PHY324 Kater Pendulum Lab

Ming Sun, Spencer Ki

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The setup of the experiment included a kater pendulum, an electronic counter timer, an oscillation sensor, a hardened steel knife edge and a very fine divided steel scale. The kater pendulum is a pendulum with two pivots and adjustable mass. The kater pendulum provided in this experiment contains two pivots, and a mass divided into three sections: a fixed main mass, an adjustable coarse mass, and an adjustable fine mass. The latter two masses are present to adjust the oscillation period with greater precision than a single movable main mass, with the coarse one used to "bracket" the area of adjustment for the fine one. The position of the fixed main mass can be controlled somewhat, by inverting the orientation of the pivots.

Two sets of period measurements were made. For the first set of measurements, the fine adjustment mass was fixed, and the coarse adjustment mass was adjusted to various positions to determine the upper and lower limits on fine adjustment mass; once the coarse position was obtained, the coarse adjustment mass was kept fixed, and the fine adjustment mass was adjusted to various positions. It is important to note that during the lab, we measured the period of the Kater pendulum in its upright position and inverted position. From this, two different relationships between period and fine adjustment mass position for each respective pivot orientation were yielded.

However, it is impractical to determine the precise equal period point. We instead plotted a pair of relationships of period versus position of the fine adjustment mass for both upright and inverted pivot orientations. The intersection point of these plots correspond to kater period, the most accurate value that can be used to determine the acceleration due to gravity, g.

Introduction

The purpose of this experiment was to determine the acceleration due to gravity, g, with great precision by measuring the period of a Kater pendulum. Heuristically, the purpose is less the literal calculation of g (which should be computationally trivial) and more the analysis of the precision errors present in measurements made on a system with many potential sources of error. Obtaining *an accurate* value of g is of much greater difficulty than obtaining the value itself.

Equations & Background Information

The relationship between the acceleration due to gravity and the characteristics of a simple pendulum are given in the equation $g = (2\pi)^2(\frac{L}{T})^2$, where L is the length of the pendulum and T is the period of its oscillation. However, no pendulum behaves perfectly as a simple pendulum in reality and there is great difficulty in accurately measuring these characteristics. In particular, L is not strictly fixed and is effectively $L_e = \frac{k^2}{r_m}$, where k is the radius of the pendulum's gyration r_m is the radius of its centre of mass relative to its single pivot point. Accordingly, nineteenth-century physicist Henry Kater devised an asymmetrical pendulum with two pivot points, increasing measurement accuracy as L can be easily measured as the fixed distance between the two points. Additionally, the position of the mass of the pendulum is adjustable, to vary period along with the orientation of the pivots.

Materials

- 1 Kater pendulum
- 1 hardened steel knife edge
- 1 electronic counter timer
- 1 flexible fixed blade (oscillation sensor)
- 1 very fine divided steel scale
- 1 cathetometer

Procedure

Fine adjustment mass fixed

1. The Kater pendulum was carefully taken out of its wooden box, and was placed pivoting on a hardened steel knife edge in an upright position
2. The electronic counter timer and the oscillation sensor were switched on
3. The electronic counter timer was set to 8 oscillations.
4. The oscillation sensor was placed beneath the kater pendulum; the oscillation sensor was placed close to the narrowed sharpened ends of the kater pendulum, this was to make sure that that electrical/optical 'ticks' were produced, so the electronic counter timer can keep track of the oscillations.

5. The fine adjustment mass was centred at 10cm on the fine measuring scale and was kept fixed.
6. The coarse adjustment mass was set to 39cm on the side of the coarse measuring scale away from the main mass.
7. Kater pendulum was titled off of its equilibrium, the angle of which the pendulum is tilted was marked down, for we had to conduct numerous trials, so we had to keep the angle constant,
8. The kater pendulum was released from its tilted position.
9. After 8 oscillations, the yellow light on the electronic counter timer lit up automatically, the reset button on the electronic counter timer was pressed.
10. The reading on the electronic counter timer was recorded.
11. Coarse adjustment mass was set at a 3cm lower position.
12. Steps 7-10 were repeated.
13. Steps 11-12 were repeated for another 8 trials.
14. The kater pendulum was placed pivoting on a hardened steel knife edge in an inverted position.
15. Steps 2-13 were repeated.
16. After 10 trials were conducted, the closest positioned that yielded the desired outcome was chosen, and further trials were performed near that position for accuracy.

Figure 1: The experimental setup. The Kater pendulum is in upright position.

Coarse adjustment mass fixed

1. Steps 1-4 from 'Fine adjustment mass fixed' were repeated.
2. Coarse adjustment mass was set to 26cm, and was kept fixed.
3. Fine adjustment mass was set to 5cm.
4. Steps 7-10 from 'Fine adjustment mass fixed' were repeated.
5. Fine adjustment mass was set 1cm lower.
6. Steps 12-16 from 'Fine adjustment mass fixed' were repeated.
7. After 6 trials were conducted, the closest positioned that yielded the desired outcome was chosen, and further trials were performed near that position for accuracy.

Results

Near-equal upright and inverted periods were obtained from the pendulum when the coarse mass was positioned with one end 26cm away from the end of the measuring track closest to the main mass. Subsequently, repeated trials detailed in the procedure above determined that the ideal positioning for the fine mass was centred at 4.70cm in the direction away from the main mass, relative to the fine measuring track.

By passing both upright and inverted observations of the varying fine mass positioning through the *curve_fit()* module of scipy's optimize module, an ideal fine mass position of 13.58cm was determined, with a corresponding period of 2.00389s was obtained.

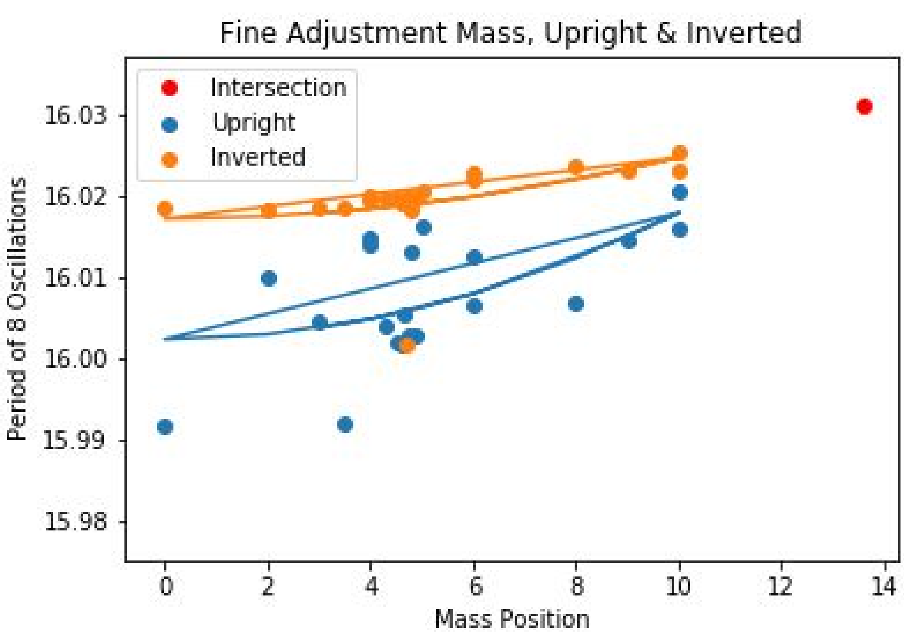

Figure 2: Fine adjustment mass plot.

Combined with a pendulum length measurement of 100.2cm and the relation $g = (2\pi)^2(\frac{L}{T})^2$, the value of acceleration due to gravity was determined to be $9.85096\frac{m}{s^2}$.

Discussion & Error Analysis

The most straightforward source of error is precision error due to measurement. The cathetometer (traveling microscope) used to measure the length of the pendulum is accurate up to $1.0 \times 10^{-4}m$, while the period recordings are accurate up to $1.0 \times 10^{-6}s$. Propagating these errors through the calculation $g = (2\pi)^2(\frac{L}{T})^2$ yields a precision error of $\pm 0.00196\frac{m}{s^2}$.

Given their general proximity to one another, one may be tempted to derive a statistical error from the collected period observations. However, as the observed variables of mass position and period are not independent, such a value would be ultimately meaningless. Regardless -- given the values in consideration -- such a deviation would be far smaller than purely the precision error.

For the large part, precision error is overshadowed by systematic experimental error. These can be separated broadly into two categories: instrumentation errors and procedural errors.

In the former category are issues related to the constructions of the various equipment.For instance, when in the equilibrium upright position (i.e.: with the main mass above the pivot-point in use), the bottom-facing needle-point deviated from being perpendicular to the ground by roughly $0.005m$, with the inverted needle-point being between $0.01m$ and $0.02m$ deviated. Also in this category of errors is the fact that the counter sensor features a detector gap of $0.01m$ in width (across the detector that is itself $0.01m$). As such, this would allow the pendulum to oscillate not only on a single axis, but somewhat ‘diagonally’ along two axes, due to possible local rotation about its length. Finally, as with all pendulums in reality, one needs to take into account the fact that the apparatus is losing energy with each oscillation (i.e.: damping).

In the case of the first problem, the measurement of period time would ultimately be unaffected, with only the position of equilibrium being slightly skewed. The second issue is more serious, as a potential diagonal oscillation across a gap of $0.01m$ in length and $0.01m$ in width could potentially cause the pendulum to be moving at a $45\circ$ skew to the correct axis. As such, each length of oscillation could potentially be underestimated by a factor of $Cos(45) = \frac{\sqrt{2}}{2} \simeq 0.525$. However, great care was taken on the part of the experimentalist to ensure that this skew and/or rotation was as minimised as possible (see Figure 3). Finally, damping was observed to be negligible in the time-scales of interest, with oscillations on both pivots being observed continuously for ten minutes with negligible change in amplitude.

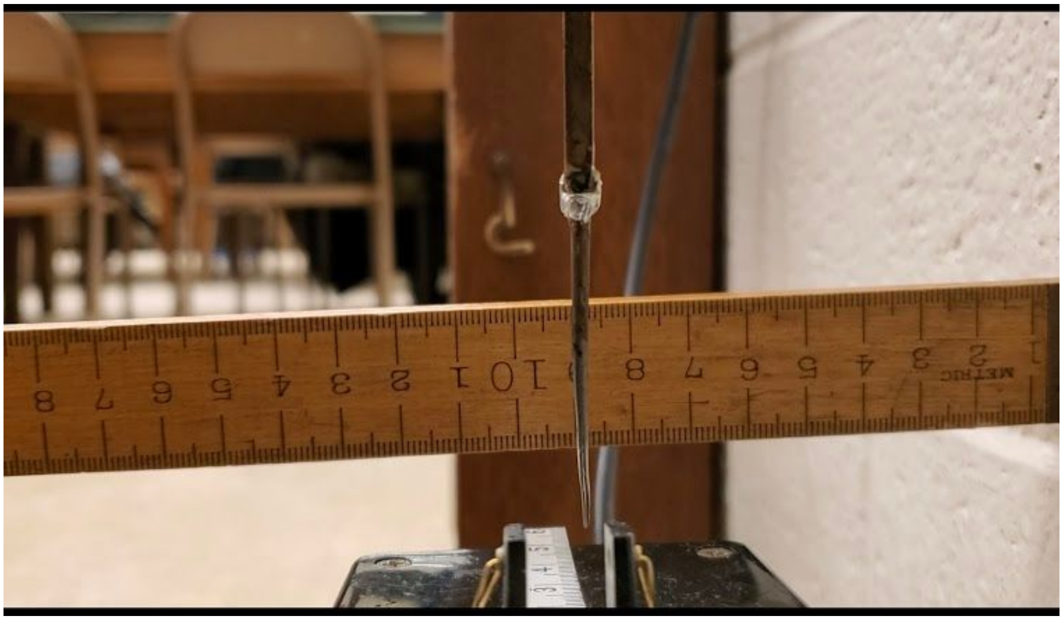

Figure 3: A still from a video recorded perpendicular to the axis of oscillation to check for axis deviation. Negligible deviation was observed.

In the case of this experiment, instrumentation error can be thought of intuitively as placing an acceptable interval around a calculated value, while procedural error can be thought of intuitively as determining that value. The problem specifically in mind here, is proper positioning of the coarse adjustment mass. Given that the purpose of the coarse mass is merely to "bracket the desired 'equal period' state," as described in the lab manual, the mass' Position vs. Period plot is a fairly crude one. Combined with the fact that measurements were only able to be taken in 1cm intervals -- the most precision capable with the coarse measuring scale on the pendulum -- meant that a position of 26cm down the scale was chosen as the aforementioned 'bracketing' position, as the upright and inverted periods for this coarse mass position were most similar out of all recorded coarse mass positions. Measurements and adjustments to fine mass positions were taken accordingly, as per the lab manual (detailed in the 'Procedure' section above).

However, when the datasets were fit to least-squares curves using the *fsolve()* function of scipy's optimize module, the theoretical intersection point of the inverted and upright coarse mass relationships was found to be approximately around the position of 30cm. This occured, despite the observed upright-inverted period difference at the 30cm position being greater than that of the 26cm one.

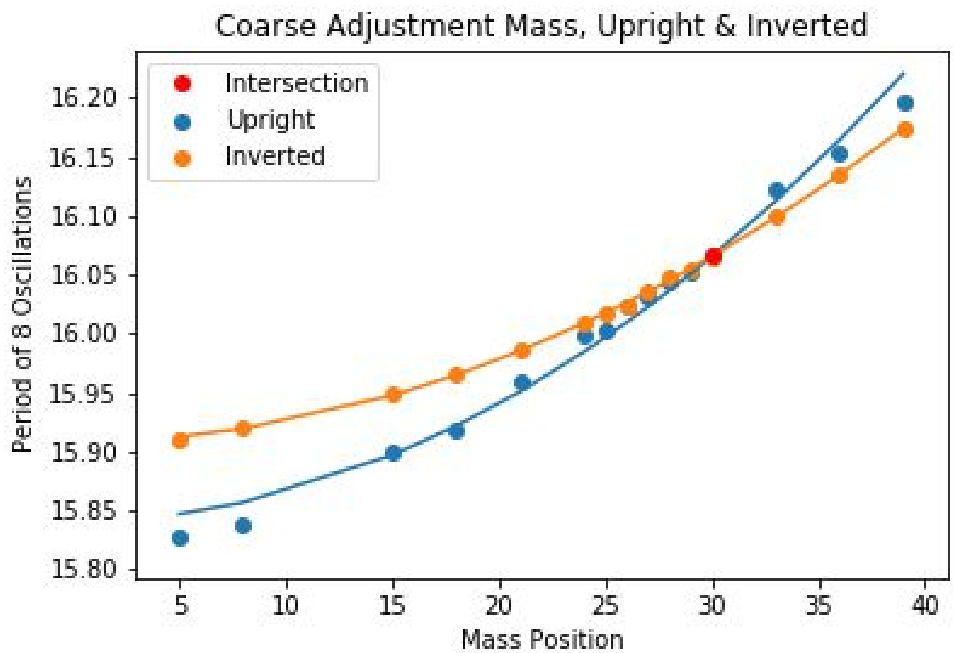

Figure 4: Coarse adjustment mass plot.

Ultimately, this discrepancy is likely to blame for the fact that the upright and inverted plots of the fine mass relations do not intersect graphically, and are projected by *fsolve()* to intersect at a point off of the fine measuring scale (i.e.: at 13.58cm when the scale ends at 10cm). By extension, this explains the deviation of the calculated value for acceleration due to gravity, $9.85096\frac{m}{s^2}$, from the known value of $9.80425\frac{m}{s^2}$ in Toronto.

Accordingly, if one were to replicate this experiment, one would be advised to rely on python analysis for the positioning of both the coarse and fine masses, not only the fine mass. Using the projected period based on a coarse mass position at 30cm, one is able to calculate a value of $g = 9.80865\frac{m}{s^2}$, which is significantly closer to the known value.

Conclusion

In conclusion, the acceleration due to gravity was calculated to be $g = 9.85096 \pm 0.00196\frac{m}{s^2}$. However, due to anomalies in analysis of the data, it is likely that the true calculated value should be closer to $g = 9.80865 \pm 0.00197\frac{m}{s^2}$.

Appendix: Relevant Code

Note: Output was omitted, as it is merely the above-included plots.

```python
 # -*- coding: utf-8 -*-
"""
Code for Kater's Pendulum Lab
"""
import numpy as np
import scipy.optimize as opt
import scipy.stats as sta
import matplotlib.pyplot as plt

#mass position and period are related via a quadratic equation.
def quadratic(x, c, b):
    return c*x**2 + b

#coarse adjustment mass positioning data.
coarse = np.loadtxt("coarse.txt", delimiter = ",", skiprows = 1, unpack = True)
coarsed = coarse[0]
coarsetop = coarse[1]
coarsebottom = coarse[2]

#curve-fitting the upright and inverted plots.
coarsetopopt, coarsetopcov = opt.curve_fit(quadratic, coarsed, coarsetop)
coarsebottomopt, coarsebottomcov = opt.curve_fit(quadratic, coarsed, coarsebottom)

#function of upright pendulum relationship.
def coarsetopquad(x):
    return coarsetopopt[0]*x**2 + coarsetopopt[1]

#function of inverted pendulum relationship.
def coarsebottomquad(x):
    return coarsebottomopt[0]*x**2 + coarsebottomopt[1]

#for use with fsolve().
def coarsediffquad(x):
    return (coarsetopopt[0] - coarsebottomopt[0])*x**2 + coarsetopopt[1] - coarsebottomopt[1]

#calculate intersection.
coarseloc = opt.fsolve(coarsediffquad, 26)
coarseperiod = coarsetopquad(coarseloc)

plt.figure(0)
plt.scatter(coarsed, coarsetop, label="Upright")
plt.scatter(coarsed, coarsebottom, label="Inverted")
plt.plot(coarsed, coarsetopquad(coarsed))
plt.plot(coarsed, coarsebottomquad(coarsed))
plt.plot(coarseloc, coarseperiod, 'ro', label="Intersection")
plt.xlabel("Mass Position")
plt.ylabel("Period of 8 Oscillations")
plt.title("Coarse Adjustment Mass, Upright & Inverted")
plt.legend()
plt.savefig("coarse.png")

#fine adjustment mass positioning data.
fine1 = np.loadtxt("fine1.txt", delimiter = ",", skiprows = 1, unpack = True)
fine1d = fine1[0]
fine1top = fine1[1]
fine1bottom = fine1[2]
```

```
#even finer positioning data.
fine2 = np.loadtxt("fine2.txt", delimiter = ",", skiprows = 1, unpack = True)
fine2d = fine2[0]
fine2top = fine2[1]
fine2bottom = fine2[2]

#combining the datasets.
finecombd = np.append(fine1d, fine2d)
finecombtop = np.append(fine1top, fine2top)
finecombbottom = np.append(fine1bottom, fine2bottom)

#curve-fitting the upright and inverted plots.
finecombtopopt, finecombtopcov = opt.curve_fit(quadratic, finecombd, finecombtop)
finecombbottomopt, finecombbottomcov = opt.curve_fit(quadratic, finecombd, finecombbottom)

#function of upright pendulum relationship.
def finecombtopquad(x):
    return finecombtopopt[0]*x**2 + finecombtopopt[1]

#function of inverted pendulum relationship.
def finecombbottomquad(x):
    return finecombbottomopt[0]*x**2 + finecombbottomopt[1]

#for use with fsolve().
def finecombdiffquad(x):
    return (finecombtopopt[0] - finecombbottomopt[0])*x**2 + finecombtopopt[1] - finecombbottomopt|

#calculate intersection.
finecombloc = opt.fsolve(finecombdiffquad, 5)
finecombperiod = finecombtopquad(finecombloc)

plt.figure(1)
plt.scatter(finecombd, finecombtop, label="Upright")
plt.scatter(finecombd, finecombbottom, label="Inverted")
plt.plot(finecombd, finecombtopquad(finecombd))
plt.plot(finecombd, finecombbottomquad(finecombd))
plt.plot(finecombloc, finecombperiod, 'ro', label="Intersection")
plt.xlabel("Mass Position")
plt.ylabel("Period of 8 Oscillations")
plt.title("Fine Adjustment Mass, Upright & Inverted")
plt.legend()
plt.savefig("fine.png")

#measured pendulum length.
L = 1.002
#calculated period.
T = finecombperiod / 8
#calculated period from course data.
cT = coarseperiod / 8

#relationship between pendulum length and period.
def grav(L, T):
    return ((2*np.pi)**2)*(L/T**2)

#calculate acceleration due to gravity.
g = grav(L, T)
#calculate acceleration due to gravity with coarse data.
```

```
cg = grav(L, cT)
#measurement error.
merr = (np.sqrt(g/((2*np.pi)**2))*np.sqrt((5e-7/finecombperiod)**2 + \
                (5e-5/L)**2))*(2*g/((2*np.pi)**2))*(2*np.pi)**2/((L/T)**2)
#measurement error with coarse data.
cmerr = (np.sqrt(g/((2*np.pi)**2))*np.sqrt((5e-7/finecombperiod)**2 + \
                (5e-5/L)**2))*(2*g/((2*np.pi)**2))*(2*np.pi)**2/((L/cT)**2)
```