**HW3 P7 Midterm**

**Test statistic and it’s PDF:**

**Code:**

%% Optimal Neyman-Pearson 2-sided hypothesis test

% Spencer Freeman, 10/21/2024

% AOE 5784, Estimation and Filtering

%

% This script solves number 7 of problem set 3 which is highy related to

% number 1-9 (Bar Shalom) of problem set 1.

% -------------------------------------------------------------------------

clear;clc;close all

disp('HW3-P7\_midterm')

%% a

alpha = .01;

P = [1 .5; ...

.5 2];

Pinv = inv(P);

e = [1; 1];

Pe = Pinv \* e;

sig\_beta = sqrt(Pe' \* P \* Pe); % variance of beta

mu\_beta = 0; % mean of beta

beta0 = -norminv(alpha/2, mu\_beta, sig\_beta); % threshold value

% create sample measurements and assess the test

thetas = -10:.01:10;

for i = 1:length(thetas)

theta = thetas(i); % signal

m = 100;%100e3; % number of samples

w = mvnrnd([0; 0], P, m)'; % random draw noise terms

z = theta \* e + w; % noisy samples

b = z' \* Pinv \* e; % test statistic for each sample

accept\_H1 = abs(b) >= beta0; % test hypothesis

pw\_beta(i) = sum(accept\_H1) / m; % detection rate (power)

Power\_beta(i) = ...

normcdf(-beta0, (theta \* e)' \* Pinv \* e, sig\_beta) + ...

1-normcdf( beta0, (theta \* e)' \* Pinv \* e, sig\_beta);

end

%% b

bs = linspace(-5, 10, 500); % beta's to evaluate

sig\_beta = sqrt(Pe' \* P \* Pe); % variance of beta

mu\_beta = 0; % mean of beta

y0 = normpdf(bs, mu\_beta, sig\_beta);

theta1 = 4;

mu\_beta = theta1\*e'\*Pe; % mean of beta

y1 = normpdf(bs, mu\_beta, sig\_beta);

%% plotting

close all

% Be sure to hand in your acquisition test statistic's formula,

% its threshold value, and its probability density functions,

% all with numerical values included where appropriate.

% CDF's of beta and eta

h = figure;

h.WindowStyle = 'Docked';

plot(thetas, pw\_beta, 'o', 'Color', "#0072BD"); hold on

plot(thetas, Power\_beta, 'LineWidth', 1.5, 'Color', "#D95319")

grid on

title('Part a')

ylabel('Power')

xlabel('Theta')

legend('Observed-Beta', 'Theory-Beta')

% PDF's for beta

h = figure;

h.WindowStyle = 'Docked';

plot(bs, y0, bs, y1)

grid on

title('Part b')

xline(beta0)

ylabel('Probability Density')

xlabel('\beta')

legend('Theta = 0', 'Theta = 4', 'Threshold \beta')

fprintf('\n\tThreshold Beta0: %f\n\t1-Sigma Beta: %f\n', beta0, sig\_beta)

**Output:**

HW3-P7\_midterm

Threshold Beta0: 2.753677

1-Sigma Beta: 1.069045

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