Learning Nonlinear Loop Invariant with Gated Continuous Logic Networks

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History of the Research

CLN2INV: Learning Loop Invariants with Continuous Logic Networks

In this paper, they propose the algorithm of learning loop invariants using CLN.
This paper generalize the model into GCLN, where "G" stands for gated.
Tool is available at
https://github.com/gryan11/cln2inv

Overview

- Introduction to learning nonlinear invariant.
- Workflow of the algorithm.
- Detailed description of the theory and techniques.
- Experimental evaluation.

Background

Definition (Loop Invariant Inference)

Given the precondition P and postcondition Q. Finding an inductive invariant I such that,

$$P \implies I, \{I \land LC\}C\{I\}, I \land \neg LC \implies Q$$

Loop invariants can be encoded in SMT. The data driven invariant inference is to find SMT formula F s.t.

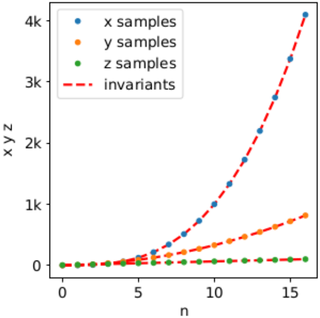
$$\forall x \in X, F(x) = True$$

Difficulties

- Large search space with high magnitude terms. e.g. terms like x^2 and x^y frow fast.
- Limitted samples. Bounds on the number of loop iterations with integer variables.
- Distinguishing sufficient inequalities.

Examples

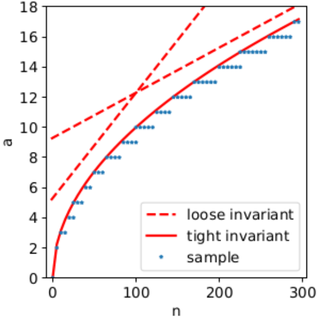
```
// pre: (a >= 0)
n=0; x=0;
y=1; z=6;
// compute cube:
while(n != a){
    n += 1;
    x += y;
    y += z;
    z += 6;
}
return x;
// post: x == a^3
```



(a) Loop for computing cubes that requires the invariant $(x = n^3) \wedge (y = 3n^2 + 3n + 1) \wedge (z = 6n + 6)$ to infer its postcondition $(x = a^3)$. A data-driven model must simultaneously learn a cubic constraint that changes by 1000s and a linear constraint that increments by 6.

Examples

```
// pre: (n >= 0)
a=0; s=1; t=1;
// compute sqrt:
while (s <= n) {
  a += 1;
  t += 2;
  s += t;
}
return a;
//post: a^2 <= n
//and n < (a+1)^2
```



(b) Loop for computing integer approximation to square root. The graph shows three valid inequality invariants, but only the tight quadratic inequality invariant ($n \geq a^2$) is sufficient to verify that the final value of a is between $\lfloor sqrt(n) \rfloor$ and $\lceil sqrt(n) \rceil$.

Ways to Resolve

- **Learning with G-CLNs**, where the gated value can be used to turn on or off the terms. Combine dropout.
- **Fractional sampling**. Relax the semantic of the loop to continuous functions.
- **Piecewise Biased Quadratic Units(PBQU)**. A way to penalizes loose fits and converges to tight contraints on data.

Making Loop Invariant Learnable

Principles:

1. Remain the meaning of the logic.
2. Continuous and smooth.
3. Increasing when unsat terms change to sat terms.

Using a differentiable logic: Basic Fuzzy Logic(BL). BL is a relaxation of FOL on continuous truth value on the interval $[0, 1]$.

- t-norms($\otimes$). Consistency: $t \otimes 1 = t, t \otimes 0 = 0$, commutative and monotonic.
- t-conorms($\oplus$), operates as disjunctions for BL and derived from DeMorgan's law with $\neg t = 1 - t$

Semantic Mapping

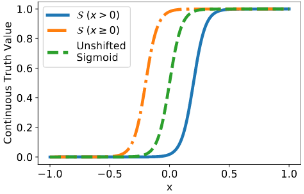
We use $\mathcal{S}$ as the mapping function and it is defined recursively.
$\mathcal{S}(F) : X \rightarrow [0, 1]$.

$$
\begin{array}{ll}
\text{Conjunction:} & \mathcal{S}(F_1 \wedge F_2) \triangleq \mathcal{S}(F_1) \otimes \mathcal{S}(F_2) \\
\text{Disjunction:} & \mathcal{S}(F_1 \vee F_2) \triangleq \mathcal{S}(F_1) \oplus \mathcal{S}(F_2) \\
\text{Negation:} & \mathcal{S}(\neg F) \triangleq 1 - \mathcal{S}(F)
\end{array}
$$

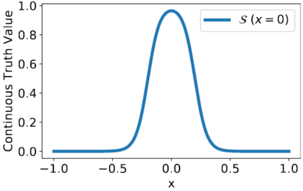
$$
\mathcal{S}(x_1 > x_2) \triangleq \frac{1}{1 + e^{-B(x_1 - x_2 - \epsilon)}}
$$

$$
\mathcal{S}(x_1 \geq x_2) \triangleq \frac{1}{1 + e^{-B(x_1 - x_2 + \epsilon)}}
$$

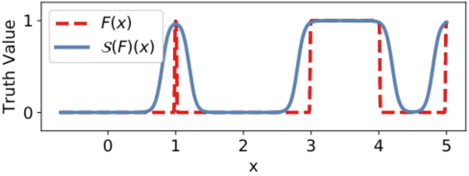
Example



(a) Plot of $\mathcal{S}(x \geq 0)$, $\mathcal{S}(x > 0)$ with sigmoid

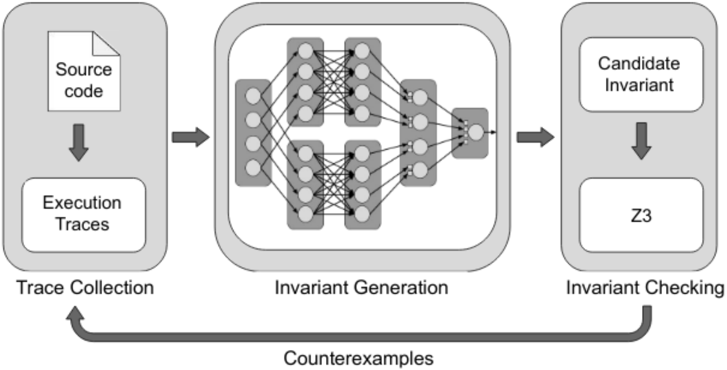


(b) Plot of $\mathcal{S}(x = 0)$ with product t-norm



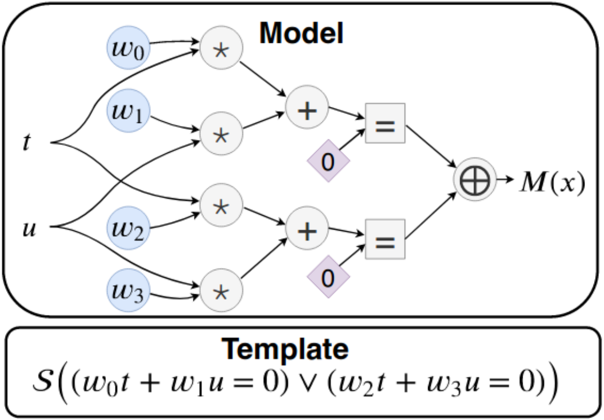
$$F(x) = (x = 1) \vee (x \geq 5) \vee (x \geq 3 \wedge x \leq 4)$$

Workflow


Source code
Execution Traces
Trace Collection
Invariant Generation
Candidate Invariant
Z3
Invariant Checking
Counterexamples

Continuous Logic Network

Example



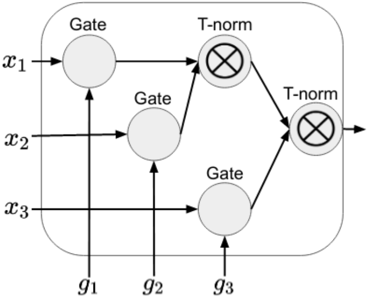
Gated Continuous Logic Network

Previous invariant learning with CLN require a preset template. To let the model fit the data automatically instead of given a template ahead, we use gated CLN.

gated t-norms:

$$T_G(x, y; g_1, g_2) = (1 + g_1(x - 1)) \otimes (1 + g_2(y - 1))$$

Likewise for gated t-conorms.



Current learning parameters: W, g_i

Learning Target

Minimize

$$\mathcal{L}(X; W, G) = \sum_{x \in X}(1 - \mathcal{M}(x; W, G)) + \lambda_1 \sum_{g_1 \in T_G}(1 - g_i) + \lambda_2 \sum_{g_i \in T'_G} g_i$$

where λ_i are normalization parameters.

Formula Extractions

Algorithm 1 Formula Extraction Algorithm.

Input: A gated CLN model $\mathcal{M}$, with input nodes $\mathcal{X} = \{x_1, x_2, ..., x_n\}$ and output node p.
Output: An SMT formula F
Procedure ExtractFormula($\mathcal{M}$)

if $p = T_G(\mathcal{M}_1, ..., \mathcal{M}_n; g_1, ..., g_n)$ **then**
  F := True
  for $i := 1$ **to** n **do**
    if $g_i > 0.5$ **then**
      $F := F \wedge ExtractFormula(\mathcal{M}_i)$
else if $p = T'_G(\mathcal{M}_1, ..., \mathcal{M}_n; g_1, ..., g_n)$ **then**
  F := False
  for $i := 1$ **to** n **do**
    if $g_i > 0.5$ **then**
      $F := F \vee ExtractFormula(\mathcal{M}_i)$
else if $p = 1 - \mathcal{M}_1$ **then**
  $F := \neg ExtractFormula(\mathcal{M}_1)$
else
  $F := BuildAtomicFormula(\mathcal{M})$

Formula Extractions

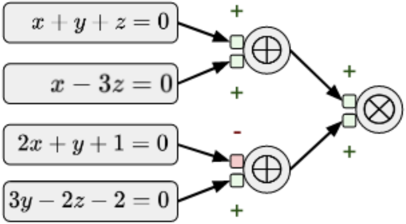
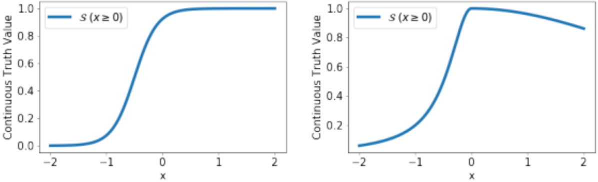


Figure 6. An instance of gated CLN. "+" means activated (g=1) and "-" means deactivated (g=0). The SMT formula learned is $(3y - 3z - 2 = 0) \wedge ((x - 3z = 0) \vee (x + y + z = 0))$.

Piecewise Construction

$$\mathcal{S}(t \geq u) \triangleq \begin{cases} \frac{c_1^2}{(t-u)^2+c_1^2} & t < u \\ \frac{c_2^2}{(t-u)^2+c_2^2} & t \geq u \end{cases}$$



(a) Plot of $\mathcal{S}(x \geq 0)$ with the CLNs' sigmoid construction.

(b) Plot of $\mathcal{S}(x \geq 0)$ with our piecewise construction.

Figure 7. Comparison of the mapping $\mathcal{S}$ on $\geq$. The hyper-parameters are $B = 5$, $\epsilon = 0.5$, $c_1 = 0.5$, and $c_2 = 5$.

Fractional Sampling

```
//pre: x = y = 0
//      /\ k >= 0
while (y < k) {
  y++;
  x += y * y * y;
}
//post: 4x == k^2
//     * (k + 1)^2
```

(a) The ps4 program in the benchmark.

x	y	y^2	y^3	y^4
0	0	0	0	0
1	1	1	1	1
9	2	4	8	16
36	3	9	27	81
100	4	16	64	256
225	5	25	125	625

(b) Training data generated without Fractional Sampling.

x	y	y^2	y^3	y^4	x_0	y_0	y_0^2	y_0^3	y_0^4
-1	-0.6	0.36	-0.22	0.13	-1	-0.6	0.36	-0.22	0.13
-0.9	0.4	0.16	0.06	0.03	-1	-0.6	0.36	-0.22	0.13
1.8	1.4	1.96	2.74	3.84	-1	-0.6	0.36	-0.22	0.13
0	-1.2	1.44	-1.73	2.07	0	-1.2	1.44	-1.73	2.07
0	-0.2	0.04	-0.01	0.00	0	-1.2	1.44	-1.73	2.07
0.5	0.8	0.64	0.52	0.41	0	-1.2	1.44	-1.73	2.07

(c) Training data generated with fractional sampling.

Optimization

Data Normalization:
Large inputs cause instability and prevent the CLN model from converging. In the implementation, the paper requires the L2-norm equals a set value l.
Weight Regularization
To avoid trivial invariant, we require the L^p-norm of the weight vector equals to a nonzero value.
Term Dropout:
Large number of terms poses diffuculties for learning. Random dropout of terms.

Experimental Evaluation: Nonlinear

Problem	Degree	# Vars	PIE	NumInv	G-CLN
divbin	2	5	-	✓	✓
cohendiv	2	6	-	✓	✓
mannadiv	2	5	✗	✓	✓
hard	2	6	-	✓	✓
sqrt1	2	4	-	✓	✓
dijkstra	2	5	-	✓	✓
cohencu	3	5	-	✓	✓
egcd	2	8	-	✓	✓
egcd2	2	11	-	✗	✓
egcd3	2	13	-	✗	✓
prodbin	2	5	-	✓	✓
prod4br	3	6	✗	✓	✓
fermat1	2	5	-	✓	✓
fermat2	2	5	-	✓	✓
freire1	2	3	-	✗	✓
freire2	3	4	-	✗	✓
knuth	3	8	-	✓	✗
lcm1	2	6	-	✓	✓
lcm2	2	6	✗	✓	✓
geo1	2	5	✗	✓	✓
geo2	2	5	✗	✓	✓
geo3	3	6	✗	✓	✓
ps2	2	4	✗	✓	✓
ps3	3	4	✗	✓	✓
ps4	4	4	✗	✓	✓
ps5	5	4	-	✓	✓
ps6	6	4	-	✓	✓

Experimental Evaluation: Ablation Study

Problem	Data Norm.	Weight Reg.	Drop-out	Frac. Sampling	Full Method
divbin	✗	✗	✓	✓	✓
cohendiv	✗	✗	✗	✓	✓
mannadiv	✗	✗	✓	✓	✓
hard	✗	✗	✓	✓	✓
sqrt1	✗	✗	✗	✓	✓
dijkstra	✗	✗	✓	✓	✓
cohencu	✗	✗	✓	✓	✓
egcd	✗	✓	✗	✓	✓
egcd2	✗	✓	✗	✓	✓
egcd3	✗	✓	✗	✓	✓
prodbin	✗	✓	✓	✓	✓
prod4br	✗	✓	✓	✓	✓
fermat1	✗	✓	✓	✓	✓
fermat2	✗	✓	✓	✓	✓
freire1	✗	✓	✓	✓	✓
freire2	✗	✓	✗	✓	✓
knuth	✗	✗	✗	✗	✗
lcm1	✗	✓	✓	✓	✓
lcm2	✗	✓	✓	✓	✓
geo1	✗	✓	✓	✓	✓
geo2	✗	✓	✓	✓	✓
geo3	✗	✓	✓	✓	✓
ps2	✓	✗	✓	✓	✓
ps3	✗	✗	✓	✓	✓
ps4	✗	✗	✓	✓	✓
ps5	✗	✗	✓	✗	✓
ps6	✗	✗	✓	✗	✓

Experimental Evaluation: Comparason to Previous CLN

Problem	Convergence Rate of CLN	Convergence Rate of G-CLN
Conj Eq	75%	95%
Disj Eq	50%	100%
Code2Inv 1	55%	90%
Code2Inv 11	70%	100%
ps2	70%	100%
ps3	30%	100%

Experimental Evaluation: Linear

Among 133 linear cases, except 9 unsovable cases, the tool solved all of the rest.