PHY224 Radius of the Earth Lab

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Abstract:

This experiment attempted to determine the radius of the Earth using a Sodin gravimeter. This was achieved by using a Sodin gravimeter to measure the downwards acceleration due to gravity as observed from at floor level from the third floor of the Burton Tower to the thirteenth floor. The relationship between gravitational acceleration and the radius of the Earth is given by;

$$\Delta g/g = -2\Delta R/R$$

For g, the gravitational acceleration at some reference level, and R, the radius of the Earth at that reference level. Rearranging this expression for the radius of the Earth, R, in terms of the quantities Δg, ΔR and the known constant g, yields an expression for the radius of the Earth. By using a plot of the observed changes in gravitational acceleration as a function of the changes in elevation and extrapolating the slope of the line of best fit (using SciPy's curve fitting module), the radius of the Earth was determined to be $6.408 \times 10^6 m \pm 2.656 \times 10^4 m$. The known value of the radius of the earth, $6.378 \times 10^6 m$, is within the margins of error.

In addition to determining the radius of the Earth, reference measurements were repeated at thirty minute intervals in order to investigate the effects of the movement of celestial bodies on the observed gravitational acceleration near Earth's surface. The fluctuations in the magnitude of gravitational acceleration observed were within the margins of error of the gravimeter. Therefore, these effects were likely negligible. This is in line with our expectations, as gravitational force at the scale of celestial bodies obeys an inverse squared relationship with respect to the distance between objects. Because these bodies are so distant, the effect was likely minimal.

Introduction:

The purpose of this experiment was to determine the radius of the Earth. This was determined by analyzing changes in the gravitational acceleration near Earth's surface at different elevations as measured by a Sodin gravimeter.

Equipment:

- W. Sodin Ltd. Gravity Meter, Model 410 with Meter Constant 0.10023

- Tape measure

Materials and Methods:

The Sodin gravimeter was set on the ground outside of the southern stairwell door on the third floor of the Burton Tower. The legs were adjusted using the adjustable screws until the apparatus was determined to be level in both pitch and yaw, as indicated by the level scales of the gravimeter. The light switch was then pulled, and the beam in the eyepiece was adjusted until it aligned with the reticle. This process was repeated for each floor between the third and thirteenth floors of the Burton Tower, returning to the third floor at approximately half hour intervals. After reaching the thirteenth floor, measurements were repeated going from the thirteenth floor back down to the third (following the same process as before, returning to the third floor at half hour intervals). This process was repeated five days later for another set of data.

Results:

Time Elapsed (minutes)	Floor	Gravimeter Measurement
0	3	813
4	4	801.5
8	5	789.9
13	6	779.7
18	7	767.1
22	8	756
26	9	746.1
33	3	815.4
51	10	733.1

56	11	719.8
62	12	708.8
68	3	814.5
75	13	696.2
79	12	708.2
84	11	719.3
88	10	732.1
93	9	744.9
98	8	756.5
104	3	814.5
109	7	767.8
116	6	780.6
120	5	791.6
125	4	802
130	3	815.3
133	0	856.9
136	1	843.2

Figure 1: Values of gravitational acceleration measured on the first day of experimentation

Time Elapsed (minutes)	Floor	Gravimeter Measurement
0	3	817
10	13	698.6
26	15	676.2
34	3	818
39	8	759.5
52	1	842.6
58	0	862.1
65	3	818.1

Figure 2: Values of gravitational acceleration measured on the second day of experimentation

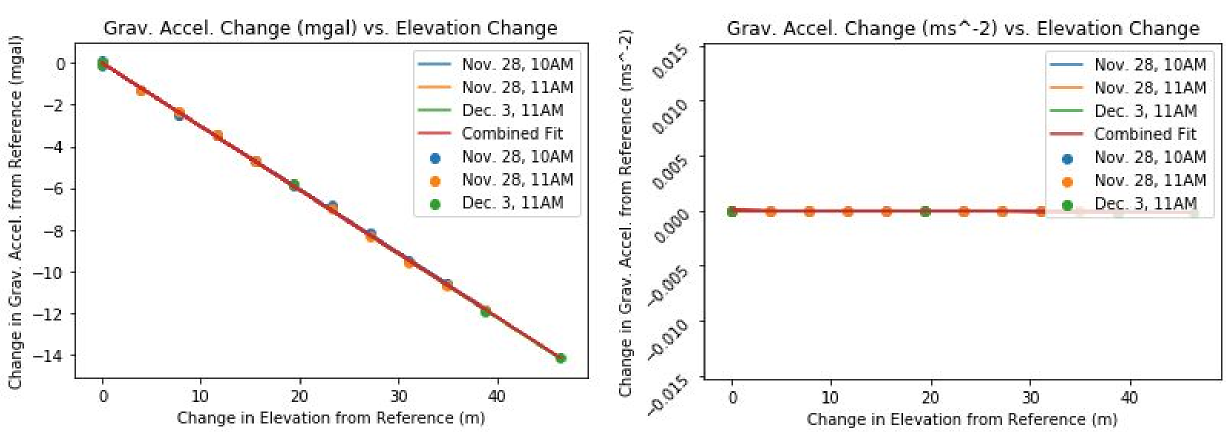

Figure 3: Plots of change in gravitational acceleration as a function of change in elevation. Vector formats available in Appendix.

Typically, gravitational acceleration obeys an inverse-square relationship with respect to distance from Earth's surface. However, because of the relatively small scale of the changes in elevation relative to the scale of the radius of the Earth, the variation in the gravitational acceleration observed at various elevations obeys the linear relation;

$$\Delta g/g = -2\Delta R/R$$

Where g is the absolute value of the acceleration due to gravity near Earth's surface from our reference level, 9.804 m/s^2, Δg is the change in the gravitational acceleration across a change in elevation, ΔR.

Treating g as a constant and rearranging the expression above for the radius of the Earth, an expression for the radius of the Earth as a function of the ratio of the changes in gravitational acceleration to the changes in elevation can be derived. By definition, the value $\Delta R/\Delta g$ represents the inverse of the slope of a plot of g as a function of elevation.
Using numpy's *curve_fit* module, the observations of change in gravitational acceleration in conjunction with measurements of change in elevation were fitted to the relationship $\Delta g = -2(g/R)\Delta R$, with *(g/R)* being the resultant parameter outputted. This was applied to four datasets: the measurements taken at 10AM on November 28, the measurements taken at 11AM on November 28, the measurements taken at 11AM on December 3, and the combined set of all measurements. Reduced Chi-Squared tests of the curve-fits indicated very close fits to the observed results.

Taking the reciprocal of the resultant parameter and multiplying it by the reference gravitational acceleration value of 9.804253 $\frac{m}{s^2}$, four values for the radius of the Earth were obtained, ranging from $6.408 \times 10^6 m$ to $6.467 \times 10^6 m$. Given the precision of the measuring devices, measurement error was negligible in the face of the statistical error of $2.656 \times 10^4 m$. With a known value of $6.378 \times 10^6 m$ for the radius of the Earth, it can be said that the measurements were accurate and that the resultant calculations are reflective of reality. Precise statistics can be seen in the code output provided in the Appendix.

Discussion:

Because the force of gravity obeys an inverse-square relationship with respect to elevation on the scale of the Earth's radius, and the scale of measurements in this experiment are relatively small, the variation of gravitational acceleration with elevation on the scale completed in this experiment obeys the relationship; $\Delta g/g = -2\Delta R/R$ (where Δg is the difference in the magnitude of gravitational acceleration between floors , g is the known value of the acceleration near Earth's surface due to gravity, ΔR is the difference in elevation between measurements of g, and R is the radius of the earth,). Rearranging this expression for R, the radius of the Earth, results in the expression $R = -2g \cdot \Delta R/\Delta g$. This expression reveals a linear relationship between the theoretical radius of the Earth and the magnitude of acceleration near Earth's surface. Interestingly, the slope of this relationship takes the form $\Delta R/\Delta g$, the inverse of the slope of a plot of gravitational acceleration as a function of elevation.

Given the lack of deviation at the extreme ends of the elevation measurements, it can be said that the shifting magnitude of the difference in mass over- and underhead during the measurements ultimately had a negligible effect on the readings. Due to the measurement having been taken somewhat in the middle of the tower (i.e.: between the third and thirteenth floors), it can be postulated that the ultimate resultant *curve_fit* parameter was equally 'weighted' at both ends, thereby evening out. Measurements taken in the basement and on the fifteenth floor were observed to still adhere to the fitted plot, supporting this postulate. Due to the magnitudes involved, factors such as sea level were not considered.

Conclusion:

The radius of the Earth is $6.408 \times 10^6 m \pm 2.656 \times 10^4 m.$ This value is in accordance with the known value of $6.378 \times 10^6 m$.

Appendix: Relevant Code & Plots as Vector Graphics