Macro II Problem Set 6

Welfare Analysis of Practical Monetary Policy Rules
within the Basic New Keyesian Model

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1 Part I. A Taylor-type Interest Rate Rule.

1.1 ...Write out the entries in the new vector $\tilde{B}_t$

A.N.S: Assuming that variations in the technology parameter a_t represent the only driving force in the economy, and are described by a stationary AR(1) process with autoregressive coefficient ρ_a, the following equality holds:

$$\begin{aligned} \hat{r}_t^n - v_t &= -\sigma\psi_{ya}^n(1-\rho_a)a_t - \phi_y\psi_{ya}^n a_t \\ &= -\psi_{ya}^n[\sigma(1-\rho_a)+\phi_y]a_t \end{aligned} \tag{1}$$

Then, we have:

$$\begin{aligned} B_T(\hat{r_t}^n - v_t) &= -B_T(\psi_{ya}^n[\sigma(1-\rho_a)+\phi_y])a_t \\ &= \tilde{B}_t a_t \end{aligned} \tag{2}$$

$$\Longrightarrow \tilde{B}_t = -B_T(\psi_{ya}^n[\sigma(1-\rho_a)+\phi_y])$$

1.2 Show how to compute $var(a_t)$?, And what is the value of $var(a_t)$?

A.N.S: For $a_t = \rho_a a_{t-1} + \varepsilon_t^a$,

$$Var(a_t) = \rho_a^2 Var(a_{t-1}) + Var(\varepsilon_t^a)$$

Assume $Var(a_t) = Var(a_{t-1})$, we can find:

$$Var(a_t) = \frac{Var(\varepsilon_t^a)}{1-\rho_a^2} = \frac{\sigma_a^2}{1-\rho_a^2} \tag{3}$$

Substitute $\sigma_a^2 = 0.00712^2$ and $\rho_a = 0.9$ into equation (**??**), we can compute

$$Var(a_t) = 2.6681 \times 10^{-4}$$

1.3 Please show how to compute $Var(\tilde{y}_t)$ and $Var(\pi_t)$ as a function of $var(a_t)$

A.N.S: Consider the DIS and NKPC system:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \tilde{B}_T a_t \tag{4}$$

where ,

$$\begin{aligned} A_T &= \begin{pmatrix} \sigma & 1-\beta\phi_\pi \\ \kappa\sigma & \kappa+\beta(\sigma+\phi_y) \end{pmatrix} \Omega \\ \tilde{B}_t &= -B_T(\psi^n_{ya}[\sigma(1-\rho_a)+\phi_y]) \\ B_T &= \begin{bmatrix} 1 \\ \kappa \end{bmatrix} \Omega \\ \kappa &= \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha}) \\ \lambda &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta \\ \Theta &= \frac{1-\alpha}{1-\alpha-\alpha\varepsilon} \end{aligned}$$

Rewrite Equation(**??**) in a recursive form:

$$\begin{aligned} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} &= A_T\{A_T \begin{bmatrix} E_t\{\tilde{y}_{t+2}\} \\ E_t\{\pi_{t+2}\} \end{bmatrix} + \tilde{B}_T a_{t+1}\} + \tilde{B}_T a_t \\ &= (A_T)^2 \begin{bmatrix} E_t\{\tilde{y}_{t+2}\} \\ E_t\{\pi_{t+2}\} \end{bmatrix} + A_T\rho_a\tilde{B}_T a_t + \tilde{B}_T a_t \\ &\vdots \quad \vdots \quad \vdots \\ &= (A_T)^\infty \begin{bmatrix} E_t\{\tilde{y}_\infty\} \\ E_t\{\pi_\infty\} \end{bmatrix} + \sum_{i=0}^{\infty}(A_T\rho_a)^i\tilde{B}_T a_t \\ &= (I_2-\rho_a A_T)^{-1}\tilde{B}_T a_t \end{aligned} \tag{5}$$

Denote 2×1 vector $C = (I_2-\rho_a A_T)^{-1}\tilde{B}_T$, thus,

$$var(\tilde{y}_t) = [C(1)]^2 var(a_t) \tag{6}$$
$$var(\pi_t) = [C(2)]^2 var(a_t) \tag{7}$$

where $C(i)$ is the $i-th$ element of C
Use Matlab , we can easily solve the system:

$$\begin{aligned} var(\tilde{y}_t) &= 1.5645\times 10^{-5} \\ var(\pi_t) &= 1.5324\times 10^{-4} \\ \begin{bmatrix} Var(\tilde{y}_t) \\ Var(\pi_t) \end{bmatrix} &= \begin{bmatrix} 1.5645\times 10^{-5} \\ 1.5324\times 10^{-4} \end{bmatrix} \end{aligned}$$

1.4 Compute the average welfare loss per period.

A.N.S: The average welfare loss per period in terms of variations of output gap and inflation is:

$$L = \frac{1}{2}\big[(\sigma + \frac{\varphi + \alpha}{1-\alpha})Var(\tilde{y}_t) + \frac{\varepsilon}{\lambda}Var(\pi_t)\big] \tag{8}$$

Substituting $Var(\tilde{y}_t)$ and $Var(\pi_t)$ into equation (**??**), we can solve the welfare loss:

$$L = 0.0041$$

1.5 Vary the values of policy reaction parameters and generate a table as follows, and fill the blank entries:

A.N.S: Repeated the problem 1.3,1.4 procedure with respect to the given ϕ_π and ϕ_y.

	Taylor Rule			
ϕ_π	**1.5**	**1.5**	**1.5**	**5**
ϕ_y	**1**	**0.125**	**0**	**0**
$\sigma(\tilde{y})$	3.6408×10^{-5}	3.0547×10^{-6}	6.8208×10^{-7}	1.5955×10^{-8}
$\sigma(\pi)$	3.5662×10^{-4}	2.9921×10^{-5}	6.6810×10^{-6}	1.5628×10^{-7}
$\sigma(L)$	0.0095	7.9396×10^{-4}	1.7728×10^{-4}	4.1469×10^{-6}

2 Part II. A Money Growth Rule.

2.1 Compute $\hat{r}_t^n$ and $\hat{y}_t^n$

From Chapter 3 Gali(2008) [1], we know the following relations:

$$\hat{y}_t^n = \psi_{ya}^n a_t \tag{9}$$
$$\hat{r}_t^n = -\sigma\psi_{ya}^n(1-\rho_a)a_t \tag{10}$$

where $\psi_{ya}^n = \frac{(1+\varphi)}{\varphi+\alpha+\sigma(1-\alpha)}$. Hence, we have:

$$var(\hat{y}_t^n) = (\psi_{ya}^n)^2 var(a_t) = 2.6681\times 10^{-4} \tag{11}$$
$$var(\hat{r}_t^n) = [\sigma\psi_{ya}^n(1-\rho_a)]^2 var(a_t) = 2.6681\times 10^{-6} \tag{12}$$

2.2 Compute $var(\Delta\zeta_t)$

Since the exogenous money demand shockζ_t is an AR(1) process,

$$var(\Delta\zeta_t) = \frac{\sigma(\zeta)}{1-\rho_\zeta^2} = 6.2016\times 10^{-5} \tag{13}$$

2.3 Show how to compute $\begin{bmatrix} var(\tilde{y}_t) & var(\pi_t) & var(\Delta\zeta_t)\end{bmatrix}'$

Following page 56 of Gali(2008), we know:

$$A_{M,0} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \quad A_{M,1} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_M \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Split the linear matrix equation into a three equations system, we have:

$$(1+\sigma\eta)\tilde{y}_t = \sigma\eta E_t(\tilde{y}_{t+1}) + \hat{l}_t^+ + \eta\hat{r}_t{}^n - \hat{y}_t{}^n \tag{14}$$
$$-\kappa\tilde{y}_t + \pi_t = \beta E_t(\pi_{t+1}) \tag{15}$$
$$-\pi_t + \hat{l}_{t-1}^+ = \hat{l}_t{}^+ - \Delta\zeta_t \tag{16}$$

Taking expectation w.r.t equation (**??**) and rearranging the equation we have,

$$\hat{l}_t{}^+ = E_t(\pi_{t+1}) + E_t(\hat{l}_{t+1}^+) - \rho_\zeta\Delta\zeta_t \tag{17}$$

Equations (14), (15) and (17) can be summarized compactly by the system:

$$\begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_t{}^+ \end{bmatrix} = \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} E_t(\tilde{y}_{t+1}) \\ E_t(\pi_{t+2}) \\ E_t(\hat{l}_{t+1}^+) \end{bmatrix} + \begin{bmatrix} -1.4 & -0.6 \\ 0 & 0 \\ 0 & -0.6 \end{bmatrix} \begin{bmatrix} a_t \\ \Delta\zeta_t \end{bmatrix} \tag{18}$$

[1] Gali, Jordi. Monetary Policy, inflation, and the Business Cycle: An introduction to the new Keynesian Framework. Princeton University Press, 2008.

Denote

$$A\begin{bmatrix}\tilde{y}_t\\ \pi_t\\ \hat{l}_t^+\end{bmatrix} = B\begin{bmatrix}E_t(\tilde{y}_{t+1})\\ E_t(\pi_{t+2})\\ E_t(\hat{l}_{t+1}^+)\end{bmatrix} + C\begin{bmatrix}a_t\\ \Delta\zeta_t\end{bmatrix} \tag{19}$$

Solve the system, we get

$$\begin{bmatrix}\tilde{y}_t\\ \pi_t\\ \hat{l}_t^+\end{bmatrix} = D\begin{bmatrix}a_t\\ \Delta\zeta_t\end{bmatrix} \tag{20}$$

Substituting the known parameters we can solve,

$$D = \begin{bmatrix}0.10496 & 0.28902\\ 0.12277 & 0.5773\\ 1.105 & 0.63406\end{bmatrix} \tag{21}$$

We have

$$var(\tilde{y}_t) = D(1,1)^2 var(a_t) + D(1,2)^2 var(\Delta\zeta_t) \tag{22}$$

$$var(\pi_t) = D(2,1)^2 var(a_t) + D(2,2)^2 var(\Delta\zeta_t) \tag{23}$$

2.4 Compute the values of $\begin{bmatrix}var(\tilde{y}_t) & var(\pi_t) & var(\Delta\zeta_t)\end{bmatrix}'$

Substituting the known parameters into equation (22) an (23), we find:

$$var(\tilde{y}_t) = 8.1197 \times 10^{-6}$$
$$var(\pi_t) = 2.469 \times 10^{-5}$$

2.5 Compute the average welfare loss per period.

Frome equation (8), we find L = 0.0075

Figure 1: Doge

3 Figure

4 Codes

4.1 Codes for Part I

```
%% ps6I_par.m This file stores the given parameter value.
beta = .99;
sigma = 1;
phi = 1;
alpha = 1/3;
epsilon = 6;
theta = 2/3;
phi_pi = 1.5;
phi_y = 0.5; % change this value to obtain various outcome
rho_v = 0.5;
rho_a = 0.9;

var_a = 0.00712^2/(1-0.9^2)
ps6_go

%% ps6I_go.m  This file undergo the procedure to calculate the value required
%Caculating parameters
phi_ya_n=(1+phi)/(phi+alpha+sigma*(1-alpha));
kappa=(sigma+(phi+alpha)/(1-alpha))*(1-theta)*(1-beta*theta)/theta*(1-alpha)/(1-alpha+a
kappa = lambda*(sigma+(phi+alpha)/(1-alpha));
Omega = 1/(sigma+phi_y+kappa*phi_pi)
AT = [sigma*Omega, (1-beta*phi_pi)*Omega; (kappa*sigma)*Omega, (kappa+beta*(sigma+phi_y
BT = [1*Omega; kappa*Omega]
residual_par = -psi_nya*(sigma*(1-rho_a)+phi_y);
BT_tilde = [-psi_nya*(sigma*(1-rho_a)+phi_y)*Omega; -kappa*psi_nya*(sigma*(1-rho_a)+phi
I = eye(2);
C = (I - rho_a*AT)^(-1)*BT_tilde
var_y = C(1)^2*var_a
```

```
var_pi = C(2)^2*var_a
WL = 0.5*((sigma+((phi+alpha)/(1-alpha)))*var_y + (epsilon/lambda)*var_pi)
```

4.2 Codes for Part II

```
%% ps6II_par.m This file stores the given parameter value.
beta = .99;
sigma = 1;
phi = 1;
alpha = 1/3;
epsilon = 6;
theta = 2/3;
phi_pi = 1.5;
phi_y = 0.5; % change this value to obtain various outcome
rho_v = 0.5;
rho_a = 0.9;

var_zeta = var_zetaepsilon^2/(1-rho_zeta^2)
var_a = 0.00712^2/(1-0.9^2)
ps6_go

%% ps6II_go.m  This file undergo the procedure to calculate the value required
%Caculating parameters

var_ny = (psi_nya)^2*var_a
var_nr = (sigma*psi_nya*(1-rho_a))^2*var_a

phi_ya_n=(1+phi)/(phi+alpha+sigma*(1-alpha));
kappa=(sigma+(phi+alpha)/(1-alpha))*(1-theta)*(1-beta*theta)/theta*(1-alpha)/(1-alpha+a
kappa = lambda*(sigma+(phi+alpha)/(1-alpha));
Omega = 1/(sigma+phi_y+kappa*phi_pi)
AM0=[1+sigma*eta  0  0
  kappa 1  0
  0  0   1  ]
  AM1 =[sigma*eta  eta   1
  0 beta  0
  0  0   1  ]
 BM = [ eta  -1  0
 0 0   0
  0 0  -1  ]

  A = 1+sigma*eta  0  0
 - kappa 1  0
  0  -1  1  ]
  B= [sigma*eta  eta   1
  0 beta  0
  0  1   1  ]
```

```
  C= [-1.4  -0.6
          0  0
         0  -0.6]
 D = [  0.10496 0.28902
 0.12277 0.5773
1.105 0.63406]
var_y = D(1,1)^2*var_a+ D(1,1)^2*var_zeta
var_pi = D(2,1)^2*var_a + D(2,2)^2*var_zeta

WL = 0.5*((sigma+((phi+alpha)/(1-alpha)))*var_y + (epsilon/lambda)*var_pi)
```