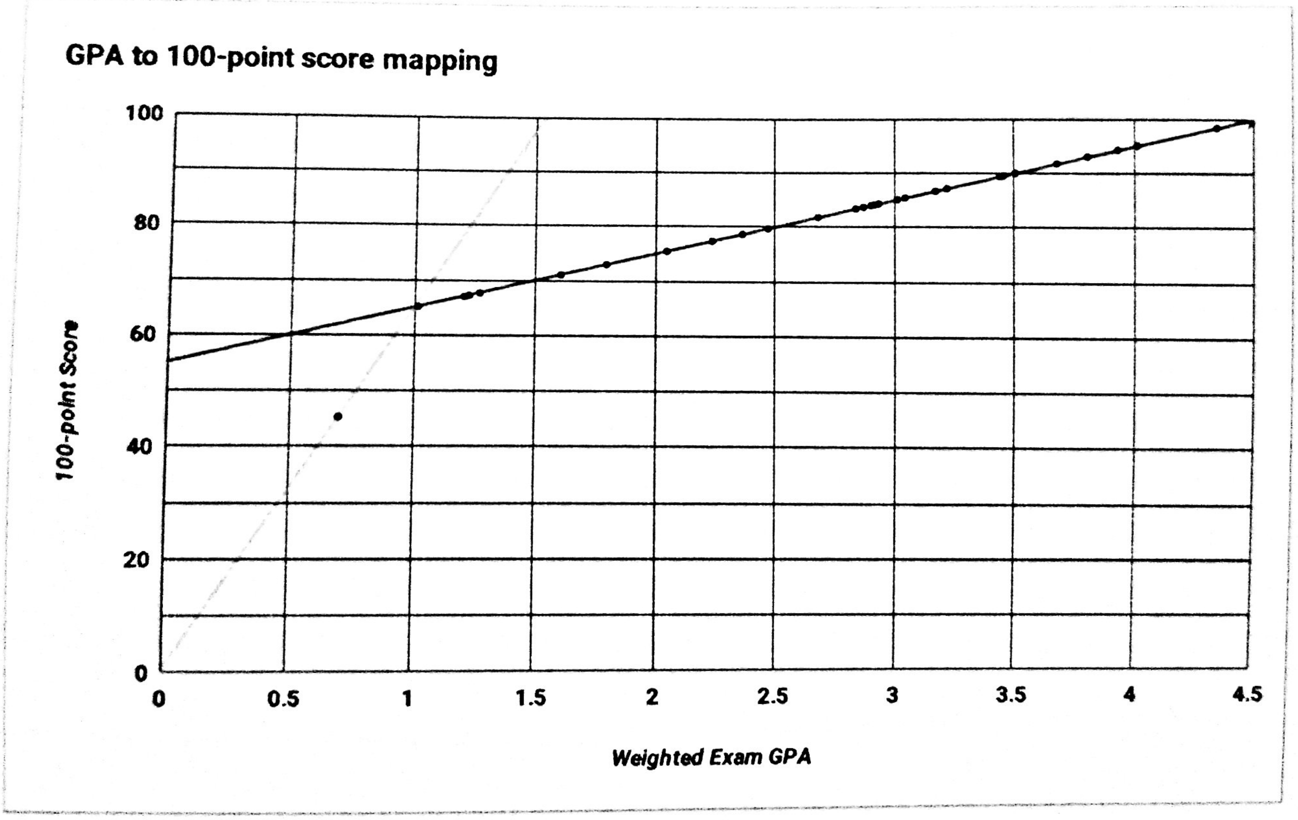
Goulette, Spencer

| | Problem Grade | Grade Points | Weight |
|---|---|---|---|
| Problem 1 | A+ | 4.50 | 19% |
| Problem 2 | A+ | 4.50 | 19% |
| Problem 3 | A+ | 4.50 | 19% |
| Problem 4 | A+ | 4.50 | 19% |
| Problem 5 | A+ | 4.50 | 5% |
| Problem 6 | A+ | 4.50 | 19% |
| Weighted Exam GPA: | | 4.50 | |

GPA > 1.0: Recorded 100-pt Score = 65.0+10.0(GPA-1.0)

GPA < 1.0: Recorded 100-pt Score = 65.0(GPA)




| Recorded 100-point max score: | 100.00 | #N/A |
|---|---|---|

(90/80/70/60 Scale)

Class Average GPA: 2.71

Class Average 100-pt score: 81.54

ECE-486 Test 2, March 9, 2019

2 Hours; Closed book;

Allowed calculator models: (a) Casio fx-115 models (b) HP33s and HP 35s (c) TI-30X and TI-36X models.

Calculators not included in this list are not permitted.

*Your solutions for time-domain expressions should all be expressed as real-valued functions.*

Name : Spencer Coulette

1. (19%) Find $x[n]$. Please simplify your result to be purely real.

$$X(z) = 6z^2 + \frac{3z^2 - \sqrt{2}z}{z^2 - 2\sqrt{2}z + 4} + \frac{-z^2 - 11z}{z^2 - 3z - 4} \qquad 2 < |z| < 4$$

$$X(z) = 6z^2 + X_1(z) + X_2(z)$$

$$X_1(z) = \frac{3z^2 - \sqrt{2}z}{z^2 - 2\sqrt{2}z + 4}$$

$$X_1(z) = 3\,\frac{z^2 - \frac{\sqrt{2}}{3}z}{z^2 - 2\sqrt{2}z + 4} \quad a = 2$$

$$X_1(z) = 3\,\frac{z^2 - 2\cos(\omega_0)z}{z^2 - 4\cos(\omega_0)z + 4} + \frac{B}{z^2 - 2\sqrt{2}z + 4}$$

$$\cos(\omega_0) = \frac{1}{\sqrt{2}}$$

$$\omega_0 = 0.7854$$

$$X_1(z) = 3\,\frac{z^2 - 2\cos(0.7854)z}{z^2 - 4\cos(0.7854)z + 4} + \frac{B}{z^2 - 2\sqrt{2}z + 4}$$

$$X_1(z) = 3\,\frac{z^2 - \sqrt{2}z}{z^2 - 2\sqrt{2}z + 4} + 2\,\frac{\sqrt{2}z}{z^2 - 2\sqrt{2}z + 4}$$

$$X_2(z) = -\left(\frac{z^2 + 11z}{z^2 - 3z - 4}\right)$$

$$X_2(z) = -z\left(\frac{z + 11}{(z-4)(z+1)}\right)$$

$$X_2(z) = -z\left(\frac{A}{z-4} + \frac{B}{z+1}\right)$$

$$A + B = 1 \qquad A = 1 - B$$

$$A - 4B = 11 \qquad 1 - B - 4B = 11$$

$$-5B = 10$$

$$A = 3 \qquad B = -2$$

$$X_2(z) = -z\left(\frac{3}{z-4} + \frac{-2}{z+1}\right)$$

$$X(z) = 6z^2 + 3\left(\frac{z^2 - \sqrt{2}z}{z^2 - 2\sqrt{2}z + 4}\right) + 2\left(\frac{\sqrt{2}z}{z^2 - 2\sqrt{2}z + 4}\right) - \frac{3z}{z-4} - \frac{2z}{z+1}$$

+

$$\boxed{x[n] = 6\delta[n+2] + 3(2)^n\cos((\pi/4)n)u[n] + 2(2)^n\sin((\pi/4)n)u[n] + 3(4)^n u[-n-1] - 2(-1)^n u[n] \quad \text{ROC } 2 < |z| < 4}$$

A+

2. (19%) A linear time-invariant discrete-time system has transfer function

$$H(z) = \frac{1 - z^2}{z - 0.9} \qquad |z| > 0.9$$

(a) Is this a causal system? (Justify using $H(z)$)

(b) Is this a stable system? (Justify using $H(z)$)

(c) Evaluate the impulse response of this system. Please provide a *real* result.

(d) Find the output of the system for $x[n] = cos((\pi/2)n)$. (NOTE: $x[n] \neq cos((\pi/2)n)u[n]$.) Please provide a *real* result.

a. This isn't a causal system since the numerator of H(z) has a higher power than the denominator. This means that future values are needed. ✓

b. This is a stable system because H(z) has a root that is less than 1 and its ROC is greater than 0.9. Also, the ROC includes the unit circle, which makes it stable.

c. $$H(z) = -\frac{z^2 - 1}{z - 0.9} = -\frac{z^2}{z - 0.9} + \frac{1}{z - 0.9}$$

$$\boxed{h[n] = -(0.9)^{n+1} u[n+1] + (0.9)^{n-1} u[n-1]}$$ ✓

d. $$x[n] = \cos((\pi/2)n) = \cos(2\pi(1/4)n) \qquad f = 1/4$$

$$H(z) = -\frac{e^{j4\pi \cdot 1/4} - 1}{e^{j2\pi \cdot 1/4} - 0.9} = -\frac{-1-1}{j - 0.9}$$

$$H(z) = \frac{2}{j - 0.9} = \frac{2}{1.345 e^{j2.304}}$$


$$H(z) = \frac{2}{1.345} e^{-j2.304}$$

$$\boxed{y[n] = \frac{2}{1.345} \cdot \cos((\pi/2)n - 2.304)}$$ ✓

A+

3. (19%) The output of a discrete-time system is found by scaling the sum of the most recent two output samples, and adding the input:

$$y[n] = \beta\,(\,y[n-1] + y[n-2]\,) + x[n]$$

For this problem, you may assume that the scale factor $\beta$ is a positive real number.

(a) Find the conditions on the parameter $\beta$ so that the system is stable.

(b) Assuming that your conditions of part 3a are satisfied, find the DC gain of this system in terms of the parameter $\beta$.

(c) For the special case $\beta = 0.5$, evaluate $Y^+(z)$ when $x[n] = u[n]$, $y[-1] = 2$, and $y[-2] = 4$. (Do *not* try to evaluate $y[n]$.)

a. $y[n] = \beta(y[n-1] + y[n-2]) + x[n]$

$y[n] - \beta y[n-1] - \beta y[n-2] = x[n]$

$Y(z) - \beta z^{-1} Y(z) - \beta z^{-2} Y(z) = X(z)$

$Y(z)(1 - \beta z^{-1} - \beta z^{-2}) = X(z)$

$Y(z) = \dfrac{1}{z^2 - \beta z - \beta} X(z)$

$H(z) = \dfrac{1}{z^2 - \beta z - \beta} \Rightarrow \left|\dfrac{\beta \pm \sqrt{\beta^2 + 4\cdot\beta}}{2}\right| < |1|$

$|\beta \pm \sqrt{\beta^2 + 4\beta}| < |2|$

$\boxed{\beta < 0.5}$

b. $H(1) = \dfrac{1}{1 - \beta - \beta} = \dfrac{1}{1 - 2\beta}$

$\boxed{\text{DC gain: } \dfrac{1}{1 - 2\beta}}$

c. $y[n] = 0.5y[n-1] + 0.5y[n-2] + x[n]$

$y[n] - 0.5y[n-1] - 0.5y[n-2] = x[n]$

$Y^+(z) - 0.5z^{-1}Y^+(z) - 0.5\cdot y[-1] - 0.5z^{-2}Y^+(z) - 0.5z^{-1}y[-1] - 0.5y[-2] = X(z)$

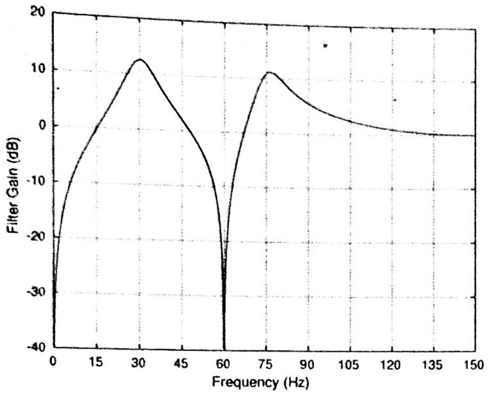
$Y^+(z)(1 - 0.5z^{-1} - 0.5z^{-2}) = \dfrac{z}{z-1} + 1 + z^{-1} + 2$

$$\boxed{Y^+(z) = \frac{z^2}{z^2 - 0.5z - 0.5}\left(\frac{z}{z-1}\right) + \frac{3z^2 + z}{z^2 - 0.5z - 0.5}}$$

Isn't stable though

A+

4. (19%) A two-stage discrete-time "biquad" filter with frequency response similar to that shown below is to be designed for a system that uses a sampling frequency of $F_s = 300$ sps. Your design should meet the stated constraints:




(a) Causal, stable filter with real coefficients.

(b) Completely eliminates DC.

(c) Completely eliminates interference at 60 Hz.

(d) Gain peaks near 30 Hz and 75 Hz.

(e) All pole magnitudes of 0.9

(f) 0 dB gain at 150 Hz.

Provide a pole-zero diagram for the filter. Give the gain value "$G$" and difference equation coefficients for each filter stage (fill in the table with your numerical results).

conjugates aren't put in. ✓

$$H(z) = G\frac{(z-1e^{j2\pi\cdot 0.2})(z-1e^{0})}{(z-0.9e^{j2\pi\cdot 0.1})(z-0.9e^{j2\pi\cdot 0.25})}$$

$$H(z) = G\,\frac{z^2-1}{z^2-1.8\cos(2\pi 0.1)z+0.81}\cdot\frac{z^2-2\cos(2\pi\cdot .2)z+1}{z^2-1.8\cos(2\pi .25)z+0.81}$$

$$H(-1) = G\,\frac{-1-1}{1+1.8\cos(2\pi\cdot 0.1)+0.81}\cdot\frac{1+2\cos(2\pi\cdot .2)+1}{1+1.8\cos(2\pi\cdot .25)+0.81}$$

$$H(-1) = G\cdot -0.8857$$

$$G = \frac{1}{-0.8857} = -1.1291 \checkmark$$

$$Y(z)+z^{-1}1.8\cos(2\pi\cdot .1)Y(z)+0.81z^{-2}Y(z) = z^{-1}X(z) - z^{-2}X(z)$$

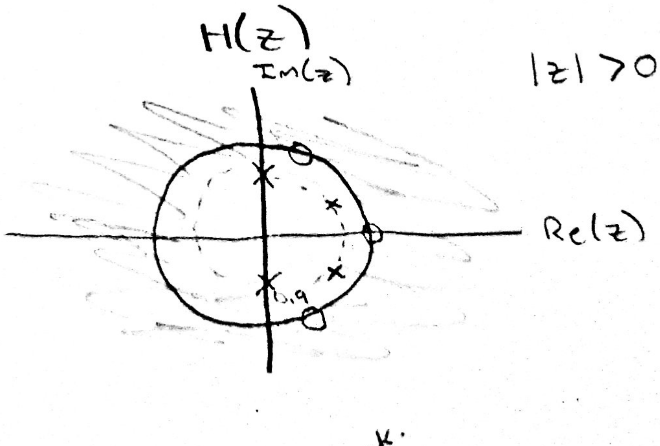
$$y[n]+1.8\cos(2\pi\cdot .1)y[n-1]+0.81\,y[n-2]$$

Enter numerical filter coefficients:

$G =$ -1.1291

| | Stage 1 | Stage 2 |
|---|---|---|
| $a_0$ | 1.0 | 1.0 |
| $a_1$ | -1.456 | 0 |
| $a_2$ | 0.81 | 0.81 |
| $b_0$ | 0 | 1 |
| $b_1$ | 1 | -0.618 |
| $b_2$ | -1 | 1 |

don't include G




$|z| > 0.9$

Nice work.

A+

5. (5%) A continuous-time signal $x_c(t)$ is sampled using a sampling frequency of 25 ksps to form a discrete-time signal $x[n]$. Noise in the measurement system introduces a random error in each sample with an RMS value of 0.2 volts. You may assume that the error associated with each sample is independent of the error associated with other samples.

To reduce the noise, an M-sample moving-average filter is used to process $x[n]$:

$$y[n] = \sum_{k=0}^{M-1} x[n-k]$$

Determine the value of $M$ required so that the noise RMS voltage for $y[n]$ is reduced to 10 mV.

$$\sigma_y^2 = \frac{\sigma_x^2}{M}$$

$$\sigma_y = \frac{\sigma_x}{\sqrt{M}}$$

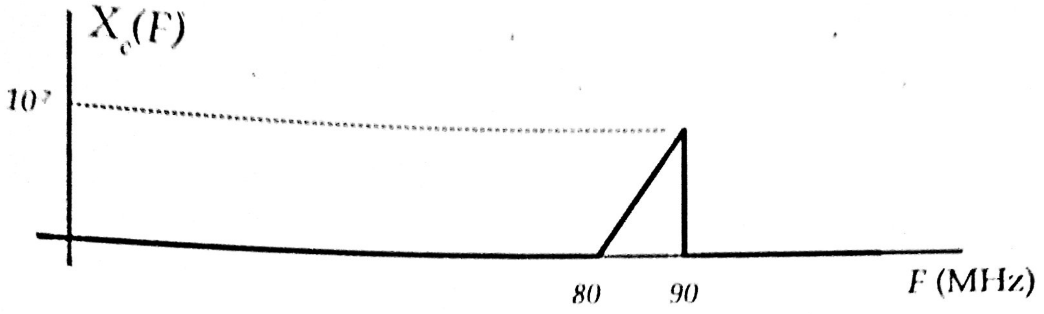
$$10m = \frac{200m}{\sqrt{M}}$$

$$M = \left(\frac{200m}{10m}\right)^2$$

$$\boxed{M = 400}$$

A+

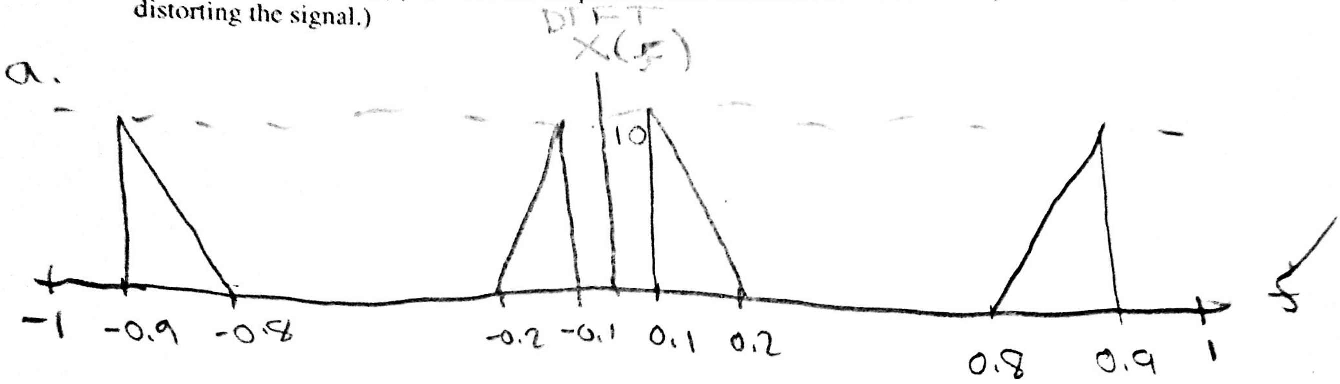
6. (10%) A continuous-time real signal $x_c(t)$ has Fourier Transform $X_c(F)$ is plotted below. (Note the the negative frequencies are not shown.) The signal has energy in the 80 MHz to 90 MHz band. The signal is to be sampled using a sampling frequency of $F_s = 100$ Msps to create a discrete time signal $x[n]$.




(a) Draw and carefully label $X(f)$ (the DTFT of $x[n]$). Show the entire range $-1 < f < 1$.

(b) Identify any other continuous-time frequency bands below 250 MHz that will alias to the same discrete-time frequencies as $x_c(t)$. (These are frequencies that would have to be removed prior to sampling to avoid distorting the signal.)

a.




b.

$F(k) = F_s k \pm (80 \text{ to } 90)$

So, 10 MHz to 20 MHz, 80 MHz to 90 MHz, 110 MHz to 120 MHz, 180 MHz to 190 MHz, 210 MHz to 220 MHz ✓

A+