# Learning Nonlinear Loop Invariant with Gated Continuous Logic Networks

Jianan Yao et. al

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# History of the Research

## CLN2INV: Learning Loop Invariants with Continuous Logic Networks

In this paper, they propose the algorithm of learning loop invariants using CLN.
This paper generalize the model into GCLN, where "G" stands for gated.

# Overview

- Introduction to learning nonlinear invariant.
- Workflow of the algorithm.
- Detailed description of the theory and techniques.
- Experimental evaluation.

# Difficulties

- Large search space with high magnitude terms. e.g. terms like $x^2$ and $x^y$ grows exponentially.
- Limitted samples. Bounds on the number of loop iterations with integer variables.
- Distinguishing sufficient inequalities.

# Ways to Resolve

- **Learning with G-CLNs**, where the gated value can be used to turn on or off the terms. Combine dropout.(Example)
- **Fractional sampling**. Relax the semantic of the loop to continuous functions.
- **Piecewise Biased Quadratic Units(PBQU)**. A way to penalizes loose fits and converges to tight contraints on data.(Example)

# Background

## Definition (Loop Invariant Inference)

Given the precondition $P$ and postcondition $Q$. Finding an inductive invariant $I$ such that,

$$P \Longrightarrow I, \{I \wedge LC\} C \{I\}, I \wedge \neg LC \Longrightarrow Q$$

Loop invariants can be encoded in SMT. The data driven invariant inference is to find SMT formula $F$ s.t.

$$\forall x \in X, F(x) = True$$

# Making Loop Invariant Learnable

Using a differentiable logic: Basic Fuzzy Logic(BL). BL is a relaxation of FOL on continuous truth value on the interval $[0, 1]$.

- t-norms($\otimes$). Consistency: $t \otimes 1 = 1, t \otimes 0 = 0$, commutative and monotonic.
- t-conorms($\oplus$), operates as disjunctions for BL and derived from DeMorgan's law with $\neg t = 1 - t$

# Semantic Mapping

Principles:

1. Remain the meaning of the logic.
2. Continuous and smooth.
3. Increasing when unsat terms change to sat terms.

We use $\mathcal{S}$ as the mapping function and it is defined recursively. $\mathcal{S}(F) : X \to [0, 1]$.

$$\begin{aligned}
&\text{Conjunction:} && \mathcal{S}(F_1 \wedge F_2) \triangleq \mathcal{S}(F_1) \otimes \mathcal{S}(F_2) \\
&\text{Disjunction:} && \mathcal{S}(F_1 \vee F_2) \triangleq \mathcal{S}(F_1) \oplus \mathcal{S}(F_2) \\
&\text{Negation:} && \mathcal{S}(\neg F) \triangleq 1 - \mathcal{S}(F) \\
&\text{Greater Than:} && \mathcal{S}(x_1 > x_2) \triangleq \frac{1}{1 + e^{-B(x_1 - x_2 - \epsilon)}} \\
&\text{Greater or Equal to:} && \mathcal{S}(x_1 \geq x_2) \triangleq \frac{1}{1 + e^{-B(x_1 - x_2 + \epsilon)}}
\end{aligned}$$

# Example

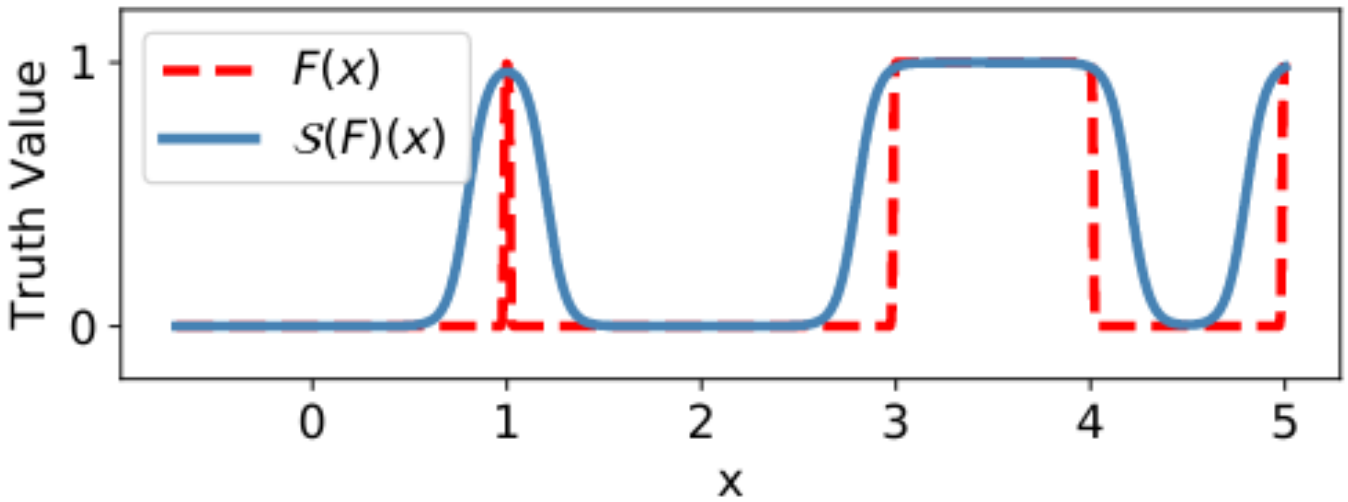
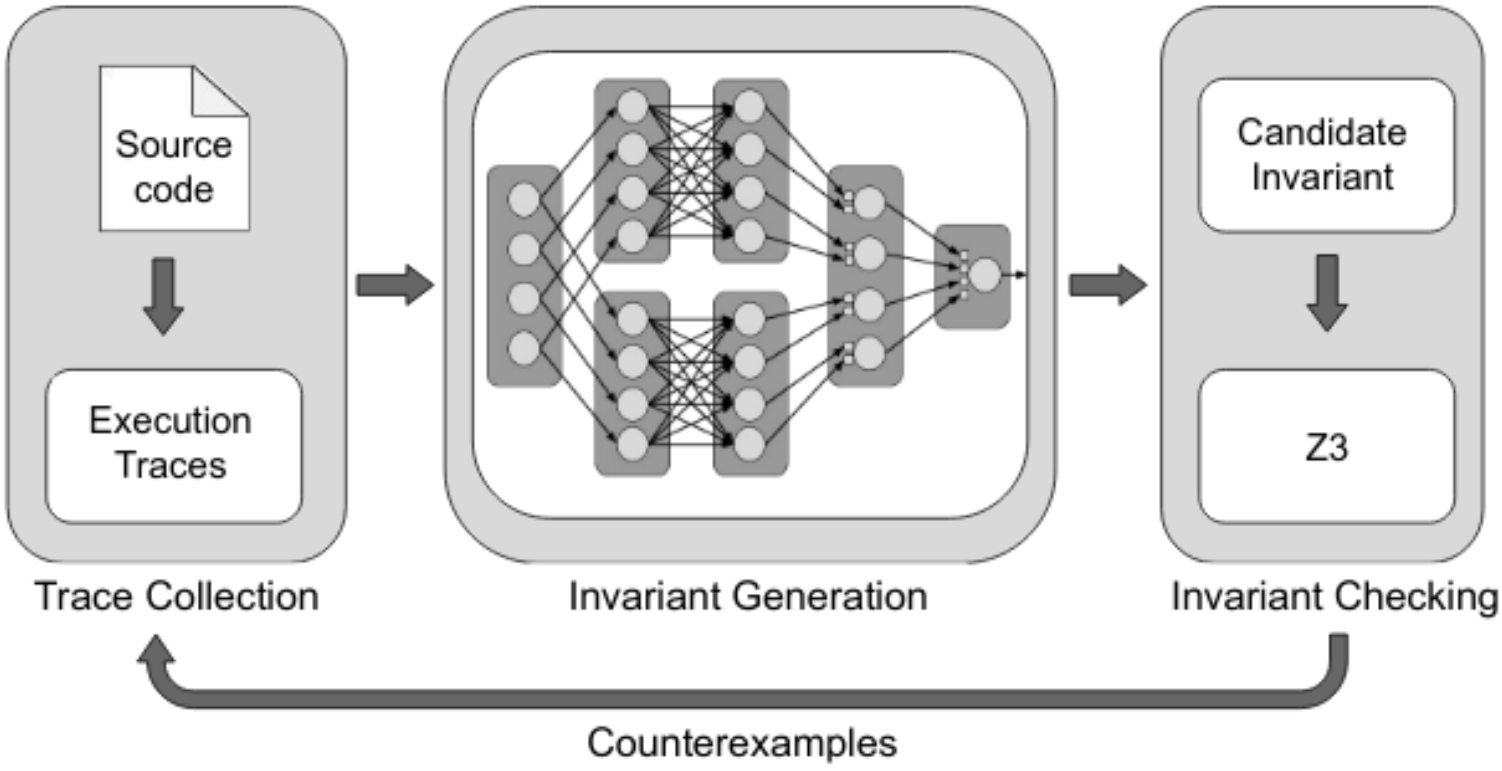


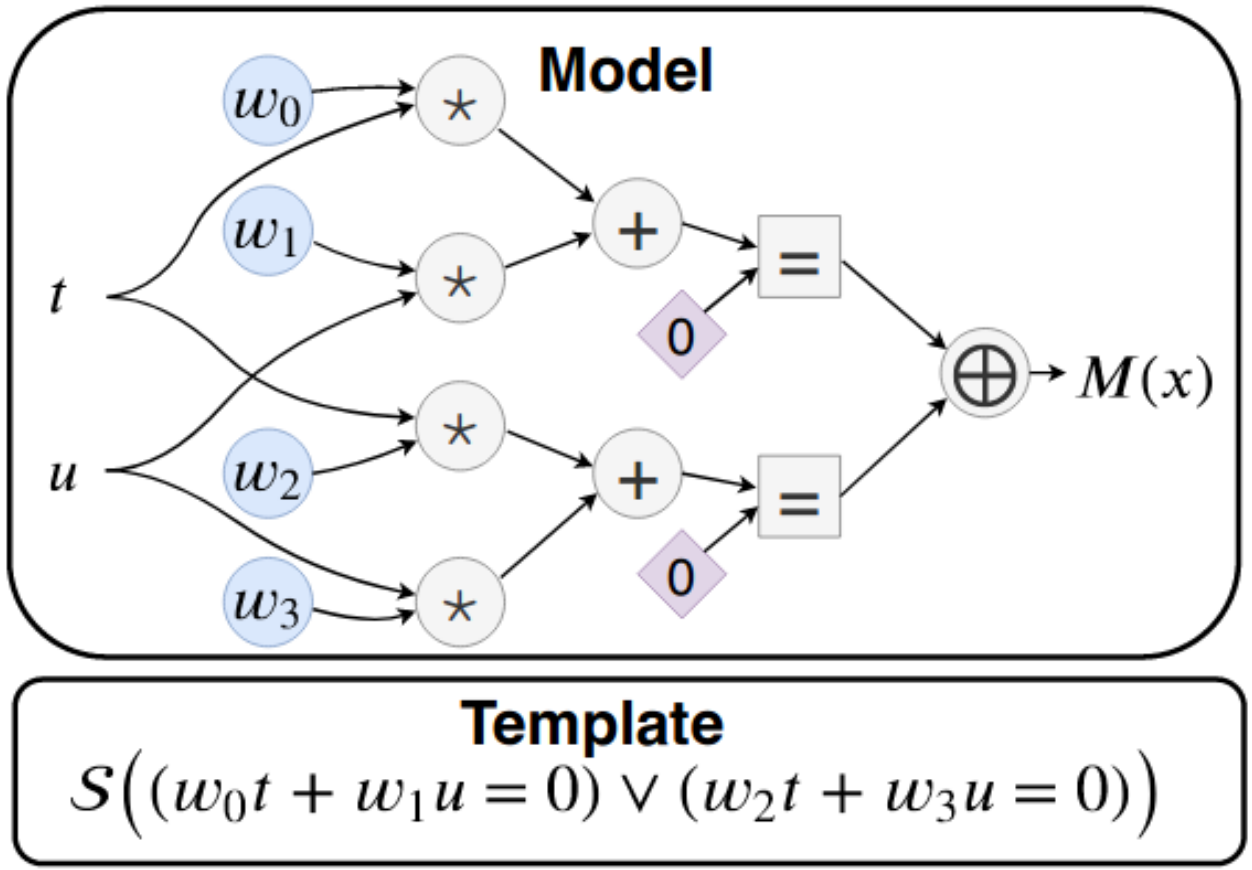

**Figure 2.** Plot of the formula $F(x) \triangleq (x = 1) \vee (x \geq 5) \vee (x \geq 2 \wedge x \leq 3)$ and its associated CLN $M(x)$.

# Workflow


Source code
Execution Traces
Trace Collection
Invariant Generation
Candidate Invariant
Z3
Invariant Checking
Counterexamples

# Continuous Logic Network

## Example



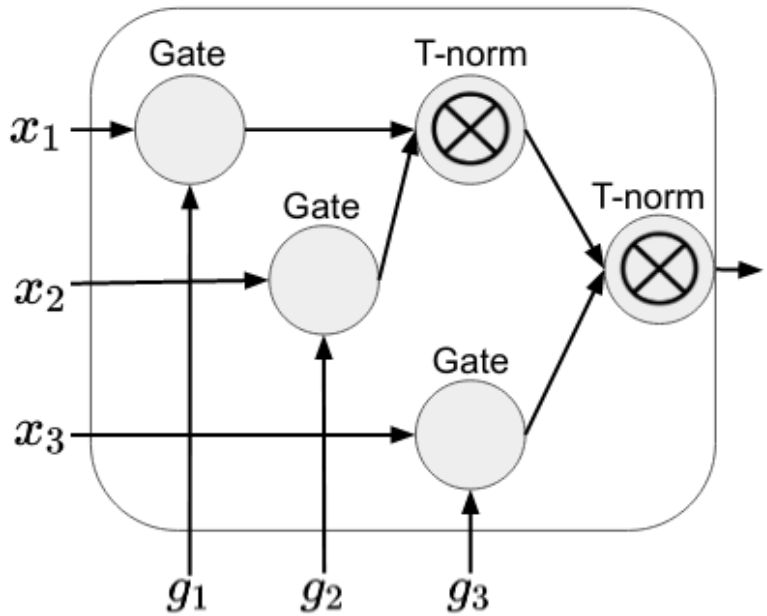
# Gated Continuous Logic Network

Previous invariant learning with CLN require a preset template. To let the model fit the data automatically instead of given a template ahead, we use gated CLN.

gated t-norms:

$$T_G(x, y; g_1, g_2) = (1 + g_1(x - 1)) \otimes (1 + g_2(y - 1))$$

Likewise for gated t-conorms.




Current learning parameters: $W, g_i$

# Learning Target

Minimize

$$\mathcal{L}(X; W, G) = \sum_{x \in X} (1 - \mathcal{M}(x; W, G)) + \lambda_1 \sum_{g_1 \in T_G} (1 - g_i) + \lambda_2 \sum_{g_i \in T'_G} g_i$$

where $\lambda_i$ are normalization parameters.

# Formula Extractions

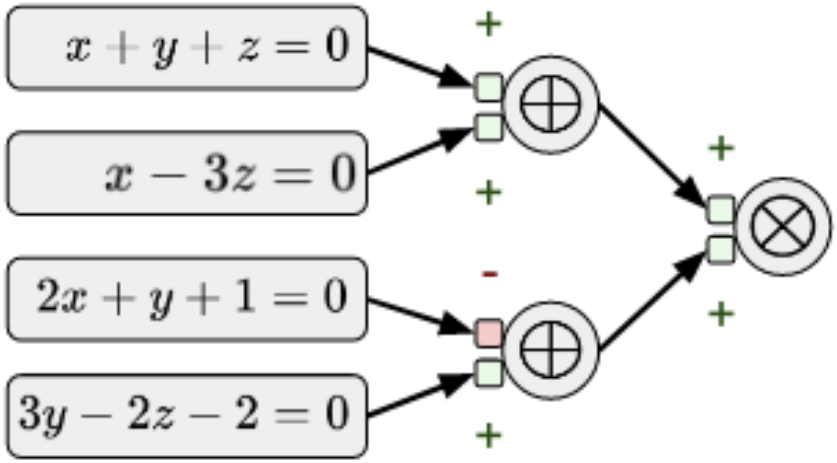



**Figure 6.** An instance of gated CLN. "+" means activated (g=1) and "-" means deactivated (g=0). The SMT formula learned is $(3y - 3z - 2 = 0) \wedge ((x - 3z = 0) \vee (x + y + z = 0))$.

# Formula Extractions

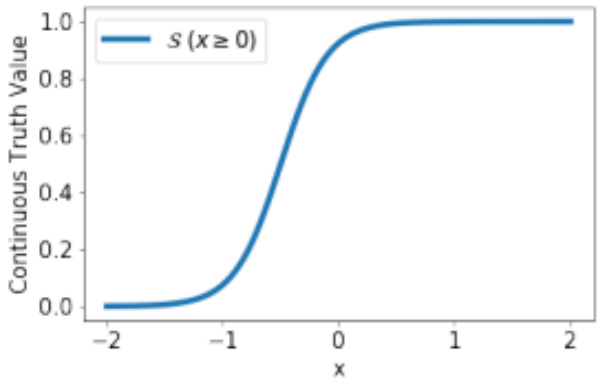
**Algorithm 1 Formula Extraction Algorithm.**

**Input:** A gated CLN model $\mathcal{M}$, with input nodes $\mathcal{X} = \{x_1, x_2, ..., x_n\}$ and output node $p$.
**Output:** An SMT formula $F$
**Procedure** ExtractFormula($\mathcal{M}$)

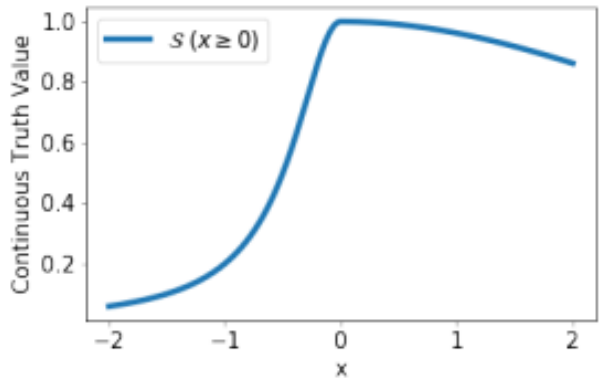
1: **if** $p = T_G(\mathcal{M}_1, ..., \mathcal{M}_n; g_1, ..., g_n)$ **then**
2: F := True
3: **for** $i := 1$ **to** $n$ **do**
4: **if** $g_i > 0.5$ **then**
5: $F := F \wedge ExtractFormula(\mathcal{M}_i)$
6: **else if** $p = T'_G(\mathcal{M}_1, ..., \mathcal{M}_n; g_1, ..., g_n)$ **then**
7: F := False
8: **for** $i := 1$ **to** $n$ **do**
9: **if** $g_i > 0.5$ **then**
10: $F := F \vee ExtractFormula(\mathcal{M}_i)$
11: **else if** $p = 1 - \mathcal{M}_1$ **then**
12: $F := \neg ExtractFormula(\mathcal{M}_1)$
13: **else**
14: $F := BuildAtomicFormula(\mathcal{M})$

# Piecewise Construction

$$\mathcal{S}(t \geq u) \triangleq \begin{cases} \frac{c_1^2}{(t-u)^2+c_1^2} & t < u \\ \frac{c_2^2}{(t-u)^2+c_2^2} & t \geq u \end{cases}$$




**(a)** Plot of $\mathcal{S}(x \geq 0)$ with the CLNs' sigmoid construction.




**(b)** Plot of $\mathcal{S}(x \geq 0)$ with our piecewise construction.

**Figure 7.** Comparison of the mapping $\mathcal{S}$ on $\geq$. The hyper-parameters are $B = 5$, $\epsilon = 0.5$, $c_1 = 0.5$, and $c_2 = 5$.

# Fractional Sampling

```
//pre: x = y = 0
//      /\ k >= 0
while (y < k) {
  y++;
  x += y * y * y;
}
//post: 4x == k^2
//    * (k + 1)^2
```

**(a)** The ps4 program in the benchmark.

| x | y | $y^2$ | $y^3$ | $y^4$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 9 | 2 | 4 | 8 | 16 |
| 36 | 3 | 9 | 27 | 81 |
| 100 | 4 | 16 | 64 | 256 |
| 225 | 5 | 25 | 125 | 625 |

**(b)** Training data generated without Fractional Sampling.

| x | y | $y^2$ | $y^3$ | $y^4$ | $x_0$ | $y_0$ | $y_0^2$ | $y_0^3$ | $y_0^4$ |
|---|---|---|---|---|---|---|---|---|---|
| -1 | -0.6 | 0.36 | -0.22 | 0.13 | -1 | -0.6 | 0.36 | -0.22 | 0.13 |
| -0.9 | 0.4 | 0.16 | 0.06 | 0.03 | -1 | -0.6 | 0.36 | -0.22 | 0.13 |
| 1.8 | 1.4 | 1.96 | 2.74 | 3.84 | -1 | -0.6 | 0.36 | -0.22 | 0.13 |
| 0 | -1.2 | 1.44 | -1.73 | 2.07 | 0 | -1.2 | 1.44 | -1.73 | 2.07 |
| 0 | -0.2 | 0.04 | -0.01 | 0.00 | 0 | -1.2 | 1.44 | -1.73 | 2.07 |
| 0.5 | 0.8 | 0.64 | 0.52 | 0.41 | 0 | -1.2 | 1.44 | -1.73 | 2.07 |

**(c)** Training data generated with fractional sampling.

# Optimization

**Data Normalization:**
Large inputs cause instability and prevent the CLN model from converging. In the implementation, the paper requires the L2-norm equals a set value $l$.

**Weight Regularization**
To avoid trivial invariant, we require the $L^p$-norm of the weight vector equals to a nonzero value.

**Term Dropout:**
Large number of terms poses diffuculties for learning. Random dropout of terms.

# Experimental Evaluation: Nonlinear

| Problem | Degree | # Vars | PIE | NumInv | G-CLN |
|---|---|---|---|---|---|
| divbin | 2 | 5 | - | ✓ | ✓ |
| cohendiv | 2 | 6 | - | ✓ | ✓ |
| mannadiv | 2 | 5 | ✗ | ✓ | ✓ |
| hard | 2 | 6 | - | ✓ | ✓ |
| sqrt1 | 2 | 4 | - | ✓ | ✓ |
| dijkstra | 2 | 5 | - | ✓ | ✓ |
| cohencu | 3 | 5 | - | ✓ | ✓ |
| egcd | 2 | 8 | - | ✓ | ✓ |
| egcd2 | 2 | 11 | - | ✗ | ✓ |
| egcd3 | 2 | 13 | - | ✗ | ✓ |
| prodbin | 2 | 5 | - | ✓ | ✓ |
| prod4br | 3 | 6 | ✗ | ✓ | ✓ |
| fermat1 | 2 | 5 | - | ✓ | ✓ |
| fermat2 | 2 | 5 | - | ✓ | ✓ |
| freire1 | 2 | 3 | - | ✗ | ✓ |
| freire2 | 3 | 4 | - | ✗ | ✓ |
| knuth | 3 | 8 | - | ✓ | ✗ |
| lcm1 | 2 | 6 | - | ✓ | ✓ |
| lcm2 | 2 | 6 | ✗ | ✓ | ✓ |
| geo1 | 2 | 5 | ✗ | ✓ | ✓ |
| geo2 | 2 | 5 | ✗ | ✓ | ✓ |
| geo3 | 3 | 6 | ✗ | ✓ | ✓ |
| ps2 | 2 | 4 | ✗ | ✓ | ✓ |
| ps3 | 3 | 4 | ✗ | ✓ | ✓ |
| ps4 | 4 | 4 | ✗ | ✓ | ✓ |
| ps5 | 5 | 4 | - | ✓ | ✓ |
| ps6 | 6 | 4 | - | ✓ | ✓ |

# Experimental Evaluation: Nonlinear

| Problem | Data Norm. | Weight Reg. | Drop-out | Frac. Sampling | Full Method |
|---|---|---|---|---|---|
| divbin | ✗ | ✗ | ✓ | ✓ | ✓ |
| cohendiv | ✗ | ✗ | ✗ | ✓ | ✓ |
| mannadiv | ✗ | ✗ | ✓ | ✓ | ✓ |
| hard | ✗ | ✗ | ✓ | ✓ | ✓ |
| sqrt1 | ✗ | ✗ | ✗ | ✓ | ✓ |
| dijkstra | ✗ | ✗ | ✓ | ✓ | ✓ |
| cohencu | ✗ | ✗ | ✓ | ✓ | ✓ |
| egcd | ✗ | ✓ | ✗ | ✓ | ✓ |
| egcd2 | ✗ | ✓ | ✗ | ✓ | ✓ |
| egcd3 | ✗ | ✓ | ✗ | ✓ | ✓ |
| prodbin | ✗ | ✓ | ✓ | ✓ | ✓ |
| prod4br | ✗ | ✓ | ✓ | ✓ | ✓ |
| fermat1 | ✗ | ✓ | ✓ | ✓ | ✓ |
| fermat2 | ✗ | ✓ | ✓ | ✓ | ✓ |
| freire1 | ✗ | ✓ | ✓ | ✓ | ✓ |
| freire2 | ✗ | ✓ | ✗ | ✓ | ✓ |
| knuth | ✗ | ✗ | ✗ | ✗ | ✗ |
| lcm1 | ✗ | ✓ | ✓ | ✓ | ✓ |
| lcm2 | ✗ | ✓ | ✓ | ✓ | ✓ |
| geo1 | ✗ | ✓ | ✓ | ✓ | ✓ |
| geo2 | ✗ | ✓ | ✓ | ✓ | ✓ |
| geo3 | ✗ | ✓ | ✓ | ✓ | ✓ |
| ps2 | ✓ | ✗ | ✓ | ✓ | ✓ |
| ps3 | ✗ | ✗ | ✓ | ✓ | ✓ |
| ps4 | ✗ | ✗ | ✓ | ✓ | ✓ |
| ps5 | ✗ | ✗ | ✓ | ✗ | ✓ |
| ps6 | ✗ | ✗ | ✓ | ✗ | ✓ |

# Experimental Evaluation: Nonlinear

| Problem | Convergence Rate of CLN | Convergence Rate of G-CLN |
|---|---|---|
| Conj Eq | 75% | 95% |
| Disj Eq | 50% | 100% |
| Code2Inv 1 | 55% | 90% |
| Code2Inv 11 | 70% | 100% |
| ps2 | 70% | 100% |
| ps3 | 30% | 100% |

# Experimental Evaluation: Linear

Among 133 linear cases, except 9 unsovable cases, the tool solved all of the rest.