Lemma 4.1.6

Clexma

February 26, 2019

Lemma 1 (4.1.6) *A flow f is a s-t path flow iff f satisfies the following conditions:*

- *If $s = t$ then*
  1. $\Sigma_{w \in out(v)} f(v, w) = \Sigma_{w \in in(v)} f(w, v)$ *for all* v
  2. $F(f)$ *is a s-t support*
- *If $s \neq t$ then*
  1. $\Sigma_{w \in out(v)} f(v, w) = \Sigma_{w \in out(v)} f(w, v)$ *for all* $v \in V - \{s, t\}$
     $\Sigma_{w \in out(s)} f(s, w) = \Sigma_{w \in out(s)} f(w, s) + 1$
     $\Sigma_{w \in out(t)} = \Sigma_{w \in out(t)} f(w, t) - 1$
  2. *F(f) is connected.*

Proof 0.1 *Prove by induction on* $n = \Sigma_{e \in E} f(e)$

- $\Rightarrow$: *By definition direction of path flow the proof is obvious.*
- $\Leftarrow$:
  1. $s = t, F(f) isconnected$*, we have the conclusion that f is a s-t path flow.*
     - *case 1 : there is a self loop from s to s.*
     - *case 2 : there is no self loop from s to s. In this case we delete an edge from v to s such that $v \neq s$.*
       * *subcase 2.1 the resulting graph is still connectedl let the new flow be f'. By the induction hypothesis f' is a path flow from s to v.*
       * *subcase 2.2 the resulting graph is not connected, in this case , there are exactly two connected components such in the resulting graph, we call them C_1 and C_2 and assume they contain s and v respectively.*
         · *sub-subcase 2.2.1: C_2 contains no edges.*
         · *sub-subcase 2.2.2: C_1 and C_2 both contain at least one edge.*