PHY324: The Cavendish Experiment

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This experiment is a re-implementation of Henry Cavendish's 1798 experiments to determine the density of the Earth, albeit with updated instrumentation and methodology. Accordingly, the end goal is to measure the gravitational constant, G, through experimental trials. The experimental set-up includes a laser on a stand rod, a pre-calibrated Leybold Gravitation Torsion Balance, a screen, and LabView motion acquisition and recording software. Over varying trials involving differing positions of lead weights, the Leybold balance-reflected laser beam is projected on the screen and its position is collected and recorded by the LabView software over a period of time. The data obtained describes the oscillations of the beam in the horizontal plane. The period of these oscillations can be used to calculate the equilibrium position of the reflected beam, and thus the equilibrium position of the Leybold balance for each trial. Together with the measured physical constants of the set-up, this information can be used to calculate the value of the gravitational constant. This was determined to be $6.5802 \times 10^{-11} \pm 1.4512 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$.

Introduction & Background

The following equations are the provided in the lab manual for this experiment:

(1) $F_g = G\frac{Mm}{x_1^2}$
(2) $\Gamma_g = G\frac{Mm}{x_1^2}2dcos\alpha_1$
(3) $G\frac{Mm}{x_1^2}2dcos\alpha_1 = k\theta_1$
(4) $T = 2\pi(\frac{I}{k})^{\frac{1}{2}}$
(5) $x = R + \frac{w}{2} - dsin\theta$

As can be seen in Figure 1 of the manual, (1) is describing the gravitational force, F_g, between the mass of each 0.015kg weight component of the Leybold Balance, *m*, and the mass of the 15kg weight that is positioned near it, *M*. x_1 refers to the distance between the respective centres of mass of both weights when the apparatus is oriented clockwise (i.e.: the torsional force is pulling the balance in a clockwise manner).

(2) follows directly from (1), where Γ_g is the total torque on the inner frame, and d is the distance from a single small weight to the axis of rotation of the balance.

(3) expands (2), describing the total torque of the inner frame with the balance at equilibrium. Equilibrium occurs due to balance by torque from the wire supporting the inner frame, $k\theta_1$. k is the torsional constant and θ_1 is the angle of twist of the supporting wire.

(4) is used to determine k based on the period of oscillation, T, as the frame approaches equilibrium. I is the moment of inertia of the inner frame and assumed to be equal to $2md^2$.

(5) is simply a geometrical description of the experimental setup when the equilibrium position is symmetric, as can be seen in Figure 1 of the manual.

Of additional importance is the background information provided by the Leybold apparatus's Directions for Use. Of primary importance is the observation that the change in the position of the reflected beam on the screen, with and without large weights, in comparison to the distance of the Leybold apparatus to the screen is geometrically proportional to the displacement of the small weights from equilibrium in the presence of large weights in comparison to the distance of the small weights to the axis of rotation. Due to the large discrepancy in magnitude between these two relationships, a small angle approximation is allowed, allowing the tangent of θ_1 to be equated approximately to θ_1.

This ultimately allows for the above equations to be rearranged to yield:

(6) $G = \frac{\pi^2 x_1^2 dS}{MT^2L}$

Where *G* is the gravitational constant, *S* is the change in position of the reflected beam on the screen, and *L* is the distance from the Leybold apparatus to the screen.

Materials

- Leybold Gravitation Torsion Balance
- Laser on rod stand
- Detector and associated electronics
- Computer with installed LabView acquisition software
- Screen
- 2 1.5kg spherical lead weights
- 1 tape measure

Procedure

Measurement Procedure for a Single Trial

1. The apparatus was checked to verify that any incidental movement is kept at a minimum.
2. The laser was plugged in and turned on.
3. The data-capture-via-digital-camera software was turned on and began collecting data.
4. Data was acquired over a period of one hour, and then exported for analysis.

Overall Experimental Procedure

1. The above measurement procedure was applied to the apparatus without the two 1.5kg lead weights attached to the frame.
2. The weights were then placed in their holders on the frame, with the holders oriented counterclockwise relative to the apparatus, and the measurement procedure was repeated.
3. The weights and holders were re-oriented to be clockwise relative to the apparatus, and the measurement procedure was repeated.
4. Additional trials of steps 1-3 of the Overall Experimental Procedure were performed as time permitted.
5. Relevant physical distances of the set-up were measured with a tape measure.

Results

Without the 1.5kg weights present, the oscillations in the horizontal plane of the reflected laser beam were of fairly uniform amplitude, as would be expected due to the lack of gravitational interference. The equilibrium position of the beam on the screen was predicted by taking the average of the position data of the oscillations, as the axis of the amplitudes would be where the position of the beam was tending towards. Thus it was determined that the equilibrium position of the beam without the presence of the large weights was $0.2408m$ from the origin on the screen.

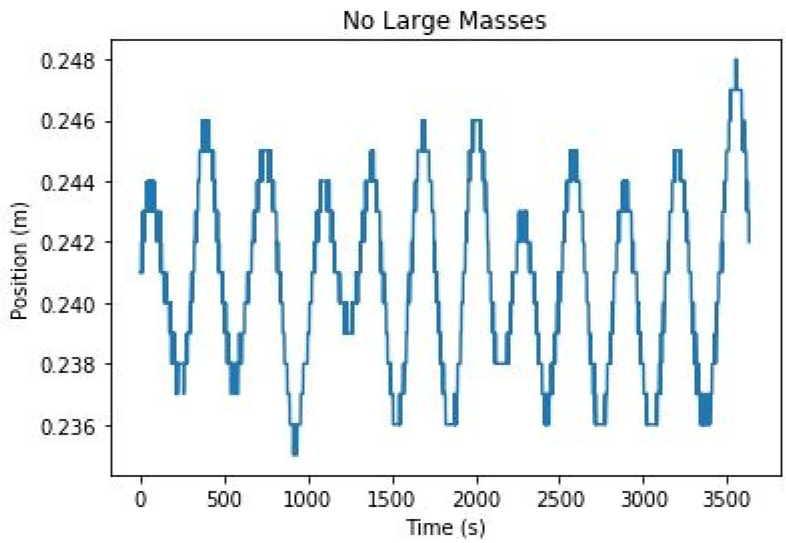

Figure 1: Oscillations of the horizontal position of the beam without 1.5kg masses present.

Similar reasoning was employed to determine the tendency of the apparatus oscillations when the 1.5kg weights were positioned counterclockwise, yielding an equilibrium position of $0.2162m$ from the origin on the screen.

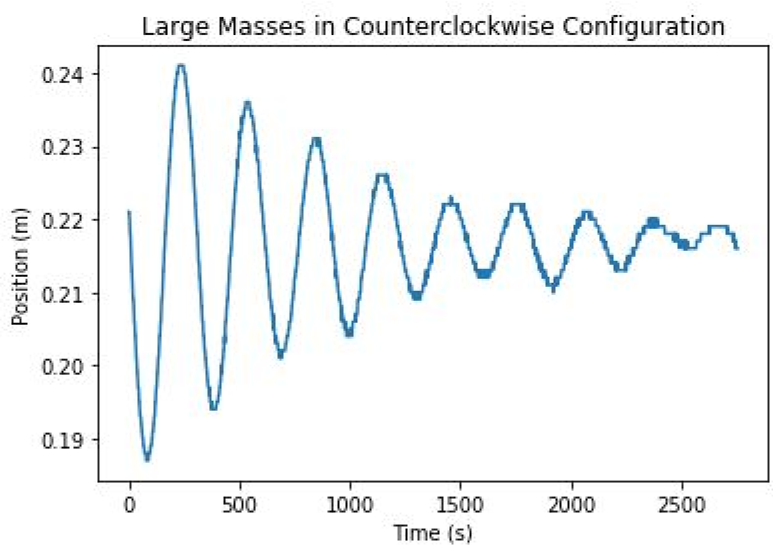

Figure 2: Oscillations of horizontal position of the beam with the 1.5kg masses in counterclockwise configuration.

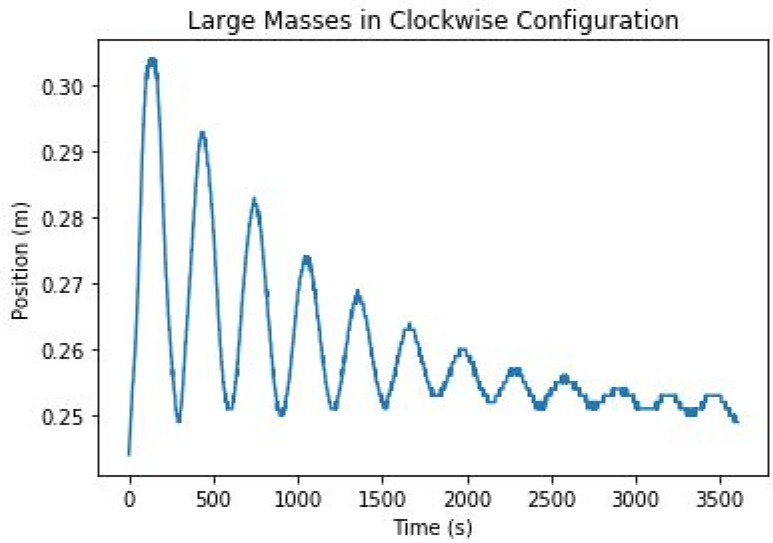

Figure 3: Oscillations of horizontal position of the beam with the 1.5kg masses in clockwise configuration.

As would be expected, similar reasoning was again employed to determine the tendency of the apparatus oscillations when the 1.5kg weights were positioned clockwise. This yielded an equilibrium position of $0.2597m$ from the origin on the screen.

To determine the period of each trend, the time-value of the first and last peaks on each graph were identified using Python. The differences between these 'start' and 'stop' times were divided by the number of cycles visible on each graph to yield a measurement of period. Using this method, the period without weights was determined to be $319.8181s$, the counterclockwise-oriented period was determined to be $310.625s$, and the clockwise-oriented period was determined to be $307.7272s$.

Additional required physical characteristics of the set-up were measured with a tape-measure. The distance between the main case of the Leybold balance (containing the small weights on the rotating axis) and the large 1.5kg weights was measured to be 0.060m for both configurations. This was used as an acceptable value for x_1, as the Leybold Directions for Use specified that "the space for this suspension in the casing has been kept as small as possible." The distance between the Leybold apparatus and the screen was measured to be 4.586m.

With the above values, (6) was used to calculate the gravitational constant, G, yielding $6.5802 \times 10^{-11} \pm 1.4512 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$ when accounting for propagation of the measurement error.

Question Responses

Question 1: Is the equilibrium position E really symmetric?

Answer: One can compare the differences in movement of the reflected beam for both a clockwise and counterclockwise orientation of the large weights to compare symmetry of the equilibrium position. A clockwise orientation of the weights results in the position of the reflected beam's predicted equilibrium moving $1.8874 \times 10^{-2} m$ from where it was without the weights present. A similar calculation reveals that a counterclockwise orientation moves the beam's predicted equilibrium position $2.4584 \times 10^{-2} m$. As these values are not equal, the conclusion is either that either E is not truly symmetric, or that the positions of the large weights relative to the apparatus, x_1 and x_2, were not truly equal as was assumed.

Question 2: The torsion pendulum is clearly a damped harmonic oscillator. Is the motion simple harmonic? What effect does the damping have on the determination of period?
Answer: Simple harmonic motion is undamped by definition, thus requiring a uniform amplitude throughout. Since damping reduces and restricts the oscillation of the system, the frequency of the oscillation decreases when the system is dampened, causing the period of the oscillation to increase. Eventually no oscillation will occur and the system will be at equilibrium. These characteristics can be observed in this experiment, and thus the motion is not simple harmonic.

Question 3: Each mass m will also be attracted by the remote second mass M. What effect does this have on your result?
Answer: As explained in the Leybold apparatus's Directions for Use, the force that each large mass exerts on the opposite small mass reduces the force that one is 'supposed to observe' by a factor of $\beta = \frac{b^3}{(b^2+4d^2)^{3/2}}$. Accordingly, this factor was calculated to be 0.1362 ± 0.0035, meaning that the experiment is operating at roughly 86.38% 'efficiency.'

Question 4: What is the dominant error in the determination of G? Does the approximation being made have an effect comparable to this dominant error? If yes, what can be done about it?
Answer: The dominant error appears to be measurement error, in particular the rough estimation of the period of oscillation. This 'graphical measurement' likely led to imprecision due to the fact that some observations were irregularly oscillating. This irregularity was likely caused by an incomplete discharge of static build up, and can be mitigated in the future by being sure to dispel all electricity prior to experimentation. In comparison, the small angle approximation and the assumption that x_1 and x_2 can be approximated as the distance from the apparatus outer casing to the large weights likely cause less imprecision in the final result.

Conclusion

In conclusion, the gravitational constant, G, was measured to be $6.5802 \times 10^{-11} \pm 1.4512 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$, placing it within acceptable distance to the true, known value of $6.6741 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$.

Appendix: Python Code

```python
# -*- coding: utf-8 -*-
"""
Code for the Cavendish Experiment Lab
"""
import numpy as np
import scipy.optimize as opt
import scipy.stats as sta
import matplotlib.pyplot as plt

"""
Unpacking Data
"""
noweights = np.loadtxt("noweights.txt", dtype = str, delimiter = "\t",\
                       skiprows = 2, unpack = True)
nwtime = np.array([])
for t in noweights[0]:
    tcorrec = float(noweights[0][0].split(":")[0])*3600 +\
    float(noweights[0][0].split(":")[1])*60 + float(noweights[0][0].split(":")[2])
    nwtime = np.append(nwtime, float(t.split(":")[0])*3600 +\
                       float(t.split(":")[1])*60 + float(t.split(":")[2]) - tcorrec)
nwposition = np.array([])
for p in noweights[1]:
    nwposition = np.append(nwposition, float(p))

ccweights = np.loadtxt("ccweights.txt", dtype = str, delimiter = "\t",\
                       skiprows = 2, unpack = True)
cctime = np.array([])
for t in ccweights[0]:
    tcorrec = float(ccweights[0][0].split(":")[0])*3600 +\
    float(ccweights[0][0].split(":")[1])*60 + float(ccweights[0][0].split(":")[2])
    cctime = np.append(cctime, float(t.split(":")[0])*3600 +\
                       float(t.split(":")[1])*60 + float(t.split(":")[2]) - tcorrec)
ccposition = np.array([])
for p in ccweights[1]:
    ccposition = np.append(ccposition, float(p))

noweights2 = np.loadtxt("noweights2.txt", dtype = str, delimiter = "\t",\
                        skiprows = 2, unpack = True)
nw2time = np.array([])
for t in noweights2[0]:
    tcorrec = float(noweights2[0][0].split(":")[0])*3600 +\
    float(noweights2[0][0].split(":")[1])*60 + float(noweights2[0][0].split(":")[2])
    nw2time = np.append(nw2time, float(t.split(":")[0])*3600 +\
                        float(t.split(":")[1])*60 + float(t.split(":")[2]) - tcorrec)
nw2position = np.array([])
for p in noweights2[1]:
    nw2position = np.append(nw2position, float(p))

cweights2 = np.loadtxt("cweights2.txt", dtype = str, delimiter = "\t",\
                       skiprows = 2, unpack = True)
c2time = np.array([])
for t in cweights2[0]:
    tcorrec = float(cweights2[0][0].split(":")[0])*3600 +\
    float(cweights2[0][0].split(":")[1])*60 + float(cweights2[0][0].split(":")[2])
    c2time = np.append(c2time, float(t.split(":")[0])*3600 +\
                       float(t.split(":")[1])*60 + float(t.split(":")[2]) - tcorrec)
c2position = np.array([])
```

```
for p in cweights2[1]:
    c2position = np.append(c2position, float(p))

ccweights2 = np.loadtxt("ccweights2.txt", dtype = str, delimiter = "\t",\
                        skiprows = 2, unpack = True)
cc2time = np.array([])
for t in ccweights2[0]:
    tcorrec = float(ccweights2[0][0].split(":")[0])*3600 +\
    float(ccweights2[0][0].split(":")[1])*60 + float(ccweights2[0][0].split(":")[2])
    cc2time = np.append(cc2time, float(t.split(":")[0])*3600 +\
                        float(t.split(":")[1])*60 + float(t.split(":")[2]) - tcorrec)
cc2position = np.array([])
for p in ccweights2[1]:
    cc2position = np.append(cc2position, float(p))

"""
Calculations of G
"""
#Period calculations
nwT = (nwtime[np.where(nwposition == 0.248)[0][-1]] -\
              nwtime[np.where(nwposition == 0.244)[0][0]])/11
ccT = (cctime[np.where(ccposition == 0.224)[0][-1]] -\
              cctime[np.where(ccposition == 0.216)[0][0]])/10
nw2T = (nw2time[np.where(nw2position == 0.243)[0][-1]] -\
                nw2time[np.where(nw2position == 0.238)[0][0]])/10
c2T = (c2time[np.where(c2position == 0.253)[0][-1]] -\
              c2time[np.where(c2position == 0.304)[0][0]])/11
cc2T = (cc2time[np.where(cc2position == 0.219)[0][-1]] -\
                cc2time[np.where(cc2position == 0.241)[0][0]])/8

#Calculation of G
b = 0.06
d = 0.05
S = np.abs(np.mean(nwposition) - np.mean(cc2position))
m = 1.5
T = cc2T
L = 4.586
G = (np.pi**2*b**2*d*S)/(m*T**2*L)

#Error calculations
be = 0.0005
Se = np.sqrt(2*0.0000005**2)
Te = 0.0005
Le = 0.0005
error = ((S*b**2)/(L*T**2))*np.sqrt(S*b**2*((Se/S)**2 + (2*be/b)**2) +\
         L*T**2*((Le/L)**2 + (2*Te/T)**2))

"""
Calculations for Questions
"""
#Symmetry of E
cS = np.abs(np.mean(nwposition) - np.mean(c2position))
ccS = np.abs(np.mean(nwposition) - np.mean(cc2position))

#Factor of other weight
beta = b**3/(b**2 + 4*d**2)**1.5
betae = beta*np.sqrt((9*be**2/b**2) + ((9*b**2*be**2)/(b**2 + 4*d**2)**2))
```

```python
"""
Plots
"""
#No Weights - Day 1
plt.figure(0)
plt.plot(nwtime, nwposition)
plt.xlabel("Time (s)")
plt.ylabel("Position (m)")
plt.title("No Large Masses")
plt.savefig("noweights.pdf")

#Counterclockwise Weights - Day 1
plt.figure(1)
plt.plot(cctime, ccposition)
plt.xlabel("Time (s)")
plt.ylabel("Position (m)")
plt.title("Large Masses in Counterclockwise Configuration")

#No Weights - Day 2
plt.figure(2)
plt.plot(nw2time, nw2position)
plt.xlabel("Time (s)")
plt.ylabel("Position (m)")
plt.title("No Large Masses")

#Clockwise Weights - Day 2
plt.figure(3)
plt.plot(c2time, c2position)
plt.xlabel("Time (s)")
plt.ylabel("Position (m)")
plt.title("Large Masses in Clockwise Configuration")
plt.savefig("cweights.pdf")

#Counterclockwise Weights - Day 2
plt.figure(4)
plt.plot(cc2time, cc2position)
plt.xlabel("Time (s)")
plt.ylabel("Position (m)")
plt.title("Large Masses in Counterclockwise Configuration")
plt.savefig("ccweights.pdf")
```