Charge to Mass Ratio of an Electron

Victor Chu (1004004043) and Spencer Ki (1003165031)

November 10, 2019

Abstract:

This experiment attempted to determine the charge to mass ratio of the electron. This was achieved by examining the radius of the circular path of an electron beam emitted in a uniform magnetic field generated by a Helmholtz coil. Electrons were emitted by a heated filament and accelerated through a specialized anode across a variable electrical potential. The path of the electron beam was visualized by the interaction between energized electrons and hydrogen gas molecules in a low-pressure bulb and measured using a self-illuminating scale and reflecting plate. The radius of seven paths were measured at ~189V while varying the magnetic field strength (by increasing the current to the Helmholtz coil). The inverse of their paths was plotted as a function of their current, and analysis with the *SciPy* Python library was used to extrapolate the external magnetic field in the room. This relationship was then manipulated and solved for the charge to mass ratio of the electron, $\frac{e}{m}$, which was found to be $3.817 \times 10^{11}\,\frac{C}{kg}$. This value is roughly twice the known value of $1.758\times 10^{11}\,\frac{C}{kg}$.

Introduction:

The purpose of this experiment was to determine the charge to mass ratio of the electron. This was determined by analyzing the radius of the path of a uniform electron beam in magnetic fields of various strengths generated by a Helmholtz coil.

Methods and Materials:

- 6.3V DC Power Source
- 0V-300V Variable Voltage DC Power Source
- Keithley 179A TRMS Multimeter
- Data Precision 2450 Digital Multimeter
- Vacuum Tube Stand (With Ports for Power Supply)
- Helmholtz Coil
- Rheostat
- 10 Cables
- Low Pressure Hydrogen Gas Bulb
- Electron Gun
- Self-Illuminated Scale and Plastic Reflector

Procedure:

The apparatus was wired as shown in the lab guide, with multimeters set to their proper settings, rheostat set to its lowest setting, and accelerating potential turned off. The accelerating potential was then turned on. The current to the coils and accelerating potential were modified until a coherent beam of electrons was observed (after ~2 minutes to wait for the electron gun filament to warm up). The accelerating potential was adjusted until a large, circular electron beam was observed. The self-illuminated scale was then turned on and aligned to be parallel with the plane of the electron beam. The scale was then moved towards the bulb until the image of the scale as seen in the reflector was observed to be in the plane of the electron beam. The current to the coils was adjusted by the rheostat and the radius of the electron beam's path was recorded. This was repeated six more times, for a total of seven measurements.

Results:

Based on the observed relationship between accelerating voltage (V), current in the Helmholtz coils (I), and radius of the observed electron beam (r), the charge to mass ratio of the electron ($\frac{e}{m}$) was found to be $3.817 \times 10^{11} \frac{C}{kg}$. This value is roughly twice the known value of $1.758 \times 10^{11} \frac{C}{kg}$.

Given the small number of measurements taken, error values were very non-uniform. This is likely due to each observation having a larger measurement error than statistical error. This variance in errors explains the small reduced chi-square statistic of 0.0575 that results from comparing the observed and the modelled relationship between the inverse of the electron beam radius to the current between the coils. Therefore from a statistical perspective, the model is likely fitting the noise of the relationship more than the actual observations.

This measurement error could have been due to over-use of the electron gun, resulting in operational wear, impacting its ability to emit high-energy electrons. In addition, misalignment between the self-illuminating measurement scale and the plane of the electron beam is unavoidable due to the imprecise nature of the adjustment controls. This would result in unaccounted-for parallax, influencing the measurement between observations.

Discussion:

Question 1:

Parallax is the error in the measurement of a quantity due to differences in the visual perception of a phenomenon when observed from different positions. In the context of this experiment, parallax would be an issue in the measurement of the radius of the circular path of the electron beam as electron beam is visualized within a spherical bulb of gas. In order to measure the radius of this closed path, the ruler must be in a plane parallel to the bulb. However, the spherical nature of the bulb limits measurements by physical scales (such as rulers) to parallel planes offset from the actual path by the radius of the bulb. This results in measurements of the radius of the radius taken in front of the bulb to differ from measurements taken behind the bulb and from the 'true' radius of the path of the electron beam. In addition, systematic error is introduced as measurements by external scales may be influenced by a nonzero angle between the plane of the path and the measuring implement. The self-illuminated scale with the plastic reflector projects a virtual image of the scale into the space behind the plastic reflector, allowing for an image of the scale of measurement to be visually “inserted” into the bulb in the same plane as the electron beam. This solves the issue of parallax as it allows for measurements of path diameter to be taken in the plane of the path (eliminating parallax due to the depth of the measurement) and reduces the difference in the planes of the path and the measuring implement.

Question 2:

While varying the accelerating voltages and current in the coils, no anomalous behavior was observed at low voltages and high currents. Although unobserved, one could postulate that sufficiently low voltages or sufficiently high currents would result in electrons exiting the gun to behave erratically (i.e.: not in the expected cylindrical motion) due to the electrons exiting the gun having less energy (and therefore, exiting the gun at a reduced velocity). Due to the linear relationship between voltage and beam radius, said low voltages would mathematically yield very small radii, likely represented by irregular movement. Similar can be said of the inversely linear relationship between current and radius, with high currents yielding very small radii. The breakdown of the mathematical relationship in the face of real values can be observed at edge cases such as these, such as how low current must be improperly represented by the model equations due to it necessitating a radius approaching infinity.

Interestingly, the linear relationship between the inverse of the observed radius and current was observed to have a negative covariance. This is an unusual statistical occurrence, and could be indicative of a mismatch between the model attempting to be applied and the true relationship. Anomalous behavior at low voltage and high current could sufficiently affect the linear relationship of the variables to yield such an occurrence.

If observed, the error of such anomalous behavior could be mitigated through auditing the observed data and making note of where the relationship between the variables begins to differ from the expected linear relationship. Constructing three different models for low accelerating voltage, high current in the coils, and ‘normal’ behavior, then taking a weighted average of the resultant charge-to-mass ratios for each model based on the collected datapoints for each could yield increased accuracy.

Question 3:

Equation (7) in the lab manual relates the inverse of the radius of a circular electron beam path as a function of the current through an n turn Helmholtz coil of radius R is:

$$\begin{aligned} \frac{1}{r} &= \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}\left[\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R} + B_e\right] \\ &= \sqrt{\frac{e}{2mV}}\left[\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R} + B_e\right] \end{aligned}$$

Where B_e is the external magnetic field present in the room.

Expressing this equation in terms of I explicitly and defining the constant K;

$$K = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n}{R}$$

$$\begin{aligned} \frac{1}{r} &= \sqrt{\frac{e}{2mV}} \left[\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R} + B_e \right] \\ &= K \sqrt{\frac{e}{2mV}} (I) + \sqrt{\frac{e}{2mV}} (B_e) \end{aligned}$$

Applying the *Optimize* module from the *SciPy* Python library to extrapolate the parameters for a line of best fit for a scatterplot of the inverse of the radius as a function of current through the Helmholtz coil yielded the linear relation;

$$\frac{1}{r} = 21.89(I) - 3.800$$

With slope $p\_opt[0] \approx 21.89$, and vertical intercept $p\_opt[1] \approx 3.800$

Equating this relation to the expression derived from equation (7) in the lab manual;

$$\begin{aligned} K \sqrt{\frac{e}{2mV}} &= p\_opt[0] \\ &\approx 21.89 \end{aligned}$$

$$\begin{aligned} \sqrt{\frac{e}{2mV}} (B_e) &= p\_opt[1] \\ &\approx 3.800 \end{aligned}$$

Substituting physical constants and properties of the Helmholtz coil;

$$\mathrm{K} \approx 7.375 \times 10^{-4}$$

From the above relations and experimental data;

$$\sqrt{\frac{e}{2mV}} = p\_opt[0]/K$$

Rearranging the second of these expressions for the external magnetic field;

$$\begin{aligned} B_e &= \frac{p_{opt[1]}}{\sqrt{\frac{e}{2mV}}} \\ &= K \frac{p_{opt[1]}}{p_{opt[0]}} \\ &\approx -1.279 \times 10^{-4}\ T \\ &= -127.9\ \mu T \\ &= -1.279\ G \end{aligned}$$

Therefore, the component of the magnetic field not produced by the Helmholtz coil was approximately 1.279 Gauss (127.9 microtesla) in the direction opposite to the direction of the Helmholtz coil's field. This is roughly double the magnetic field strength of the Earth's magnetic field at the 50th parallel (near the latitude of Montreal), and roughly 40 times weaker than the strength of the magnetic field of a typical fridge magnet.

Question 4:

When bringing a new cell phone (Samsung Galaxy S10) near the electron beam (with electrons accelerated across a potential difference of 189.6 V and 1.300A of current running through the Helmholtz coil), no deflection was observed. However, bringing an older phone (Samsung Galaxy S7) near the beam, a nearly imperceptible change in the radius ($<$ 1mm) was observed.

These weak magnetic fields were likely the magnetic fields due to electrical activity within the phone (such as the magnetic fields due to wires in the phone). The observed discrepancy in the effects the different devices had on the electron beam was likely due to newer phones being more energy efficient, drawing less current from their batteries as they operate. While technically observed for the older model phone, the deflection was nearly imperceptible, and only observed when bringing the phone right up to the bulb. This change was not able to be measured with any significance using the self-illuminated scale and reflector, as the change was less than the smallest reading on the self-illuminated scale. This change was likely insignificant as the magnetic field of these devices is theoretically multiple orders of magnitude smaller than the external magnetic field due to Earth (or other sources of magnetic fields in the building). In addition, the Biot-Savart law expression for the magnetic field due to current through a straight wire indicates an inverse proportionality between the magnetic field and the distance from a wire to the point at which it is being measured. Thus, at the distance they were kept during the experimentation, it is unlikely that these devices had any effect on the experimental reading.

Question 5:

From Equation (7) in the lab manual;

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}\left[\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n I}{R} + B_e\right]$$

Defining k and I_0 as;

$$k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}$$

$$I_0 = -\frac{B_e}{k}$$

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\left[\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}\right]\frac{1}{\sqrt{V}}\left(I + \frac{B_e}{\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}}\right)$$
$$= \sqrt{\frac{e}{m}}k\left(\frac{1}{\sqrt{V}}\right)\left(I + \frac{B_e}{k}\right)$$
$$\frac{1}{r} = \sqrt{\frac{e}{m}}k\left(\frac{I - I_0}{\sqrt{V}}\right)$$
$$= \sqrt{\frac{e}{m}}\frac{k}{\sqrt{V}}\ (I - I_0)$$

Isolating this expression for the ratio of the charge of an electron to the mass of an electron in terms of the current through the Helmholtz coil;

$$\sqrt{\frac{e}{m}} = \frac{\sqrt{V}}{r \cdot k}\left(\frac{1}{I - I_0}\right)$$
$$\frac{e}{m} = \frac{V}{r^2 \cdot k^2}\left(\frac{1}{I - I_0}\right)^2$$

From *Question 3*, the external magnetic field, B_e, was;

$$B_e = K\frac{p_{opt[1]}}{p_{opt[0]}}$$
$$\approx -1.279 \times 10^{-4}\ T$$

And from experimental data and *Question 3*;

$$I_0 = -\frac{B_e}{k}$$
$$\approx 0.2454$$

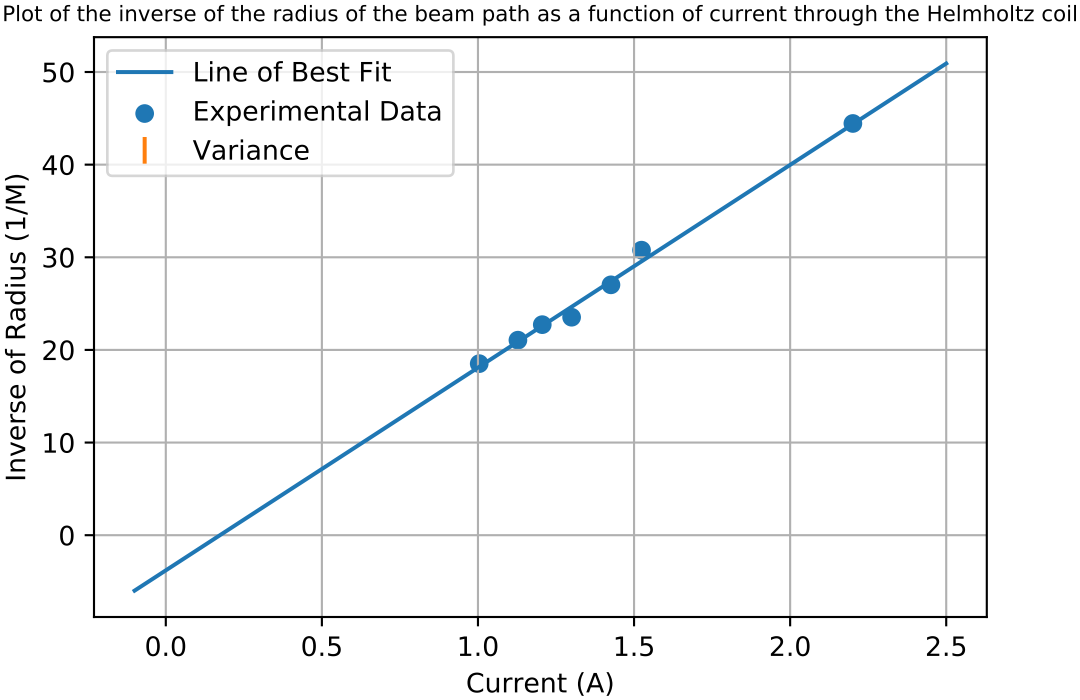
Using this value in the equation for the charge to mass ratio of the electron derived above and experimental data;

$$\frac{e}{m} \approx 3.817 \times 10^{11}\,\frac{C}{kg}$$

Therefore, the charge to mass ratio of the electron is approximately $3.817 \times 10^{11}\,\frac{C}{kg}$. This result is roughly twice the expected value of $1.758\times 10^{11}\,\frac{C}{kg}$, and well within the margins of error for this experiment.

<u>Conclusion:</u>

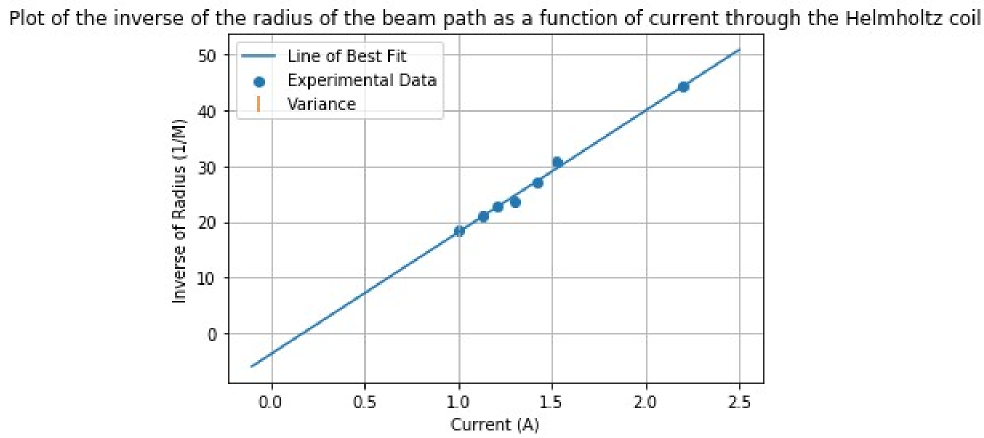
The charge to mass ratio of an electron is approximately $3.817 \times 10^{11}\,\frac{C}{kg}$.


Plot of the inverse of the radius of the beam path as a function of current through the Helmholtz coil
Line of Best Fit
Experimental Data
Variance
Inverse of Radius (1/M)
Current (A)
0
10
20
30
40
50
0.0
0.5
1.0
1.5
2.0
2.5

```python
import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize as sp
import scipy.stats as sta

#reusing my average function from before
def average(container):
    avg = np.sum(container)/len(container)
    return(avg)

#propagation of error modules
def adsubprop(err_x, err_y): #err_x is the error in the argument x
    err_adsub=np.sqrt(err_x**2+err_y**2)
    return(err_adsub)

def muldivprop(x, y, z, err_x, err_y):#z is the final result of multiplication/division of x and y
    err_muldiv=z*(np.sqrt((err_x/x)**2+(err_y/y)**2))
    return(err_muldiv)

def exprop(exponent, x, err_x):
    err_exp=exponent*(x**exponent)*(err_x/x)
    return(err_exp)

#defines a model function for the curve fit to use
def model_function(x, a, b):
    return a*x+b

#defining constants
perm =  4e-7*np.pi #universal constant
turns = 130 #given constant
R = 15.85*0.01 #radius of the coil
K = ((4/5)**(3/2))*perm*turns/R #constant; B-field/current (coil constant)
k = K/np.sqrt(2) #coil constant

#loading data
data = np.loadtxt('Fixed-Voltage-Charge-to-Mass.txt', skiprows = 1, unpack = True)
vdata = data[0] #
idata = data[1] #amperes
radius = (data[2]/2)*0.01 #radius of the electron beam path
inv_r = 1/radius #1/radius

#error for V is recorded as the standard deviation in V
#error for I is the reading error
#reading error for the ruler was recorded as +/- 0.5mm = 0.005m
#this value was used as the instrument error was given at 0.04%, an order of
#magnitude below reading error.
err_i = data[3]
err_v = np.std(vdata)
err_r = 0.0005

B = ((4/5)**(3/2))*((perm*turns*idata)/radius)

#curve fitting of 1/r as a linear function of current by Equation 7
p_opt, p_cov = sp.curve_fit(model_function, idata, inv_r,(1,0), err_i, True)

#Derived in discussion
Bext = p_opt[1]*K/p_opt[0]
```

```
var_invr = p_cov[0,1]

#plotting
plt.figure()
plt.plot(np.arange(-0.1,2.6,0.1), p_opt[0]*np.arange(-0.1,2.6,0.1)+p_opt[1],\
         label = 'Line of Best Fit')
plt.grid(True)
plt.scatter(idata, inv_r, label = "Experimental Data")
plt.errorbar(idata, inv_r, yerr = var_invr, ls = 'none', label = 'Variance')
plt.ylabel("Inverse of Radius (1/M)")
plt.xlabel("Current (A)")
plt.title("Plot of the inverse of the radius of the beam path as a function of " +\
          "current through the Helmholtz coil", fontsize = 8)
plt.legend()
plt.savefig("ChuKi I vs r-1.pdf")

#calculation of charge/mass by the method described in the lab report;
i0 = -Bext/k
idiff = idata-i0
cm = average((vdata/((radius*k)**2))*((1/(idiff)**2)))

#propagation of error
#k is constants multiplied, propagating no error
#treating i0 as constant
err_invert_r = exprop(-1, radius, err_r)
err_invert_r_sq = exprop(2, 1/radius, err_invert_r)
err_diff_i = adsubprop(0, err_i)
err_invert_diff_i = exprop(-1, idiff, err_diff_i)
err_invdiff_i_sq = exprop(2, 1/(idiff**2), err_invert_diff_i)
z = inv_r**2*1/k**2
err_a = muldivprop(inv_r**2, 1/k**2, z, err_invert_r_sq, 0)
err_b = muldivprop(z, vdata, z*vdata, err_a, err_v)
err_tot = muldivprop(z*vdata, 1/idiff**2, z*vdata*1/idiff**2, err_b, err_invdiff_i_sq)

#reduced chi-square calculation
fit = sta.chisquare(inv_r, p_opt[0]*idata+p_opt[1])[0]/2

#print-out
print("The final, calculated charge-to-mass ratio of an electron is")
print(cm)
print("The observed errors for each datapoint are:")
print(err_tot)
print("The resultant reduced chi-square statistic is")
print(fit)
```

```
In [19]: runfile('C:/Users/ASUS/Desktop/PHY224/11. Charge to Mass Ratio/Charge-To-Mass.py', wdir='C:/Users/ASUS/Desktop/PHY224/11. Charge to Mass Ratio')
The final, calculated charge-to-mass ratio of an electron is
381655155578.06757
The observed errors for each datapoint are:
[8.83081803e+09 8.58788513e+09 9.80744219e+09 8.38933316e+09
 9.96795718e+09 1.25562221e+10 1.59615396e+10]
The resultant reduced chi-square statistic is
0.05752940770473313
```




```
In [20]:
```