On Multiphase-Linear Ranking Functions

Xie Li

May 7, 2020

Contributions

- Equivalence of different classes of ranking function.
- Algorithms for converting between ranking functions.
- Complete solution for ranking functions on integer.
- Depth bound and iteration bound for MΦRF.

Single Path Linear Constraint Loop

Example

$$\texttt{while}\ (x \geq -z)\ \texttt{do}\ x' = x + y,\ y' = y + z,\ z' = z - 1$$

$$\texttt{while}\ (x_2 - x_1 \leq 0,\ x_1 + x_2 \geq 1)\ \texttt{do}\ x_2' = x_2 - 2x_1 + 1,\ x_1' = x_1$$

Definition (SLC)

$$\mathit{while}\ (B\mathbf{x} \leq \mathbf{b})\ \mathit{do}\ A\begin{pmatrix}\mathbf{x}\\ \mathbf{x}'\end{pmatrix} \leq \mathbf{c}$$

$$A'' = \begin{pmatrix}B & 0\\ A\end{pmatrix} \qquad \mathbf{c}'' = \begin{pmatrix}\mathbf{b}\\ \mathbf{c}\end{pmatrix}$$

$$A''\mathbf{x}'' \leq \mathbf{c}''$$

Ranking Functions

Definition (Linear Ranking Function(LRF))

$f(x_1, \ldots, x_n) = a_1 x_1 + \ldots a_n x_n + a_0$, such that

- $f(\mathbf{x}) \geq 0$ for any $\mathbf{x}$ satisfies the loop constraints.
- $f(\mathbf{x}) - f(\mathbf{x}') \geq 1$ for any transition from $\mathbf{x}$ to $\mathbf{x}'$.

Example

$$\texttt{while } (x - 1 > 0)\texttt{do } x' = x - 1$$

Its LRF: $f(x) = x - 1$

Example: Multiphase Ranking Function

Problem: LRF is not strong enough for all loops.

Example

$$\texttt{while } (x > -z)\texttt{do } x' = x + y, y' = y + z, z = z - 1$$