HWI | Spencer Freeman | 9/15/24

1-7) $\underline{x} \in \mathbb{R}^{n_x}$, $E[\underline{x}] = \bar{\underline{x}}$, $cov(\underline{x}) = P_{xx}$

$\underline{y} \in \mathbb{R}^{n_y}$, $E(\underline{y}) = \bar{\underline{y}}$, $cov(\underline{y}) = P_{yy}$

$\underline{z} = A\underline{x} + B\underline{y} + \underline{c}$

$\underline{z} \in \mathbb{R}^{n_z}$, $E[\underline{z}] = \bar{\underline{z}}$, $cov(\underline{z}) = P_{zz}$

① $E[\underline{z}] = E[A\underline{x} + B\underline{y} + \underline{c}] = E[A\underline{x}] + E[B\underline{y}] + E[\underline{c}]$

$E[\underline{z}] = \underbrace{A\,E[\underline{x}] + B\,E[\underline{y}]}_{\text{linearity of expectation}} + \underline{c}$ } $\underline{c}$: constant vector

$E[z] = \boxed{A\bar{\underline{x}} + B\bar{\underline{y}} + \underline{c} = \bar{\underline{z}}}$

$cov(\underline{z}) = P_{zz} = E[(\underline{z} - \bar{\underline{z}})(\underline{z} - \bar{\underline{z}})^T] =$

$P_{zz} = E[(A\underline{x} + B\underline{y} + \underline{c} - A\bar{\underline{x}} - B\bar{\underline{y}} - \underline{c})(\dots)^T] =$

$P_{zz} = E[(A(\underline{x} - \bar{\underline{x}}) + B(\underline{y} - \bar{\underline{y}})(\dots)^T] =$

$P_{zz} = E[A(\underline{x} - \bar{\underline{x}})(A(\underline{x} - \bar{\underline{x}}))^T + A(\underline{x} - \bar{\underline{x}})(B(\underline{y} - \bar{\underline{y}}))^T + \dots$
$B(\underline{y} - \bar{\underline{y}})(A(\underline{x} - \bar{\underline{x}}))^T + B(\underline{y} - \bar{\underline{y}})(B(\underline{y} - \bar{\underline{y}}))^T]$

linearity of expectation

$P_{zz} = E[A(\underline{x} - \bar{\underline{x}})(\underline{x} - \bar{\underline{x}})^T A^T + A(\underline{x} - \bar{\underline{x}})(\underline{y} - \bar{\underline{y}})^T B^T + \dots$
$B(\underline{y} - \bar{\underline{y}})(\underline{x} - \bar{\underline{x}})^T A^T + B(\underline{y} - \bar{\underline{y}})(\underline{y} - \bar{\underline{y}})^T B^T]$

$$\boxed{P_{zz} = A P_{xx} A^T + A P_{xy} B^T + B P_{yx} A^T + B P_{yy} B^T}$$

② $\boxed{A \in \mathbb{R}^{n_z \times n_x},\ B \in \mathbb{R}^{n_z \times n_y},\ \underline{c} \in \mathbb{R}^{n_z}}$