and Sphere. So for Square the will have x, y axis, for Disc also the point will have x, y axis and for sphere the point will have x, y, z axis since it is in three dimensional. As each sensor has omnidirectional radio ability, coverage range of a sensor can be represented by a disk around that point with radius 'r', so the area will be 'πr^2' and also we consider that all the sensors have same signal strength, that is, a disk with radius 'r' around each sensor. The below figure 1 shows random points or sensors in a circular geometry having a uniform radio around them.

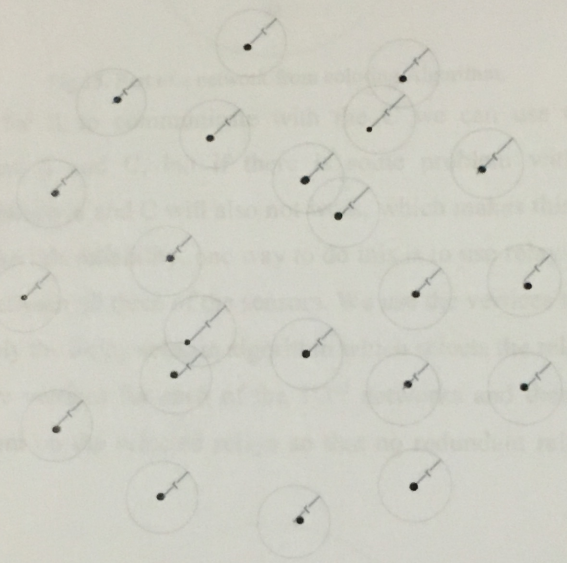

Fig 14. Radio around the random points in a circular geometry.

We can directly apply the maximum independent set algorithm to remove the redundant sensors and cover the entire geometry with the less number of sensors, therefore this reduces the cost of investment. But there is a disadvantage with this method is that we only get one single network which makes it less robust since there are no supporting networks to handle the load. So, for reason we use coloring algorithm which generates set networks which makes the complete network robust. Smallest last vertex ordering algorithm is a step that is used to achieve the coloring algorithm efficiently. The coloring algorithm generates many independent networks, but all the networks we get does not have same number of vertices in each network, that is, the coverage is not same in all the networks. Only the first 1/3rd networks have same and dominant network coverage when compared to other networks since the number of vertices in all of these networks is almost all the same, this is shown in the color plot of the graphs presented in the coming sections

of this work. We can guarantee the signal coverage with these networks, but these networks may not be reliable. If a part of a network that is obtained is as shown in Fig 2. In this figure if A, B, C

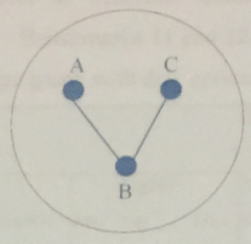

Fig 15. Part of a network from coloring Algorithm.

are sensors, in order for A to communicate with the C we can use the sensor B for the communication between A and C, but if there is some problem with sensor B then the communication link between A and C will also not work, which makes this network not reliable. Now we want to improve this reliability, one way to do this is to use relays which are capable of providing connection between all three of the sensors. We use the vertices that are not in the 1\3rd of the color sets and apply the Relay selectin algorithm which selects the relays or vertices that are connected three or more vertices for each of the 1\3rd networks and then apply the maximum independent set algorithm on the selected relays so that no redundant relay is selected. In the below figure,

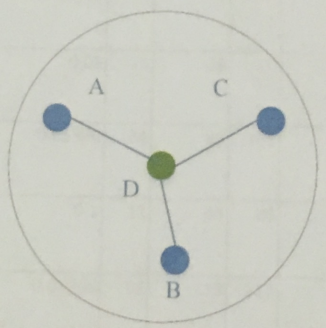

Fig 16. Part of a network with relays

Sensor D acts as a relay between all the three sensors A, B, C. As seen in the figure 3 it is more reliable than the network in the figure 2. Like this we get the bipartite backbone subgraphs which form a robust, reliable and less investment cost wireless sensor networks.

2.2 Benchmark Result Summary Table

The below table gives an overview summary of benchmarks with different geometries square, disk and sphere. Benchmarks 11 and 12 are chosen as of sizes N = 500 for a small graph and N = 80000 for a large graph with disk geometry respectively.

	INPUT			OUTPUT							
	N	Target Avg Degree	Geometry	Threshold R	Degree Min	Avg	Max	Terminal Clique Size	Number of Colors	Number of Primary Sets	Vertex % in Primary
0	1000	40	Square	0.114	10	37	64	23	24	8	
1	1000	40	Square	0.11283	8	36	55	24	24	8	48.1
2	4000	40	Square	0.05641	11	37	66	26	27	9	52.55
3	4000	60	Square	0.06909	18	56	89	36	36	12	49.475
4	16000	100	Square	0.04460	27	96	141	50	52	17	44.78125
5	32000	150	Square	0.03862	37	145	203	65	74	24	43.83125
6	4000	40	Disk	0.05	14	38	60	23	24	8	46.575
7	8000	100	Disk	0.05590	34	95	132	46	53	17	44.8125
8	4000	40	Sphere *	0.1	11	45	86	34	36	12	62.9750
9	16000	100	Sphere *	0.07905	32	113	217	76	80	26	62.2937
10	64000	150	Sphere *	0.04841	53	171	369	111	129	43	70.55
11	500	40	Disk	0.14142	14	35	57	24	25	8	47
12	80000	250	Disk	0.02795	94	244	310	95	112	37	42.09

Table 1. Benchmark Result Summary Table

2.3 DISPLAY AND PLOTS

2.3.1 Benchmark 1

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
1000	40	Square	106 (64 sensors+42 relay sensors)	94 (59sensors+35 relay sensors)	86 (60sensors+26 relay sensors)	128	107	81	0.1128	35

Fig 17. RGG Representing Maximum degree Vertex and minimum degree vertex

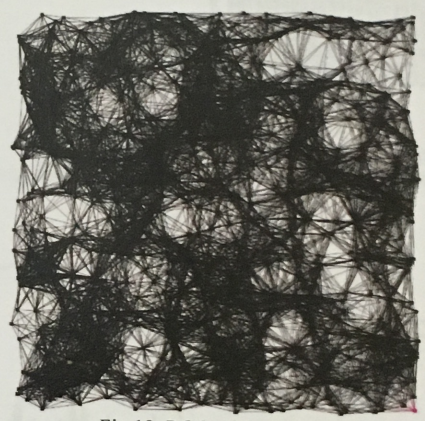

Fig 18. RGG with all the edges

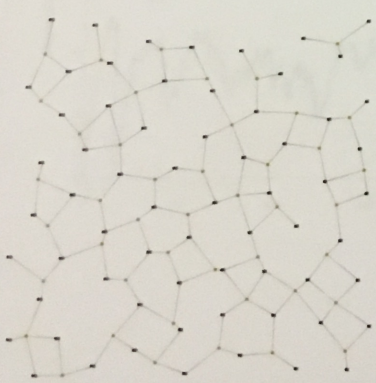

Fig 19. 1st Bipartite Subgraph

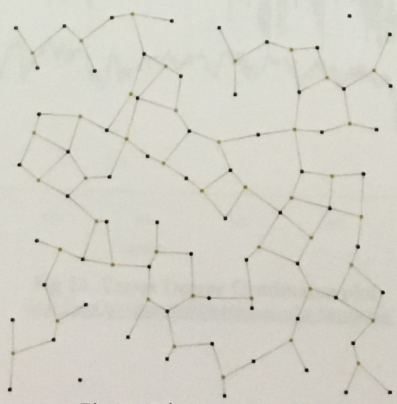

Fig 20. 2nd Bipartite Subgraph

Fig 21. 3rd Bipartite Subgraph

In the above subgraphs green color square represents relay sensor and the black color square represents a normal sensor.

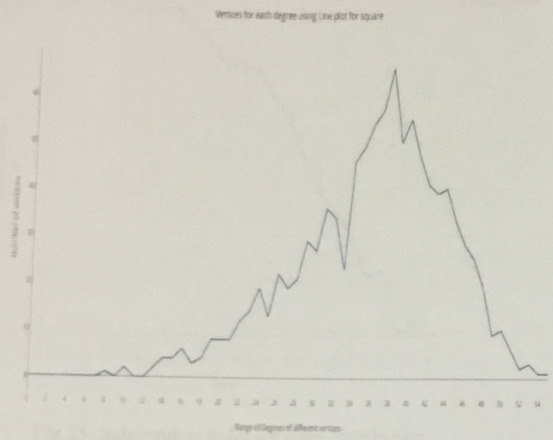

Fig 22. Degree Distribution line plot
https://plot.ly/~chandan9/682/vertices-for-each-degree-using-line-plot-for-square/

Fig 23. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/683/vertices-for-each-degree-using-bar-graph-for-square/

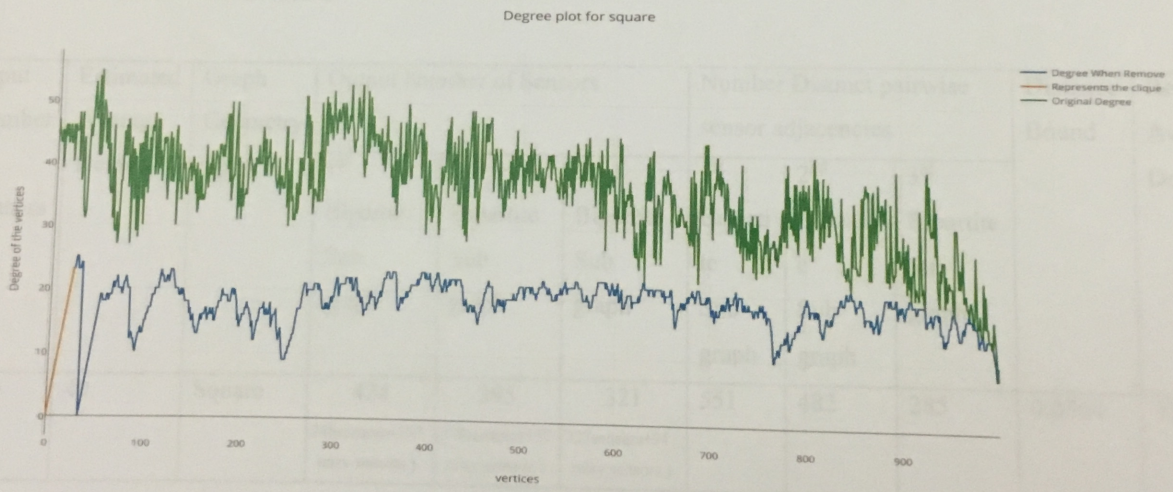

Fig 24. Vertex Degree Distribution plot
https://plot.ly/~chandan9/684/degree-plot-for-square/

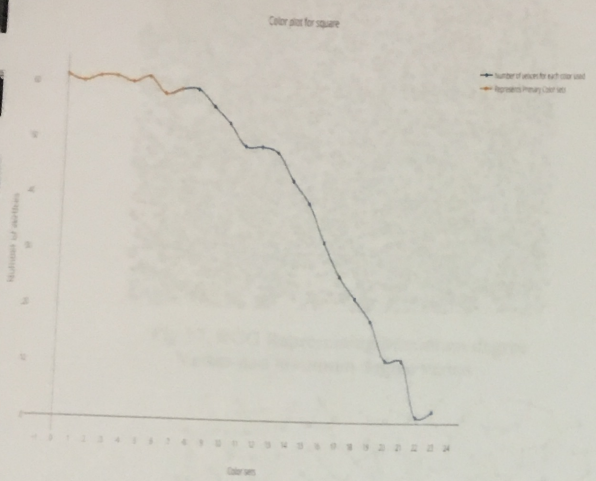

Fig 25. Independent color set sizes distribution

https://plot.ly/~chandan9/685/color-plot-for-square/

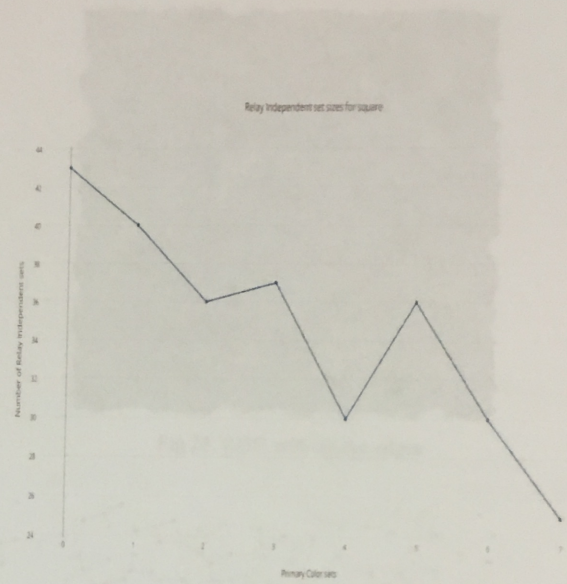

Fig 26. Relay Independent set size distribution for primary color sets

https://plot.ly/~chandan9/687/relay-independent-set-sizes-for-square/

2.3.2 Benchmark 2

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
4000	40	Square	424 (245sensors+179 relay sensors)	395 (238sensors+157 relay sensors)	321 (227sensors+94 relay sensors)	551	482	285	0.0564	37

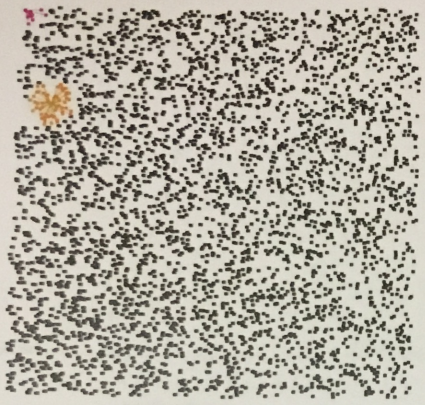

Fig 27. RGG Representing Maximum degree Vertex and minimum degree vertex

Fig 28. RGG with all the edges

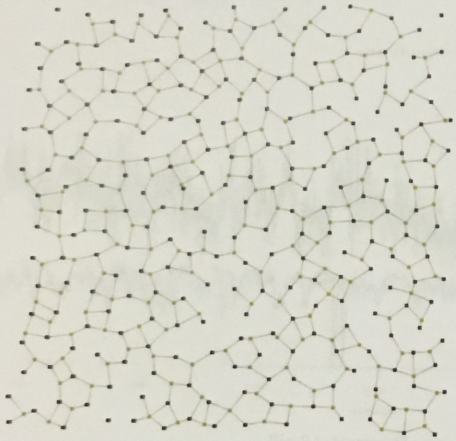

Fig 29. 1st Bipartite Subgraph

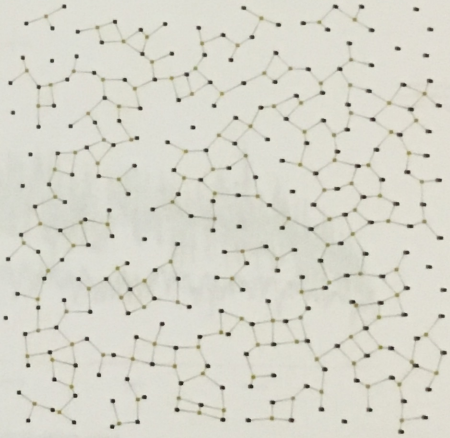

Fig 30. 2nd Bipartite Subgraph

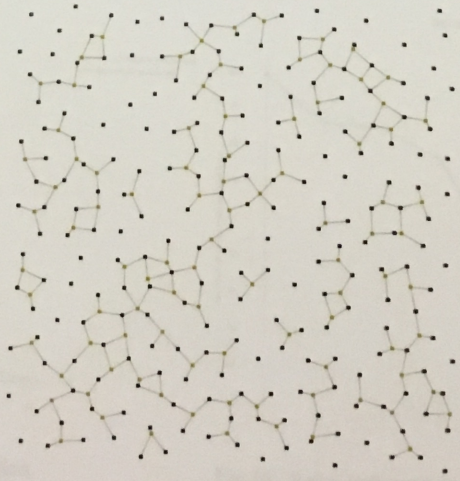

Fig 31. 3rd Bipartite Subgraph

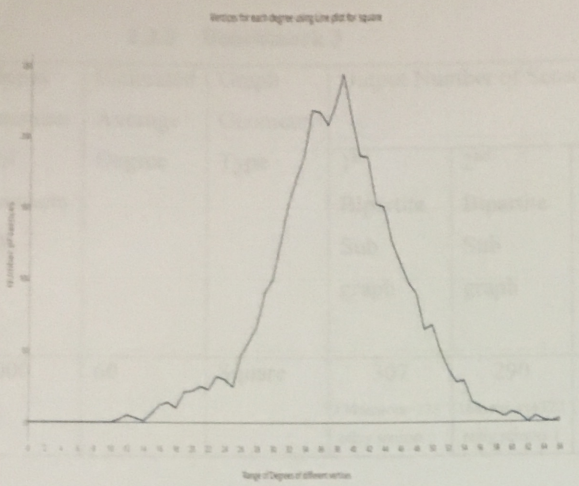

Fig 32. Degree Distribution line plot
https://plot.ly/~chandan9/1038/vertices-for-each-degree-using-line-plot-for-square/

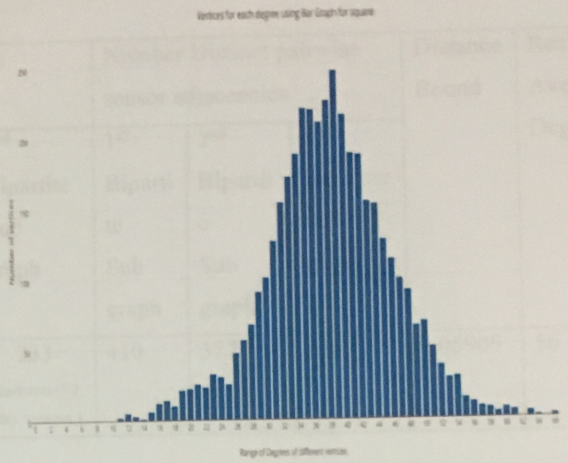

Fig 33. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1045/vertices-for-each-degree-using-bar-graph-for-square/

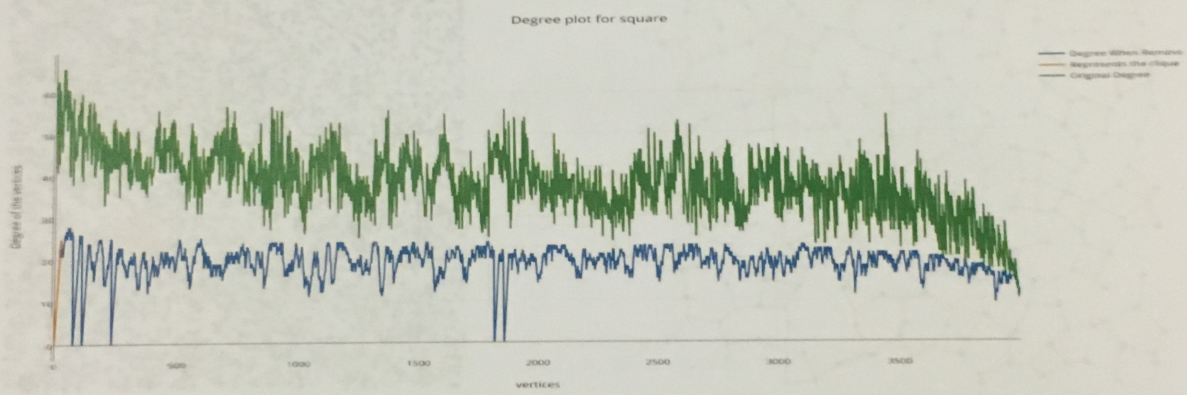

Fig 34. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1035/degree-plot-for-square/

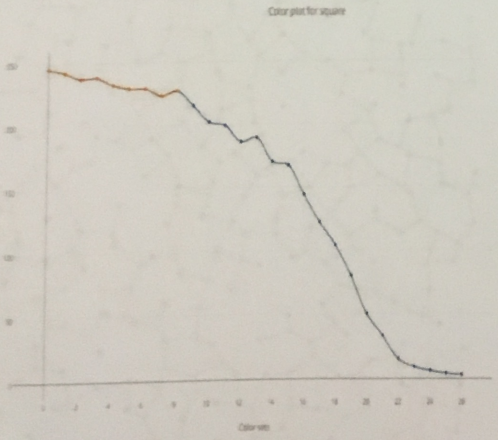

Fig 35. Independent color set sizes distribution
https://plot.ly/~chandan9/1036/color-plot-for-square/

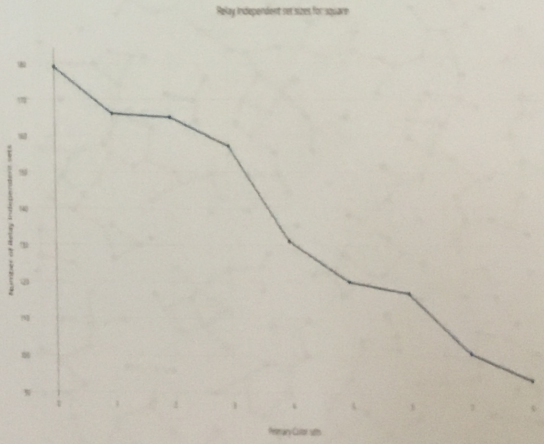

Fig 36. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1043/relay-independent-set-sizes-for-square/

2.3.3 Benchmark 3

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
4000	60	Square	307 (174sensors+133 relay sensors)	290 (168sensors+122 relay sensors)	233 (156sensors+77 relay sensors)	410	377	240	0.06909	56

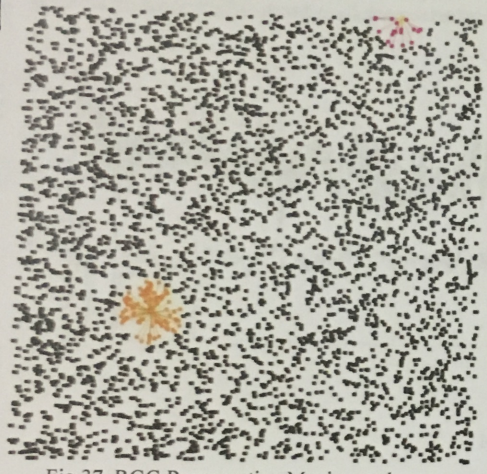

Fig 37. RGG Representing Maximum degree Vertex and minimum degree vertex

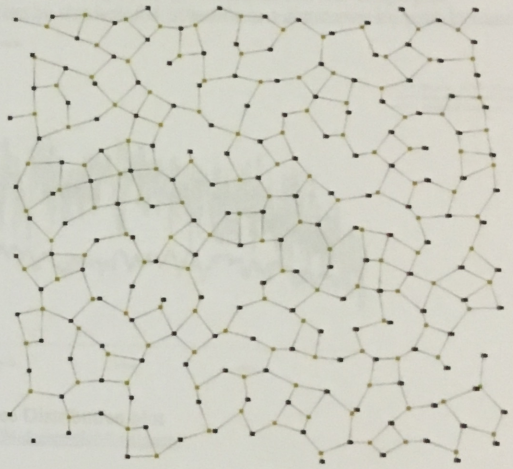

Fig 38. 1st Bipartite Subgraph

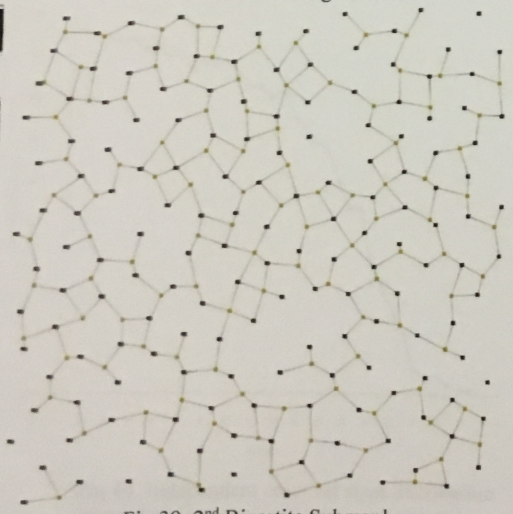

Fig 39. 2nd Bipartite Subgraph

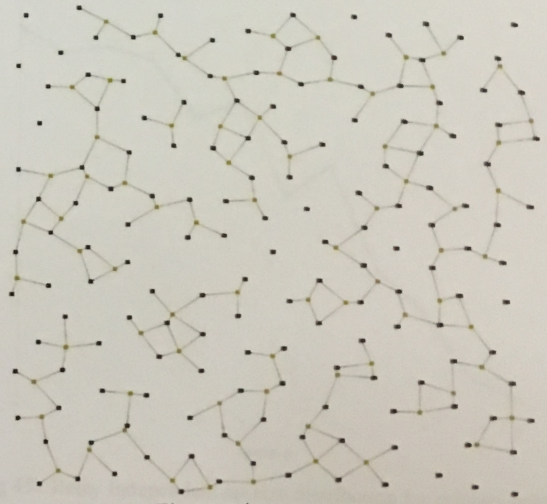

Fig 40. 3rd Bipartite Subgraph

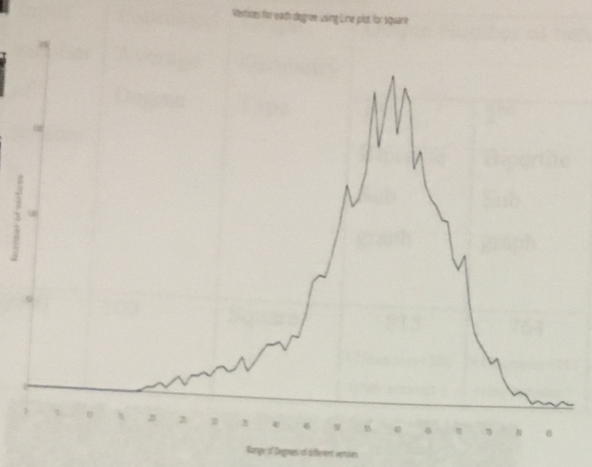

Fig 41. Degree Distribution line plot
https://plot.ly/~chandan9/1076/vertices-for-each-degree-using-line-plot-for-square/

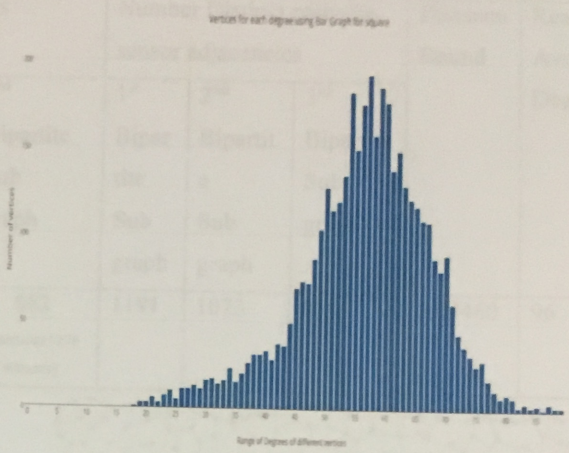

Fig 42. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1078/vertices-for-each-degree-using-bar-graph-for-square/

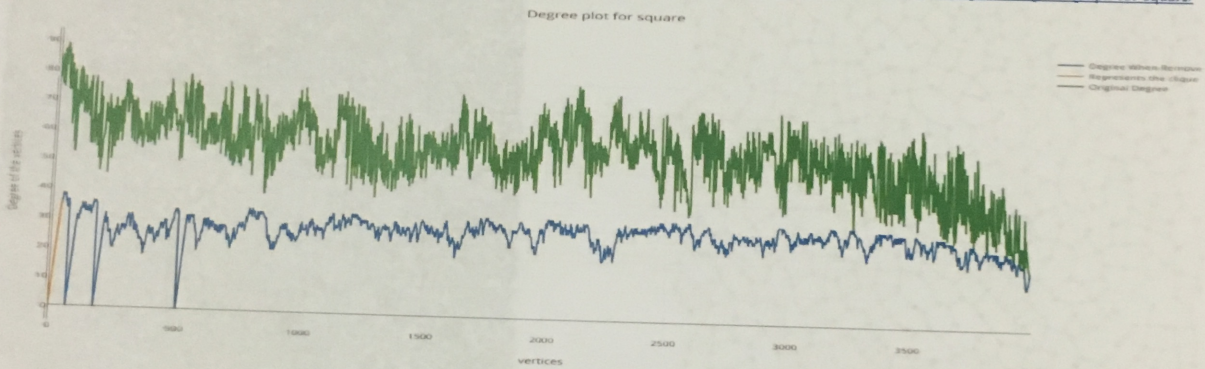

Fig 43. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1094/degree-plot-for-square/

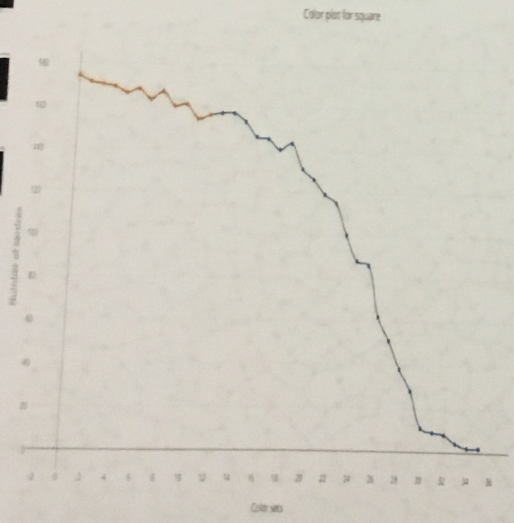

Fig 44. Independent color set sizes distribution
https://plot.ly/~chandan9/1084/color-plot-for-square/

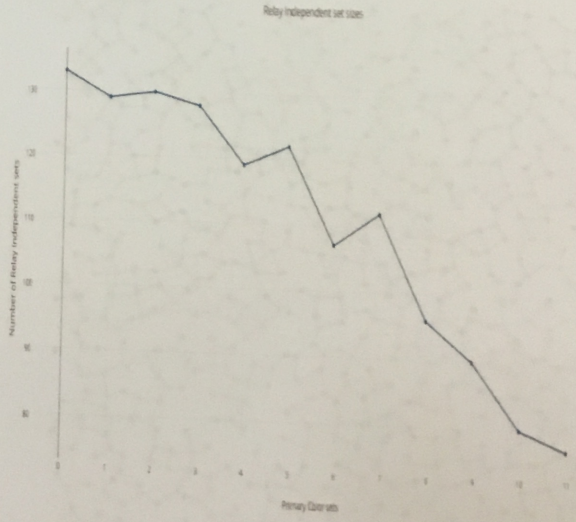

Fig 45. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1089/relay-independent-set-sizes/

2.3.4 Benchmark 4

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
16000	100	Square	815 (435sensors+380 relay sensors)	764 (419sensors+345 relay sensors)	682 (406sensors+276 relay sensors)	1191	1073	858	0.04460	96

Fig 46. RGG Representing Maximum degree Vertex and minimum degree vertex

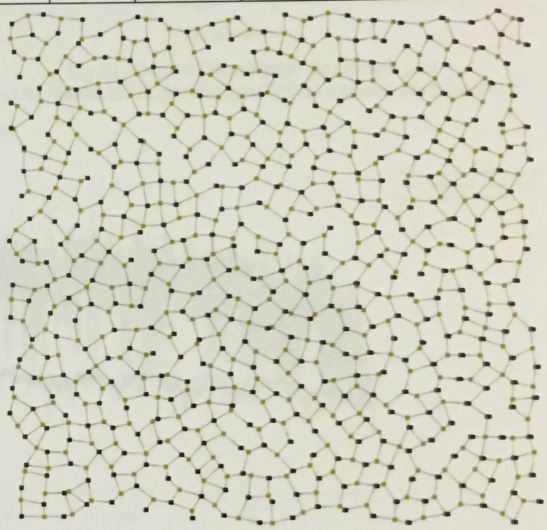

Fig 47. 1st Bipartite Subgraph

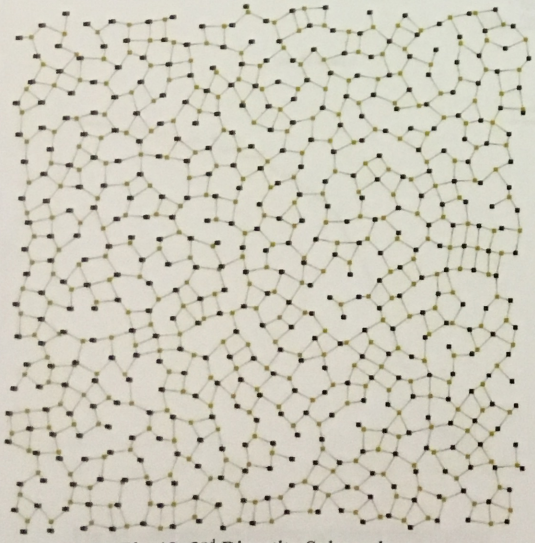

Fig 48. 2nd Bipartite Subgraph

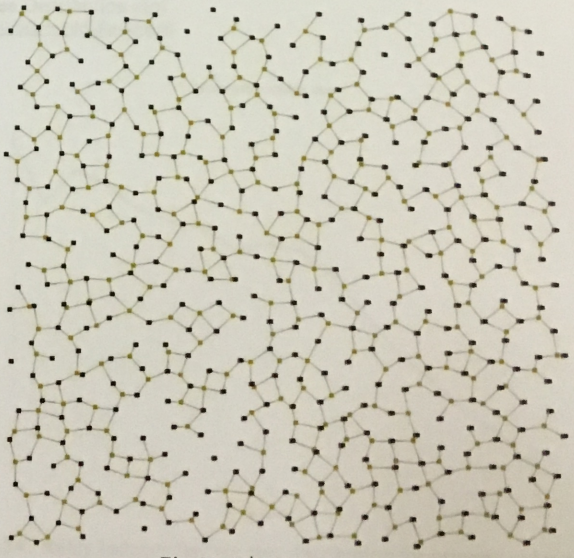

Fig 49. 3rd Bipartite Subgraph

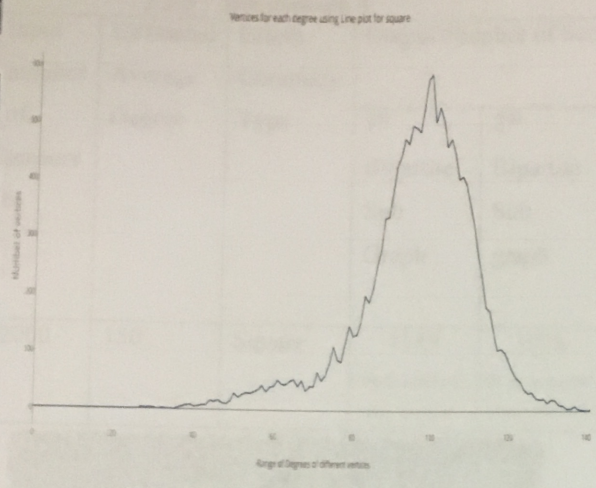

Fig 50. Degree Distribution line plot
https://plot.ly/~chandan9/1138/vertices-for-each-degree-using-line-plot-for-square/

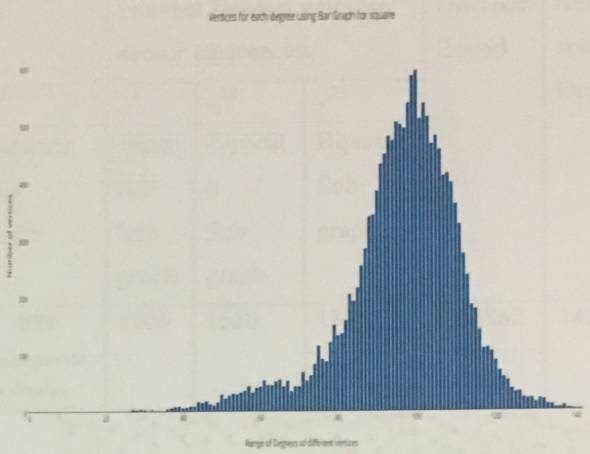

Fig 51. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1139/vertices-for-each-degree-using-bar-graph-for-square/

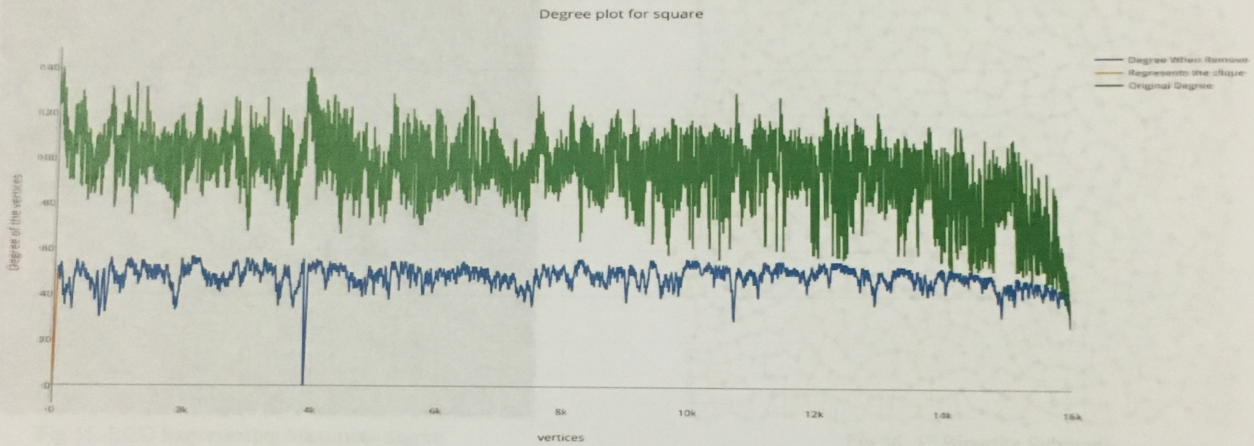

Fig 52. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1140/degree-plot-for-square/

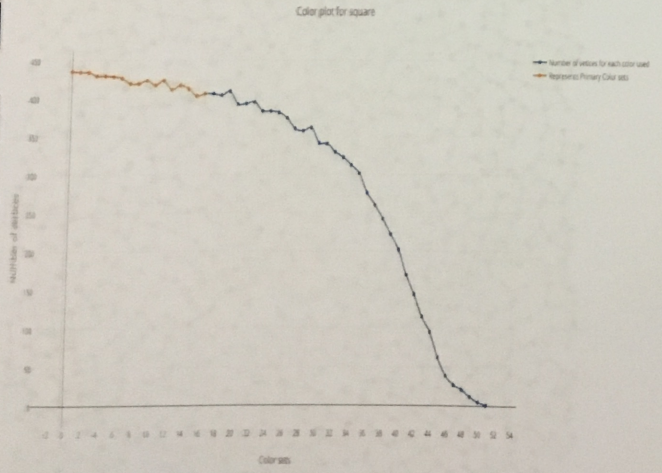

Fig 53. Independent color set sizes distribution
https://plot.ly/~chandan9/1111/color-plot-for-square/

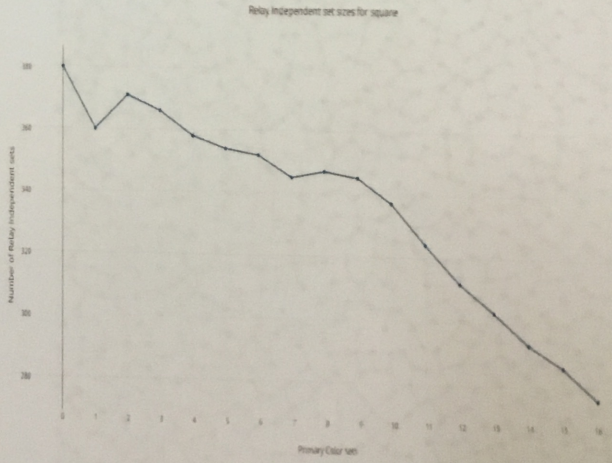

Fig 54. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1143/relay-independent-set-sizes-for-square/

2.3.5 Benchmark 5

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
32000	150	Square	1149 (604sensors+545 relay sensors)	1078 (587sensors+491 relay sensors)	984 (556sensors+418 relay sensors)	1706	1550	1324	0.03862	145

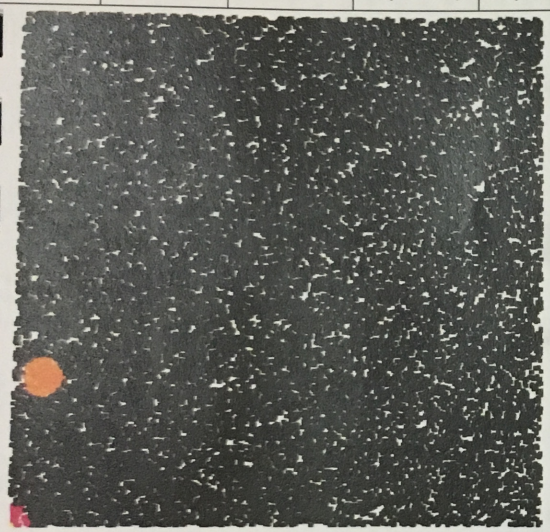

Fig 55. RGG Representing Maximum degree Vertex and minimum degree vertex

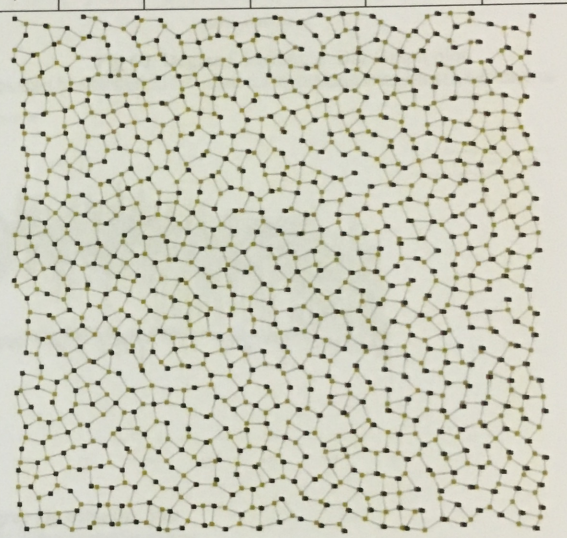

Fig 56. 1st Bipartite Subgraph

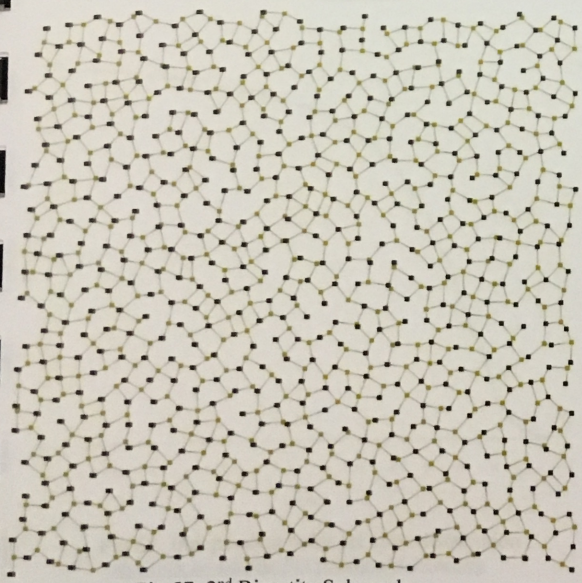

Fig 57. 2nd Bipartite Subgraph

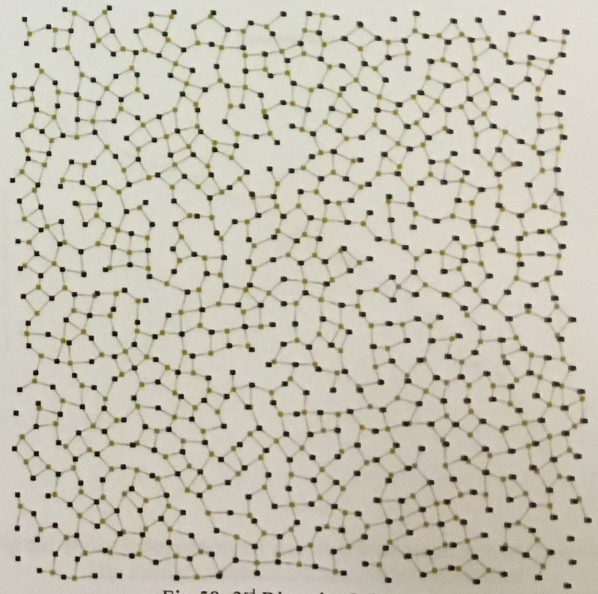

Fig 58. 3rd Bipartite Subgraph

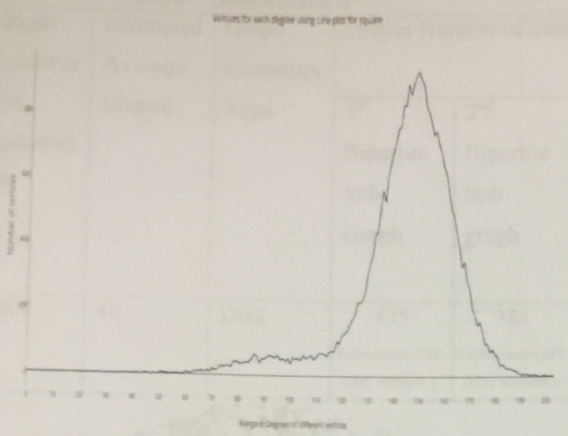

Fig 59. Degree Distribution line plot
https://plot.ly/~chandan9/1154/vertices-for-each-degree-using-line-plot-for-square/

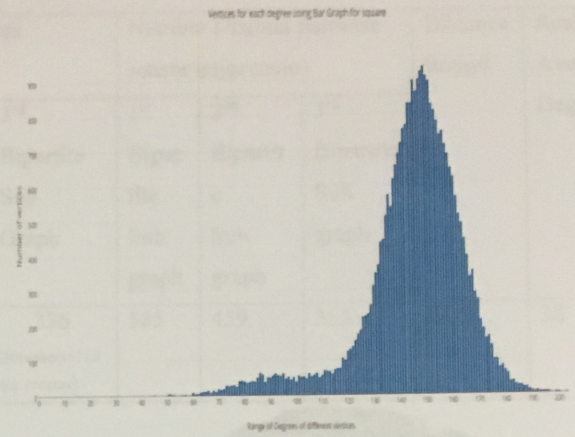

Fig 60. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1157/vertices-for-each-degree-using-bar-graph-for-square/

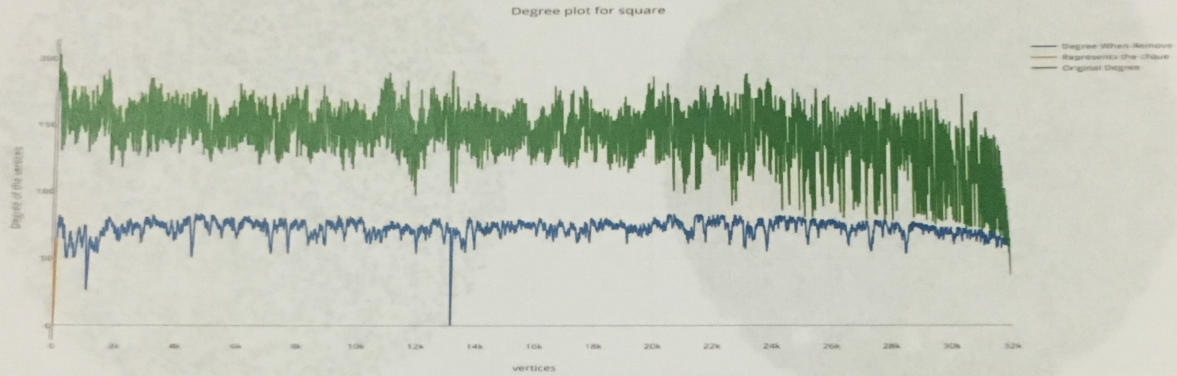

Fig 61. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1158/degree-plot-for-square/

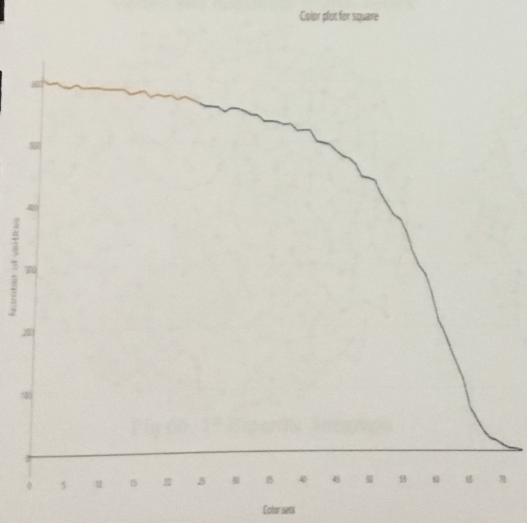

Fig 62. Independent color set sizes distribution
https://plot.ly/~chandan9/1159/color-plot-for-square/

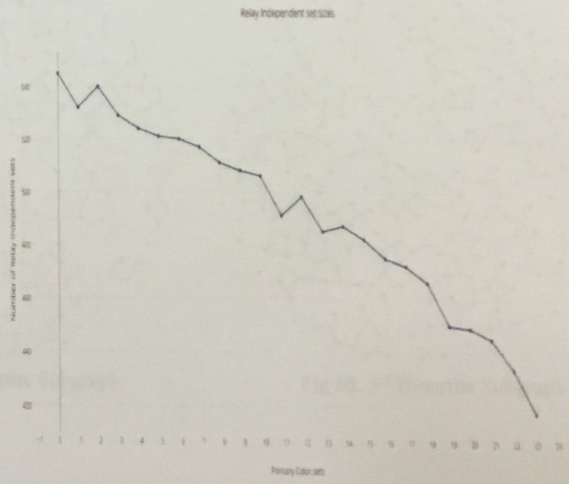

Fig 63. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1218/relay-independent-set-sizes/

2.3.6 Benchmark 6

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub Graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
4000	40	Disk	435 (245sensors+190 relay sensors)	384 (235sensors+149 relay sensors)	336 (220sensors+116 relay sensors)	585	459	353	0.05	38

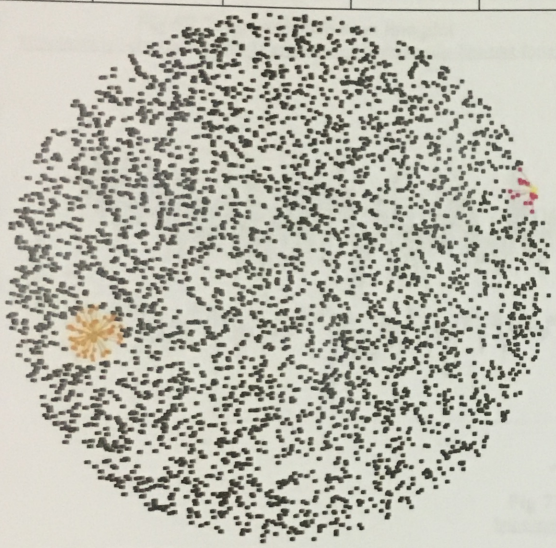

Fig 64. RGG Representing Maximum degree Vertex and minimum degree vertex

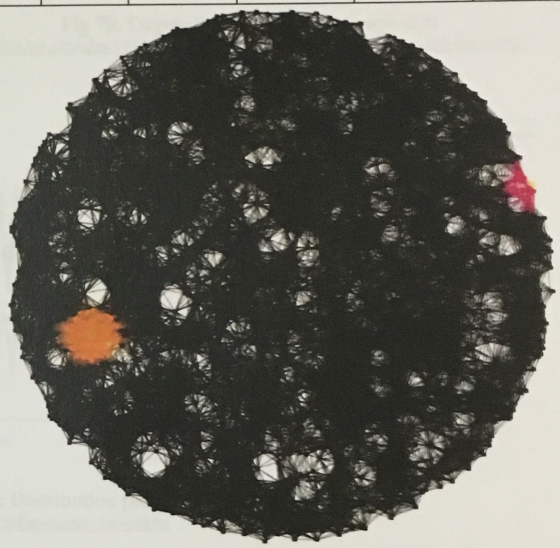

Fig 65. RGG with all the edges

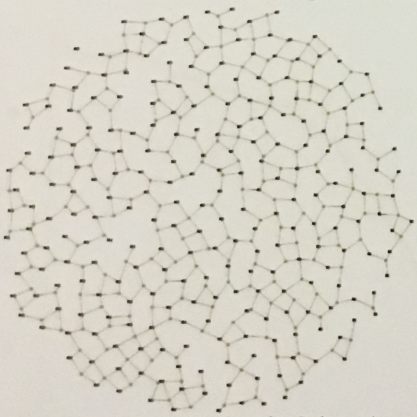

Fig 66. 1st Bipartite Subgraph

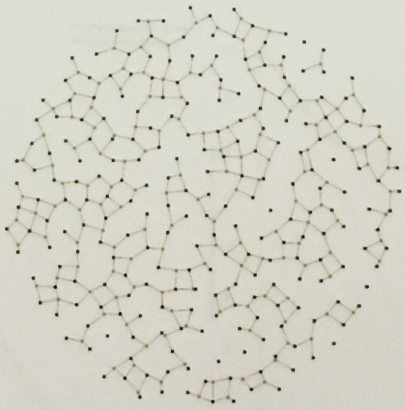

Fig 67. 2nd Bipartite Subgraph

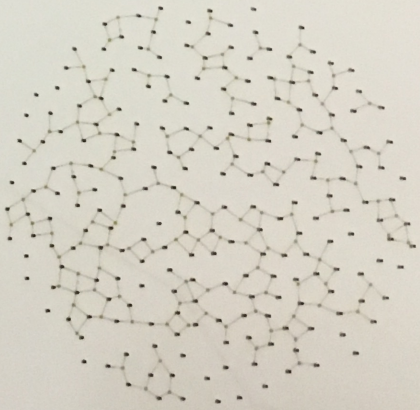

Fig 68. 3rd Bipartite Subgraph

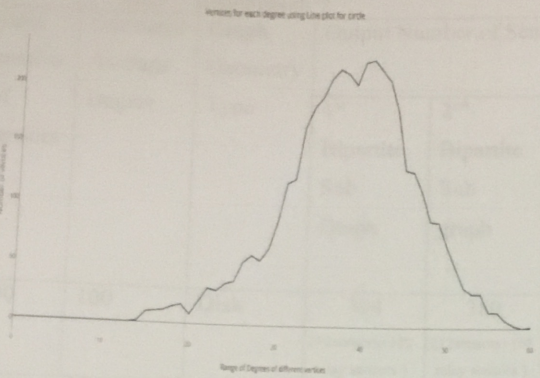

Fig 69. Degree Distribution line plot
https://plot.ly/~chandan9/1443/vertices-for-each-degree-using-line-plot-for-circle/

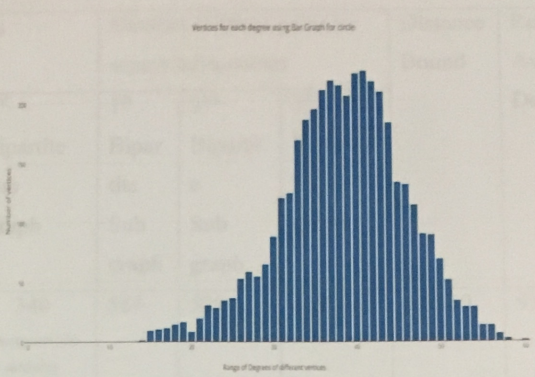

Fig 70. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1444/vertices-for-each-degree-using-bar-graph-for-circle/

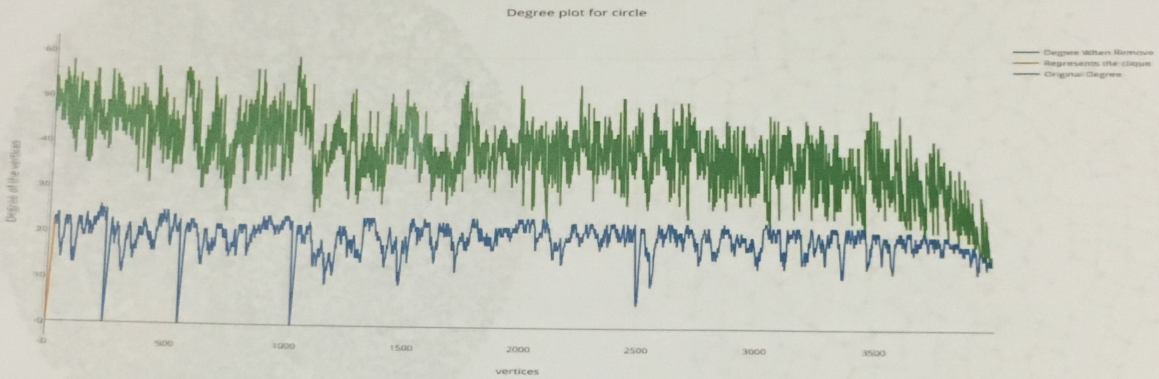

Fig 71. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1445/degree-plot-for-circle/

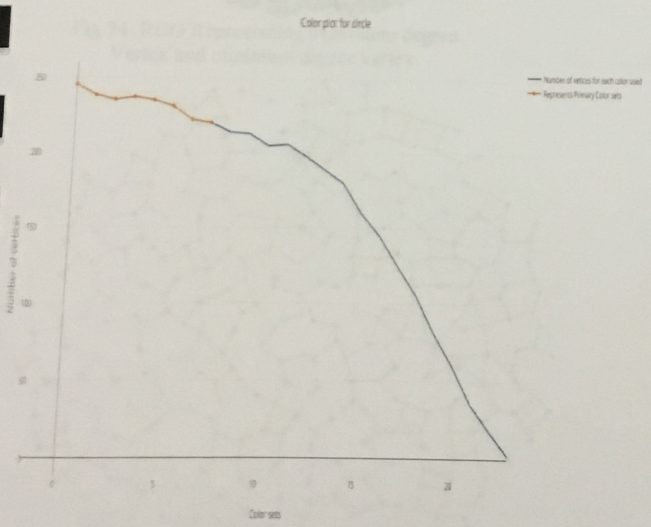

Fig 72. Independent color set sizes distribution
https://plot.ly/~chandan9/1446/color-plot-for-circle/

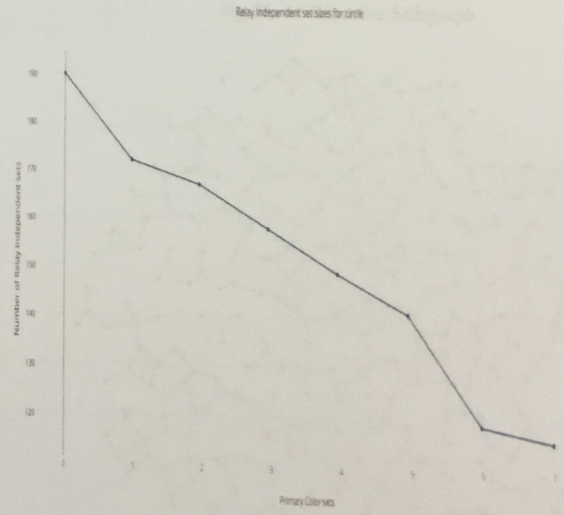

Fig 73. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1448/relay-independent-set-sizes-for-circle/

2.3.7 Benchmark 7

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub Graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
8000	100	Disk	394 (212sensors+182 relay sensors)	380 (212sensors+168 relay sensors)	340 (204sensors+136 relay sensors)	564	520	419	0.0559	95

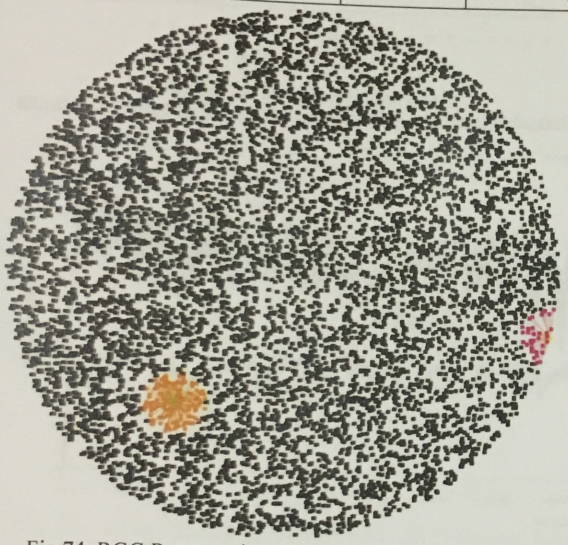

Fig 74. RGG Representing Maximum degree Vertex and minimum degree vertex

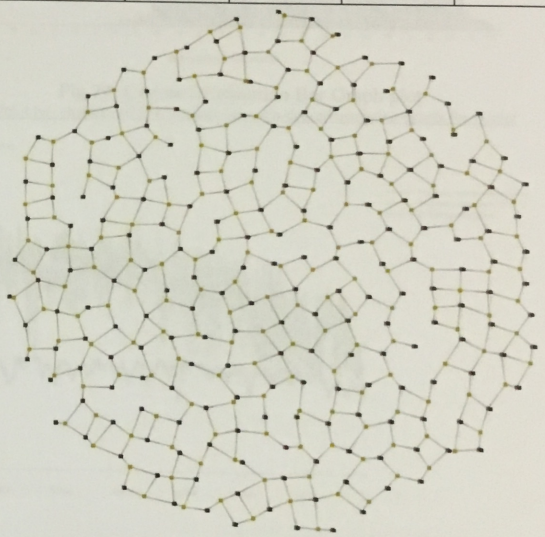

Fig 75. 1st Bipartite Subgraph

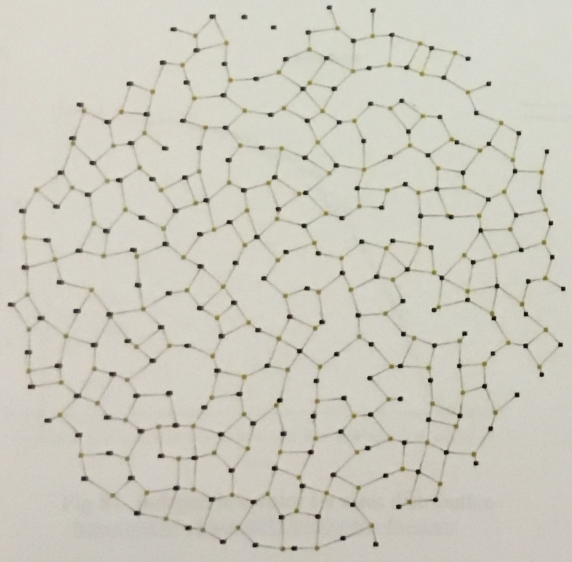

Fig 76. 2nd Bipartite Subgraph

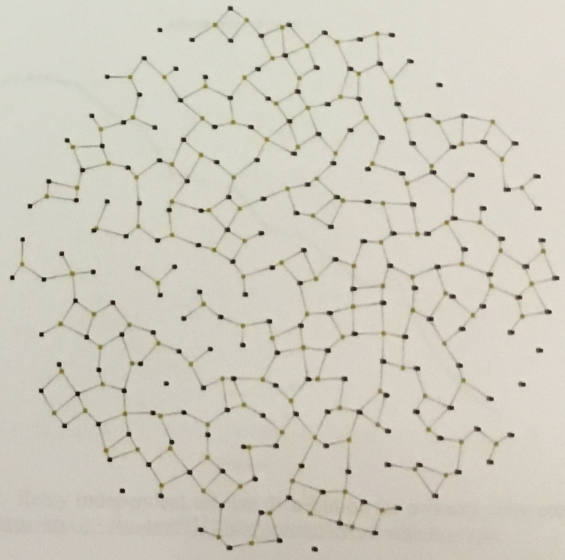

Fig 77. 3rd Bipartite Subgraph

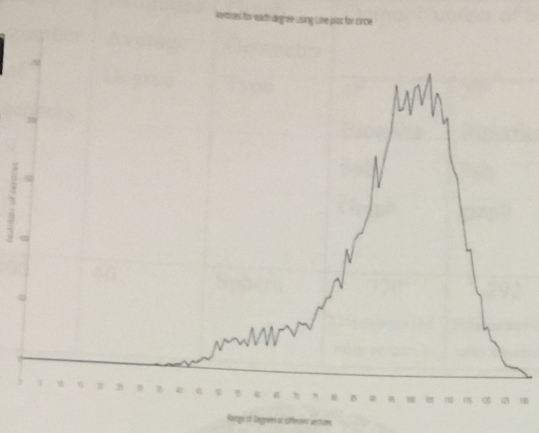

Fig 78. Degree Distribution line plot
https://plot.ly/~chandan9/1536/vertices-for-each-degree-using-line-plot-for-circle/

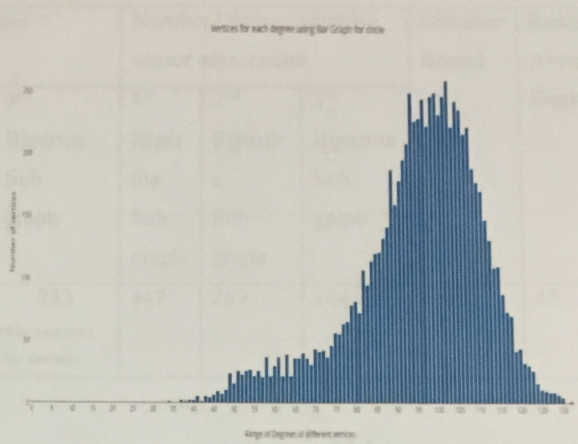

Fig 79. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1537/vertices-for-each-degree-using-bar-graph-for-circle/

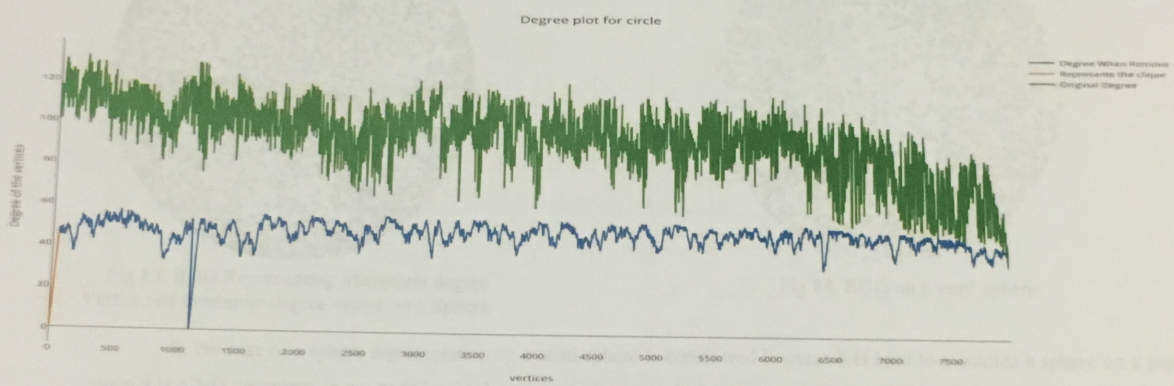

Fig 80. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1538/degree-plot-for-circle/

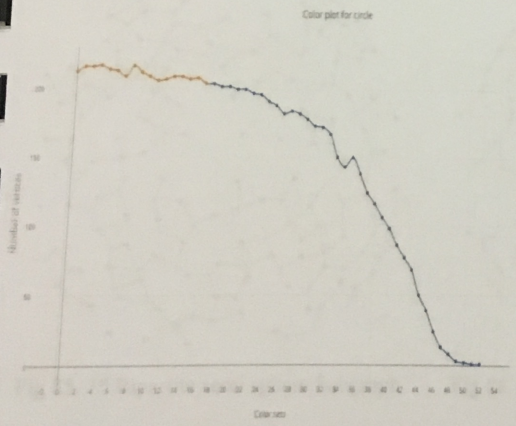

Fig 81. Independent color set sizes distribution
https://plot.ly/~chandan9/1539/color-plot-for-circle/

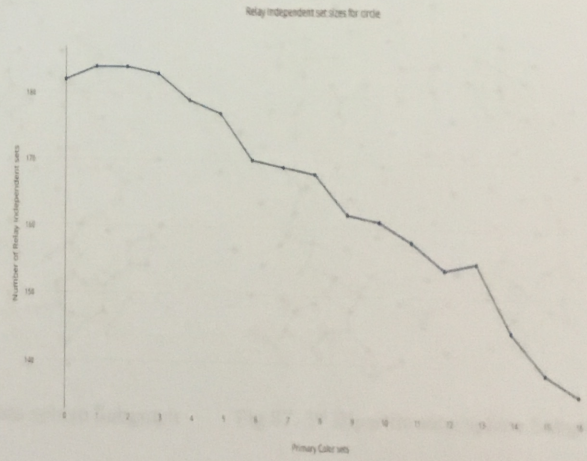

Fig 82. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1541/relay-independent-set-sizes-for-circle/

2.3.8 Benchmark 8

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
4000	40	Sphere	370 (226sensors+144 relay sensors)	292 (205sensors+87 relay sensors)	233 (186sensors+47 relay sensors)	447	269	144	0.1	45

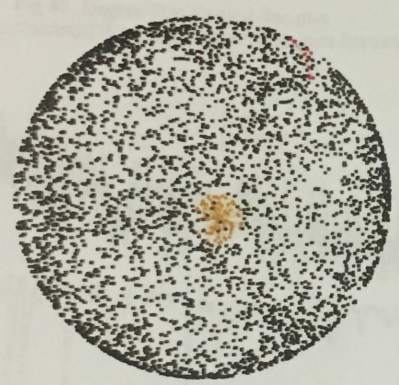

Fig 83. RGG Representing Maximum degree Vertex and minimum degree vertex on a Sphere

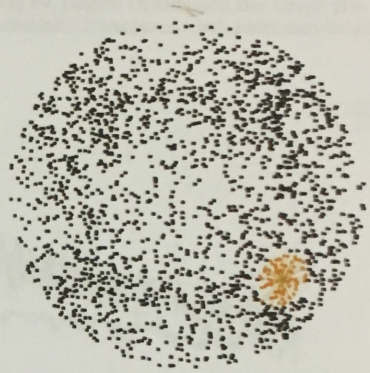

Fig 84. RGG on a semi sphere

In the case of a sphere representation on a semi sphere is considered because it is hard to visualize a sphere on a paper since it is a 3-D geometry, in my code I used the concept of rotation in 3-D space so that I can view the complete sphere, only for representation in the report I used the semi sphere which is easy to represent on paper.

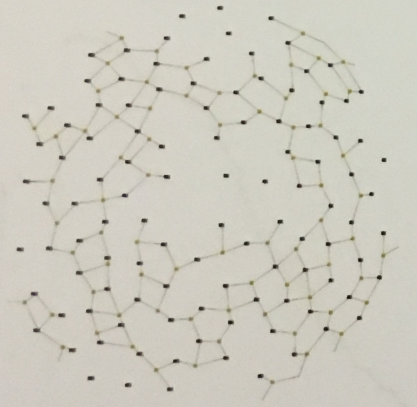

Fig 85. 1st Bipartite semi sphere Subgraph

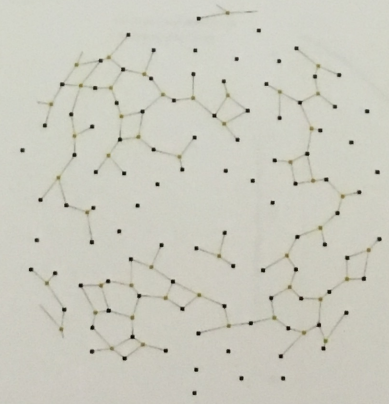

Fig 86. 2nd Bipartite semi sphere Subgraph

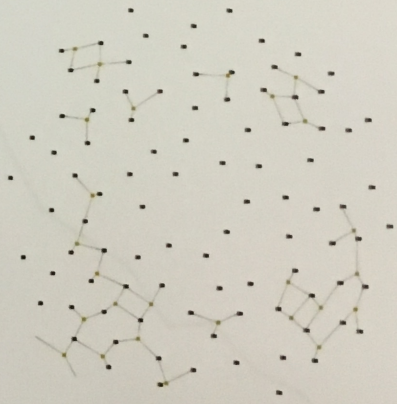

Fig 87. 3rd Bipartite semi sphere Subgraph

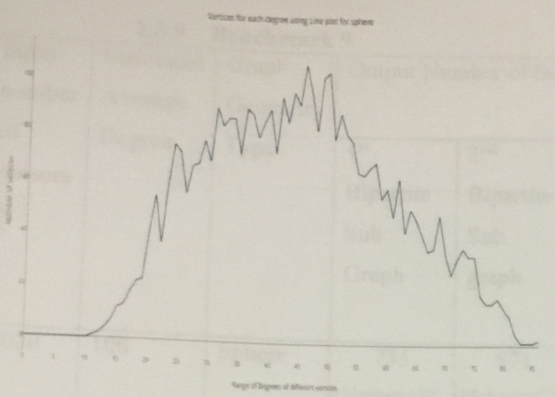

Fig 88. Degree Distribution line plot
https://plot.ly/~chandan9/1748/vertices-for-each-degree-using-line-plot-for-sphere/

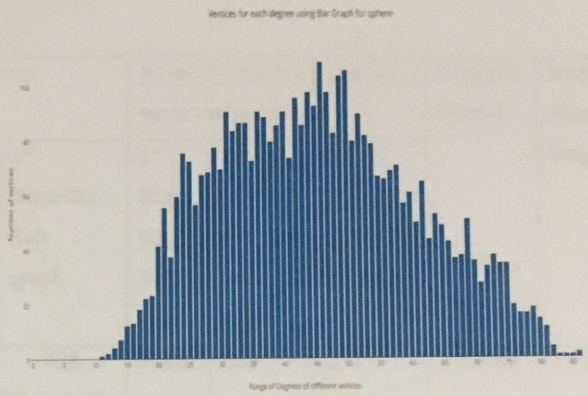

Fig 89. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1749/vertices-for-each-degree-using-bar-graph-for-sphere/

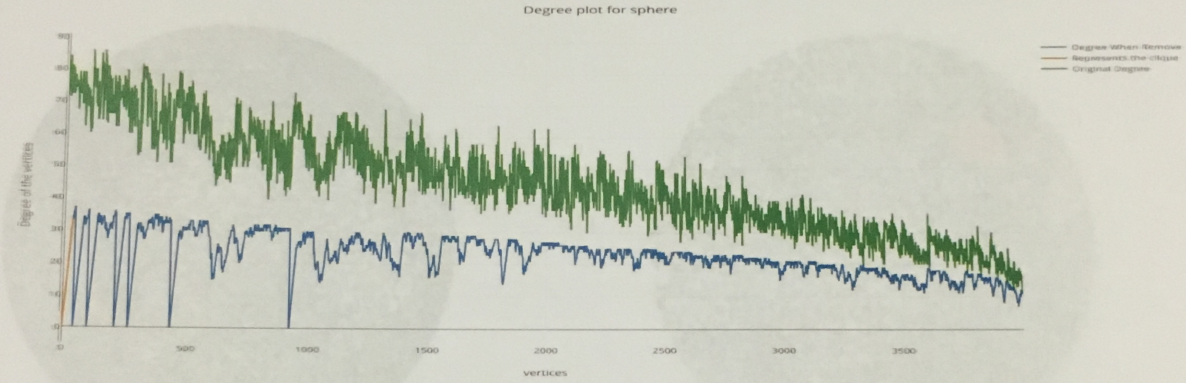

Fig 90. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1750/degree-plot-for-sphere/

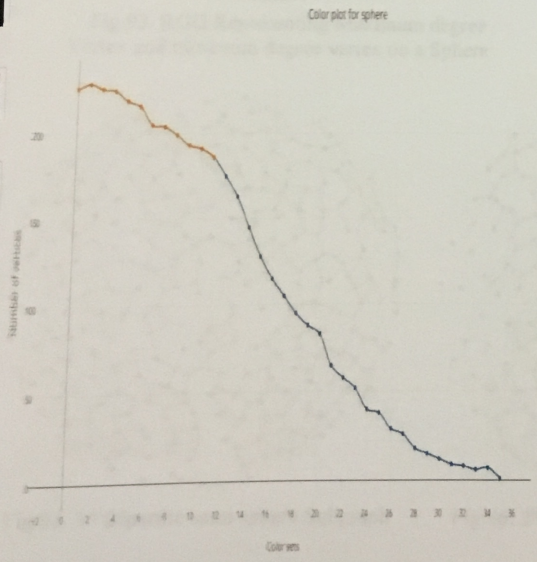

Fig 91. Independent color set sizes distribution
https://plot.ly/~chandan9/1751/color-plot-for-sphere/

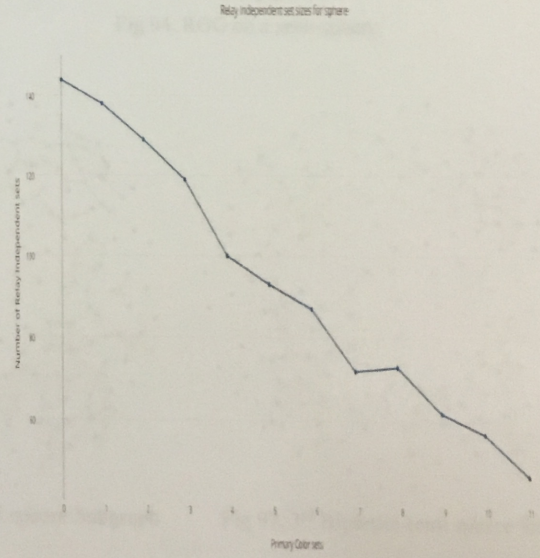

Fig 92. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1753/relay-independent-set-sizes-for-sphere/

2.3.9 Benchmark 9

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
6000	100	Sphere	735 (412sensors+323 relay sensors)	571 (392sensors+179 relay sensors)	428 (340sensors+88 relay sensors)	1016	560	272	0.07905	113

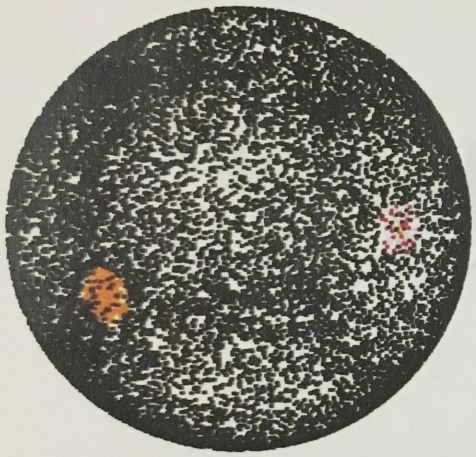

Fig 93. RGG Representing Maximum degree Vertex and minimum degree vertex on a Sphere

Fig 94. RGG on a semi sphere

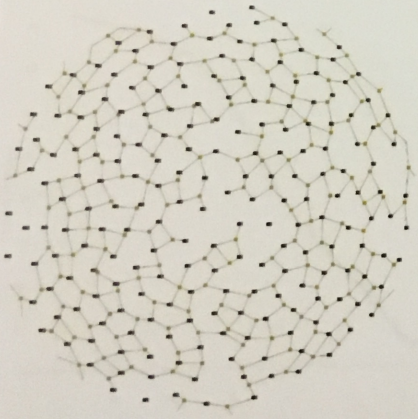

Fig 95. 1st Bipartite semi sphere Subgraph

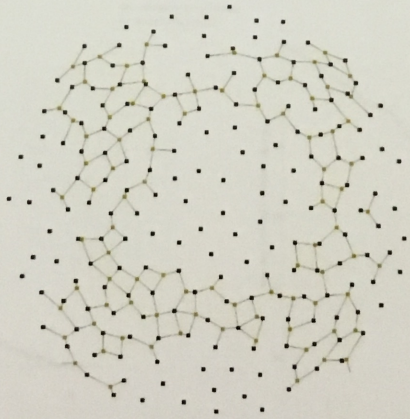

Fig 96. 2nd Bipartite semi sphere Subgraph

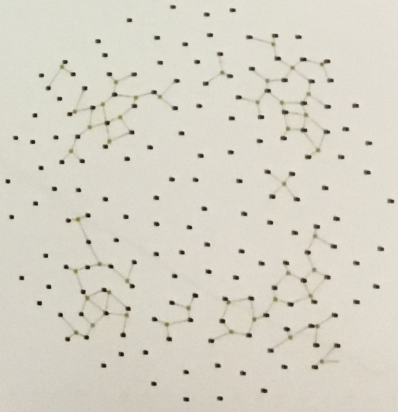

Fig 97. 3rd Bipartite semi sphere Subgraph

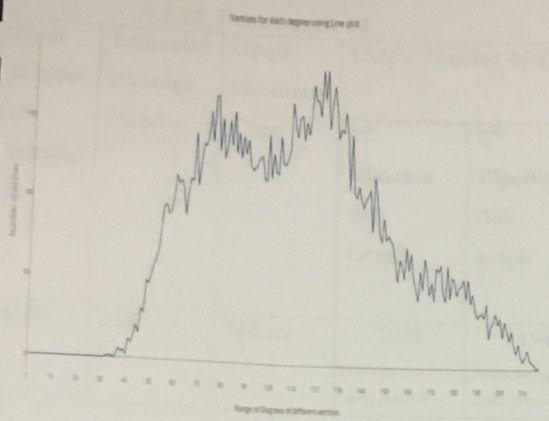

Fig 98. Degree Distribution line plot
https://plot.ly/~chandan9/1808/vertices-for-each-degree-using-line-plot/

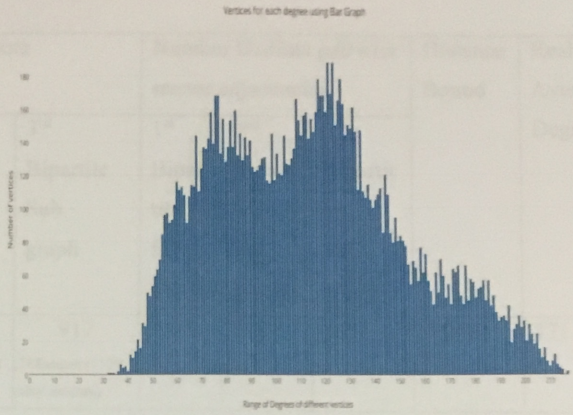

Fig 99. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1809/vertices-for-each-degree-using-bar-graph/

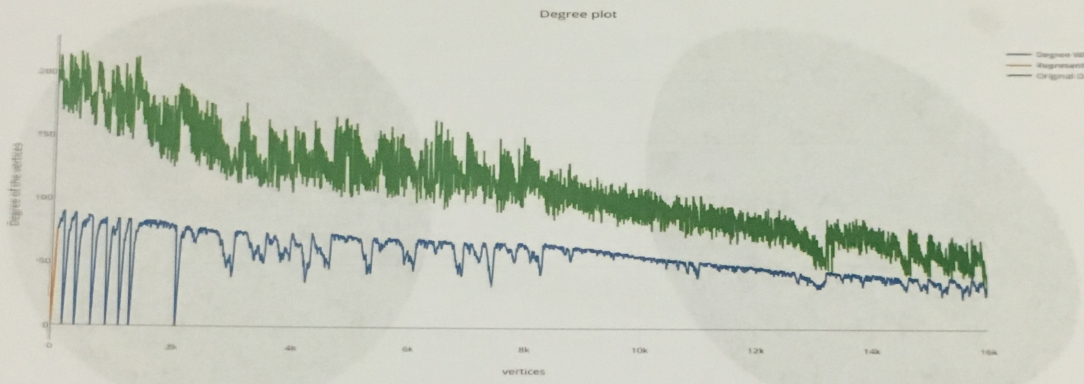

Fig 100. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1810/degree-plot/

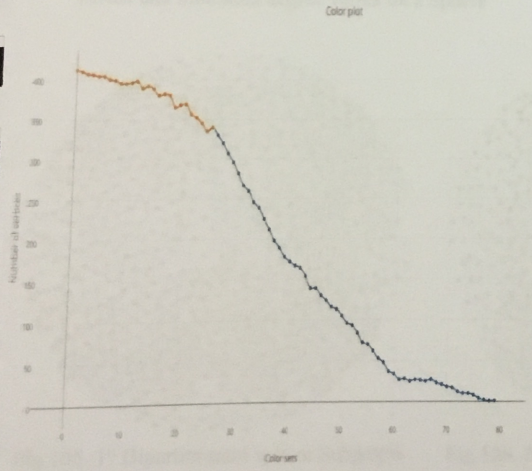

Fig 101. Independent color set sizes distribution
https://plot.ly/~chandan9/1811/color-plot/

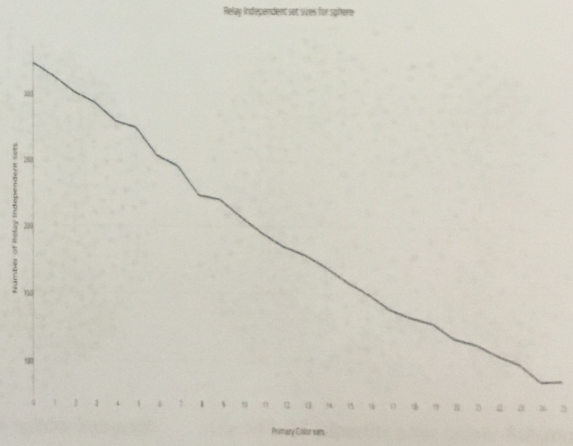

Fig 102. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/1943/relay-independent-set-sizes-for-sphere/

2.3.10 Benchmark 10

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
4000	150	Sphere	1928 (1157sensors+771 relay sensors)	1454 (1083sensors+371 relay sensors)	917 (788sensors+129 relay sensors)	2438	1182	405	0.0484	171

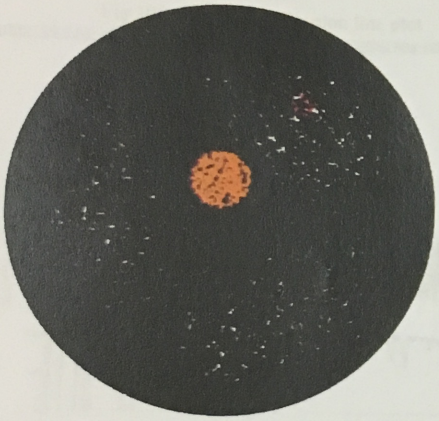

Fig 103. RGG Representing Maximum degree Vertex and minimum degree vertex on a Sphere

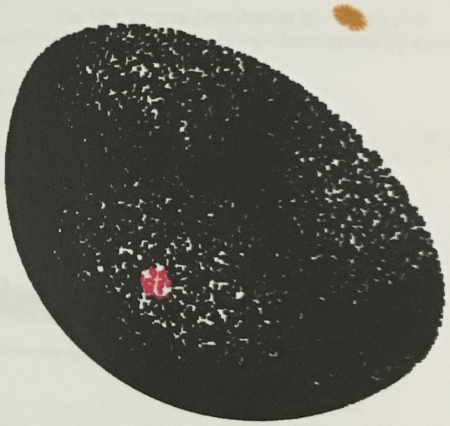

Fig 104. RGG on a semi sphere

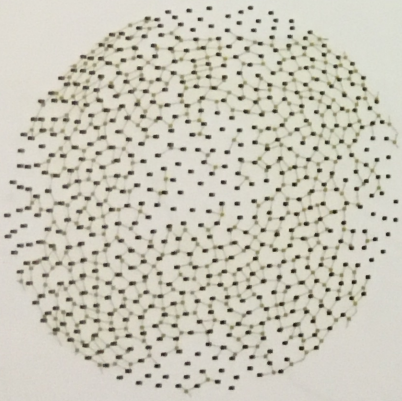

Fig 105. 1st Bipartite semi sphere Subgraph

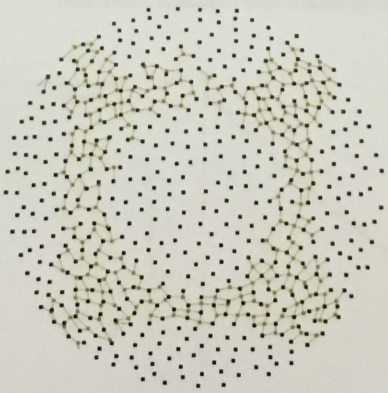

Fig 106. 2nd Bipartite semi sphere Subgraph

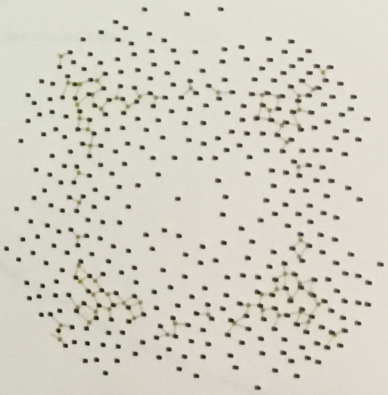

Fig 107. 3rd Bipartite semi sphere Subgraph

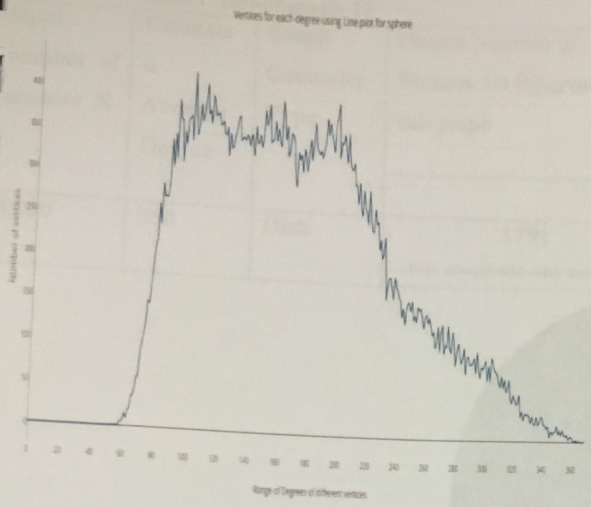

Fig 108. Degree Distribution line plot
https://plot.ly/~chandan9/1866/vertices-for-each-degree-using-line-plot-for-sphere/

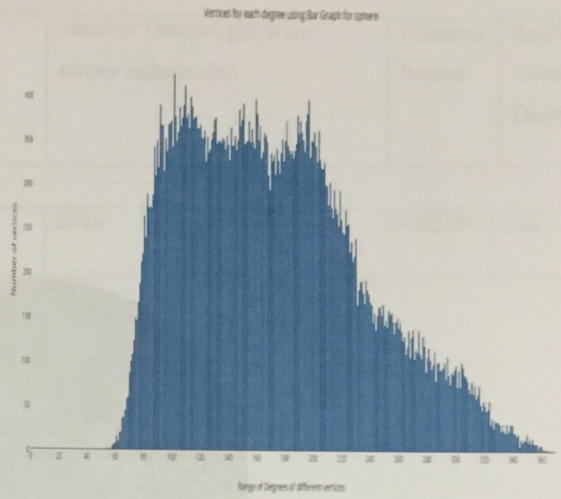

Fig 109. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/1867/vertices-for-each-degree-using-bar-graph-for-sphere/

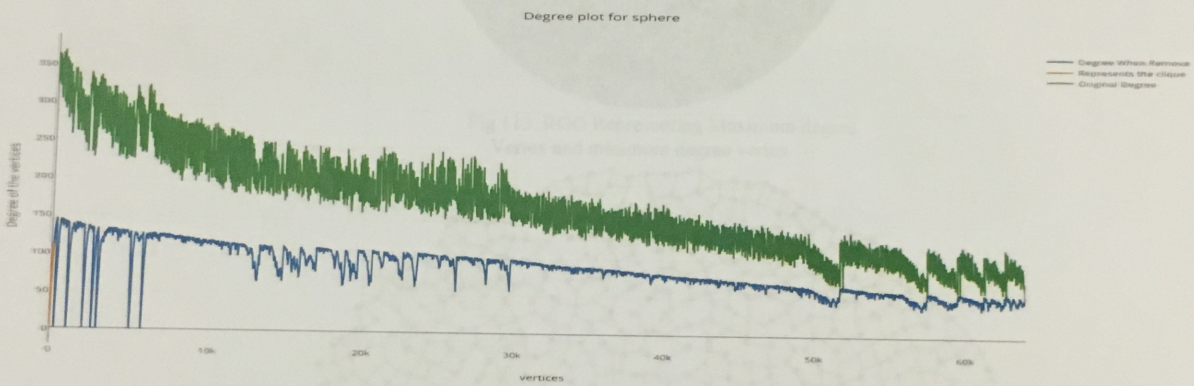

Fig 110. Vertex Degree Distribution plot
https://plot.ly/~chandan9/1868/degree-plot-for-sphere/

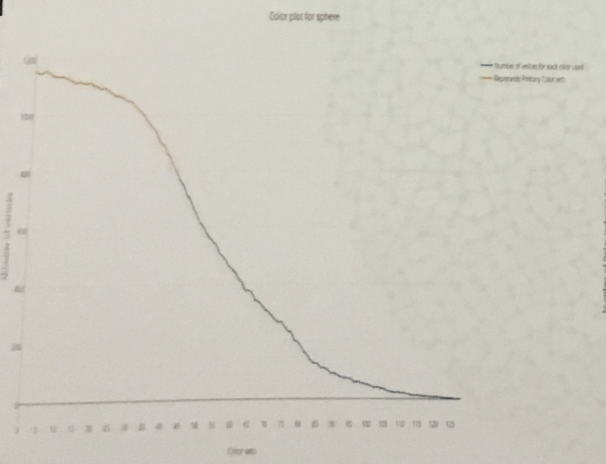

Fig 111. Independent color set sizes distribution
https://plot.ly/~chandan9/1869/color-plot-for-sphere/

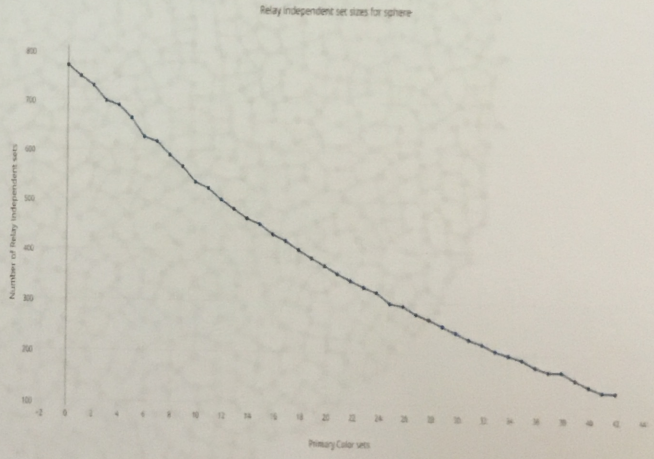

Fig 112. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/2000/relay-independent-set-sizes-for-sphere/

2.3.11 Benchmark 11

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors 1st Bipartite sub graph	Number Distinct pairwise sensor adjacencies	Distance Bound	Realized Average Degree
80000	250	Disk	1791 (926 sensors+865 relay sensors)	2749	0.0279	244

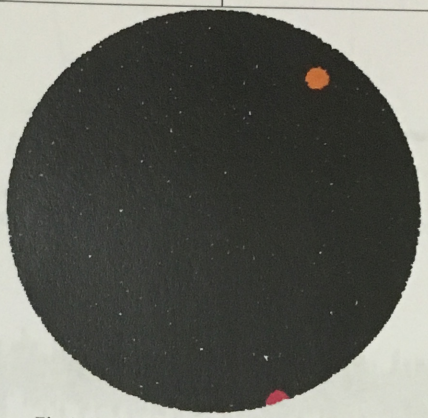

Fig 113. RGG Representing Maximum degree Vertex and minimum degree vertex

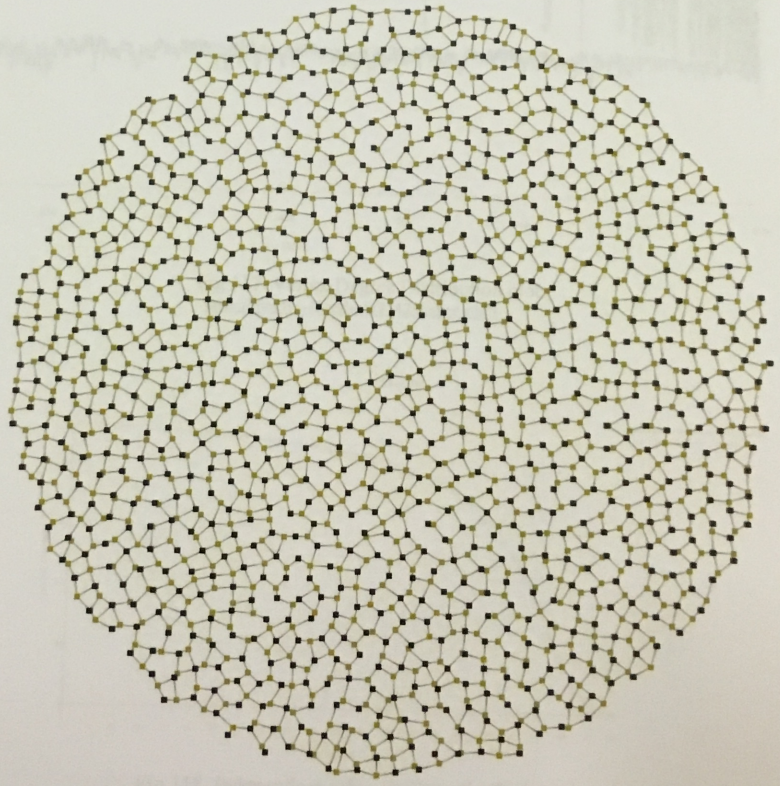

Fig 114. 1st Bipartite Subgraph

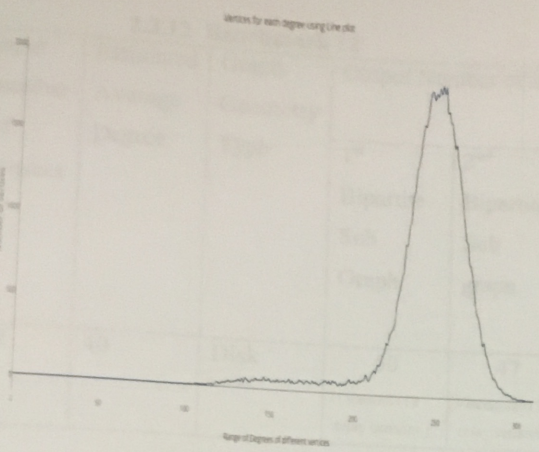

Fig 115. Degree Distribution line plot
https://plot.ly/~chandan9/2140/vertices-for-each-degree-using-line-plot/

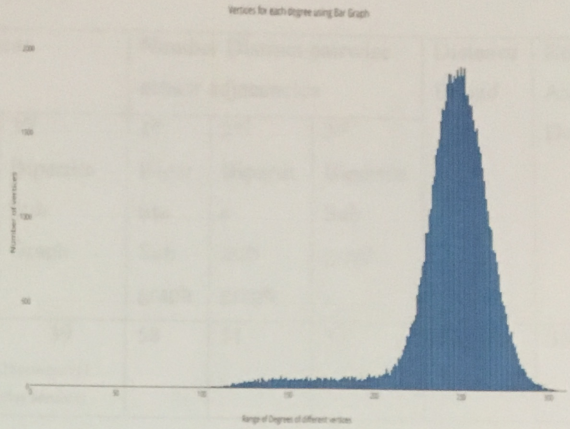

Fig 116. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/2141/vertices-for-each-degree-using-bar-graph/

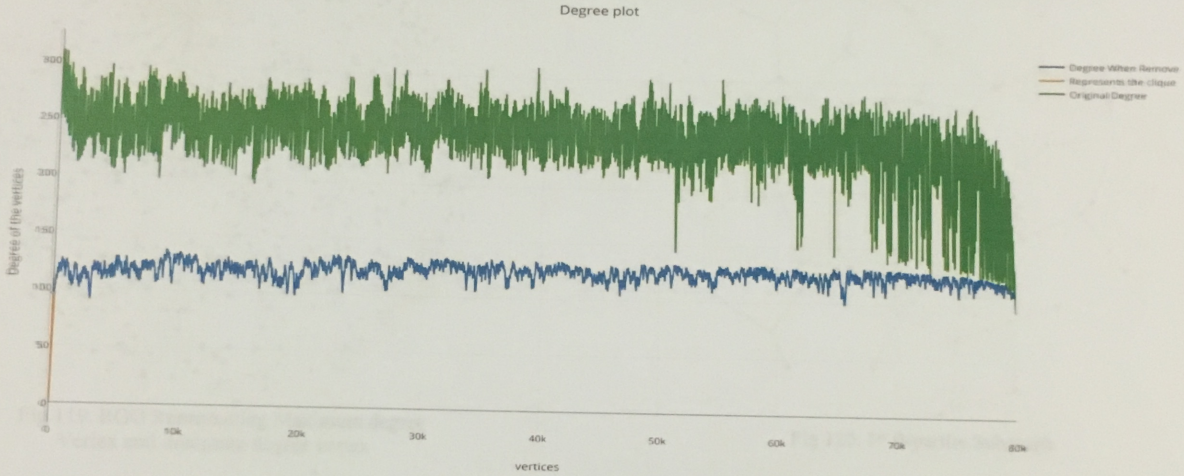

Fig 117. Vertex Degree Distribution plot
https://plot.ly/~chandan9/2142/degree-plot/

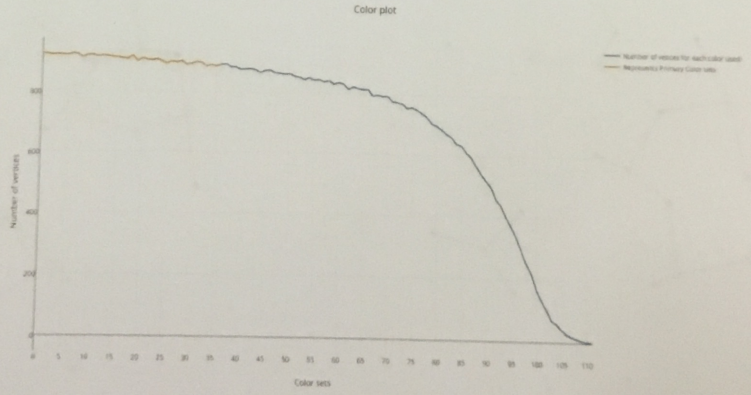

Fig 118. Independent color set sizes distribution
https://plot.ly/~chandan9/2143/color-plot/

2.3.12 Benchmark 12

Input number of sensors N	Estimated Average Degree	Graph Geometry Type	Output Number of Sensors			Number Distinct pairwise sensor adjacencies			Distance Bound	Realized Average Degree
			1st Bipartite Sub Graph	2nd Bipartite Sub graph	3rd Bipartite Sub Graph	1st Bipartite Sub graph	2nd Bipartite Sub graph	3rd Bipartite Sub graph		
00	40	Disk	50 (31sensors+19 relay sensors)	47 (30sensors+17 relay sensors)	39 (28sensors+11 relay sensors)	58	51	33	0.1414	35

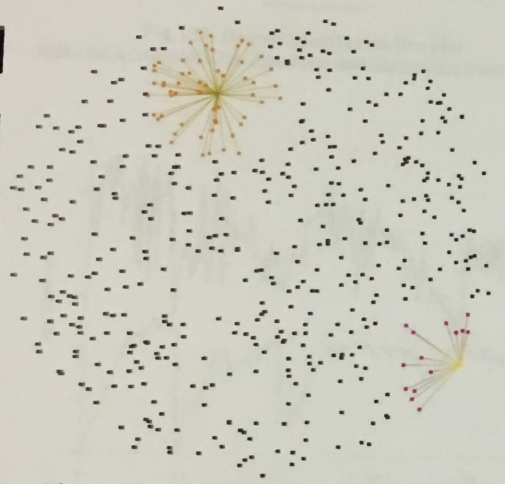

Fig 119. RGG Representing Maximum degree Vertex and minimum degree vertex

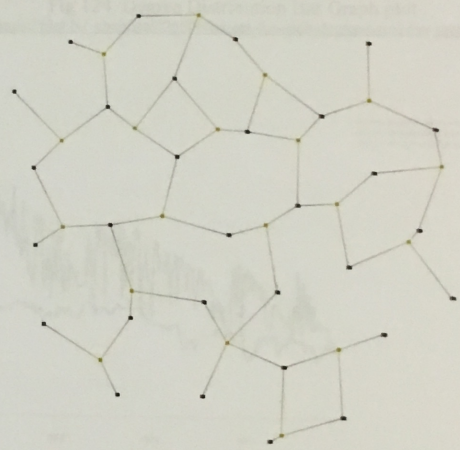

Fig 120. 1st Bipartite Subgraph

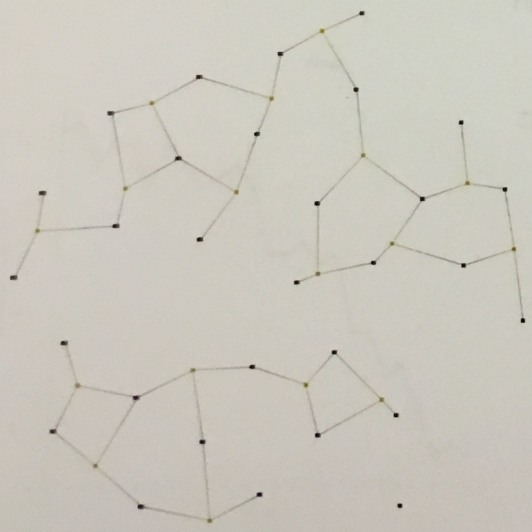

Fig 121. 2nd Bipartite Subgraph

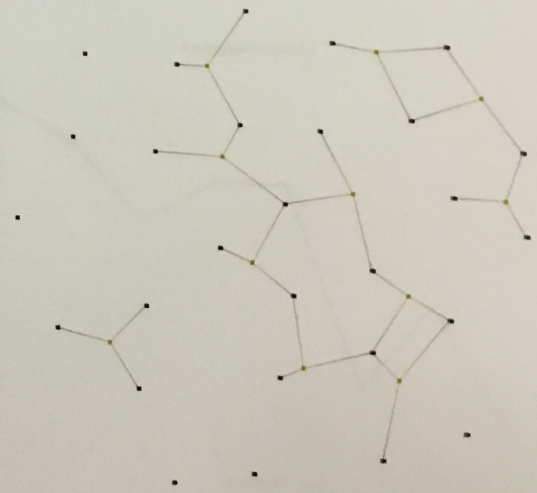

Fig 122. 3rd Bipartite Subgraph

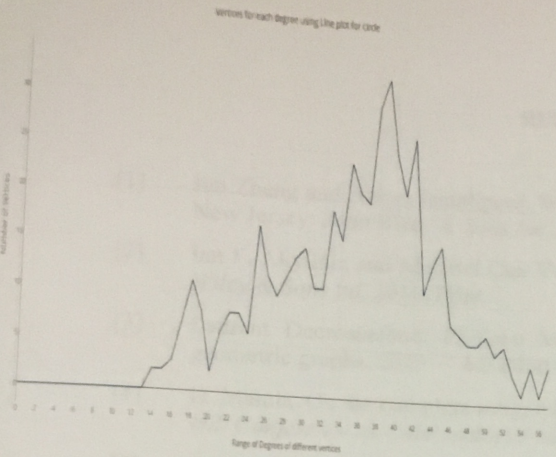

Fig 123. Degree Distribution line plot
https://plot.ly/~chandan9/2641/vertices-for-each-degree-using-line-plot-for-circle/

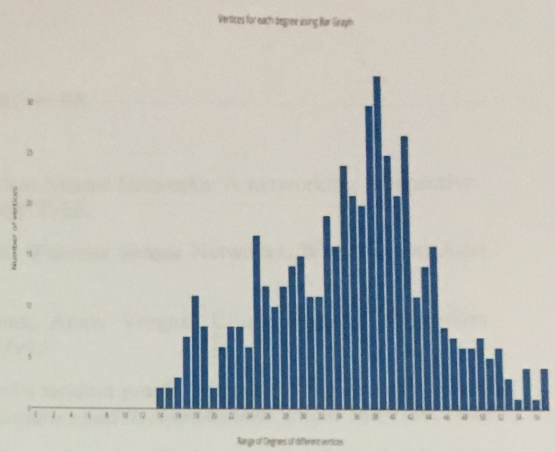

Fig 124. Degree Distribution Bar Graph plot
https://plot.ly/~chandan9/2650/vertices-for-each-degree-using-bar-graph/

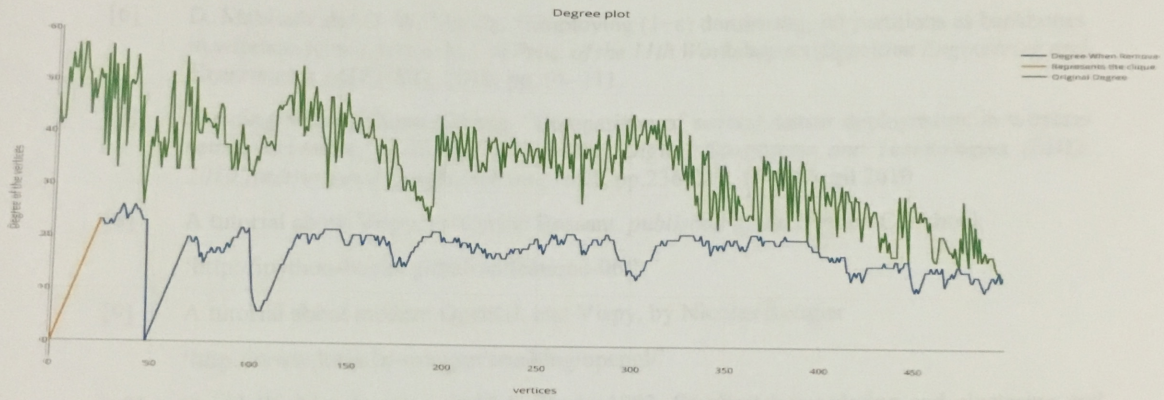

Fig 125. Vertex Degree Distribution plot
https://plot.ly/~chandan9/2671/degree-plot/

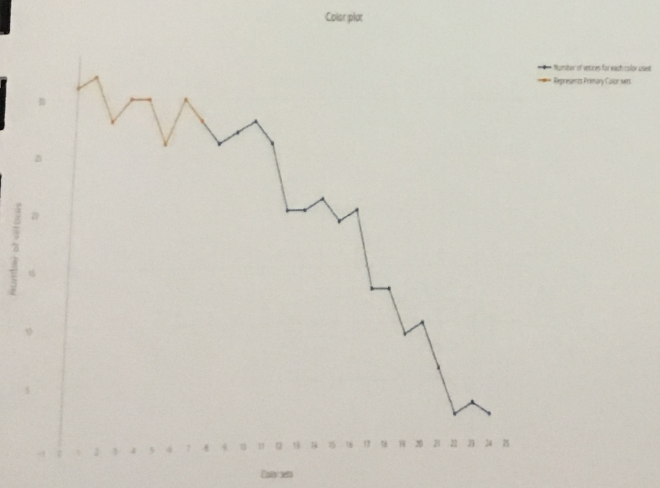

Fig 126. Independent color set sizes distribution
https://plot.ly/~chandan9/2673/color-plot/

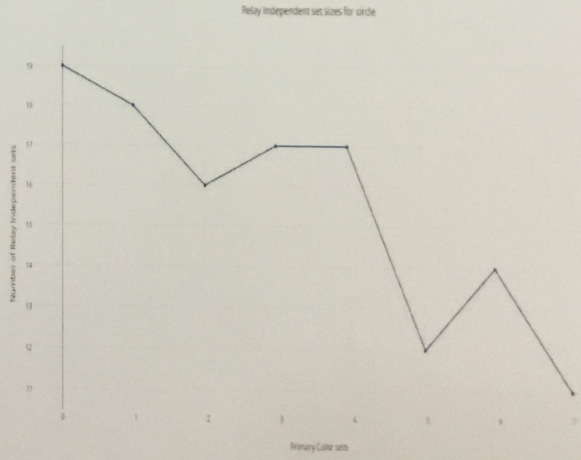

Fig 127. Relay Independent set size distribution for primary color sets
https://plot.ly/~chandan9/2695/relay-independent-set-sizes-for-circle/

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