SVMRanker: A General Termination Aalysis Framework of Loop Programs via SVM Presentation & Demonstration

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Outline

1. Introduction to Ranking Functions
2. Overview of SVMRanker
3. Demonstration of the Tool in Command Line.

Single Path Linear Constraint Loop

Example

$$\texttt{while}\ (x \geq -z)\ \texttt{do}\ x' = x + y,\ y' = y + z,\ z' = z - 1$$

Let $B = (-1, 0, 1)$, $\mathbf{x} = (x, y, z)^T, \mathbf{b} = 0$.
Let $\mathbf{x}'' = (x, y, z, x', y', z')$,

$$A = \begin{bmatrix} 1 & 1 & 0-1 & 0 & 0 & \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \tag{1}$$

and $\mathbf{c} = (0, 0, 1)^T$

Definition (SLC)

$$\textit{while}\ (B\mathbf{x} \leq \mathbf{b})\ \textit{do}\ A \begin{pmatrix} \mathbf{x} \\ \mathbf{x}' \end{pmatrix} \leq \mathbf{c}$$

$$A'' = \begin{pmatrix} B & 0 \\ & A \end{pmatrix} \qquad \mathbf{c}'' = \begin{pmatrix} \mathbf{b} \\ \mathbf{c} \end{pmatrix}$$

$$A''\mathbf{x}'' \leq \mathbf{c}''$$

Ranking Functions

Definition (Single Linear Ranking Function(LRF))

$f(x_1, \ldots, x_n) = a_1 x_1 + \ldots a_n x_n + a_0$, such that

- $f(\mathbf{x}) \geq 0$ for any $\mathbf{x}$ satisfies the loop constraints.
- $f(\mathbf{x}) - f(\mathbf{x}') \geq 1$ for any transition from $\mathbf{x}$ to $\mathbf{x}'$.

Example

$$\texttt{while } (x - 1 > 0)\texttt{do } x' = x - 5$$

LRF: $f(x) = ax + b$.

- $ax + b \geq 0 \Rightarrow x \geq -\frac{b}{a} = 1$.
- $ax + b - (ax' + b) = a(x - x') = 5a \Rightarrow 5a \geq 1$

A possible SLRF: $f(x) = x - 1$

Limitation of SLRF

$$\texttt{while } (q > 0)\texttt{do } q' = q - y, y' = y + 1$$

Assume there is a LRF for this loop, say $f(q, y) = a_1 q + a_2 y + b$

$$f(q, y) - f(q', y') = a_1 y + a_2$$

Since y is not bounded, we cannot guarantee $\Delta f(q, y, q', y') > 0$
The loop does not has a SLRF, however, it does terminate.
We still wish to use q for ranking function, but to distinguish different "phase" of q base on either $y \geq 0$ or $y < 0$

Nested RF

Definition (Nested Ranking Function)

A tuple $\langle f_1, \ldots, f_d \rangle$ is a nested ranking function for T if the following requirements are satisfied for all $\mathbf{x}'' \in T$

$$\begin{aligned} f_d(\mathbf{x}) &\geq 0 \\ (\Delta f_i(\mathbf{x}'') - 1) + f_{i-1}(\mathbf{x}) &\geq 0 \qquad \text{for all } i = 1, \ldots, d. \end{aligned}$$

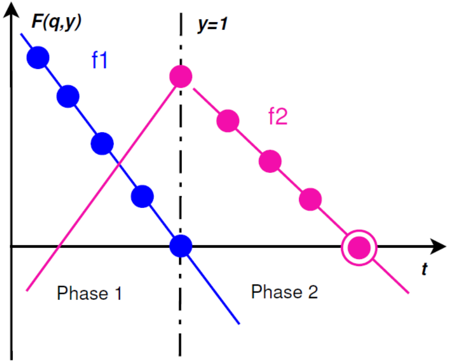
Let $f_0 = 0$.

Example: Nested RF

$$\begin{aligned} f_d(\mathbf{x}) &\geq 0 \\ (\Delta f_i(\mathbf{x}'') - 1) + f_{i-1}(\mathbf{x}) &\geq 0 \qquad \text{for all } i = 1, \dots, d. \end{aligned}$$

$$\texttt{while } (q > 0)\texttt{do } q' = q - y, y' = y + 1$$

- Above loop has Nested RF $\langle 1 - y, q + 1 \rangle$



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Linear Loop Program

Definition

A linear loop program $\mathtt{LOOP}(x, x')$ is a binary relation defined by a formula with the free variables x and x' of the form

$$\bigvee_{i \in I} \left(A_i \binom{x}{x'} \leq b_i \ \wedge \ C_i \binom{x}{x'} < d_i \right)$$

for some finite index set I.

Example

$\mathtt{while}\ (q > 0)\{\mathtt{if}\ (y > 0) : q' = q - y - 1; \mathtt{else}\ : q' = q + y - 1\}$

can be represented by

$$\begin{aligned} &(q > 0 \ \wedge \ y > 0 \ \wedge \ y' = y \ \wedge \ q' = q - y - 1) \\ \vee\, &(q > 0 \ \wedge \ y \leq 0 \ \wedge \ y' = y \ \wedge \ q' = q + y - 1) \end{aligned}$$

Limitation of Nested RF

Example

$$\texttt{while} \ \ (q > 0 \vee y > 0)$$

$$\{\texttt{if} \ \ (y > 0) : y' = y - 1; q' = q; \texttt{else} \ \ : q' = q - 1\}$$

This program does not have a nested ranking function for we require $f_d \geq 0$ but the guard is $q > 0 \vee y > 0$.
Howerver, this loop does terminate. Then we use a "multi-phase" ranking function $\langle y, q \rangle$ to prove the termination.

$$\begin{aligned} f_d(\mathbf{x}) &\geq 0 \\ (\Delta f_i(\mathbf{x}'') - 1) + f_{i-1}(\mathbf{x}) &\geq 0 \qquad \text{for all } i = 1, \ldots, d. \end{aligned}$$

Multiphase Ranking Function

Definition

Given a set of transitions $T \subseteq \mathbb{Q}^{2n}$, we say $\langle f_1, \ldots, f_d \rangle$ is a multiphase ranking function for T if for every $\mathbf{x}'' \in T$, there is an index $i \in [1, d]$, s.t.

$$\begin{aligned} \forall j \leq i \ . \ \Delta f_j(\mathbf{x}'') &\geq 1 \,, \\ f_i(\mathbf{x}) &\geq 0 \,, \\ \forall j < i \ . \quad f_j(\mathbf{x}) &\leq 0 \,. \end{aligned}$$

We say that $\mathbf{x}''$ is ranked by f_i(for the minimal).

Example: Multiphase Ranking Function

$$\texttt{while } (x > -z)\texttt{do } x' = x + y, y' = y + z, z = z - 1$$

Attempt to use a ranking function that has several phases:
$\langle z + 1, y + 1, x \rangle$

x	y	z	$z+1$	$y+1$	x
1	1	1	**2**	2	1
2	2	0	**1**	3	2
4	2	-1	**0**	3	4
6	1	-2	-1	**2**	6
7	-1	-3	-2	**0**	7
6	-4	-4	-3	-3	**6**
2	-8	-5	-4	-7	**2**
-6	-13	-6	-5	-12	-6

Example: Multiphase Ranking Function

$$\texttt{while}\ \ (x > -z)\texttt{do}\ \ x' = x + y, y' = y + z, z' = z - 1$$

$$\langle z + 1, y + 1, x \rangle$$

$\mathbf{x}''$ is ranked by f_k when $i = k$. In this example, $f_1(x, y, z) = z + 1$, $f_2(x, y, z) = y + 1$ and $f_3(x, y, z) = x$

$$\begin{aligned} \forall j \le i \ . \ \Delta f_j(\mathbf{x}'') &\ge 1\,, \\ f_i(\mathbf{x}) &\ge 0\,, \\ \forall j < i \ . \quad f_j(\mathbf{x}) &\le 0\,. \end{aligned}$$