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A new measure using intuitionistic fuzzy set theory and its application to edge detection

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Abstract

In this paper, a new attempt has been made using Attanassov's intuitionistic fuzzy set theory for image edge detection. Intuitionistic fuzzy set takes into account the uncertainty in assignment of membership degree known as hesitation degree. Also a new distance measure, called intuitionistic fuzzy divergence, has been proposed. With this proposed distance measure, edge detection is carried out, and the results are found better with respect to the previous methods.

Keywords: Intuitionistic fuzzy set; Membership degree; Hesitation degree; Intuitionistic fuzzy divergence; Edge detection

1. Introduction

The theory of fuzzy set proposed by Zadeh [19] in 1965 has gained much importance in various fields of signal and image processing in recent times [4,7,8,12,18]. In fuzzy set theory, a degree of membership is assigned to each element, where the degree of non-membership is just automatically equal to 1 minus the degree of membership. However, human being who expresses the degree of membership of a fuzzy set very often does not express the corresponding degree of non-membership as the complement to 1.

With the above observation, Atanassov [1–3] introduced the concept of intuitionistic fuzzy set that is meant to reflect the fact that the degree of non-membership is not always equal to 1 minus degree of membership, but there may be some hesitation degree. Another generalization of fuzzy set theory is the concept of interval-valued fuzzy set, introduced by Sambuc [15]. Sometimes these two sets are mathematically equivalent but have arisen from different backgrounds [5,13]. Yet, there is a strong relationship between interval-valued fuzzy sets and intuitionistic fuzzy sets. Szmidt and Kacpryzk [16,17] proposed some distance measures between intuitionistic fuzzy sets that are the generalization of the Hamming distance and the Euclidean distance. Grzegorzewski [10] suggested Hamming distance and Euclidean distance based on Hausdorff metric using intuitionistic fuzzy sets. Li and Cheng [14] introduced the similarity measures between two intuitionistic fuzzy sets and applied them in pattern recognition.

In this paper, a new attempt has been made to use the intuitionistic fuzzy set theory in image edge detection. A new definition of distance measure, called intuitionistic fuzzy divergence (IFD), is also proposed. Unlike fuzzy divergence that considers only the membership degree [7,8]; IFD takes into account the membership degree, the non-membership degree, and the hesitation degree. To demonstrate the applicability of the proposed IFD in practice, it has been used here for edge detection. The results using the new measure are found better with respect to the previous approaches in the literature.

The rest of the paper is organized as follows. In Section 2, intuitionistic fuzzy sets and some related distance measures are described. Section 3 contains the proposed distance measure. Section 4 briefly describes about edge detection and its related work. Section 5 provides the experimental model. Section 6 displays and discusses the results. Finally, conclusion is drawn in Section 7.

2. Intuitionistic fuzzy sets

In this section the concept of intuitionistic fuzzy set and some of the related distance measures are described.

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2.1. The concept

A fuzzy set A in a finite set $X = \{x_1, x_2, \ldots, x_n\}$ may be represented mathematically as

$$A = \{(x, \mu_A(x)|x \in X\},$$

where the function $\mu_A(x)$: $X \to [0,1]$ is measure of belongingness or degree of membership of an element x in the finite set X. Thus, automatically the measure of non-belongingness is $1 - \mu_A(x)$.

Attanassov suggested a generalization of fuzzy sets, called intuitionistic fuzzy sets.

An intuitionistic fuzzy set A in a finite set X may be mathematically represented as

$$A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}, \tag{1}$$

where the functions $\mu_A(x)$, $\nu_A(x)$: $X \to [0,1]$ are, respectively, the membership degree and the non-membership degree of an element x in a finite set X with the necessary condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

It can be easily observed that, every fuzzy set is a particular case of intuitionistic fuzzy set:

$$A = \{(x, \mu_A(x), 1 - \mu_A(x))|x \in X\}.$$

Recently, Szmidt and Kacpryzk stressed the necessity of taking into consideration a third parameter $\pi_A(x)$, known as the intuitionistic fuzzy index or hesitation degree, which arises due to the lack of knowledge or 'personal error' (Attanassov [3]) in calculating the distances between two fuzzy sets. So, with the introduction of hesitation degree, an intuitionistic fuzzy set A in X may be represented as

$$A = \{(x, \mu_A(x), \nu_A(x), \pi_A(x))|x \in X\}$$

with the condition

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1 \tag{2}$$

In fuzzy set, non-membership value is equal to 1-membership value or the sum of membership degree and non-membership value is equal to 1. This is logically true. But in real world, this may not be true as human being may not express the non-membership value as 1-membership value. So logical negation is not equal to practical negation. This is due to the presence of uncertainty or hesitation or the lack of knowledge in defining the membership function. This uncertainty is named as hesitation degree. Thus the summation of three degrees, i.e., membership, non-membership, and hesitation degree is 1.

It is obvious that

$$0 \le \pi_A(x) \le 1, \quad \text{for each } x \in X.$$

2.2. Some related distance measures

In many practical and theoretical problems, in order to find the difference between two objects, the knowledge of distance between two fuzzy sets is necessary. Szmidt and Kacprzyk introduced some popular distance measures between two intuitionistic fuzzy sets A and B that take into account the membership degree μ, the non-membership degree ν, and the hesitation degree (or intuitionistic fuzzy index) π in $X = \{x_1, x_2, \ldots, x_n\}$. Some of the intuitionistic fuzzy distance measures are as follows:

Intuitionistic Hamming distance:

$$H_{\mathrm{IFS}}(A,B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|).$$

Intuitionistic Euclidean distance:

$$E_{\mathrm{IFS}}(A,B) = \sqrt{\sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2} + (\pi_A(x_i) - \pi_B(x_i))^2].$$

Grzegorzewski introduced Hamming distance and Euclidean distance based on Hausdorff metric, which are given as follows:

Hamming distance :

$$d_{\mathrm{H}}(A,B) = \sum \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}.$$

Euclidean distance :

$$d_{\mathrm{E}}(A,B) = \sqrt{\sum_{i=1}^{n} [\max(\mu_A(x_i) - \mu_B(x_i))^2}, (\nu_A(x_i) - \nu_B(x_i))^2].$$

3. The newly proposed distance measure

In our work, using intuitionistic fuzzy set, a new distance measure called intuitionistic fuzzy divergence (IFD) is proposed, where the three parameters, namely, the membership degree, the non-membership degree, and the hesitation degree (or intuitionistic fuzzy index), are considered.

Let $A = \{(x, \mu_A(x), \nu_A(x) \,|x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x) \,|x \in X\}$ be two intuitionistic fuzzy sets. Considering the hesitation degree, the interval or range of the membership degree of the two intuitionistic fuzzy sets A and B may be represented as $\{(\mu_A(x), (\mu_A(x) + \pi_A(x))\}$, $\{(\mu_B(x), \mu_B(x) + \pi_B(x))\}$ where $\mu_A(x)$, $\mu_B(x)$ are the membership degrees and $\pi_A(x), \pi_B(x)$ are the hesitation degrees in the respective sets, with $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ and $\pi_B(x) = 1 - \mu_B(x) - \nu_B(x)$. The interval is due to the hesitation or the lack of knowledge in assigning membership values. The distance measure has been proposed here taking into account the hesitation degrees.

In an image of size $M \times M$ with L distinct gray levels having probabilities $p_0, p_1, \ldots, p_{L-1}$, the exponential entropy is defined as $H = \sum_{i=0}^{L-1} p_i \, e^{1-p_i}$.

In fuzzy cases, the fuzzy entropy of an image A of size $M \times M$ is defined as [8,9]:

$$H(A) = \frac{1}{n(\sqrt{e}-1)} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} [(\mu_A(a_{ij})\, \mathrm{e}^{1-\mu_A(a_{ij})} + (1-\mu_A(a_{ij})\, \mathrm{e}^{\mu_A(a_{ij})} - 1] \tag{3}$$

where $n = M^2$, $i, j = 0, 1, 2, \ldots, M-1$, and $\mu_A(a_{ij})$ is the membership degree of the (i,j)th pixel a_{ij} in the image A.

For two images A and B, at the (i,j)th pixels (i.e., at pixels a_{ij} and b_{ij}), the amount of information between the membership degrees of images A and B is given as follows:

(i) due to $m_1(A)$ and $m_1(B)$, i.e., $\mu_A(a_{ij})$ and $\mu_B(b_{ij})$ of the (i,j)th pixels:

$$\mathrm{e}^{\mu_A(a_{ij})}/\mathrm{e}^{\mu_B(b_{ij})} \quad \text{or} \quad \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})}$$

(ii) due $m_2(A)$ and $m_2(B)$, i.e., $\mu_A(a_{ij}) + \pi_A(b_{ij})$ and $\mu_B(a_{ij}) + \pi_B(b_{ij})$ of the (i,j)th pixels:

$$\mathrm{e}^{\mu_A(a_{ij})+\pi_A(a_{ij})}/\mathrm{e}^{\mu_B(b_{ij})+\pi_B(b_{ij})}$$

Corresponding to the fuzzy entropy, the divergence between images A and B due to $m_1(A)$ and $m_1(B)$ may be given as

$$D_1(A,B) = \sum_i \sum_j (1 - (1-\mu_A(a_{ij}))\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})} - \mu_A(a_{ij})\, \mathrm{e}^{\mu_B(b_{ij})-\mu_A(a_{ij})}) \tag{4}$$

Similarly, the divergence of B against A is:

$$D_1(B,A) = \sum_i \sum_j 1 - (1-\mu_B(b_{ij}))\, \mathrm{e}^{\mu_B(b_{ij})-\mu_A(a_{ij})} - \mu_B(b_{ij})\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})} \tag{5}$$

So, the total divergence between the pixels a_{ij} and b_{ij} of the images A and B due to $m_1(A)$ and $m_1(B)$ is

$$\begin{aligned} \mathrm{Div} - m_1(A,B) &= D_1(A,B) + D_1(B,A) \\ &= \sum_i \sum_j 2 - (1-\mu_A(a_{ij}) + \mu_B(b_{ij}))\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})} \\ &\quad - (1-\mu_B(b_{ij}) + \mu_A(a_{ij}))\, \mathrm{e}^{\mu_B(b_{ij})-\mu_A(a_{ij})} \end{aligned} \tag{6}$$

Likewise, the total divergence between the pixels a_{ij} and b_{ij} of the images A and B due to $m_2(A)$ and $m_2(B)$ is:

$$\begin{aligned} &\mathrm{Div} - m_2(A,B) \\ &= \sum_i \sum_j (2 - [1 - (\mu_A(a_{ij}) - (\mu_B(b_{ij})) + (\pi_B(b_{ij}) \\ &\quad - \pi_A(a_{ij}))]\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})-((\pi_B(b_{ij})-\pi_A(a_{ij}))} \\ &\quad - [1 - (\pi_B(b_{ij}) - \pi_A(a_{ij})) + (\mu_A(a_{ij}) \\ &\quad - \mu_B(b_{ij}))]\, \mathrm{e}^{\pi_B(b_{ij})-\pi_A(a_{ij})-((\mu_A(a_{ij})-\mu_B(b_{ij}))} \end{aligned} \tag{7}$$

Thus, the overall intuitionistic fuzzy divergence, IFD, between the images A and B by adding Eqs. (6) and (7), is

$$\begin{bmatrix}0&b&a\\0&b&a\\0&b&a\end{bmatrix} \begin{bmatrix}a&a&a\\0&0&0\\b&b&b\end{bmatrix} \begin{bmatrix}a&a&b\\a&b&0\\b&0&0\end{bmatrix} \begin{bmatrix}b&b&b\\0&0&0\\a&a&a\end{bmatrix} \begin{bmatrix}b&a&a\\0&b&a\\0&0&b\end{bmatrix} \begin{bmatrix}b&a&0\\b&a&0\\b&a&0\end{bmatrix} \begin{bmatrix}a&0&b\\a&0&b\\a&0&b\end{bmatrix} \begin{bmatrix}0&0&0\\b&b&b\\a&a&a\end{bmatrix}$$

$$\begin{bmatrix}a&a&a\\b&b&b\\0&0&0\end{bmatrix} \begin{bmatrix}a&b&0\\a&b&0\\a&b&0\end{bmatrix} \begin{bmatrix}0&0&0\\a&a&a\\b&b&b\end{bmatrix} \begin{bmatrix}0&a&b\\0&a&b\\0&a&b\end{bmatrix} \begin{bmatrix}b&b&b\\a&a&a\\0&0&0\end{bmatrix} \begin{bmatrix}b&0&a\\b&0&a\\b&0&a\end{bmatrix} \begin{bmatrix}b&0&0\\a&b&0\\a&a&b\end{bmatrix} \begin{bmatrix}0&0&b\\0&b&a\\b&a&a\end{bmatrix}$$

Fig. 1. Set of sixteen 3×3 templates.

defined as

$$\begin{aligned} \mathrm{IFD}(A,B) &= \mathrm{Div} - m_1(A,B) + \mathrm{Div} - m_2(A,B) \\ &= \sum_i \sum_j 2 - [1 - \mu_A(a_{ij}) + \mu_B(b_{ij})]\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})} \\ &\quad - [1 - \mu_B(b_{ij}) + \mu_A(a_{ij})]\, \mathrm{e}^{\mu_B(b_{ij})-\mu_A(a_{ij})} \\ &\quad + (2 - [1 - (\mu_A(a_{ij}) - (\mu_B(b_{ij})) + (\pi_B(b_{ij}) \\ &\quad - \pi_A(a_{ij}))]\, \mathrm{e}^{\mu_A(a_{ij})-\mu_B(b_{ij})-((\pi_B(b_{ij})-\pi_A(a_{ij}))} \\ &\quad - [1 - (\pi_B(b_{ij}) - \pi_A(a_{ij})) + (\mu_A(a_{ij}) \\ &\quad - \mu_B(b_{ij}))]\, \mathrm{e}^{\pi_B(b_{ij})-\pi_A(a_{ij})-((\mu_A(a_{ij})-\mu_B(b_{ij}))} \end{aligned} \tag{8}$$

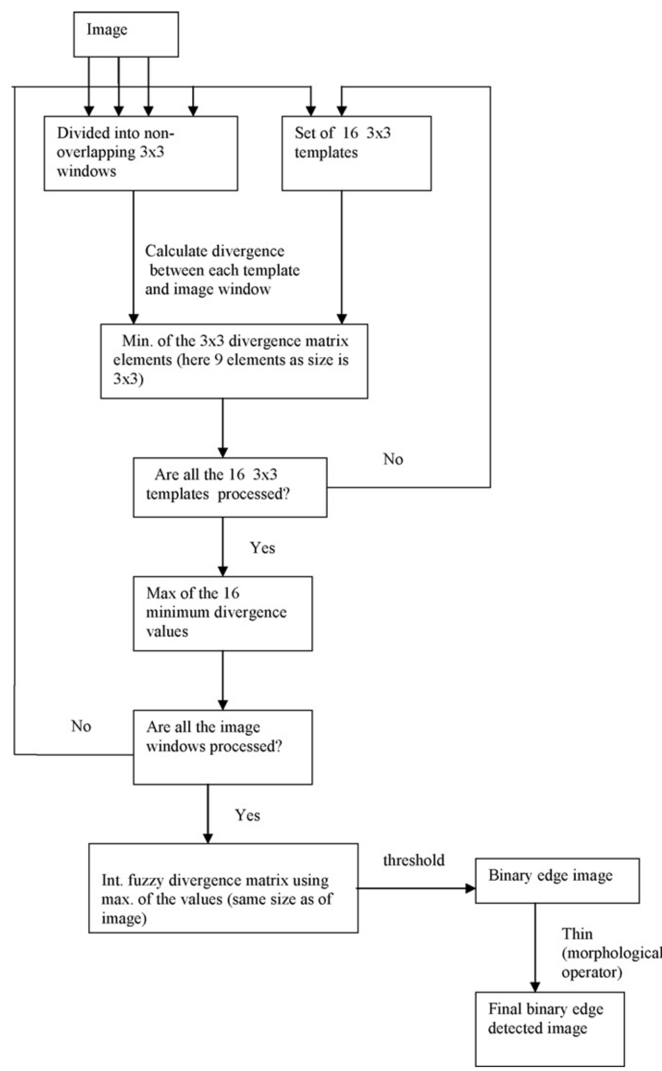

Fig. 2. Block diagram of the algorithm.

With the above formulation of our proposed distance measure, experiment on image edge detection using IFD is carried out in the following section.

In the following, the image edge detection procedure using IFD is presented.

4. Background on edge detection

An edge is a property of an individual pixel and is calculated from the functional behavior of an image in the neighborhood of a pixel. The edge serves in simplifying the analysis of images by drastically reducing the data to be processed and preserving the useful structural information about object boundaries.

There are many fuzzy and crisp edge detection methods in literature (see, e.g. [7,12,18]). But as the area of intuitionistic fuzzy set in image processing is just beginning to develop; there is hardly any related work in the literature. Some of the related work in fuzzy set theory is mentioned. Lopera et al. [11] used Jensen-Shannon divergence of gray level histogram obtained by sliding a double window over an image for edge detection. Tao and Thomson [18] used gradient approximations as input variables. In their approach two fuzzy sets, large and small, were used as linguistic variables and 16 fuzzy rules from 16 contour structures were obtained. Kenneth et al. [12] used fuzzy edge detector for edge detection. They used several fuzzy templates and then edge detect an image using a similarity measure.

5. Experimental model for edge detection

For edge detection, a set of 16 fuzzy templates each of size 3×3, representing the edge profiles of different types, have been used as shown in Fig. 1.

The choice of templates is crucial which reflects the type and direction of edges. The templates are the examples of the edges, which are also the images. a, b, and 0 represent the pixels of the edge templates, where the values of a and b have been chosen by trial and error method. It is to be noted that, the size of the

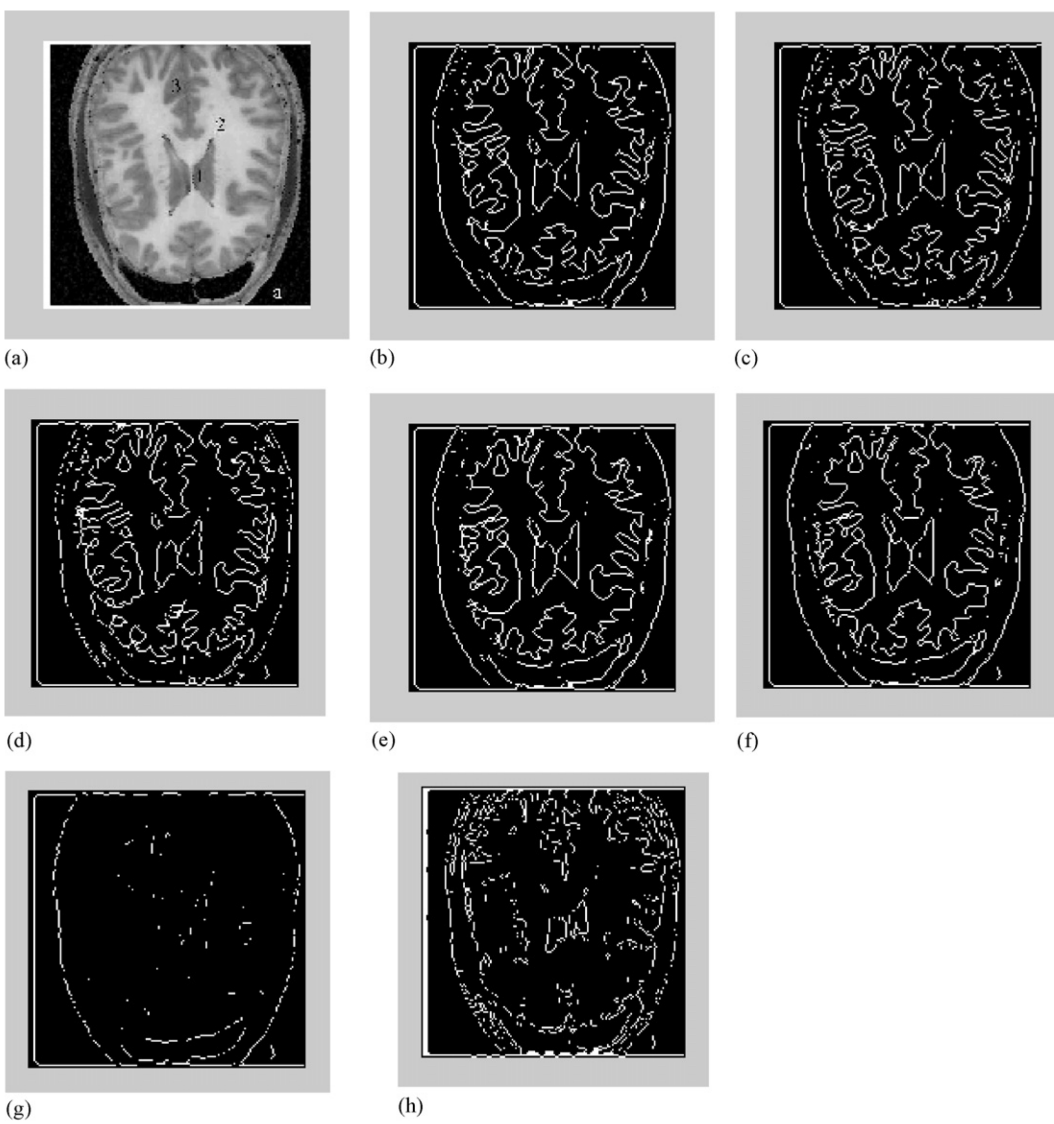

Fig. 3. 'Brain' image. (a) Brain image, (b) $c_t = 0.2$, th = 0.11, (c) $c_t = 0.15$, th = 0.11, (d) $c_t = 0.1$, th = 0.11, (e) $c_t = 0.3$, th = 0.11, (f) $c_t = 0.4$, th = 0.11, (g) $c_t = 0.45$, th = 0.24 and (h) $a = 0.25$, $b = 0.7$, $c_t = 0.2$, th = 0.08.

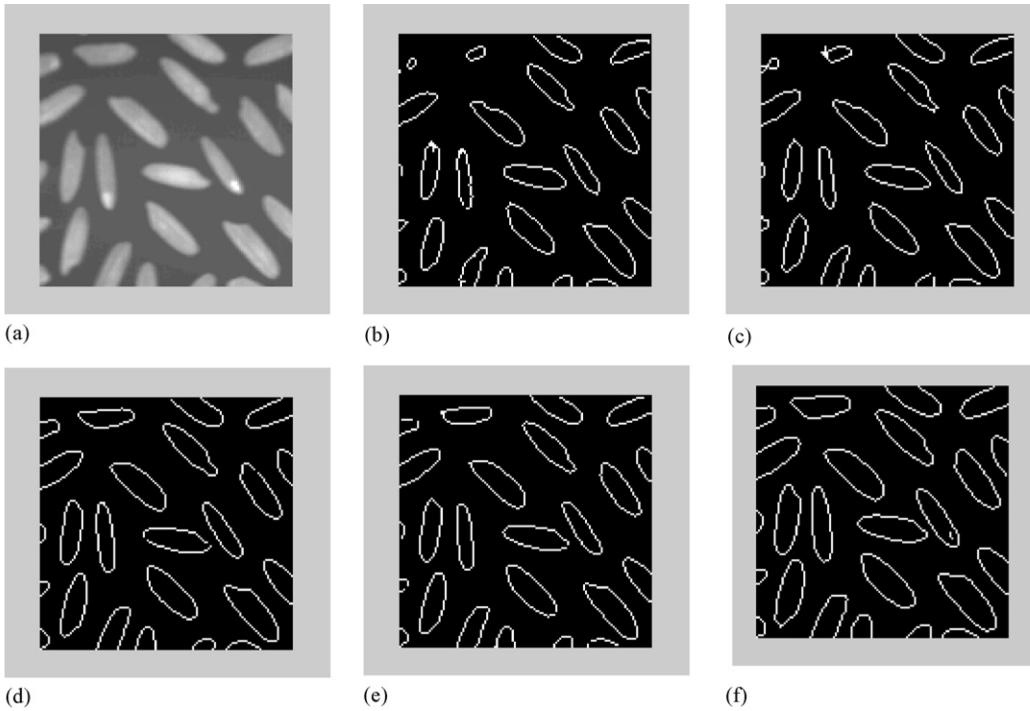

Fig. 4. Rice image. (a) Rice image, (b) $c_t = 0.5$, th = 0.41, (c) $c_t = 0.35$, th = 0.40, (d) $c_t = 0.05$, th = 0.40, (e) $c_t = 0.2$, th = 0.40 and (f) $a = 0.25$, $b = 0.7$, $c_t = 0.2$, th = 0.4.

templates should be less than that of an image. The center of each template is placed at each pixel position (i,j) over a normalized image. The IFD measure at each pixel position (i,j) in the image, where the template was centered, IFD(i,j), is calculated between the image window (same size as that of the template) and the template using the max–min relationship, as given below:

$$\mathrm{IFD}(i,j) = \max_{N}[\min_{r}(\mathrm{IFD}(A,B)] \tag{9}$$

The IFD between A and B, IFD(A,B), is calculated by finding the IFD between each of the elements a_{ij} and b_{ij} of image window A and of template B using Eq. (8), and is given as

$$\begin{aligned}\mathrm{IFD}(a_{ij}, b_{ij}) = {} & (2 - [1 - \mu_A(a_{ij}) + \mu_B(b_{ij})]\, \mathrm{e}^{\mu_A(a_{ij}) - \mu_B(b_{ij})} \\ & - [1 - \mu_B(b_{ij}) + \mu_A(a_{ij})]\, \mathrm{e}^{\mu_B(b_{ij}) - \mu_A(a_{ij})}) \\ & + (2 - [1 - (\mu_A(a_{ij}) - (\mu_B(b_{ij})) \\ & + (\pi_B(b_{ij}) - \pi_A(a_{ij}))]\, \mathrm{e}^{\mu_A(a_{ij}) - \mu_B(b_{ij}) - ((\pi_B(b_{ij}) - \pi_A(a_{ij}))} \\ & - [1 - (\pi_B(b_{ij}) - \pi_A(a_{ij})) + (\mu_A(a_{ij}) \\ & - \mu_B(b_{ij}))]\, \mathrm{e}^{\pi_B(b_{ij}) - \pi_A(a_{ij}) - ((\mu_A(a_{ij}) - \mu_B(b_{ij}))}) \end{aligned} \tag{10}$$

IFD(a_{ij},b_{ij}) is the IFD between each element in the template (b_{ij}) and those in image window (a_{ij}). N = number of templates, r = number of elements in the square template. IFD(i,j) is calculated for all pixel positions of the image. Finally IFD matrix, the same size as that of image, is formed with values of IFD(i,j) at each point of the matrix. This IFD matrix is thresholded and thinned to get an edge-detected image. The threshold selection is manually adjusted for getting the final edge-detected result (Fig. 2).

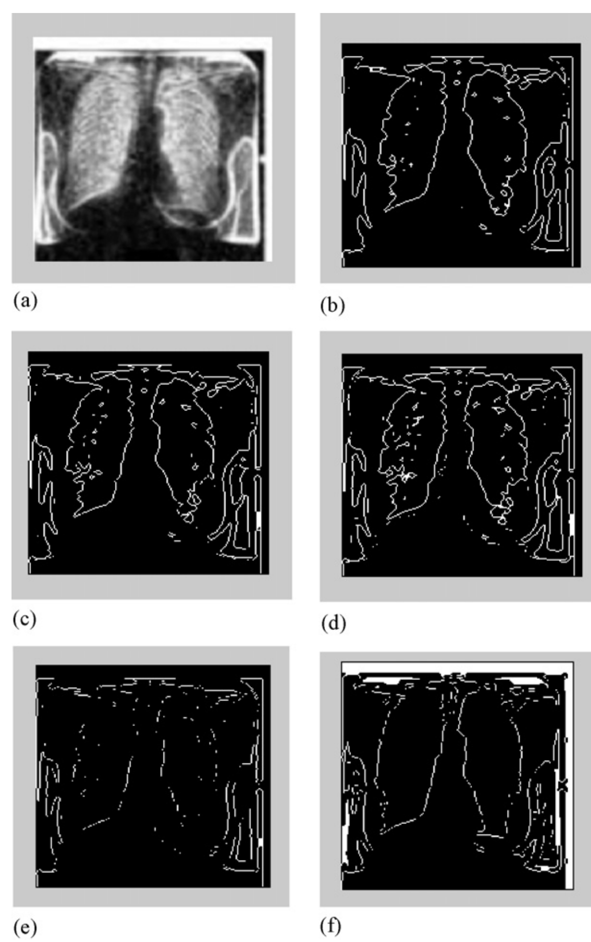

Fig. 5. 'Lung' image. (a) Lung image, (b) $c_t = 0.2$, th = 0.20, (c) $c_t = 0.3$, th = 0.20, (d) $c_t = 0.35$, th = 0.20, (e) $c_t = 0.4$, th = 0.27 and (f) $a = 0.25$, $b = 0.7$, $c_t = 0.2$, th = 0.25.

5.1. Calculation of membership degree, non-membership degree, and intuitionistic fuzzy index

(i) Membership degree

In our experiment, the image A represents the chosen window in the test image, which is of the same size as that of the template, and the image B is the template. Normalized values of the (i,j)th pixel of image A are the membership degrees, $\mu_A a_{ij}$, while the values of the template are the membership degrees of the template pixels, $\mu_B b_{ij}$. Due to the intuitionistic characteristics, the membership degrees of images A and B may take the values in the interval $\{\mu_A(a_{ij}), (\mu_A(a_{ij}) + \pi_A(a_{ij}))\}$, and $\{\mu_B(b_{ij}), (\mu_B(b_{ij}) + \pi_B(b_{ij}))\}$, respectively.

(ii) Non-membership degree

From (2) we know that $\pi_A(a_{ij}) + \mu_A(a_{ij}) + \nu_A(a_{ij}) = 1$, i.e., the non-membership degree = 1 − membership degree − hesitation degree.

(iii) Hesitation degree or intuitionistic fuzzy index

For calculating the intuitionistic fuzzy index, i.e., to know the hesitation degree, we have assumed

$$\text{Intuitionistic fuzzy index} = c * (1 - \text{membership}). \tag{11}$$

where c is a hesitation constant (also known as intuitionistic fuzzy constant). The value of c should be such that the (2) holds.

5.2. The algorithm

- *Step 1*. Form 16 edge-detected templates with values 'a', 'b'.
- *Step 2*. Apply the edge templates over the image by placing the center of each template at each point (i,j) over the normalized image.
- *Step 3*. Calculate the intuitionistic fuzzy divergence (IFD) between each elements of each template and the image

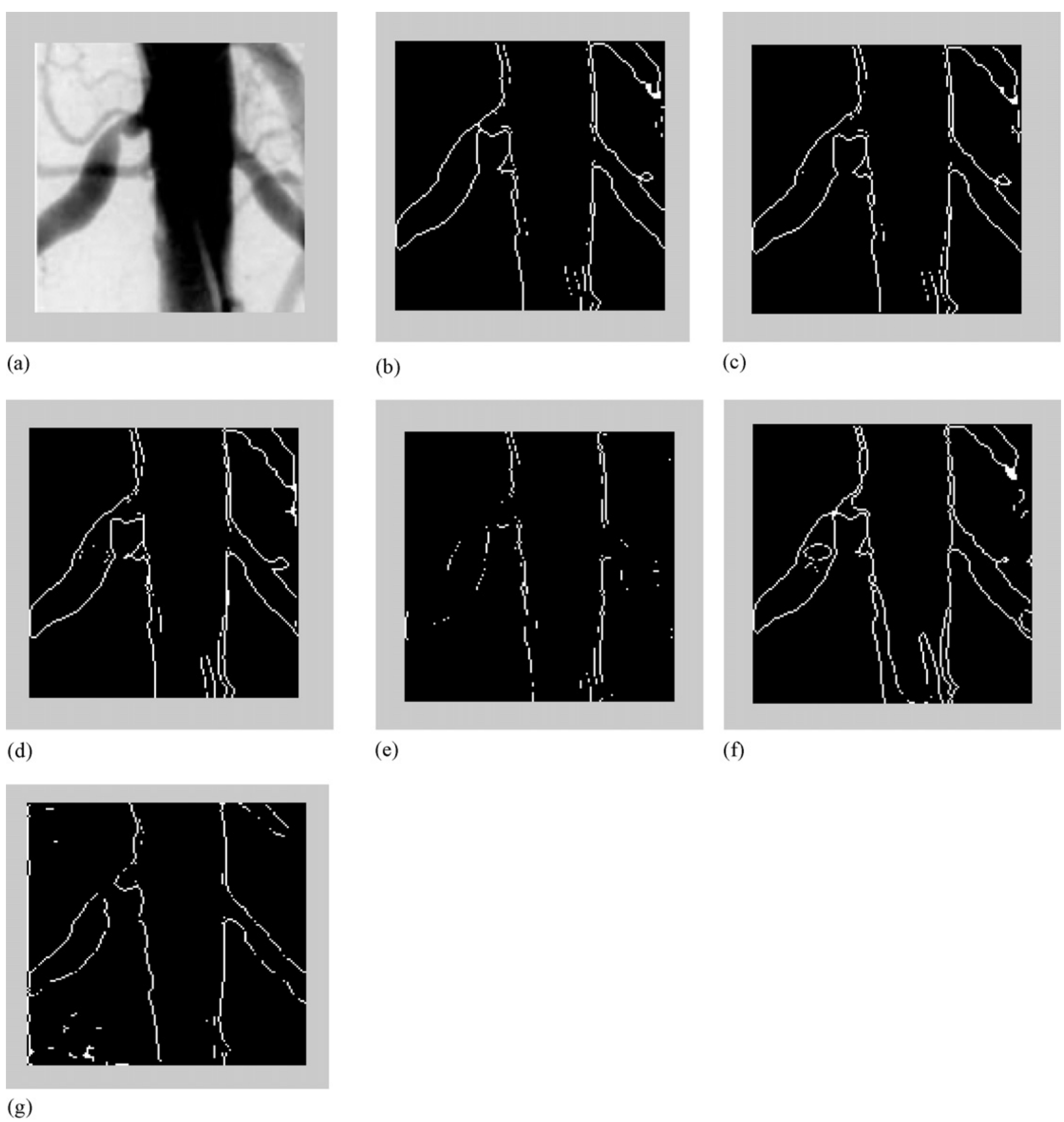

Fig. 6. 'Aorta' image. (a) 'Aorta' image, (b) $c_t = 0.2$, th = 0.25, (c) $c_t = 0.25$, th = 0.25, (d) $c_t = 0.35$, th = 0.30, (e) $c_t = 0.4$, th = 0.45, (f) $c_t = 0.3$, th = 0.25 and (g) $a = 0.25$, $b = 0.7$, th = 0.2, $c_t = 0.3$.

window (same size as that of template) and choose the minimum IFD value.

- *Step 4*. Choose the maximum of all the 16 (total no. of templates) minimum intuitionistic fuzzy divergence values.
- *Step 5*. Position the maximum value at the point where the template was centered over the image.
- *Step 6*. For all the pixel positions (considering the border pixels by taking the mirror values of the image), the max–min value has been selected and positioned.

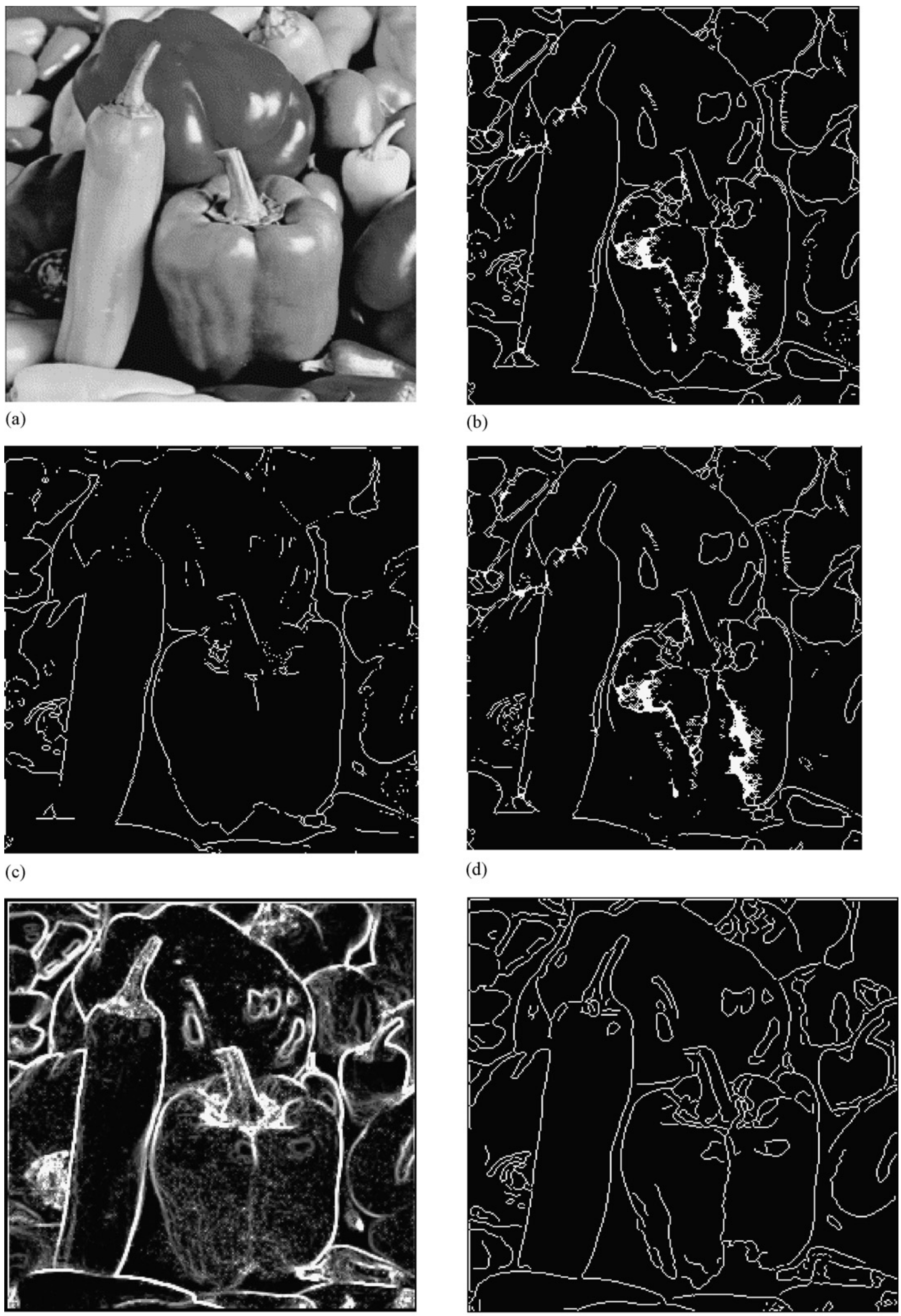

Fig. 7. 'Pepper' image. (a) Image, (b) th = 0.17, $c_t = 0.3$, $c_i = 0.1$, (c) th = 0.28, $c_t = 0.4$, $c_i = 0.1$, (d) th = 0.13, $c_t = 0.2$, $c_i = 0.1$, (e) Berceiki and (f) Canny, th = 0.15, S.D. = 1.

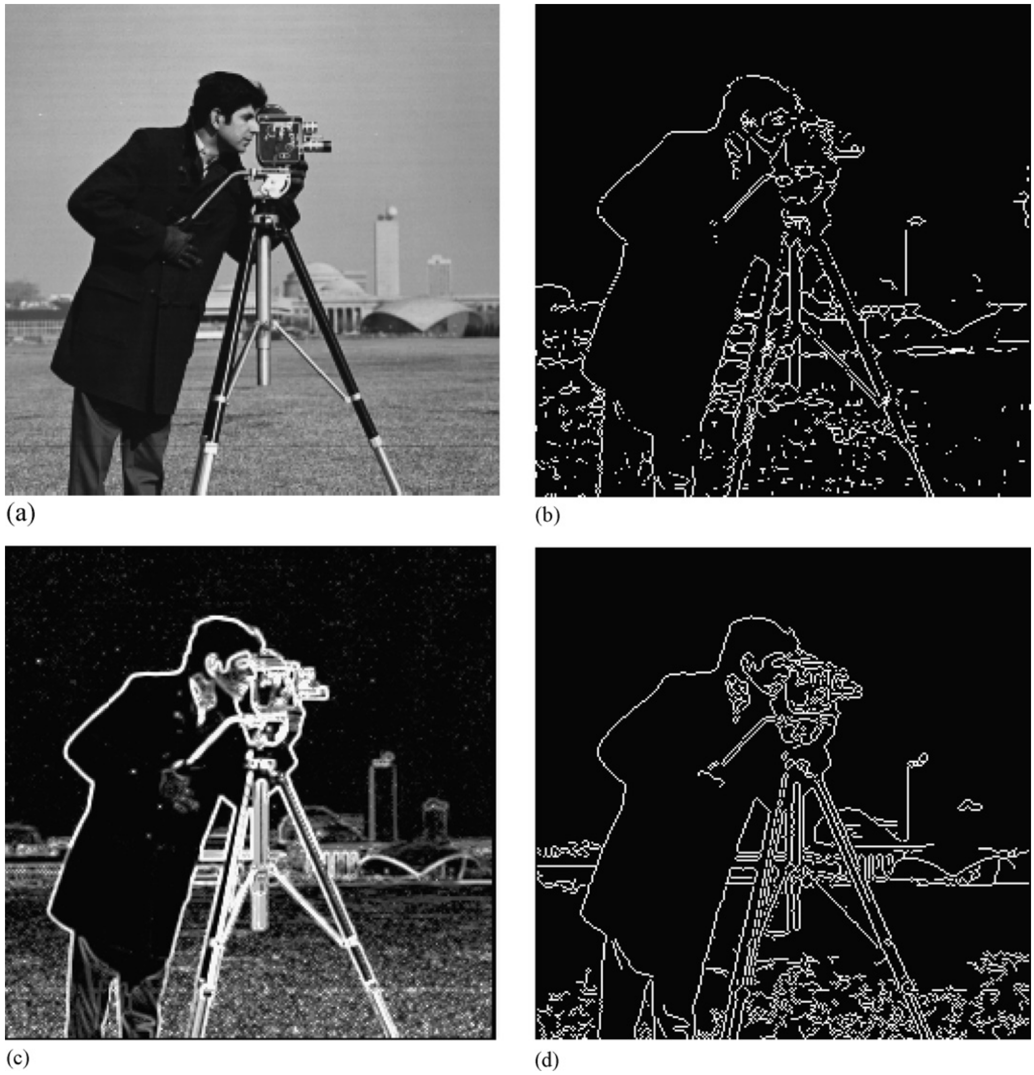

Fig. 8. 'Cameraman' image. (a) Image, (b) $c_t = 0.4$, $c_i = 0.2$, th = 0.19, (c) Berceiki and (d) Canny, th = 0.2, S.D. = 0.2.

- *Step 7*. A new intuitionistic divergence matrix has been formed.
- *Step 8*. Threshold the intuitionistic divergence matrix and thin.
- *Step 9*. An edge-detected image is obtained.

6. Results and discussion

The values of a and b in the edge templates have been chosen here by trial and error. Results with $a = 0.25$, $b = 0.7$ are shown for each images, but the edge-detected results are not at all good as shown in the Figs. 3–6. The best combination is $a = 0.3$, $b = 0.8$ and with these values, the edge-detected results are found better. It is further observed that, on increasing the number of templates, there is no remarkable change in the edge detection results and that when decreasing the number of templates, many edges are found to be missing. With 16 templates edge-detected results are found good. The value of r, the number of elements in the square template, in (8) is $3^2 = 9$.

Results with different values of hesitation constant c or consequently different values of hesitation degree have been shown in Figs. 3–8. The results using our proposed method have been compared with Canny's method [6] and Becerikli and Karan's method [4] in Figs. 7 and 8. Normally medical images or natural images are not well illuminated and therefore, the edge detection technique cannot extract the edges as a whole; rather fragments of edges are extracted. For analyzing purpose, ground truth edge-detected image is needed. There are many edge detection operators. However, obtaining a ground-truth edge-detected image is very difficult. The only way to compare the results is by visual inspection.

Hesitation constant c has been introduced to the edge templates and also to the images. For the edge templates and image, the hesitation constants are marked as c_t and c_i, respectively, and these are varied to obtain a good edge-detected image. In the experiment, c_i is kept fixed for each image and c_t is varied. In each of the images in Figs. 3–8, c_t and th denote, respectively, the edge template hesitation constant and the threshold value.

(a) '*Brain*' *image*. CT scanned 'Brain' image of size 316 × 316 containing the gray and white matter is shown in Fig. 3(a). The IFD matrix as mentioned above has been thresholded and then thinned. The results with varying values of c_t, are shown. The hesitation constant for image is $c_i = 0.05$. The values of c_t are mentioned at in each figure. The results with $c_t = 0.3$ and 0.4 have been found better, as shown in Fig. 3(e) and (f), where the outer edges are clearly detected. The gray matter is clearly extracted using our proposed method, with appropriate c_t, implies that the edge detection is completely dependent on the choice of c_t. Fig. 3(h) is the result with different values of $a = 0.25$, $b = 0.7$, where the edges are not properly detected.

(b) '*Rice*' *image*. Rice image of size 120 × 120 is shown in Fig. 4(a). The hesitation constant for image is $c_i = 0.05$. Edge-detected results with varying values of c_t are shown. Better edge-detected results are obtained in Fig. 4(d) and (e) with c_t less than 0.3, where the seeds are properly extracted. Fig. 4(f) is the result with $a = 0.25$, $b = 0.7$.

(c) '*Lung*' *image* of size 189 × 189 shown in Fig. 5(a). The hesitation constant for image is $c_i = 0.05$. Edge-detected results with varying values of c_t are shown. With $c_t = 0.3$ and above, in Fig. 5(b)–(d) are giving a better result where the outer edges are clearly detected and no false edges are shown. Fig. 5(f) shows, the result with different values of $a = 0.25$, $b = 0.7$ where the edges are not properly detected.

(d) '*Aorta*' *image* is of size 290 × 290 shown in Fig. 6(a). The hesitation constant for image is $c_i = 0.05$. Results with varying values of c_t are shown in Fig. 6(b)–(f). But the result with $c_t = 0.3$ in Fig. 6(f) is better where the main artery and also other veins are detected. Fig. 6(g) is the result with $a = 0.25$, $b = 0.7$ where the edges are not properly detected.

For comparison with other methods, two results are shown. As intuitionistic fuzzy set in edge detection is a new attempt and the novelty lies at this point, so comparison has been done with only crisp and fuzzy methods. The results are found better.

(e) '*Pepper*' *image* of size 215 × 290 is shown in Fig. 7(a). The results have been compared with a Canny's method and Becerikli's method. The hesitation constant for image is $c_i = 0.1$. Results with different values of hesitation constants, c_t, are shown. Better result is obtained in Fig. 7(b) with $c_t = 0.3$, where all the edges are satisfactorily detected. Fig. 7(e) is the result using Becerikli's method. Fig. 7(f) is the result using Canny's method.

(f) '*Cameraman*' *image* of size 256 × 256 is shown in Fig. 8(a). The hesitation constant for image is $c_i = 0.2$. Fig. 8(b) is the result using proposed method where $c_t = 0.4$. Fig. 8(c) is the result using Becerikli's method. Fig. 8(d) is the result using Canny's method.

In our opinion, the results are found better due to the use of intuitionistic fuzzy set. It takes into account the uncertainty in the assignment of the membership degrees. The membership degree is set with the change in the hesitation degree and so the edge-detected results also vary with it. Thus with the change in hesitation degree, good edge-detected image is obtained.

7. Conclusion and future work

In this paper, the novelty lies in the use of intuitionistic fuzzy set theory in image edge detection. Also, a new distance measure called intuitionistic fuzzy divergence has been proposed. This measure has been applied on images for edge detection. Experimental studies reveal that, for edge detection the result is completely dependant on the selection of hesitation constant and thereby be the hesitation degree (also called the intuitionistic fuzzy index). The proposed method detects the dominant edges clearly, while removing the unwanted edges. In our opinion, as the uncertainty has been considered in the assignment of the membership degrees due to the intuitionistic fuzzy set, the results are found better. Future research will be to obtain much better result taking into account the uncertainty.

References

[1] K.T. Atanassov, Intuitionistic Fuzzy Sets, Theory, and Applications, Series in Fuzziness and Soft Computing, Phisica-Verlag, 1999.
[2] K.T. Atanassov, Intuitionistic fuzzy set, Fuzzy Sets Syst. (1986) 87–97.
[3] K.T. Atanassov, S. Stoeva, Intuitionistic fuzzy set, in: Polish Symposium on Interval and Fuzzy Mathematics, Poznan, (1993), pp. 23–26.
[4] Y. Becerikli, T.M. Karan, A new fuzzy approach to edge detection, Lecture Notes Comput. Sci. (LNCS) 3512 (2005) 943–951.
[5] H. Bustince, P. Burillo, Vague sets are intuitionistic fuzzy sets, Fuzzy Sets Syst. 79 (1996) 403–405.
[6] J. Canny, Computational approach to edge detection, IEEE Trans. Pattern Anal. Mach. Intell. 8 (6) (1986) 679–698.
[7] T. Chaira, Image Segmentation and Color Retrieval—A Fuzzy and Intuitionistic Fuzzy Set Theoretic Approach, PhD Thesis, Indian Institute of Technology, Kharagpur, India, 2004.
[8] T. Chaira, A.K. Ray, Segmentation using fuzzy divergence, Pattern Recog. Lett. 24 (12) (2003) 1837–1844.
[9] J. Fan, W. Xie, Distance measure and induced fuzzy entropy, Fuzzy Sets Syst. 104 (1999) 305–314.
[10] P. Grzegorzewski, Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on Hausdorff metric, Fuzzy Sets Syst. 148 (2004) 319–328.
[11] J. Gómez Lopera, N. Ilhami, P. Luque Escamilla, J. Martínez Aroza, R. Román Roldán, Improved entropic edge detection, in: Proceedings of the 10th international Conference of Image Analysis and Processing, Venice, Italy, (1999), pp. 180–184.
[12] H.L. Kenneth, N. Ho, Ohnishi, FEDGE—fuzzy edge detection by fuzzy categorization and classification of edges, Fuzzy Logic Artif. Intell. (1995) 182–196.
[13] W. Guo-Jun, H. Ying-Yu, Intuitionistic fuzzy sets and L-fuzzy sets, Fuzzy Sets Syst. 110 (2000) 271–274.
[14] D. Li, C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognition, Pattern Recog. Lett. 23 (1–3) (2002) 221–225.
[15] R. Sambuc, Fonctions phi floues, Application a l' Aide au diagnostic en pathologie thyroidienne, PhD thesis, University of Marsielle, France, 1975.
[16] E. Szmidt, J. Kacprzyk, Entropy for intuitionistic fuzzy set, Fuzzy Sets Syst. 118 (2001) 467–477.
[17] E. Szmidt, J. Kacprzyk, Distance between intuitionistic fuzzy set, Fuzzy Sets Syst. 114 (2000) 505–518.
[18] T.W. Tao, W.E. Thomson, A fuzzy IF-THEN approach to edge detection, in: Proceedings of the Second IEEE International Conference on Fuzzy Systems, 1993, pp. 1356–1360.
[19] L.A. Zadeh, Concept of a linguistic variable and its application to approximate reasoning, I, II, III, Informat. Sci. 8 (1985), pp. 199–249, 301–357, 43–80.