

On Multiphase-Linear Ranking Functions

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Contributions

- ▶ Equivalence of different classes of ranking function.
- ▶ Algorithms for converting between ranking functions.
- ▶ Complete solution for ranking functions on integer.
- ▶ Depth bound and iteration bound for $M\Phi RF$.

Single Path Linear Constraint Loop

Example

`while` $(x \geq -z)$ `do` $x' = x + y, y' = y + z, z' = z - 1$

`while` $(x_2 - x_1 \leq 0, x_1 + x_2 \geq 1)$ `do` $x'_2 = x_2 - 2x_1 + 1, x'_1 = x_1$

Definition (SLC)

while $(B\mathbf{x} \leq \mathbf{b})$ *do* $A \begin{pmatrix} \mathbf{x} \\ \mathbf{x}' \end{pmatrix} \leq \mathbf{c}$

$$A'' = \begin{pmatrix} B & 0 \\ A \end{pmatrix}$$

$$\mathbf{c}'' = \begin{pmatrix} \mathbf{b} \\ \mathbf{c} \end{pmatrix}$$

$$A''\mathbf{x}'' \leq \mathbf{c}''$$

Ranking Functions

Definition (Linear Ranking Function(LRF))

$f(x_1, \dots, x_n) = a_1x_1 + \dots a_nx_n + a_0$, such that

- ▶ $f(\mathbf{x}) \geq 0$ for any \mathbf{x} satisfies the loop constraints.
- ▶ $f(\mathbf{x}) - f(\mathbf{x}') \geq 1$ for any transition from \mathbf{x} to \mathbf{x}' .

Example

`while (x - 1 > 0)do x' = x - 1`

Its LRF: $f(x) = x - 1$

Example: Multiphase Ranking Function

Problem: LRF is not strong enough for all loops.

Example

`while ($x > -z$)do $x' = x + y, y' = y + z, z = z - 1$`

$f(x, y, z) = a_1x + a_2y + a_3z + b$

y cannot be used for non-existence of its lower bound.

$f(x, y, z) = x + z$

Problem?

Example: Multiphase Ranking Function