4-7) ① Simplify (4.3.3-1) for a time-invariant system, that is, $F(i)=F$, $G(i)=G$.

$$\underline{x}(k)=\left[\prod_{j=0}^{k-l-1} F(k-1-j)\right]\underline{x}(l)+\sum_{i=l}^{k-1}\left[\prod_{j=0}^{k-i-2} F(k-1-j)\right]\cdot\ldots$$

$$[G(i)\underline{u}(i)+\underline{v}(i)]$$

(4.3.3-1)

② Find a close-form solution, similar to 4.3.3-1, for the covariance 4.3.4-7 assuming $F(k)=F$, $\Gamma(k)=\Gamma$, $Q(k)=Q$.

$$P_{xx}(k+1)=F(k)P_{xx}(k)F(k)'+\Gamma(k)Q(k)\Gamma(k)'$$

(4.3.4-7)

①
$$\prod_{j=0}^{k-l-1} F(k-1-j)=F(k-1-(0))F(k-1-(1))\cdot\ldots F(k-1-(k-l-1))$$

$$F=\text{¢}: \hookrightarrow = F^{k-l}$$

$$\prod_{j=0}^{k-i-2} F(k-1-j)=F(k-1-(0))\cdot\ldots F(k-1-(k-i-2))$$

$$F=\text{¢}: \hookrightarrow = F^{k-i-1}$$

$$\sum_{i=l}^{k-1} F^{k-i-1}=\underbrace{F^{k-l-1}+F^{\overbrace{k-(l+1)-1}^{k-l-2}}+\ldots F^{\overbrace{k-(k-1)-1}^{0}}}_{\text{geometric series } i=0,1,2\ldots k-l-1}$$

$\downarrow$

$$\sum_{i=l}^{k-1} F^{k-i-1}=(I-F)^{-1}(I-F^{k-l-1})$$

$$\boxed{\underline{x}(k)=F^{k-l}\underline{x}(l)+(I-F)^{-1}(I-F^{k-l-1})[G\underline{u}(i)+\underline{v}(i)]}$$

② $P_{xx}(k+1) = F P_{xx}(k) F^T + \Gamma Q \Gamma^T$

$$P_{xx}(\overbrace{k+1-1}^{k}) = F P_{xx}(k-1) F^T + \Gamma Q \Gamma^T$$

$$P(k+1) = F[F P(k-1) F^T + \Gamma Q \Gamma^T] F^T + \Gamma Q \Gamma^T$$

$$P(\overbrace{k+1}^{k}) = F^2 P(\overbrace{k-1}^{l}) F^{T^2} + F^1 \overbrace{\Gamma Q \Gamma^T}^{T} F^{T^1} + \overbrace{\Gamma Q \Gamma^T}^{T}$$

$$\boxed{P(k) = F^{k-l} P(l) F^{T^{k-l}} + \sum_{i=0}^{k-l-1} F^i T F^{T^i}}$$