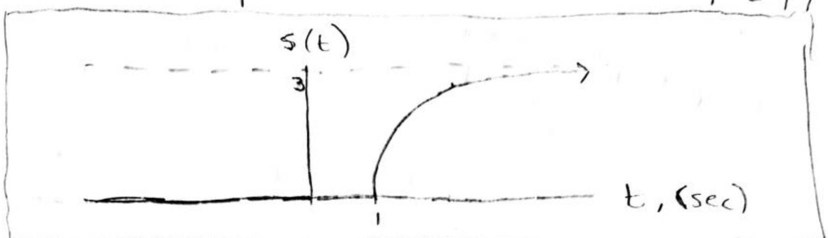
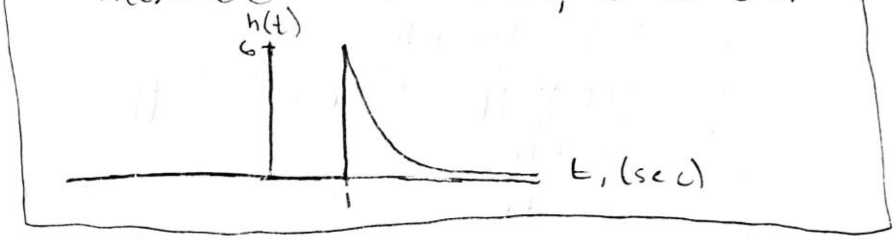
ECE 314
HW #2
Spencer Goulette   09/27/18

1. a.



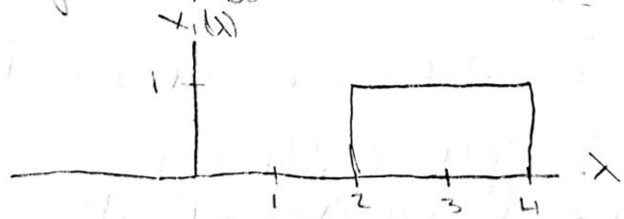
b.

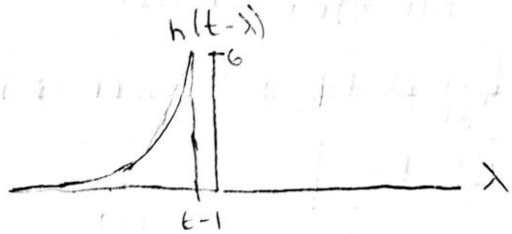
$$h(t) = \frac{d}{dt}[s(t)]$$
$$h(t) = \frac{d}{dt}[3 - 3e^{-2(t-1)}]$$
$$h(t) = \frac{d}{dt}[3 - 3e^{-2t}e^{2}]$$
$$h(t) = 6e^{-2(t-1)} \text{ for } t \geq 1, \ 0 \text{ for } t < 1$$



c.

$$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda)\, d\lambda$$





$y(t) = 0$ for $t < 3$
$$y(t) = \int_{2}^{t-1} 6e^{2\lambda} e^{-2t+2}\, d\lambda \text{ for } 3 \leq t \leq 5$$
$$y(t) = e^{-2t+2} \frac{6e^{2\lambda}}{2}\Big|_{2}^{t-1} = e^{-2t+2}\left[\frac{6e^{2t-2}}{2} - \frac{6e^{4}}{2}\right]$$
$$y(t) = 3 - 3e^{-2t+6} \text{ for } 3 \leq t \leq 5$$
$$y(t) = \int_{2}^{4} 6e^{2\lambda} e^{-2t+2}\, d\lambda = e^{-2t+2} \frac{6e^{2\lambda}}{2}\Big|_{2}^{4}$$
$$y(t) = 3e^{-2t+10} - 3e^{-2t+6}, \text{ for } t > 5$$

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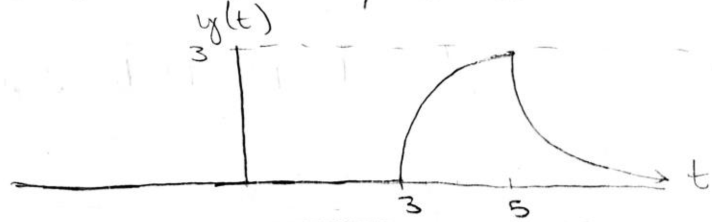
$$y(t) = \begin{cases} 0, & t < 3 \\ 3 - 3e^{-2t+6}, & 3 \le t \le 5 \\ 3e^{-2t+10} - 3e^{-2t+6}, & t > 5 \end{cases}$$



d. To be B.I.B.O. Stable $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ and $|x(t)| < B$

$|x(t)| \le 1$ for all t

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} 6e^{-2(t-1)}\,dt$$

$$= \frac{6e^{-2t+2}}{-2}\Big|_{1}^{\infty} = 0 + 3 = 3 < \infty$$

Since $|x(t)|$ is bounded and $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$, this is B.I.B.O. Stable

e. This is causal because $h(t) = 0$ for $t < 0$.

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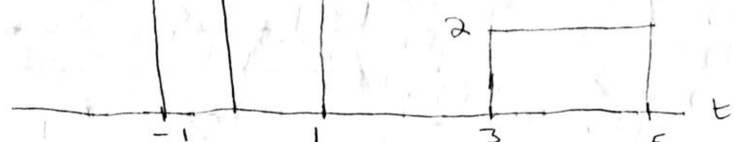
a. $x(t-t_0) = x(t) * \delta(t-t_0)$

$$x(t-t_0) = \int_{-\infty}^{\infty} x(t-\lambda)\,\delta(\lambda - t_0)\,d\lambda$$

Since the impulse occurs at t_0, λ in $x(t-\lambda)$ is replaced with t_0

$$\boxed{x(t-t_0) = x(t-t_0)}$$

b.



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3. a. $\ddot{y} + 6\dot{y} + 34y = 0$

$$s^2 + 6s + 34 = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$s = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(34)}}{2(1)}$$

$\boxed{s_1 = -3 + j5,\ s_2 = -3 - j5}$

So, it is underdamped since $b^2 < 4mk$

$$y(t) = e^{-3t}(C_1\cos(5t) + C_2\sin(5t))$$

$$\dot{y}(t) = -3e^{-3t}(C_1\cos(5t) + C_2\sin(5t)) + e^{-3t}(-5C_1\sin(5t) + 5C_2\cos(5t))$$

$$y(0) = 5 = e^0(C_1\cos(0) + C_2\sin(0))$$

$$C_1 = 5$$

$$\dot{y}(0) = 0 = -3e^0((5)\cos(0) + C_2\sin(0)) + e^0(-5(5)\sin(0) + 5C_2\cos(0))$$

$$\dot{y}(0) = 0 = -15 + 5C_2$$

$$C_2 = 3$$

$$\boxed{y(t) = e^{-3t}(5\cos(5t) + 3\sin(5t))}$$

Matlab Plot for 3a at bottom!

b. $\ddot{y} + 6\dot{y} + 9y = 0$

$$s^2 + 6s + 9 = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$s = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$$

$\boxed{s_1 = s_2 = -3}$, since $b^2 = 4mk$ it is critically damped

$$y(t) = e^{-3t}(C_1 + C_2 t)$$

$$\dot{y}(t) = -3e^{-3t}(C_1 + C_2 t) + C_2 e^{-3t}$$

$$y(0) = 5 = e^0 \cdot C_1$$

$$C_1 = 5$$

$$\dot{y}(0) = 0 = -3e^0 \cdot C_2(0) + C_2 + -3C_1$$

$$C_2 = 3C_1 \Rightarrow C_2 = 15$$

$$\boxed{y(t) = e^{-3t}(5 + 15t)}$$

Matlab Plot for 3b at bottom!

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3. c. $\ddot{y} + 7\dot{y} + 10y = 0$

$s^2 + 7s + 10 = 0$

$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$

$s = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}$

$\boxed{s_1 = -2,\ s_2 = -5 \text{, and it is overdamped since } b^2 > 4mK}$

$y(t) = C_1 e^{-2t} + C_2 e^{-5t}$

$\dot{y}(t) = -2C_1 e^{-2t} + -5C_2 e^{-5t}$

$y(0) = 5 = C_1 + C_2 \Rightarrow \dot{y}(0) = -2C_1 + -5C_2 = 0$

$C_2 = \frac{-2C_1}{5}$

$C_1 - \frac{2C_1}{5} = 5$

$\frac{3}{5} C_1 = 5 \Rightarrow C_1 = \frac{25}{3}$

$C_2 = -\frac{10}{3}$

$\boxed{y(t) = \frac{25}{3} e^{-2t} - \frac{10}{3} e^{-5t}}$

Matlab plot for 3c at bottom!

4. $y(t) = 3e^{-5t} \cos(30t)$

$\dot{y}(t) = -15e^{-5t} \cos(30t) - 90e^{-5t} \sin(30t)$

$\ddot{y}(t) = 75e^{-5t} \cos(30t) + 900e^{-5t} \sin(30t) - 2700e^{-5t} \cos(30t)$

$m\ddot{y} + b\dot{y} + ky = 0$

$m \cdot -2625\cos(30t) + m \cdot 900\sin(30t) + b \cdot -15\cos(30t) + b \cdot -90\sin(30t) + k \cdot 3\cos(30t) = 0$

$m900\cancel{\sin(30t)} + b \cdot -90\cancel{\sin(30t)} = 0$

$\frac{m}{b} = \frac{90}{900} = \frac{1}{10} \Rightarrow$

picked $m = 3 \Rightarrow b = 30$

$-3 \cdot 2625\cos(30t) + 30 \cdot -5\cos(30t) + k \cdot 3\cos(30t) = 0$

$-2675 + k = 0$

$k = 2675$

$$\boxed{\begin{array}{l} 3\ddot{y} + 30\dot{y} + 2675y = 0 \\ \text{Initial Conditions} \\ y(0) = 3 \\ \dot{y}(0) = -15 \end{array}}$$