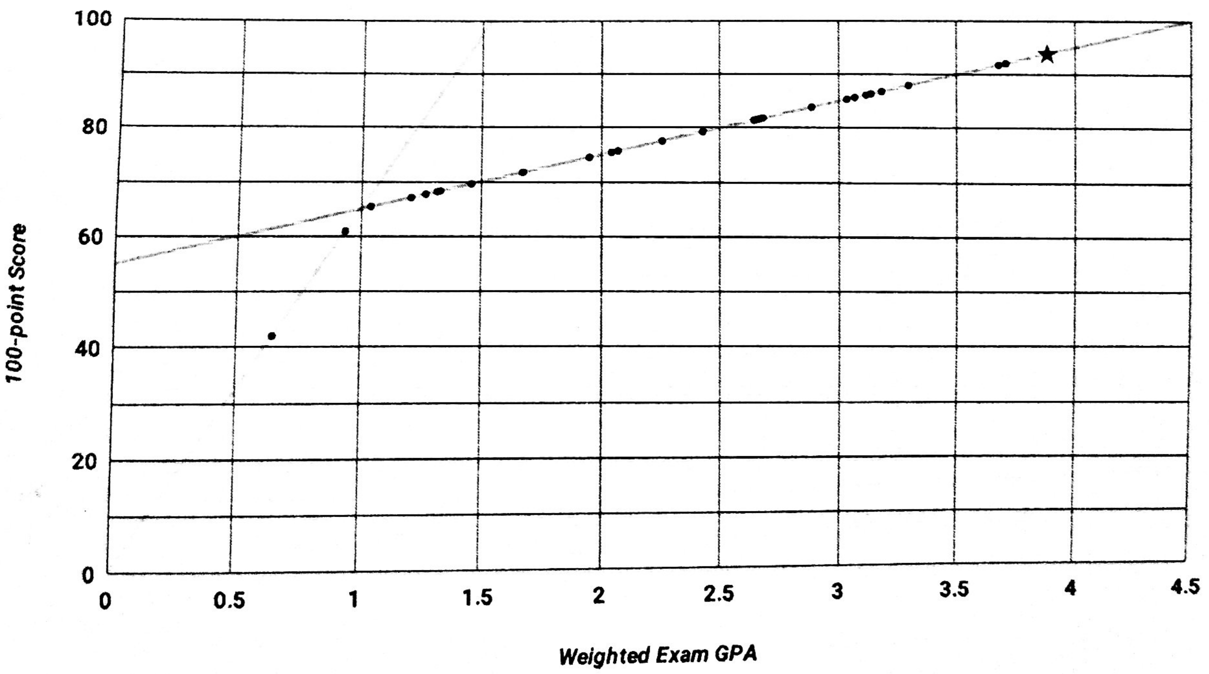
Goulette, Spencer

	Problem Grade	Grade Points	Weight
Problem 1	A-	3.67	15%
Problem 2	A-	3.67	19%
Problem 3	A+	4.50	19%
Problem 4	B	3.00	19%
Problem 5	A+	4.50	18%
Problem 6	A	4.00	10%
Weighted Exam GPA:		3.88	

GPA > 1.0: Recorded 100-pt Score = 65.0+10.0(GPA-1.0)

GPA < 1.0: Recorded 100-pt Score = 65.0(GPA)

GPA to 100-point score mapping



Recorded 100-point max score:	**93.82**	**A**

(90/80/70/60 Scale)

Class Average GPA: 2.32

Class Average 100-pt score: 77.43

ECE-486 Test 3, April 25, 2019

2 Hours; Closed book;

Allowed calculator models: (a) Casio fx-115 models (b) HP33s and HP 35s (c) TI-30X and TI-36X models. Calculators not included in this list are not permitted.

Name : Spencer Goulette

1. (15%) A second-order continuous-time Butterworth low-pass filter with −3 dB cutoff frequency Ω_0 radians per second has transfer function

$$H(s) = \frac{\Omega_0^2}{s^2 + \sqrt{2}\Omega_0 s + \Omega_0^2}$$

Use the bilinear transform method to design a second-order low-pass discrete-time filter with −3 dB cutoff frequency of $f_0 = 0.1$ cycles per sample. Provide the numerical filter coefficients in the table provided.

$\Omega_0 = \tan(\pi f_0)$

$\Omega_0 = \tan(\pi 0.1)$

$\Omega_0 = 0.325$ ✓

$s = \frac{z-1}{z+1}$

$$H(z) = \frac{0.325^2}{\left(\frac{z-1}{z+1}\right)^2 + \sqrt{2}\,0.325\left(\frac{z-1}{z+1}\right) + 0.325^2}$$ ✓

$$H(z) = \frac{(z+1)^2}{(z+1)^2}\,\frac{0.325^2}{\left(\frac{z-1}{z+1}\right)^2 + \sqrt{2}\,0.325\left(\frac{z-1}{z+1}\right) + 0.325^2}$$

$$H(z) = \frac{0.325^2(z+1)^2}{(z-1)^2 + \sqrt{2}\,0.325(z-1)(z+1) + 0.325^2(z+1)^2}$$

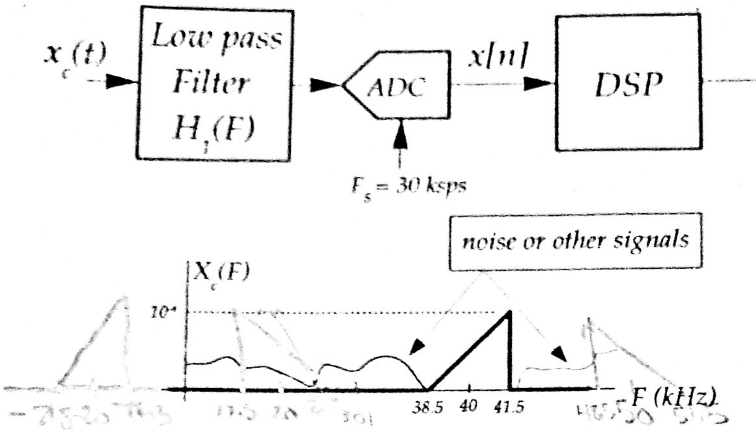
$$H(z) = \frac{0.325^2 z^2 + 2\cdot 0.325^2 z + 0.325^2}{z - 1 + \sqrt{2}\,0.325 z^2 - \sqrt{2}\,0.325 + 0.325^2 z^2 + 2\cdot 0.325^2 z + 0.325^2}$$

$$H(z) = \frac{0.325^2 z^2 + 2\cdot 0.325^2 z + 0.325^2}{(0.325^2 + \sqrt{2}\,0.325)z^2 + (1 + 2\cdot 0.325^2)z + (0.325^2 - 1 - \sqrt{2}\,0.325)}$$

	Value
a_0	0.565 ✗
a_1	1.211 ✗
a_2	−1.354 ✗
b_0	0.106
b_1	0.211
b_2	0.106

A−

2. (19%) An infrared (IR) remote control involves pulsing IR light at a 40 kHz rate to carry digital information. The IR light intensity is detected and used as an input $x_c(t)$ to a digital receiver as shown below. For this problem, assume the the input has the spectrum as shown.



(a) Specify the pass-band(s) and stop-band(s) edges for the continuous-time filter $H_c(F)$ to avoid any aliasing onto the $38.5 < F < 41.5$ kHz band of frequencies. Make the filter transition band as wide as possible.

(b) Draw and carefully label the discrete-time spectrum $|X(f)|$. Show the region $-0.5 < f < 0.5$. (No need to show the noise/other signals portion of the spectrum.)

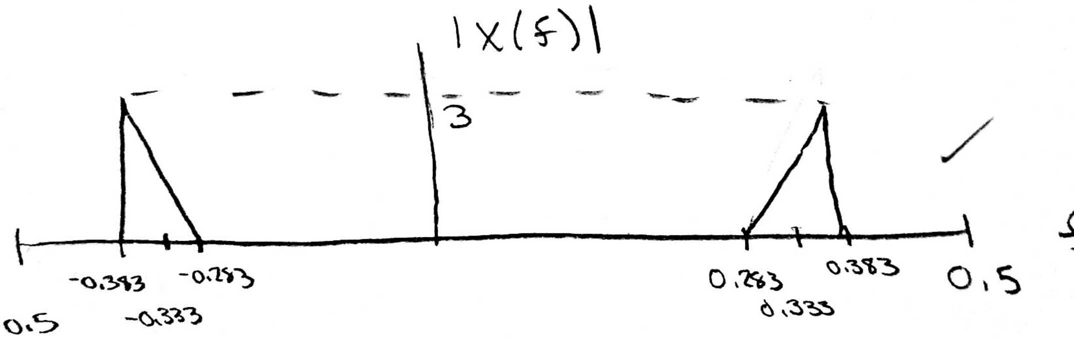
(c) Is it possible to use a sampling frequency below 20 ksps such that the response associated with the $38.5 < F < 41.5$ kHz band will be centered at $f = 0.25$ without aliasing? If not, explain why. If so, give an example of a possible sampling frequency.

a. Stopband : 38.5kHz < |F| < 41.5kHz
Passband

Passband : 21.5kHz < |F| < 48.5kHz ✗

Need two stop bands: |F| < 21.5 and |F| > 48.5

b.



c. No, this is because the sampling frequency must be at least 1/2 of the incoming signal frequency

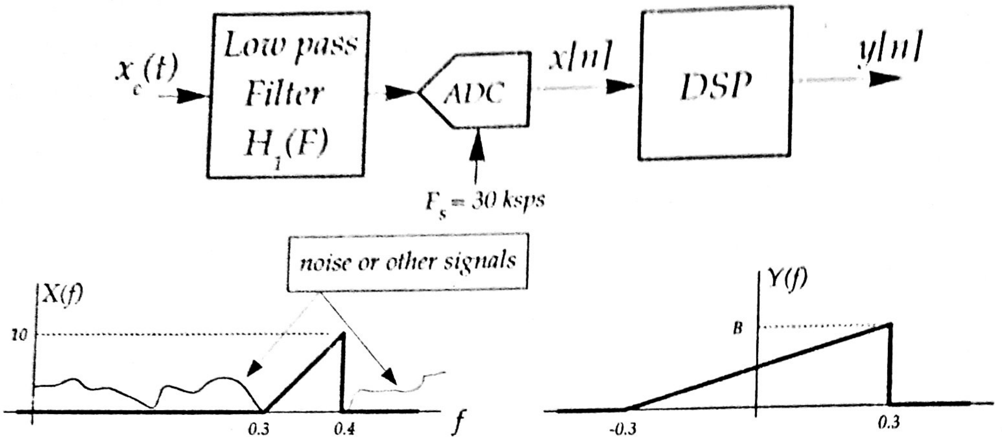
$$f = \frac{\pm F}{F_s} \pm k$$

$$0.25 = \frac{40k}{F_s} \pm k$$

$k = 2 \Rightarrow F_s = 17.778$ ksps

A

3. (19%) The IR receiver from problem 2 is repeated below. Assume that the discrete time spectrum $X(f)$ is as shown. (Note that this is NOT the correct result for problem 2!) The "DSP" portion of the design is intended to produce the output spectrum $Y(f)$ as shown on the right. Note that the sampling frequency associated with the spectrum on the right must be different from that on the left.



Draw a block diagram for the DSP processing block. Specify the numeric value of any required *discrete-time* frequency constants, and provide the band-edge specifications of any filters in your design (again, use *discrete-time* frequency specifications!). Make filter transition bands as wide as possible while preventing aliasing. Give numerical values for any interpolation or decimation steps, and provide a numerical value for the constant "B" shown in the plot of $Y(f)$.

$F_1 = 30\text{ksps}$ $\quad F_2 = 30\text{ksps}$ $\quad F_3 = 5\text{ksps}$ $\quad F_4 = 5\text{ksps}$

$x[n] \rightarrow (+) \rightarrow \boxed{H_2(f)} \rightarrow \boxed{\downarrow 6} \rightarrow \boxed{H_3(f)} \rightarrow y[n]$

$e^{-j2\pi f_0 n}$

$f_0 = 0.35$ ✓

H_2: Passband: $|f| < 0.05$ ✓

Stopband: $|f| > 0.117$ ✓

H_3: Passband: $|f| < 0.3$

$B = 1.667 = \frac{10}{6}$ ✓

4. (19%) Design a $M = 5$ coefficient linear phase filter which has a DC gain of 1.0, and completely eliminates discrete-time frequencies of $f = 0.25$ and $f = 0.5$.

Provide the following for your design:

- The complete impulse response for the filter.
- The real-valued transfer function $H_r(\omega)$. (Simplify this result so that it contains no imaginary terms.)
- Find the filter output $y[n]$ when the input signal is $x[n] = \cos(0.1n)$. (Please provide a "closed-form", real answer -- no summations, integrals or imaginary terms.)

DC gain of 1.0, eliminates f=0.25 and f=0.5

$$H(z) = \frac{z^4}{(z - e^{j2\pi 0.25})(z - e^{-j2\pi 0.25})(z - e^{j2\pi 0.5})(z - e^{-j2\pi 0.5})}$$

$$H(z) = \frac{z^4}{(z^2 + 1)(z^2 + 2 + 1)}$$

$$H(z) = \frac{z^4}{z^4 + 4z^2 + 3}$$

$0 = 2A\cos(2\pi) + 2B\cos(\pi) + C$

$0 = 2A\cos(\pi) + 2B\cos(\pi/2) + C$

$2A = C$

$C = 2$

$A = -1$

$2 + 2 \quad B = 2$

↑ Better!

Need to scale for DC gain = 1

$$\boxed{h[n] = \{\ldots, 0, -1, 2, 2, 2, -1, 0, \ldots\}}$$

$$H(\omega) = e^{-j2\omega}\left(-1e^{j2\omega} + 2e^{j\omega} + 2 + 2e^{-j\omega} - 1e^{-j2\omega}\right)$$

$$\boxed{H_r(\omega) = -2\cos(2\omega) + 4\cos(\omega) + 2}$$

ran out of time

- $\omega = 0.1$

$$y[n] = |H_r(0.1)| \cos(0.1n + \angle H_r(0.1))$$

$y[n]$

$0 = 2A\cos(2\pi) + 2B\cos(\pi) + C$

$0 = 2A\cos(\pi) + 2B\cos(\pi/2) + C$

$1 = 2A + 2B + C$

$C = 1 - 2A - 2B$

$h[n] = \{\ldots, 0, 1/5, 2/5, 2/5, 2/5, 1/5, 0, \ldots\}$

$H_r(\omega) = 2/5\cos(2\omega) + 4/5\cos(\omega) + 2/5$

$0 = 2A\cos(2\pi) + 2B\cos(\pi) + 1 - 2A - 2$

$0 = 2A\cos(\pi) + 2B\cos(\pi/2) + 1 - 2A$

$0 = -2A + 1 - 2A - 2B$

$2B = 1 - 4A \Rightarrow B = \frac{1 - 4A}{2}$

$0 = 2A\cos(2\pi) + \cos(\pi) - 4A\cos(\pi) + 1 - 2A -$

$0 = 2A - 1 + 4A + 1 - 2A - 1/2 - 2A$

$0 = 2A - 1/2$

$A = 1/4 \quad C = 2/5$

$B = 2/5$

5. (18%) Let $x[n]$ and $h[n]$ be defined by:

$$x[n] = \{\ldots\ 0,\ 0,\ 1,\ 2,\ \underset{\uparrow}{3},\ 4,\ 5,\ 6,\ 0,\ 0,\ \ldots\} = \begin{cases} n+3 & n = -2, -1, \ldots, 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$h[n] = \{\ldots\ 0,\ 0,\ \underset{\uparrow}{7},\ 8,\ 9,\ 10,\ 0,\ 0,\ \ldots\} = \begin{cases} n+7 & n = 0, 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$

DFTs are to be used to evaluate $y[n] = x[n] * h[n]$. To do this, length N vectors $\vec{x}$ and $\vec{h}$ are to be initialized using values of $x[n]$ and $h[n]$. The N-point DFTs of these vectors provide $X[k]$ and $H[k]$, and $Y[k] = X[k]H[k]$. he inverse N-point DFT of $Y[k]$ provides an output vector $\vec{y}$.

- What are the constraints on the vector length N?
- How should the vector $\vec{x}$ be initialized for $n = 0, \ldots, N-1$?
- How should the vector $\vec{h}$ be initialized for $n = 0, \ldots, N-1$?
- How is the output sequence $y[n]$ determined from the output vector $\vec{y}$?

- N shouldn't be greater than len(x[n]) + len(h[n]) − 1
So, N shouldn't be greater than 9. ✓
- The vector $\vec{x}$ should be initialized 1,2,3,4,5,6 and then padded with zeros until a size of N is met.
- The vector $\vec{h}$ should be initialized by having the first four elements be 7, 8, 9, 10, and then the rest padded with zeros until you get to N size
- Y[n] is determined from the output vector $\vec{y}$ by starting two elements later in the sequence. So y[n] is the nth + 2 element in $\vec{y}$

A+

6. (10%) A music audio signal is known to have peak voltage 3 V and RMS voltage 0.4 V. The signal is to be sampled using a sampling frequency of 48 ksps. Find the minimum number of ADC bits if a quantization signal-to-noise ratio of at least 90 dB is to be maintained.

Signal Power = $\frac{(RMS)^2}{?}$ ~~X~~ Signal Power $= (RMS)^2$ SNR = $\frac{\text{Signal Power}}{\text{Signal Noise}}$

Signal Noise $= \left(\frac{2V_{max}}{2^n}\right)^2 / 12$

$$SNR = 10\log10\left(\frac{RMS^2}{\left(\frac{2V_{max}}{2^n}\right)^2/12}\right) = 10\log10\left(\frac{RMS^2}{2}\frac{2^{2n}\,12}{(2V_{max})^2}\right)$$

$$2^{2n} = \frac{10^{\left(\frac{90}{10}\right)}\cdot 8V_{max}^2}{12RMS^2} = \frac{10^9\cdot 8\cdot 9}{12\;0.4^2} = 3.75\cdot 10^{10}$$

$$2n > 34 \Rightarrow n > 17$$

$$\boxed{n = 18}$$

A