CSE 7350/5350 D.W. Matula	Homework Set # 3	Due date: 29 October 2013

Total Points: 65

Greedy Paradigm
Applications

Grading Policy:
Unless otherwise stated all problems are 10 points each. Partial credit is available on all problems. Homework submitted after the due date may incur a 10 point penalty on that homework set. (Distance students add one day to deadlines.)

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1. [FIFO Scheduling](15 points) Consider the greedy first-come first-serve coloring of interval graphs, where the n intervals all start and stop at integral times 0, 1, 2, …, t for t ≤ n. Intervals that overlap only at end points are *not* to be considered to yield edges in the interval graph. Describe an O (n) procedure for first building the *event list data structure* from the list of intervals assumed given by start and stop times. The event list is a temporally ordered sequence with each interval occurring twice. The first occurrence of each interval is determined by its start time and the second occurrence by its stop time. Then describe and analyze an implementation of the coloring procedure employing the event list data structure that operates in O(n) time and space that colors with a minimum number of colors. Provide a walk-through of your coloring algorithm for the graph with the event list **ZWXTZVYYXQWUVSUTRSRQ**.

Provide an algorithm determining the degrees of all vertices in O(n) time from the event list for the interval graph. Walkthrough your algorithm for the above graph (provide a drawing of the graph).

Reference: Text Problem 16.1-4, p. 422

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2. [Matching Substrings] A matching substring pair of length k in a binary bit string $b_1, b_2 \ldots b_n$ is a pair $b_i b_{i+1} \ldots b_{i+k-1} = b_j b_{j+1} \ldots b_{j+k-1}$, with i not equal to j, of identical k-bit substrings.

Bonus!

(a) Determine a maximum length binary string with no matching substrings of length 4.

(b) Describe the suffix tree algorithm for determining a longest matching substring pair and give its time and space complexity. Apply the algorithm to find the longest binary string match for the first 62 bits of

$$\pi = 3.11037\ 55242\ 10264\ 30215_8.$$

Reference: See notes on web page.

3. [Graph Degree Structures] Describe data and record structures for vertex ordering and vertex or edge coloring (or labeling) and a suitably greedy graph search algorithm to solve each of the following problems in the time bound indicated. Illustrate each algorithm with vertex or edge coloring (or labeling) on a graph or tree designed to teach your algorithm. The graph (or tree) should have at least 18 vertices and a maximum degree of at least 4. The graph should be connected with a minimum degree 3.

i) Find a maximum independent set in a tree in time O (|V|).

ii) Find some "k-connected" pair of vertices u, v in the graph having k = min {degree (u), degree (v)} edge disjoint distinctly colored (labeled) paths between u and v in time O (|V|+ |E|) using maximum adjacency search. Identify all the sets of pairwise k-edge-connected vertices using the "k-1 cycle" observation discussed in class.

Reference: For (ii) see web page Project Description for Maximum Adjacency Search and the example walkthrough.

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4. [GCD] Find the greatest common divisor GCD (40902, 24140) by each of the following algorithms, showing the intermediate steps. Convert the input arguments to binary for parts (b) and (c).

a) Euclid's Algorithm employing division in decimal and illustrating the successive division invariants $x = q \cdot y + r$.
b) Binary normalization shift-and-subtract.
c) Binary parity "right-normalize" and subtract.
d) "Ternary parity right-normalize" and subtract using decimal.

5. [Selecting Multiple Random Integers] Given a random number generator that delivers a uniform normalized binary single precision random number, you wish to efficiently select a sequence of uniform random integers over the range {1, 2, …k} from this one random choice using the "multiply and integer, fraction partition" procedure. How many integer choices should be possible with round-off not causing significant bias? Discuss what will happen if the procedure continues to provide more integers, then answer the same questions for a normalized binary double precision random number.

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NOTE: to appreciate round-off solve the problem for k=2 and 4 first before solving for general integers k.

6. [**Random Geometric Graphs: Smallest Last Ordering**] Implement the smallest last ordering algorithm applicable to graphs of up to 10,000 vertices. For the RGG's G (200, 0.16) and G (1600, 0.06) considered in problem set #2:

(i) Plot the original degrees of the vertices in smallest last order for both graphs.

(ii) Plot the degree when deleted list in smallest last order for both graphs,

(iii) Display the remaining graphs after 10%, 20%, 50%, 80%, and 90%, of the vertices in smallest last order are removed. For G (1600, 0.06) display only the vertex sets.

(iv) [5 bonus points] Do (i), (ii), (iii) for a random graph on the sphere with n = 400 and r = 0.25 as described in problem set #2. No