PHY324: Thermoelectricity Lab

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This experiment is an exploration behind the principles governing a Thermo-Electric Cooler, and is focused on determining the physical constants characteristic to the particular apparatus at hand. Physical laws of interest to this exploration are the Seebeck effect, the Peltier effect, and Ohmic heating. The properties of Thermo-Electric Coolers were explored, and verified to behave as expected. For this particular experimental set-up, the thermal conductance of the system, K_d, was determined to be 2.3392 ± 0.3315; the Seebeck coefficient of the apparatus, S_d, was determined to be $14.6190 \pm 0.0031\,\frac{mV}{K}$; and the resistance across the thermoelectric cooler, R_d, was determined to be $R_d = 0.3189 \pm 0.0002\Omega$. Computational exercises determined a hypothetical minimum T_{in} of -10.9532℃ under the ambient conditions present during the observations, an optimum drive current of $0.2048 \pm 1.0282 \times 10^{-8}A$, and a hypothetical minimum T_{in} of -22.4449℃ under idealised conditions specified in an exercise.

Introduction

The purpose of this experiment is to observe the varying relationship between electrical power input and the ambient air temperature as affected by the thermoelectric cooler provided. By understanding this relationship, one may derive the thermal conductance, K_d, between the two reservoirs of the thermoelectric cooler, as well as, K_{hs}, between the output heat sink and the ambient air. After conducting exercises 1 (regarding passive heat flow) and 2b (regarding cooling) of the experiment, select results can be used to determine the thermal conductance of the system, K_d, the Seebeck coefficient of the apparatus, S_d, and the resistance across the thermoelectric cooler, R_d.

Materials

- 1 Thermo-Electric Cooler (TEC)
- 2 multimeters in ammeter configuration
- 1 multimeter in voltmeter configuration
- 1 Variac autotransformer
- 1 current balance
- 1 double-reading thermometer
- 1 set of filter capacitors
- 1 stopwatch
- 1 tube of thermal paste
- Wires with banana clip endings

Procedure

1. The experiment was set-up as specified in Figure 4 of the lab manual.
    a. The TEC and Variac autotransformer were plugged into the power outlet.
    b. The current balance was plugged into the Variac transformer and connected to the resistor of the TEC.
    c. An ammeter under AC settings was connected in series to the circuit in Step b.
    d. An ammeter under DC settings was connected in parallel to the positive ports of the TEC.
    e. A voltmer was connected in parallel to one negative port and one positive port of the TEC.
    f. The filter capacitors were connected to the positive port of the DC-calibrated ammeter and the negative port of the voltmeter.
2. A system check was performed, as specified in the lab manual.
    a. The TEC was switched on while the Variac transformer was not. The system was observed to transfer heat as expected.

   b. With an observed temperature difference across the TEC, the Variac transformer was switched on and set to a power output of 5W. The temperature increased across the TEC as expected.
   c. The Variac transformer was switched off, and the thermal reservoir was allowed to reach $0°C$. The thermal reservoir continued to cool beyond this point as expected.
   d. Step c was replicated with the Variac transformer on and set to 1W. The temperature change still behaved as expected.
   e. As all physical systems behaved as expected throughout the system check, the experimental set-up was judged to be functioning optimally and the system was allowed to return to ambient temperature before experimental trials commenced.
3. One of the wires connecting the TEC to the DC-calibrated ammeter was disconnected, to put the TEC in Operating Mode 2 as specified by the lab manual.
4. Fourteen values of input power through the Variac transformer were varied between 0W to 5W, and the resultant steady state temperatures of the heat sink, heat reservoir, and surrounding ambient air were recorded. The system was allowed to return to ambient temperature prior to the next trial.
5. The DC-calibrated ammeter was re-connected as it was prior to Step 4, and the Variac transformer was switched off to put the TEC in Operating Mode 1 as specified by the lab manual.
6. The TEC was operated without interference for 15 minutes, and 28 measurements of voltage, DC current, and input and output temperatures were made over this timeframe.
7. Immediately after Step 6 (i.e.: without returning the system to ambient temperature) the Variac transformer was switched on and 21 observations were made of the effect that different input powers (ranging from 0W to 10W) had on the system.
   a. For each different input power, a steady state was approximated by allowing the system to ‘slow’ its change in temperature. The DC current, AC current, DC voltage, and input and output temperatures were recorded for each approximated steady state.
   b. The Seebeck voltage of each approximated steady state was observed by abruptly switching the TEC off and making note of the DC voltage value that is measured before continuously running-off of the system.
8. The TEC system was returned to Operating Mode 1 (as specified in Step 5) and allowed to run without interference or input from the Variac transformer. The coldest temperature achievable in the heat reservoir was recorded.

Results

Passive Heat Flow Experiment

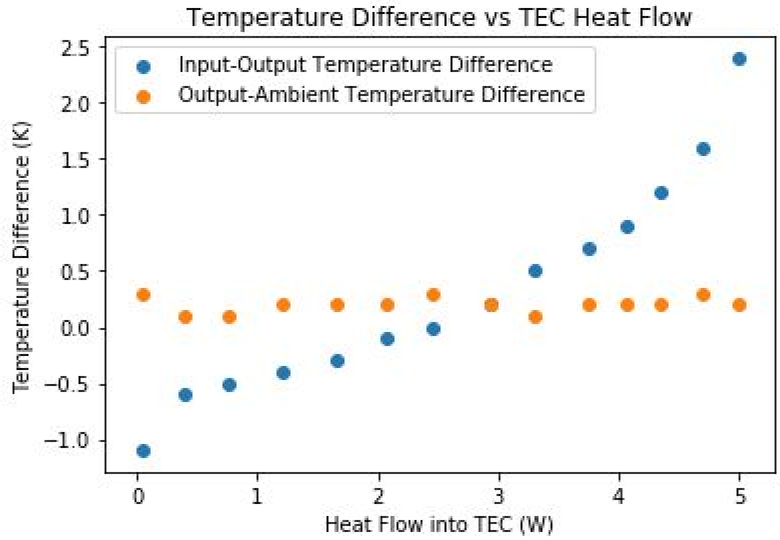

Figure 1: $T_{in} - T_{out}$ and $T_{out} - T_0$ as a function of P_{in}.

Figure 1 graphically presents the observations made during Step 4 of the procedure. Given that $P_K = K_d \Delta T$, as specified in Figure 5 of the lab manual, the thermal conductance across the TEC should be the slope of a linear input-output temperature difference line. While the relevant comparison plot in Figure 1 isn't precisely linear, a fair estimate of slope can be made after dropping the first and last observations (as they exhibit characteristics of data outliers). This yields a rough change of $2.5K$ (from roughly $-0.5K$ to roughly $2.0K$) over $5W$, which corresponds to an approximate K_d of 2.

Similar physical reasoning can be applied to the thermal conductance between the heat sink and the ambient air, as -- despite the change in medium -- the laws of heat transfer themselves do not change. Within allowable error, it appears that the relevant comparison plot in Figure 1 has a linear slope of zero, and thus a thermal conductance of zero. This fits the expectation that neither the TEC nor the surrounding ambient air are affecting each other to a noticeable degree while the system is powered on.

Cooling Experiment A

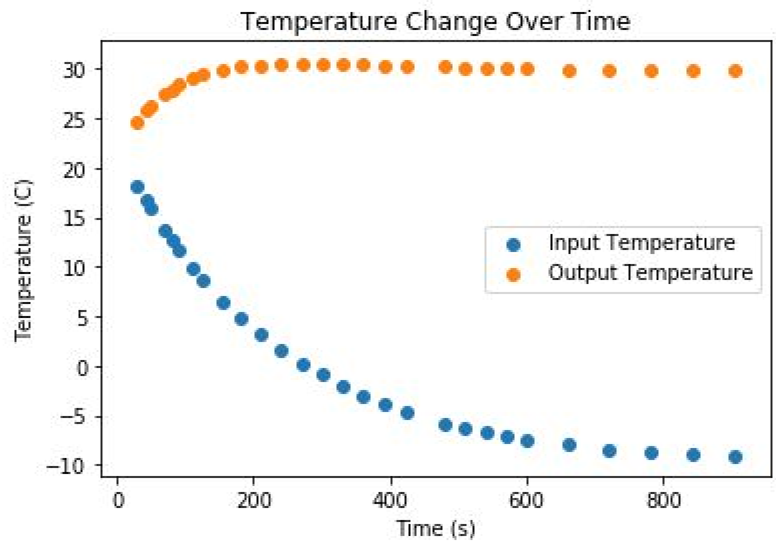

Figure 2: Input and output temperature comparison as a function of time.

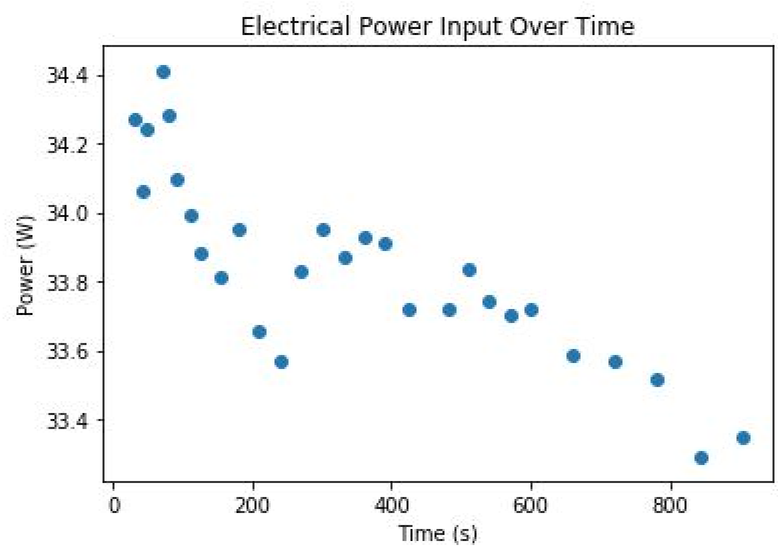

Figure 3: P_d as a function of time.

Figures 2 and 3 graphically present the observations made during Step 6 of the procedure.

Cooling Experiment B

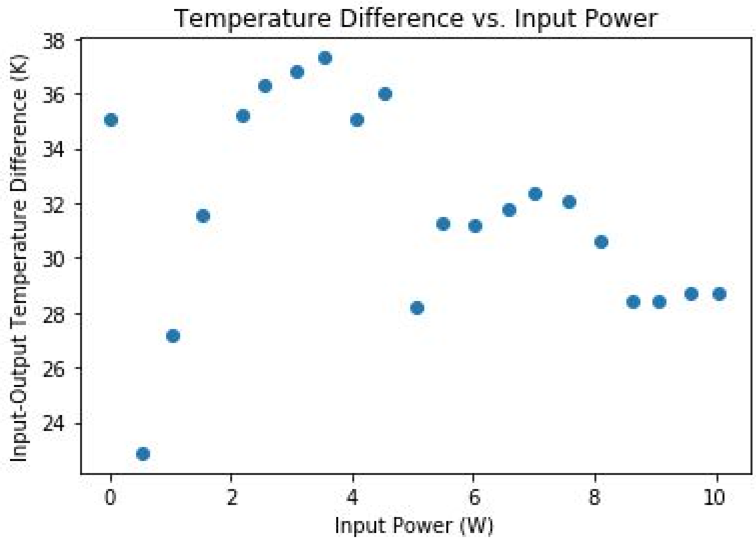

Figure 4: $T_{in} - T_{out}$ as a function of P_{in}.

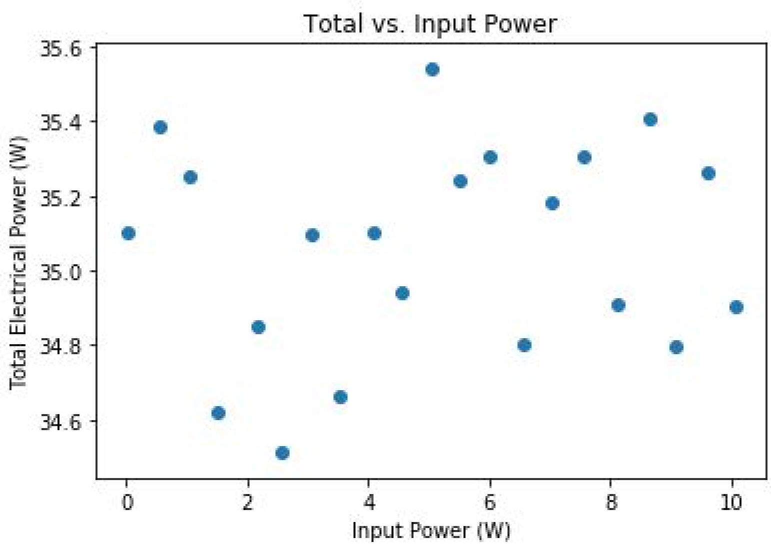

Figure 5: P_d as a function of P_{in}.

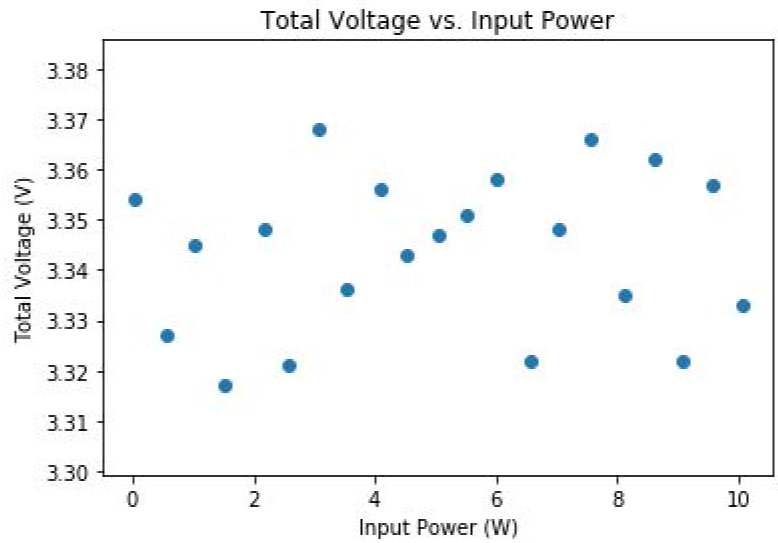

Figure 6: V_d as a function of P_{in}.

Figures 4, 5, and 6 graphically present the observations made during Step 7 of the procedure.

Calculation of Thermal Conductance of the System, Seebeck Coefficient, and TEC Resistance

The thermal conductance of the system was calculated by applying the *curve_fit* function of Scipy's *optimize* module to observations made during Cooling Experiment B (Step 7 of the procedure) related by $P_K = K_d \Delta T$, as specified in Figure 5 of the lab manual. This yielded a thermal conductance value of 2.3392 ± 0.3315, which fits with the rough estimate of $K_d = 2$ made in the presentation of results on the Passive Heat Flow Experiment above.

The Seebeck Coefficient of the system was calculated from the equation $P_s = V_s I_d = S_d \Delta T I_d$ given in Figure 5 of the lab manual. This can be rearranged into the relation $V_s = S_d \Delta T$, with both the varying Seebeck voltages and input and output temperatures having been observed during the experiment. Applying similar computational methods as for K_d, S_d was calculated to be $14.6190 \pm 0.0031 \frac{mV}{K}$.

Finally, the resistance across the TEC was calculated from the basic relationship that $V_d = I_d R_d$. With both the voltage and current of the direct current across the TEC resistor observed, application of the same computational method yielded $R_d = 0.3189 \pm 0.0002\Omega$.

Calculation of Hypothetical Lowest Achievable Temperature of the Thermal Reservoir

Using Operating Mode 1 as specified in the lab manual allows one to set $P_{in} = (T_{out} - \Delta T) S_d I_d - \frac{1}{2} R_d I_d^2 - K_d \Delta T$ equal to $P_d = V_d I_d = S_d \Delta T I_d + I_d^2 R_d$. Substituting the previously determined constants and rearranging allows one to yield a the temperature of the heat reservoir as a function of the temperature of the heat sink: $T_{in} = -\frac{T_{out} S_d K_d + \frac{3}{2} R_d I_d^2}{2 S_d I_d K_d}$.

As can be seen in Figure 2, both input and output temperature observations behave asymptotically, but it stands to reason that T_{out} is only constrained thus due to the presence of the ambient air at an equilibrium temperature around it. Therefore, one can consider this ambient temperature to be the asymptote that T_{out} tends towards. Using the experimentally observed value of the ambient temperature, $21.4℃$, as the input to the above function, one determines a hypothetical minimum T_{in} of $-10.9532℃$.

This value fits with the minimum value of T_{in} that was observed during the experiment, $-11.0℃$.

Calculation of Optimum Drive Current

Given that $P_{out} = P_{in} + P_d$, computationally maximizing P_{out} must begin by summing $P_{in} = (T_{out} - \Delta T)S_d I_d - \frac{1}{2}R_d I_d^{\ 2} - K_d \Delta T = (T_{in})S_d I_d - \frac{1}{2}R_d I_d^{\ 2} - K_d(T_{out} - T_{in})$ and $P_d = V_d I_d = S_d(T_{out} - T_{in})I_d + I_d^{\ 2}R_d$, expanded thus to accommodate the constraints of the exercise. By substituting in the above-determined physical characteristics of the system, and set $T_{out} = 35°C$ as specified by the exercise, one develops a formula where P_{out} can be considered a single-variable function of T_{in}, with the optimum drive current as a single unknown variable. By again using Scipy's *optimize* module, 'feeding' this function with the observations made during Cooling Experiment A (Step 6 of the procedure) and the constraints on T_{in} specified by the exercise, one obtains a value of $0.2048 \pm 1.0282 \times 10^{-8} A$ for the optimum drive current.

Calculation of Hypothetical Lowest Achievable Temperature Using Optimum Input Current

Using the previously determined function for T_{in} as a function of T_{out}, one may substitute the previously used I_d for the calculated optimum drive current, and the ambient air temperature for the $35℃$ given in the exercise. The resultant hypothetical minimum temperature for T_{in} is $-22.4449℃$.

Error Analysis

Most errors present in calculated final values were propagated from the reading errors of the various instrumentation. Measurements of voltage (in volts) and current (in amperes) were recorded to the third decimal place, measurements of time (in seconds) were recorded to the second decimal place, and measurements of temperature (in degrees Celsius) and specifically the Seebeck voltage (in millivolts) were recorded to one decimal place. The reading error for each was considered to be the next smallest decimal place set to a value of 5, as is convention. Where appropriate, noticeably wide fluctuations in measurement replaced the reading error for particular measurements.

Precautions taken to minimise error included measuring ambient temperature far away from the thermal influence of the TEC, and using significant amounts of thermal paste to promote heat conductivity.

Conclusion

The properties of Thermo-Electric Coolers were explored, and verified to behave as expected. For this particular experimental set-up, the thermal conductance of the system, K_d, was determined to be 2.3392 ± 0.3315; the Seebeck coefficient of the apparatus, S_d, was determined to be $14.6190 \pm 0.0031 \frac{mV}{K}$; and the resistance across the thermoelectric cooler, R_d, was determined to be $R_d = 0.3189 \pm 0.0002\Omega$. Further computational exercises determined a hypothetical minimum T_{in} of -10.9532℃ under the ambient conditions present during the observations, an optimum drive current of $0.2048 \pm 1.0282 \times 10^{-8} A$, and a hypothetical minimum T_{in} of -22.4449℃ under idealised conditions specified in an exercise.

Appendix: Python Code

Note: Output is omitted as it is merely composed of the graphs already included above.

```python
# -*- coding: utf-8 -*-
"""
Code for Thermoelectricity Lab
"""
import numpy as np
import scipy.optimize as opt
import matplotlib.pyplot as plt

"""
Passive Heat Flow Experiment
"""
phf = np.loadtxt("passiveheatflow.txt", delimiter = ",", skiprows = 1, unpack = True)
phfi = phf[0]
phfierr = phf[1]
phftin = phf[2]
phftinerr = phf[3]
phftout = phf[4]
phftouterr = phf[5]
t0 = 21.4

phfpin = phfi * 5
phfpinerr = phfierr * 5
phftinphftoutdiff = phftin - phftout
phftinphftoutdifferr = np.sqrt(phftinerr**2 + phftout**2)
phftoutt0diff = phftout - t0

plt.figure(0)
plt.scatter(phfpin, phftinphftoutdiff, label="Input-Output Temperature Difference")
plt.scatter(phfpin, phftoutt0diff, label="Output-Ambient Temperature Difference")
plt.xlabel("Heat Flow into TEC (W)")
plt.ylabel("Temperature Difference (K)")
plt.title("Temperature Difference vs TEC Heat Flow")
plt.legend()

"""
Cooling Experiment A
"""
cooling1 = np.loadtxt("cooling1day1.csv", delimiter = ",", skiprows = 1, unpack = True)

cl1time = cooling1[0]
cl1i = cooling1[1]
cl1v = cooling1[2]
cl1tin = cooling1[4]
cl1tout = cooling1[3]
Pd = cl1i * cl1v

plt.figure(1)
plt.scatter(cl1time, cl1tin, label="Input Temperature")
plt.scatter(cl1time, cl1tout, label="Output Temperature")
plt.xlabel("Time (s)")
plt.ylabel("Temperature (C)")
plt.title("Temperature Change Over Time")
plt.legend()

plt.figure(2)
plt.scatter(cl1time, Pd)
plt.xlabel("Time (s)")
```

```
plt.ylabel("Power (W)")
plt.title("Electrical Power Input Over Time")

"""
Cooling Experiment B
"""
cooling2 = np.loadtxt("cooling2.csv", delimiter = ",", skiprows = 1, unpack = True)

cl2dci = cooling2[0]
cl2aci = cooling2[1]
cl2dcv = cooling2[2]
cl2sv = cooling2[3]
cl2tin = cooling2[5]
cl2tout = cooling2[4]

cl2pin = cl2aci * 5
cl2tempdiff = cl2tout - cl2tin
cl2pd = cl2dci * cl2dcv

plt.figure(3)
plt.scatter(cl2pin, cl2tempdiff)
plt.xlabel("Input Power (W)")
plt.ylabel("Input-Output Temperature Difference (K)")
plt.title("Temperature Difference vs. Input Power")

plt.figure(4)
plt.scatter(cl2pin, cl2pd)
plt.xlabel("Input Power (W)")
plt.ylabel("Total Electrical Power (W)")
plt.title("Total vs. Input Power")

plt.figure(5)
plt.scatter(cl2pin, cl2dcv)
plt.xlabel("Input Power (W)")
plt.ylabel("Total Voltage (V)")
plt.title("Total Voltage vs. Input Power")

"""
Calculation of Thermal Conductance of the System,
Seebeck Coefficient, and TEC Resistance
"""
def P(T, K):
    return K*T

kd_opt, kd_cov = opt.curve_fit(P, phftinphftoutdiff, phfpin)
sd_opt, sd_cov = opt.curve_fit(P, cl2tempdiff, cl2sv)
rd_opt, rd_cov = opt.curve_fit(P, cl2dci, cl2dcv)

Kd = kd_opt
Sd = sd_opt
Rd = rd_opt

Kderr = kd_cov
Sderr = sd_cov
Rderr = rd_cov

"""
```

```
Calculation of Hypothetical Lowest Achievable Temperature of the Thermal Reservoir
"""
Id = np.mean(cl2dci)
Iderr = 0.0005
def Tin(tout, i):
    return -(tout*(Sd*i+Kd)+1.5*Rd*i**2)/(2*Sd*i+Kd)

min_temp = Tin(t0, Id)

"""
Calculation of Optimum Drive Current
"""
def Pout(tin, i):
    return (tin)*Sd*i - 0.5*Rd*i**2-Kd*(35-tin) + Sd*(35-tin)*i + i**2*Rd

i_opt, i_cov = opt.curve_fit(Pout, 5, cl1i*cl1v)

OI = i_opt
OIerr = i_cov

"""
Calculation of Hypothetical Lowest Achievable Temperature Using Optimum Input Current
"""
new_min_temp = Tin(35, OI)
```