Discussion

May 16, 2020

Limitation of Nested Ranking Functions

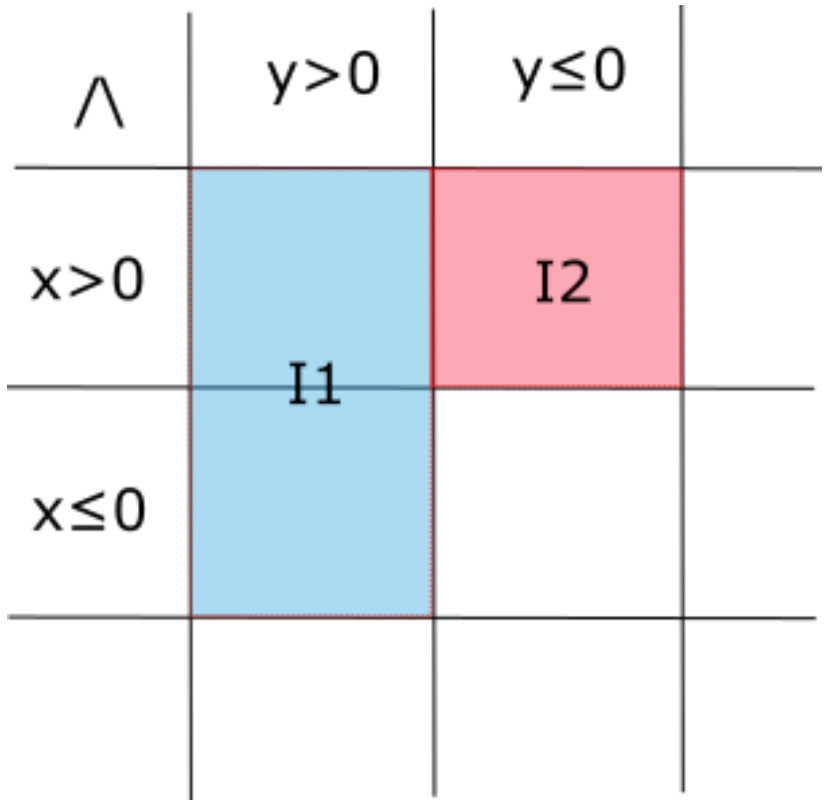
Example

$$\texttt{while } (x > 0 \vee y > 0)$$

$$\{\texttt{if } (y > 0) : y' = y - 1; x' = x; \texttt{else } : x' = x - 1; \}$$

Blocks

Idea: use terms of the loop guard to split the space into *blocks*.



Cyclic Dependency and Irregular Split

Example

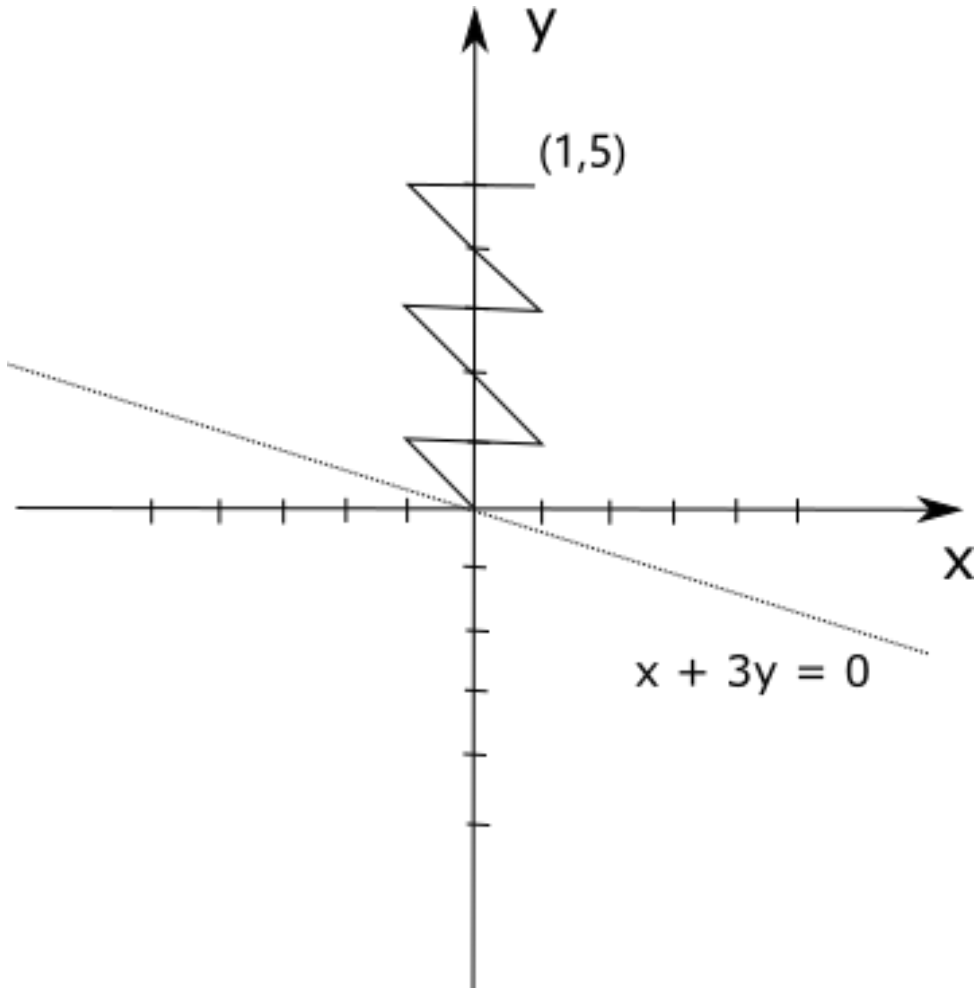
$$\texttt{while } (x > 0 \vee y > 0)$$

$$\{\texttt{if } (y > 0) : y' = y - 2; x' = x; \texttt{else } : y' = y + 1; x' = x - 1; \}$$

Start with $x = 1, y = 5$
A possible single linear ranking function $f(x, y) = x + 3y$.
This example demonstrates the blocks may be split by the ranking function.
i.e. in last example the regular split is just a coincidency.

Blocks

Use the terms of a loop



Multiphase Cyclic Dependency

Example

$$\texttt{while } (x > 0 \vee y > 0)$$

$$\{\texttt{if } (y > 0) : x' = x - 2z; y' = y; z' = z + 1;$$

$$\texttt{else } : x' = x + z; y' = y - 1; z' = z; \}$$

Start from $x = 1, y = 8, z = -2$.

Guess f_1 for Multiphase Ranking Function

Idea: although some ranking functions are not the borders of the blocks, we still wish to use the guard to guess the phases.
After a guess, conjunct the negation of the guard into the loop for nest guess.
The examble below shows that this idea is not feasible.

Example

$$\texttt{while } (x > 0 \vee y > 0)$$
$$\{\texttt{if } (y > 0) : y' = y - 1; x' = 5;$$
$$\texttt{else } : x' = x - 1; y = 5; \}$$

The loop is not terminating but...

Attempt to Merge the blocks

Idea: if we find the jump-in-and-out situations of the execution between two blocks, we consider it as one.
Problem: One cannot tell it is the problem of the inability of multiphase ranking function for this loop, or it is really that these two blocks should be merged.