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Millikan Experiment

Introduction

In this experiment, we are replicating R.A. Millikan's 1913 experiment where he observed the various forces behaving on a drop of oil in an electric field. The purpose of this experiment is to show that electric charge exists as integer multiples of the charge on a single electron, e; also, by the end of the experiment, a numerical value for e will also be determined.

The setup of the experiment includes an atomizer, rubber bulb, chamber, power supply, camera attached to a microscope, data tracking software, and 50 Excel files. The atomizer contains oil, when the rubber bulb connected to the atomizer was squeezed, oil droplets then enter the chamber. The camera captures the movements of the droplets, the output image is then sent to the computer, and through the tracking software 'Millikan Experiment', we were able to see a 'snowfall' of dots of different sizes and brightness; while the Excel files were used to store the positional data. The chamber is where the experiment takes place. Inside the chamber, we can control the movements of the oil droplet by varying voltage, resulting in differing electrical fields. The contained isolation of the chamber minimizes the effect that outside factors have on the motion of the oil droplet.

There are two given methods that can be used to observe the charge acting on an oil droplet. Method 1 is based on the relation $Q = const_1 \frac{v_t^{3/2}}{V_{stop}}$ which is relevant when quickly halting an oil drop in free-fall and then allowing it to continue to fall. One measures V_{stop}, the voltage that can stop a moving droplet and making it float, as well as the droplet's terminal velocity, v_t, for use in the above equation. Using Method 2, one uses the relation $Q = const_2 (v_t + v_2) \frac{v_t^{1/2}}{V_{up}}$ which is relevant when the user-controlled electrical field is causing the oil droplet to ascend at a constant velocity. The 'ascending' voltage, V_{up}, and the resultant constant velocity, v_2 will be recorded. Assuming that this trial occurs after a successful experiment using Method 1, there should already be a known precise value for v_t, the terminal fall velocity of the droplet.

Position and time data were collected during the experiment.

Background Information

Individual droplets of oil released into the charged plate chamber's electric field via an atomiser have four main forces acting upon them. Classically, there is the force of gravity on the droplet, given by $F_{grav} = m_{oil} g$, and the upward force of buoyancy as the falling droplet displaces aire, given by $F_b =- m_{air}g$. Additionally, there is the force generated by the electric field, given by $F_E = QE$, and the viscosity of flow as described by Stokes' resistance force, $F_{drag} \approx 6\pi r\eta v$. During the experiment, the input voltage can be controlled, thus allowing for variation in the strength of the electric field. Depending on chosen input voltage, an oil droplet might: 'float' in midair due to a balance of forces in; this scenario can be described mathematically by the equation $g(m_{oil} - m_{air}) - QV_{stop}/d = 0$. If the oil droplet is moving downwards at constant, terminal velocity, net force is zero and this scenario is described by: $g(m_{oil} - m_{air}) - 6\pi r\eta v_t = 0$. Finally, a sufficiently strong electric field can accelerate the oil droplet upwards, described via the equation: $ma =- g(m_{oil} - m_{air}) + QE - 6\pi r\eta v$.

Materials

- Atomizer
- Rubber bulb
- Camera attached to a microscope
- Power supply
- Chamber
- computer containing data tracking software 'Millikan Experiment', and 50 Excel files

Procedure

1. Lights were turned off in the room.
2. The connection between the positive terminal and upper socket of the chamber was checked.
3. Power supply was switched on.
4. 50 Excels were prepared.
5. Data tracking software 'Millikan Experiment' was opened.
6. 'Upper Gain' was adjusted to its top position.
7. 'Lower Gain' was used to adjust brightness of the display.
8. The voltage was set to its maximum position.
9. Rubber bulb was squeezed a few times. From the main display, we saw 'snowfall' patterns of dots of different sizes and brightness. An oil droplet that was slowly moving upward was selected.
10. Microscope was adjusted to focus on the selected dot.
11. The voltage was adjusted to stop the selected dot from moving. The required applied voltage is known as the stop voltage, and was recorded.
12. An Excel file was selected, 'Track Image' and 'Save Data' was clicked. The voltage was adjusted to zero, the positional movement of the selected droplet was tracked for 5 or more seconds.
13. The voltage was adjusted back to its stop voltage.

14. The voltage was increased, so that the selected oil droplet rises at a constant velocity. The positional movement was tracked for 5 or more seconds.
15. The Excel file was saved.
16. Steps 9-12 were repeated for another 49 trials.

Derivation of Equations

Exercise 1

$g(m_{oil} - m_{air}) - 6\pi r \eta v_t = 0$

$r = \frac{gm}{6\pi\eta v_t}$, $where\ m = \rho V,\ where\ \rho = \rho_o - \rho_a,\ V = \frac{4}{3}\pi r^3$

$r = \frac{4\pi r^3 g\rho}{18\pi\eta v_t}$

$r = \frac{2r^3 g\rho}{9\eta v_t}$

$r = \sqrt{\frac{9\eta v_t}{2g\rho}}$

Since $g(m_{oil} - m_{air}) - Q\frac{V_{stop}}{d} = 0$, then $Q\frac{V_{stop}}{d} = 6\pi r\eta v_t$

$Q = \frac{6\pi\eta d r v_t}{V_{stop}}$

$Q = \frac{6\pi\eta d v_t}{V_{stop}}\left(\sqrt{\frac{9\eta v_t}{2g\rho}}\right)$

$Q = \left(\frac{18\pi\eta^{\frac{3}{2}} d}{\sqrt{2g\rho}}\right)\frac{v_t^{\frac{3}{2}}}{V_{stop}}$

$Q = (const_1)\frac{v_t^{\frac{3}{2}}}{V_{stop}}$

$Q = (2.0 \times 10^{-10})\frac{v_t^{\frac{3}{2}}}{V_{stop}}\ C$

Exercise 2

$g(m_{oil} - m_{air}) - 6\pi r \eta v_t = 0$

$r = \sqrt{\frac{9\eta v_t}{2g\rho}}$

Given: $ma = -g(m_{oil} - m_{air}) + Q\frac{V}{d} - 6\pi r\eta v_t$, and $a = 0$, I know that

$g(m_{oil} - m_{air}) - Q\frac{V_{stop}}{d} + 6\pi r\eta v_t = 0$

$6\pi r\eta v_t = Q\frac{V}{d} - 6\pi r\eta v_t$

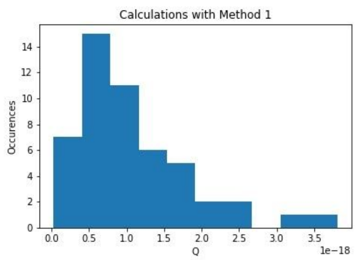
$Q = \frac{6\pi\eta d(v_t + v_2)}{V} r = \frac{6\pi\eta d(v_t + v_2)}{V}\left(\sqrt{\frac{9\eta v_t}{2g\rho}}\right)$

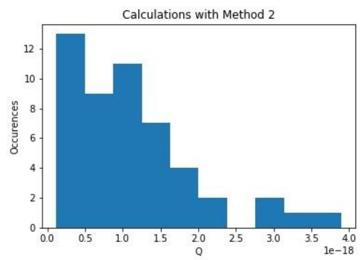
$Q = \left(\frac{18\pi\eta^{\frac{3}{2}} d}{\sqrt{2g\rho}}\right)(v_t + v_2)\frac{v_t^{\frac{1}{2}}}{V}$

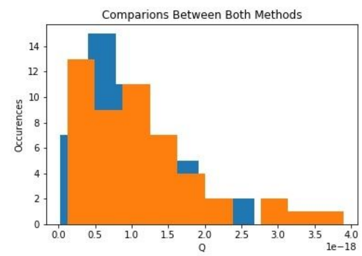
$Q = (const_2)(v_t + v_2)\frac{v_t^{\frac{1}{2}}}{V}$

$Q = (2.0 \times 10^{-10})(v_t + v_2)\frac{v_t^{\frac{1}{2}}}{V}\ C$

Results







From left to right: Q values calculated from Method 1 (Figure 1), Q values calculated from Method 2 (Figure 2), Both histograms overlain (Figure 3).

Due to the varied nature of the data collected, plotting histograms was preferable to determining the greatest common divisor of the dataset. Method 1 calculations yielded a minimum Q value of $2.4939 \times 10^{-20} C$ and a maximum Q value of $3.8084 \times 10^{-18} C$. These values were $1.199 \times 10^{-19} C$ and $3.8922 \times 10^{-18} C$ respectively for the dataset from Method 2. The calculated average margins of error were $\pm 1.7942 \times 10^{-20} C$ and $\pm 1.1050 \times 10^{-20} C$ respectively.

Although taken independently these values are far from Millikan's historical value of $1.5924 \times 10^{-19} C$ and the current known value of $1.6022 \times 10^{-19} C$, the median values of both datasets -- $9.2891 \times 10^{-19} C$ for Method 1 and $9.2444 \times 10^{-19} C$ -- are roughly the correct values multiplied by a quantised integer value of 6, correcting statistically. Dividing these medians by 6 yields values of $1.5482 \times 10^{-19} C$ and $1.5407 \times 10^{-19} C$ respectively.

Discussion and Questions

Regarding reading error, the largest source were observations of voltage, due to the supplied meter only reading values to the nearest value of 10 volts. The according error of 5 volts greatly overshadowed any reading error on the part of pixel length and frame number, therefore rendering velocity error negligible in calculations involving voltage. Other calculations requiring velocity error utilised its statistical error (i.e.: standard deviation of the set) due to the accuracy of the measuring materials.

By dividing the propagated error of the Q datasets one obtains respective mean errors of $\pm 2.9902 \times 10^{-21} C$ and $\pm 1.842 \times 10^{-21} C$ for the values of $1.5482 \times 10^{-19} C$ and $1.5407 \times 10^{-19} C$. Averaging both values yields a calculated charge value of $1.5445 \times 10^{-19} \pm 2.4160 \times 10^{-21} C$. While this interval does not strictly include Millkan's calculated value, it is reasonable in the face of two imprecisions: the difficulty in judging the constancy in the velocity of the oil droplets solely by the human eye, and the large reading error due to the large reading intervals on the voltmeter.

1. Estimate the radius of typical droplets in your experiment.

Answer: Since $r = \sqrt{\frac{9\eta v_t}{2g\rho}}$, each terminal velocity obtained from 50 trials of the experiment was used to calculate a different radius. These values ranged between $3.5622 \times 10^{-7}m$ and $1.8958 \times 10^{-6}m$. Then, the mean radius of a drolet in our experiment was found to be $1.1346 \times 10^{-6}m$ with the median radius being $1.0466 \times 10^{-6}m$.

2. Is the buoyant force significant in this experiment?

Answer: The buoyant force is not particularly significant in this experiment. The buoyant force always acts upwards, opposite to the force of gravity. Although it is a factor to be considered in every trial, the magnitude discrepancy between the density of air and the density of oil essentially renders the buoyant force as a statistical error of gravitational force.

3. Does the experiment work better with smaller or larger radii? Why?

Answer: The Pearson correlation between radius and charge calculated via Method 1 was calculated to be 0.83900093. A similar value was obtained for Method 2 with 0.83811249. While this is a significant correlation, it really only indicates that larger droplets are prone to having larger charges. A more meaningful metric can be found in the correlation between r and the respective errors of each charge calculation. This turns out to be 0.36874525 for Method 1 and 0.70436851 for Method 2.

Conclusion

In conclusion, the charge value was calculated to be $1.5445 \times 10^{-19} \pm 2.4160 \times 10^{-21}C$.

```
# -*- coding: utf-8 -*-
"""
Code for Analysis of the Millikan Oil-Drop Experiment
"""
import pandas as pd
import numpy as np
import scipy.signal as sig
import matplotlib.pyplot as plt

"""
The following block of code was used to analse the individual excel files
containing the paths of the oil droplets. ".xlsx" was replaced by the filename
in question, with X,Y being the observed boundaries on terminal velocity
movement and A,B being the observed boundaries on upwards movement. All data
was saved as part of "millkian.txt".

data = pd.read_excel(".xlsx", na_values = "#NV").fillna(0).to_numpy(np.int64).flatten()
time = np.linspace(0, (len(data)-1), len(data))

plt.figure(0)
plt.scatter(time, data)
plt.xlabel("Frame")
plt.ylabel("Pixels from Top")

print(sig.argrelextrema(data, np.less))
print(sig.argrelextrema(data, np.greater))

vt = 10*(data[X]-data[Y])/(520*(time[X]-time[Y]))
v2 = 10*(data[A]-data[B])/(520*(time[A]-time[B]))
print("vt: " + str(vt))
print("v2: " + str(v2))
"""
"""
Unpacking the Data
"""
data = np.loadtxt("millikan.txt", delimiter = ",", skiprows = 1, unpack = True)
stop_volt = data[1]#in volts
up_volt = data[2]#in volts
term_velo = data[3]*10**-3#in m/s
up_velo = data[4]*10**-3#in m/s
C = 2.0232e-10#Calculated constant. No uncertainty as calculated from known values.
volt_err = 5#Reading error of voltage measurements
#Due to discrepancy in size compared to above, velocity reading error is negligble.

"""
Calculation of Q
"""
#Calculations of Q via Method 1
method1 = np.array([])
for i in range(len(term_velo)):
    method1 = np.append(method1, C*term_velo[i]**1.5/stop_volt[i])
method1err = (5/stop_volt)*method1#Propagation of error for method 1.
#Calculations of Q via Method 2
method2 = np.array([])
for j in range(len(term_velo)):
    method2 = np.append(method2, C*(term_velo[j]+up_velo[j])*term_velo[j]**0.5/up_volt[j])
method2err = (5/up_volt)*method2#Propagation of error for method 2.
```

```python
"""
Calculation of Radius
"""
#All values given in the lab manual.
n = 1.827e-5
g = 9.8
p = 875.3 - 1.204

r = np.sqrt(9*n*term_velo/(2*g*p))#As derived in the report.

np.corrcoef(r, method1)
np.corrcoef(r, method2)
np.corrcoef(r, method1err)
np.corrcoef(r, method2err)
"""
Plots
"""

plt.figure(0)
plt.hist(method1)
plt.xlabel("Q")
plt.ylabel("Occurences")
plt.title("Calculations with Method 1")
plt.savefig("1.jpg")

plt.figure(1)
plt.hist(method2)
plt.xlabel("Q")
plt.ylabel("Occurences")
plt.title("Calculations with Method 2")
plt.savefig("2.jpg")

plt.figure(2)
plt.hist(method1)
plt.hist(method2)
plt.xlabel("Q")
plt.ylabel("Occurences")
plt.title("Comparions Between Both Methods")
plt.savefig("3.jpg")
```