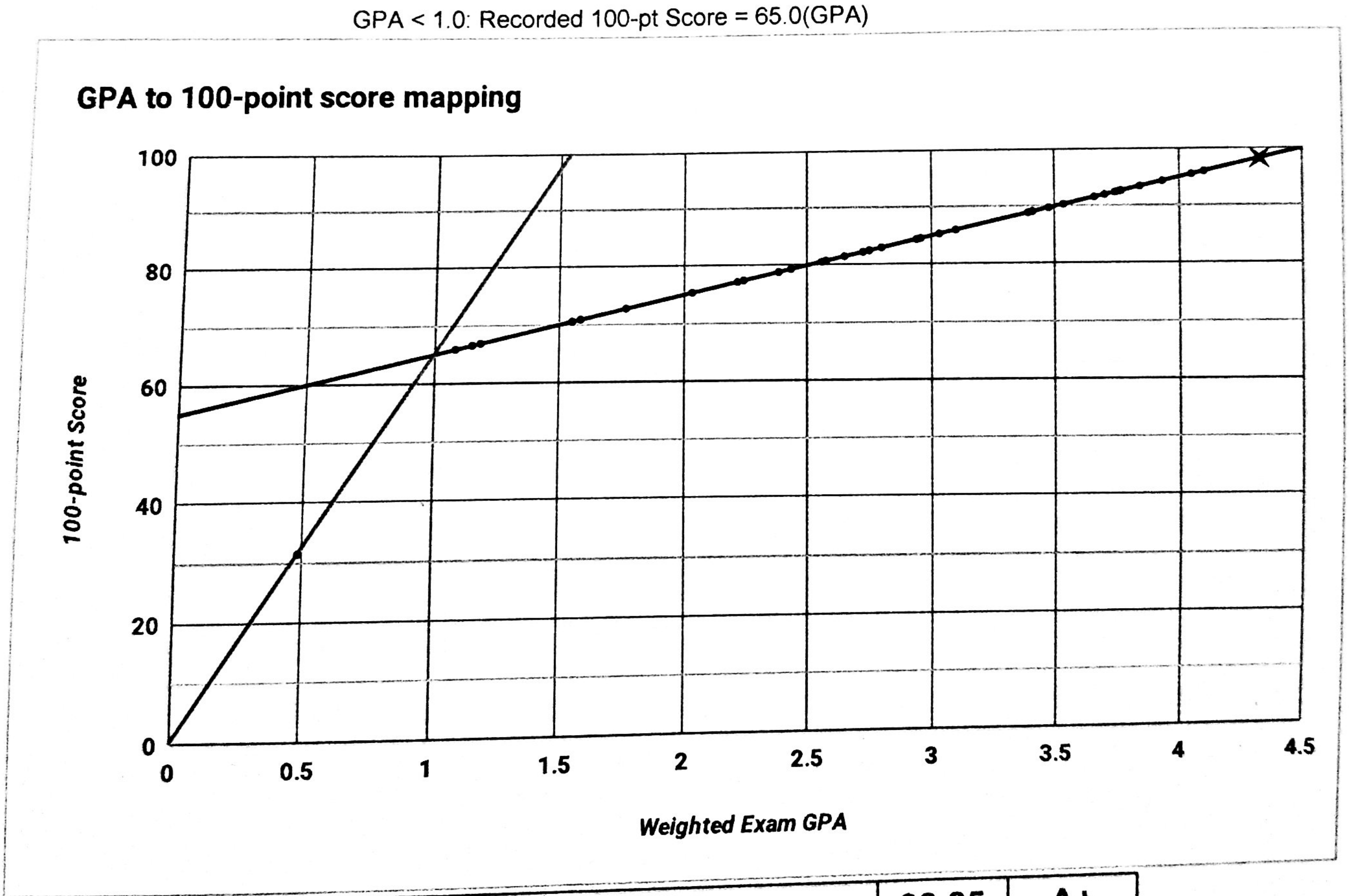
Goulette,Spencer J

	Problem Grade	Grade Points	Weight
Problem 1	A	4.00	15%
Problem 2	A+	4.50	10%
Problem 3	A	4.00	20%
Problem 4	A+	4.50	15%
Problem 5	A+	4.50	20%
Problem 6	A+	4.50	20%
Weighted Exam GPA:		4.33	

GPA > 1.0: Recorded 100-pt Score = 65.0+10.0(GPA-1.0)

GPA < 1.0: Recorded 100-pt Score = 65.0(GPA)



Recorded 100-point max score:	98.25	A+

(90/80/70/60 Scale)

Class Average GPA: 2.89

Class Average 100-pt score: 83.17

ECE-314 Test 2: Oct 30, 2018

2 Hours; Closed book;

Allowed calculator models: (a) Casio fx-115 or fx-991 models (b) HP33s and HP 35s (c) TI-30X and TI-36X models. Calculators not included in this list are not permitted.

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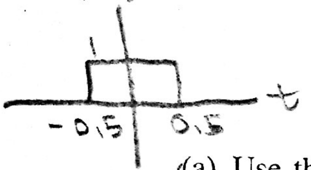
p(t) 1. (15%) In class we proved the transform pair provided below:



$$p(t) = \begin{cases} 1 & |t| < 0.5 \\ 0 & \text{elsewhere} \end{cases} \qquad P(f) = \frac{\sin(\pi f)}{\pi f}$$

√ (a) Use the time shift and time scale properties of the Fourier transform to find $X(f)$, where

$$x(t) = 3p(2t - 10) + 6p(10(t - 0.2))$$

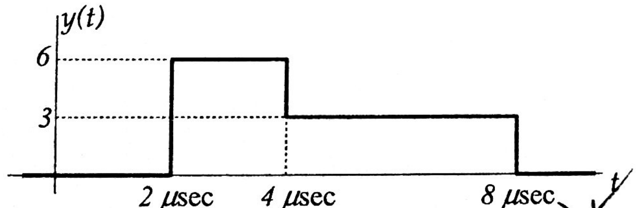
$$x(t) = 3p(2(t-5)) + 6p(10(t-0.2))$$

$$X(f) = 3\cdot\frac{1}{2}P(f/2)e^{-j2\pi f(5)} + 6\cdot\frac{1}{10}P(f/10)e^{-j2\pi f(0.2)}$$

$$X(f) = \frac{3}{2}P(f/2)e^{-j10\pi f} + \frac{6}{10}P(f/10)e^{-j0.4\pi f}$$

$$\boxed{X(f) = \frac{3}{2}\,\frac{\sin(\frac{\pi f}{2})}{\pi f/2}e^{-j10\pi f} + \frac{6}{10}\,\frac{\sin(\frac{\pi f}{10})}{\pi f/10}e^{-j0.4\pi f}}$$ ✓

√ (b) Use the time shift and time scale properties of the Fourier transform to find $Y(f)$, where $y(t)$ is illustrated in the plot below.



$$y(t) = 6p(\tfrac{1}{2}\cdot 10^6(t - 3\cdot 10^{-6})) + 3p(\tfrac{1}{4}\cdot 10^6(t - 6\cdot 10^{-6}))$$ ✓

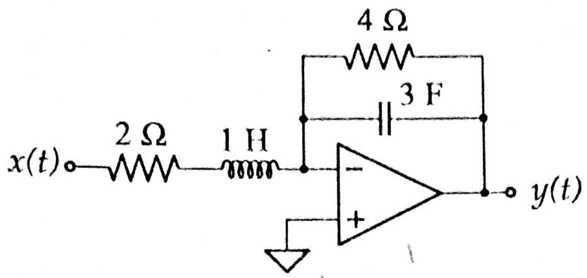
$$Y(f) = 6\cdot\frac{1}{\frac{1}{2}\cdot 10^6}P(f/\tfrac{1}{2}\cdot 10^6)e^{-j2\pi f(3\cdot 10^{-6})} + 3\cdot\frac{1}{\frac{1}{4}\cdot 10^6}P(f/\tfrac{1}{4}\cdot 10^6)e^{-j2\pi f\, 6\cdot 10^{-6}}$$

$$Y(f) = 12\cdot 10^{-6}P(\frac{f}{500\cdot 10^{3}})e^{-j6\cdot 10^{-6}\pi f} + 12\cdot 10^{-6}P(\frac{f}{250\cdot 10^{3}})e^{-j12\cdot 10^{-6}\pi f}$$

$$Y(f) = 12\cdot 10^{-6}\frac{\sin(\frac{\pi f}{500\cdot 10^{3}})}{\frac{\pi f}{500\cdot 10^{3}}}e^{-j6\cdot 10^{-6}\pi f} + 12\cdot 10^{-6}\frac{\sin(\frac{\pi f}{250\cdot 10^{3}})}{\frac{\pi f}{250\cdot 10^{3}}}e^{-j12\cdot 10^{-6}\pi f}$$

A

2. (10%) Find the system gain $H(s)$ for the circuit illustrated below.



$$H(s) = \frac{-1}{2 + Ls} \cdot \frac{1}{\frac{1}{4} + Cs}$$

$$H(s) = \frac{-1}{2 + Ls} \cdot \frac{1}{\frac{1+4Cs}{4}}$$

$$H(s) = \frac{-1}{2 + Ls} \cdot \frac{4}{1+4Cs}$$

$$H(s) = \frac{-4}{(2+Ls)(1+4Cs)}$$

$$H(s) = \frac{-4}{(2+s)(1+12s)}$$

$$\boxed{H(s) = \frac{-4}{12s^2+25s+2}}$$

A+

3. (20%) A similar (but not identical) circuit to that of problem 2 has gain given by

$$H(s) = \frac{10}{1 + 5s + 4s^2}$$

In the following, please simplify any results to give a real expression:

(a) Find the DC gain of the circuit.

(b) Evaluate the impulse response of the circuit.

(c) Find the system output $y(t)$ when the input signal is $x(t) = 1 + \cos(t/2)$.

3. a. $H(0) = \frac{10}{1+5(0)+4(0)}$

$\boxed{H(0) = 10\,\frac{V}{V}}$

b. $10\,\frac{1}{1+5s+4s^2} \Rightarrow 10\,\frac{1}{(4s+1)(s+1)}$

$10\left[\frac{A}{(4s+1)} + \frac{B}{(s+1)}\right]$

$1 = A(s+1) + B(4s+1)$

$0 = A + 4B$

$A = -4B$

$1 = -4B + B$

$1 = -3B$

$B = -\frac{1}{3} \Rightarrow A = \frac{4}{3}$

$H(s) = 10\left[\frac{4/3}{4s+1} + \frac{-\frac{1}{3}}{s+1}\right] = 10\left[\frac{4/12}{s+1/4} + \frac{-1/3}{s+1}\right]$

$\boxed{h(t) = 10\left[4/12\, e^{-1/4t} - 1/3\, e^{-t}\right]}$

c. $x(t) = 1 + \cos(t/2)$

$H(0) = 10\ V/V$

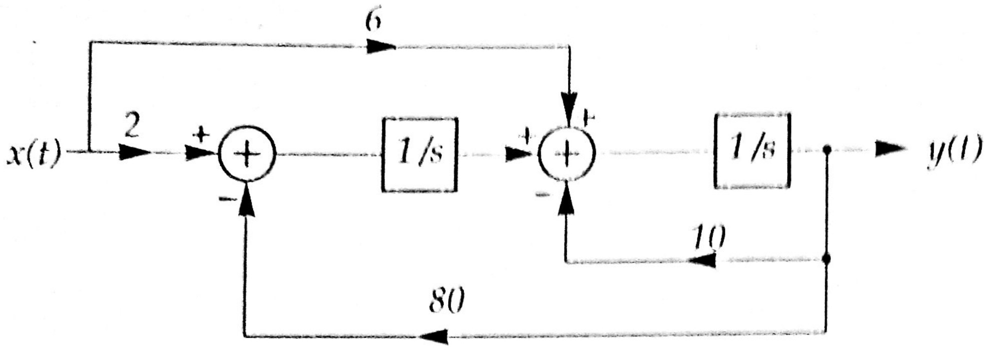
$H(j/2) = \frac{10}{1 + j\frac{5}{2} + -4/2} = \frac{10}{-1 + j\frac{5}{2}}$

$y(t) = 10 + \frac{10}{-1 + j\frac{5}{2}} \cos(t/2)$ complex?

$y(t) = 10 + 3.71 \cos(t/2 - 1.19)$

A

√ 4. (15%) Find the system gain $H(s)$ for the circuit illustrated below.



$$Y(s) = \frac{1}{s}\left(6X(s) - 10Y(s) + \frac{1}{s}(2X(s) - 80\right.$$

$$Y(s) = \frac{6}{s}X(s) - \frac{10}{s}Y(s) + \frac{2}{s^2}X(s) - \frac{80}{s^2}Y(s)$$

$$Y(s) + \frac{10}{s}Y(s) + \frac{80}{s^2}Y(s) = \frac{6}{s}X(s) + \frac{2}{s^2}X(s)$$

$$Y(s)\left[1 + \frac{10}{s} + \frac{80}{s^2}\right] = X(s)\left[\frac{6}{s} + \frac{2}{s^2}\right]$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{\frac{6}{s} + \frac{2}{s^2}}{1 + \frac{10}{s} + \frac{80}{s^2}} \cdot \frac{s^2}{s^2}$$

$$\boxed{H(s) = \frac{6s + 2}{s^2 + 10s + 80}}$$

A+

✓ 5. (20%) A system is governed by differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13y(t) = \frac{dx}{dt} + 6x(t)$$

(a) Find the system gain function $H(s)$ for this system.

(b) Evaluate the impulse response of the system.

a. $H(s)s^2\cancel{e^{st}} + 6H(s)s\cancel{e^{st}} + 13H(s)\cancel{e^{st}} = s\cancel{e^{st}} + 6\cancel{e^{st}}$

$H(s)\left[s^2+6s+13\right] = s+6$

$$\boxed{H(s) = \frac{s+6}{s^2+6s+13}}$$

b. $H(s) = \dfrac{s+6}{(s+3)^2+4}$

$H(s) = \dfrac{s+3}{(s+3)^2+4} + \dfrac{3}{(s+3)^2+4}$

$H(s) = \dfrac{s+3}{(s+3)^2+4} + \dfrac{2\cdot 3/2}{(s+3)^2+4}$

$H(s) = \dfrac{s+3}{(s+3)^2+4} + \dfrac{3}{2}\,\dfrac{2}{(s+3)^2+4}$

Laplace

$$\boxed{h(t) = e^{-3t}\cos(2t)u(t) + \frac{3}{2}e^{-3t}\sin(2t)u(t)}$$

A+

6. (20%) In the following, assume that $x(t)$ has units of volts. Include appropriate units for all answers. The system of problem 5 is driven by a periodic signal $x(t)$. The Fourier series representation of $x(t)$ is given by

$$x(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n) e^{j2nt}$$

(a) Find the DC component of the system input.

(b) What is the period of the signal $x(t)$?

(c) What is the Fourier *transform* of $x(t)$?

(d) Find the power in the first harmonic of the system input signal $x(t)$.

(e) Give the Fourier series representation of the system *output* signal $y(t)$.

(f) Find the power in the first harmonic of the system output $y(t)$.

a. $C_0 = \left(\frac{1}{2}\right)^0 \cos(0.2\pi(0))$

$\boxed{C_0 = 1\,V}$

b. $e^{j2\pi nft} = e^{j2nt}$

$\pi f = 1$

$\boxed{T = \pi \text{ sec}}$

c. $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2nt}$ $\quad C_n = \left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n)$

$X(f) = \sum_{n=-\infty}^{\infty} C_n \delta(f - nf_0)$

$\boxed{X(f) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n)\, \delta\left(f - \frac{n}{\pi}\right)}$

d. $P_1 = |C_{-1}|^2 + |C_1|^2$

$P_1 = \left|\left(\frac{1}{2}\right)\cos(-0.2\pi)\right|^2 + \left|\left(\frac{1}{2}\right)\cos(0.2\pi)\right|^2$

$P_1 = |0.4045|^2 + |.4045|^2 = 0.327\,V^2$

$\boxed{P_1 = 327\,mV^2}$

At

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e. $Y(f) = H(f)\,X(f)$

$Y(f) = H(j2n)\,X(f)$

$$Y(f) = \frac{j2n+6}{-4n^2+j12n+13} \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n)\,\delta\left(f-\frac{n}{\pi}\right)$$

$$Y(f) = \frac{6+j2n}{13-4n^2+j12n} \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n)\,\delta\left(f-\frac{n}{\pi}\right)$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} \left(\frac{6+j2n}{13-4n^2+j12n}\right)\left(\frac{1}{2}\right)^{|n|} \cos(0.2\pi n)\,e^{j2nt}}$$

f. $P_1 = |c_{-1}|^2 + |c_1|^2$

$$P_1 = \left|\frac{6-j2}{9-j2\ j12}\left(\frac{1}{2}\right)\cos(-0.2\pi)\right|^2 + \left|\frac{6+j2}{9+j2}\left(\frac{1}{2}\right)\cos(0.2\pi)\right|^2$$

$$P_1 = 2\,\frac{40}{85}\left(\frac{1}{4}\right)\cos^2(0.2\pi)$$

$$\boxed{P_1 = 154\ mV^2}$$

close enough!