CSE 7350/5350

D.W. Matula

Homework Set # 4
Divide-and-Conquer

[60 points] **Due date: 19 November 2015**

1. [Recurrences]

a) Use induction to verify the candidate solution to each of the following recurrence equations.

i. $t_n = t_{n-1} + 6$ for $n > 1$ $t_1 = 2$.

The candidate solution is $t_n = 6n - 4$.

Also find the solution if the recurrence is $t_n = t_{n-1} + 8$ with the same boundary conditions.

ii. $t_n = t_{n-1} + n$ for $n > 1$ $t_1 = 1$.

The candidate solution is $t_n = ((n+1)^2 - (n+1))/2$

b) Solve the following recurrence equations using substitution (text Section 4.1).

i. $t_n = t_{n-1} + n^2$ for $n > 1$ $t_1 = 2$.

ii. $t_n = 3\, t_{n-1} + 2^n$ for $n > 1$ $t_1 = 1$.

2. [Recurrences]

(a) Simplify the master theorem (text Theorem 4.1) for solving recurrences of the form (where a, b, c are positive integers greater than unity):

$$T(n) = a\, T(n/b) + cn.$$

(b) Give an asymptotic formula for each of the following recurrence equations, using big "Θ" rather than just big "O" for better results when possible. Show the values for the next five values of T(n) in each case.

b1. $T(n) = 4T(n-4) + 1$, $T(1) = T(2) = T(3) = T(4) = 2$.

b2. $T(n) = 3T(\lfloor n/4 \rfloor) + 2n$, $T(0) = 1$.

b3. $T(n) = 4T(\lfloor n/2 \rfloor) + 2\, n^2$, $T(1) = 1$.

b4. $T(n) = 1/n + T(n-1)$, $T(1) = 2$.

3. The following recurrence equation gives the expected number of comparisons for Quicksort, given that the "pivot element" is selected uniformly at random from the list:

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$$T(n) = (n-1) + \frac{1}{n}\sum_{i=0}^{n-1}(T(i) + T(n-1-i)), \quad T(0) = 0.$$

(a) Let $S(n) = \sum_{i=0}^{n-1}(T(i) + T(n-1-i))$. Give Dual recurrence equations expressing T(n) in terms of S(n), and S(n) in terms of S(n-1) and T(n-1).

(b) Evaluate S(n) and T(n) for n = 1, 2, …, 10.

(c) What are the time and space requirements for computing T(n)?

4. [Polynomial and Matrix Multiplication]

a) Text problem 30-1, p. 920. (a and b only)
b) Text Problem 4.2-1, p. 82.
c) Text Problem 4.2-6, p.83.

5. [Weighted Median] Text exercise 9-2 (a, b, c only), p. 225.

6. [Points in space] Describe how to solve by divide-and-conquer in O (n lg n) time either the convex hull problem [text section 33.3] or the Voroni diagram problem. Clearly describe the combination method as that is the primary step in the solution process. Clearly describe the special terminal case solution when the recurrence reduces to a very small problem.