HW3 | Spencer Freeman | AOE 5784 | 12/17/24

(8)

From BS 1-9:

$\beta = \underline{z}^T P^{-1} \underline{e}$, accept H_1 if $|\beta| \geq \beta_0$

$$\underbrace{q(\underline{z}, \theta_1)}_{\text{likelihood ratio}} = \frac{1}{2}\underline{z}^T P^{-1}\underline{z} - \frac{1}{2}(\underline{z} - \theta_1 \underline{e})^T P^{-1}(\underline{z} - \theta_1 \underline{e})$$

alternate:

$p(\underline{z}|H_1) = p[\underline{z} | \hat{\theta}_1(\underline{z})]$, $\hat{\theta}_1(\underline{z})$ is optimal estimate of θ_1, $\hat{\theta}_1(\underline{z})$ **maximizes** $p(\underline{z}|\theta_1)$

$$\frac{dq}{d\theta_1} = 0$$

$$q(\underline{z}, \theta_1) = \theta_1 \underline{z}^T P^{-1}\underline{e} - \frac{\theta_1^2}{2}\underline{e}^T P^{-1}\underline{e}$$

$$\frac{dq}{d\theta_1} = \underline{z}^T P^{-1}\underline{e} - \theta_1 \underline{e}^T P^{-1}\underline{e} = 0, \quad \hat{\theta}_1 = \frac{\underline{z}^T P^{-1}\underline{e}}{\underline{e}^T P^{-1}\underline{e}}$$

$$q(\underline{z}, \hat{\theta}) = \frac{\underline{z}^T P^{-1}\underline{e}}{\underline{e}^T P^{-1}\underline{e}}(\underline{z}^T P^{-1}\underline{e}) - \left(\frac{\underline{z}^T P^{-1}\underline{e}}{\underline{e}^T P^{-1}\underline{e}}\right)^2 \frac{\underline{e}^T P^{-1}\underline{e}}{2}$$

$$q(\underline{z}, \hat{\theta}_1) = \frac{(\underline{z}^T P^{-1}\underline{e})^2}{\underline{e}^T P^{-1}\underline{e}} - \frac{(\underline{z}^T P^{-1}\underline{e})^2}{\underline{e}^T P^{-1}\underline{e}}\frac{1}{2} = \frac{1}{2}\frac{(\underline{z}^T P^{-1}\underline{e})^2}{\underline{e}^T P^{-1}\underline{e}}$$

$$q(\underline{z}, \hat{\theta}_1) = \frac{1}{2}\frac{\beta^2}{\underline{e}^T P^{-1}\underline{e}}, \quad \text{accept } H_1 \text{ if } q(\underline{z}, \hat{\theta}_1) > q_0$$

$$q_0 = \frac{1}{2}\frac{\beta_0^2}{\underline{e}^T P^{-1}\underline{e}}$$

$$\boxed{(\underline{e}^T P^{-1}\underline{z})^2 \geq \beta_0^2 \leftrightarrow \beta^2 \geq \beta_0^2 \qquad |\beta| \geq \beta_0}$$