P6)

$$J(\underline{x},k) = \sum_{j=1}^{k} (\underline{z}_j - H_j \underline{x})^T R_j^{-1} (\underline{z}_j - H_j \underline{x})$$

$$J(\underline{x},k) = \sum_{j=1}^{k} \underline{z}_j^T R_j^{-1} \underline{z}_j - \underline{z}_j^T R_j^{-1} H_j \underline{x} - \underline{x}^T H_j^T R_j^{-1} \underline{z}_j + \underline{x}^T H_j^T R_j^{-1} H_j \underline{x}$$

$$J(\underline{x},k) = \sum_{j=1}^{k} \underline{x}^T H_j^T R_j^{-1} H_j \underline{x} - 2\underline{z}_j^T R_j^{-1} H_j \underline{x} + \underline{z}_j^T R_j^{-1} \underline{z}_j$$

define: $R = \begin{bmatrix} R_1 & \cdots & 0 \\ \vdots & R_2 & \vdots \\ 0 & \cdots & R_k \end{bmatrix}$, $H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_k \end{bmatrix}$, $\underline{z} = \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \vdots \\ \underline{z}_k \end{bmatrix}$

$$\sum_{j=1}^{k} \underline{z}_j^T R_j^{-1} \underline{z}_j = \underline{z}^T R^{-1} \underline{z}, \quad \sum_{j=1}^{k} -2\underline{z}_j^T R_j^{-1} H_j \underline{x} = -2\underline{z}^T R^{-1} H \underline{x}$$

$$\sum_{j=1}^{k} \underline{x}^T H_j^T R_j^{-1} H_j \underline{x} = \underline{x}^T H^T R^{-1} H \underline{x}$$

①

redefined: $J(\underline{x},k) = \underline{x}^T (H^T R^{-1} H) \underline{x} - 2(\underline{z}^T R^{-1} H) \underline{x} + (\underline{z}^T R^{-1} \underline{z})$

$\updownarrow$ equal

prove equal: $J_p(\underline{x},k) = (\underline{x} - \hat{\underline{x}}(k,\underline{z}^k))^T \hat{P}^{-1}(k,\underline{z}^k)(\underline{x} - \hat{\underline{x}}(k,\underline{z}^k)) + J(\hat{x}(k,\underline{z}^k),k)$

$$J_p(\underline{x},k) = \underline{x}^T \hat{P}^{-1} \underline{x} - \underline{x}^T \hat{P}^{-1} \hat{\underline{x}} - \hat{\underline{x}}^T \hat{P}^{-1} \underline{x} + \hat{\underline{x}}^T \hat{P}^{-1} \hat{\underline{x}} + J(\hat{x},k)$$

$$J_p(\underline{x},k) = \underline{x}^T \hat{P}^{-1} \underline{x} - 2\hat{\underline{x}}^T \hat{P}^{-1} \underline{x} + \hat{\underline{x}}^T \hat{P}^{-1} \hat{\underline{x}} + J(\hat{\underline{x}},k)$$

$$\hat{\underline{x}} = (H^T R^{-1} H)^{-1} H^T R^{-1} \underline{z} = \hat{P} H^T R^{-1} \underline{z}, \quad \hat{P} = (H^T R^{-1} H)^{-1}$$

$$\hat{\underline{x}}^T \hat{P}^{-1} \hat{\underline{x}} = (\hat{P} H^T R^{-1} \underline{z})^T \overbrace{\hat{P}^{-1} \hat{P}}^{I} H^T R^{-1} \underline{z}$$

$$= \underline{z}^T R^{-1T} H \hat{P}^T H^T R^{-1} \underline{z}$$

$$= \underline{z}^T R^{-1T} \overbrace{H H^{-1}}^{I,\ I} R^T \overbrace{H^{-1T} H^T}^{I} R^{-1} \underline{z}$$

$$\hat{\underline{x}}^T \hat{P}^{-1} \hat{\underline{x}} = \underline{z}^T R^{-1} \underline{z}$$

$$\hat{\underline{x}}^T \hat{P}^{-1} = (\hat{P} H^T R^{-1} \underline{z})^T \hat{P}^{-1}$$

$$= (H^T R^{-1} \underline{z})^T \overbrace{\hat{P}^T \hat{P}^{-1}}^{I \text{ (symmetric)}}$$

$$\hat{\underline{x}}^T \hat{P}^{-1} = \underline{z}^T R^{-1} H$$

$$\hat{P}^{-1} = H^T R^{-1} H$$

$$J_p(\underline{x}, k) = \underline{x}^T (H^T R^{-1} H) \underline{x} - 2(\underline{z}^T R^{-1} H) \underline{x} + (\underline{z}^T R^{-1} \underline{z}) + J(\hat{\underline{x}}, k)$$

refer to 1 to prove coefficients equality.