HW6	Spencer Freeman	AOE 5784	11/18/24

3)

$$\underline{x}^*(k) = F^{-1}(k)\left[\underline{x}^*(k+1) - G(k)\underline{u}(k) - \Gamma(k)\underline{v}^*(k)\right]$$

equivalent

$$\textcircled{A}\quad \underline{x}^*(k) = \hat{\underline{x}}(k) + P(k)F^T(k)\bar{P}^{-1}(k+1)\left[\underline{x}^*(k+1) - \bar{\underline{x}}(k+1)\right]$$

$$F^{-1}(k)\left[\underline{x}^*(k+1) - G(k)\underline{u}(k) - \Gamma(k)\underline{v}^*(k)\right] - F^{-1}(k)F(k)\hat{\underline{x}}(k) + \hat{\underline{x}}(k)$$

$$F^{-1}(k)\left[\underline{x}^*(k+1) - G(k)\underline{u}(k) - \Gamma(k)\underline{v}^*(k) - F(k)\hat{\underline{x}}(k)\right] + \hat{\underline{x}}(k)$$

$$\underline{v}^*(k) = Q(k)\Gamma^T(k)\bar{P}^{-1}(k+1)\left[\underline{x}^*(k+1) - \bar{\underline{x}}(k+1)\right]$$

↳ MAP Kalman Filter derivation

$$-F(k)\hat{\underline{x}}(k) - G(k)\underline{u}(k) = -\bar{\underline{x}}(k+1)$$ → state mean propagation

$$F^{-1}(k)\left[\underline{x}^*(k+1) - \bar{\underline{x}}(k+1) - \Gamma(k)Q(k)\Gamma^T(k)\bar{P}^{-1}(k+1)(\underline{x}^*(k+1) - \bar{\underline{x}}(k+1))\right] + \hat{\underline{x}}(k)$$

$$F^{-1}(k)\left[(I - \Gamma(k)Q(k)\Gamma^T(k)\bar{P}^{-1}(k+1))(\underline{x}^*(k+1) - \bar{\underline{x}}(k+1))\right] + \hat{\underline{x}}(k)$$

$$\bar{P}(k+1) = F(k)P(k)F^T(k) + \Gamma(k)Q(k)\Gamma^T(k)$$

↳ Kalman filter covariance propagation

$$I - (\bar{P}(k+1) - F(k)P(k)F^T(k))\bar{P}^{-1}(k+1) = +F(k)P(k)F^T(k)\bar{P}^{-1}(k+1)$$

$$F^{-1}(k)\left[F(k)P(k)F^T(k)\bar{P}^{-1}(k+1)\right](\underline{x}^*(k+1) - \bar{\underline{x}}(k+1)) + \hat{\underline{x}}(k)$$

$$\boxed{P(k)F^T(k)\bar{P}^{-1}(k+1)\left[\underline{x}^*(k+1) - \bar{\underline{x}}(k+1)\right] + \hat{\underline{x}}(k) = \underline{x}^*(k) = \textcircled{A}}$$