underline{\tilde{x}}(k+1|k)\underline{w}^T(k+1)] = 0,$$ measurement noise uncorrelated with current state

$$P(k+1|k+1) = (I - W(k+1)H(k+1))\, E[\tilde{x}(k+1|k)\tilde{x}^T(k+1|k)]\, (\ldots$$
$$I - W(k+1)H(k+1))^T + \ldots$$
$$W(k+1)\, E[\, \underline{w}(k+1)\, \underline{w}^T(k+1)\,]\, W(k+1) \rightarrow$$ expectation is linear

$$\boxed{\begin{aligned} P(k+1|k+1) &= (I - W(k+1)H(k+1))\, P(k+1|k)\, (I - W(k+1)H(k+1))^T + \ldots \\ &\quad W(k+1)\, R(k+1)\, W^T(k+1) \quad \checkmark \end{aligned}}$$

$W(k+1)$ arbitrary gain matrix, no assumptions