

Discussion

May 20, 2020

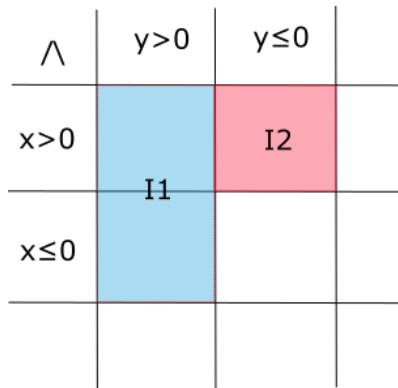
Limitation of Nested Ranking Functions

Example

```
while ( $x > 0 \vee y > 0$ )  
{if ( $y > 0$ ) :  $y' = y - 1$ ;  $x' = x$ ; else :  $x' = x - 1$ ;}
```

Blocks

Idea: use terms of the loop guard to split the space into *blocks*.



Cyclic Dependency and Irregular Split

Example

while $(x > 0 \vee y > 0)$

{if $(y > 0) : y' = y - 2; x' = x; \mathbf{else} : y' = y + 1; x' = x - 1; \mathbf{}}$

Start with $x = 1, y = 5$

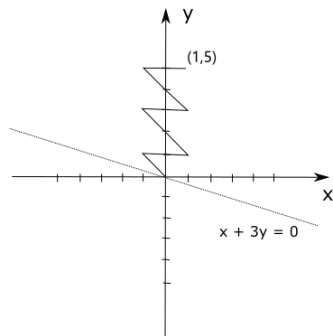
A possible single linear ranking function $f(x, y) = x + 3y$.

This example demonstrates the blocks may be split by the ranking function into halves.

i.e. the regular split of last example is just a coincidence.

Blocks

Use the terms of a loop



Multiphase Cyclic Dependency

Example

```
while ( $x > 0 \vee y > 0$ )  
{if ( $y > 0$ ) :  $x' = x - 2z$ ;  $y' = y$ ;  $z' = z + 1$ ;  
  else :  $x' = x + z$ ;  $y' = y - 1$ ;  $z' = z$ ; }
```

Start from $x = 1, y = 8, z = -2$.

Idea1: Guess f_1 for Multiphase Ranking Function

Idea: although some ranking functions are not the borders of the blocks, we still wish to use the guard to guess the phases. Put the guessed f_1 into SVMRanker to solve a ranking function for the first phase.

After a guess, conjunct the negation of the guard into the loop for next guess.

The example below shows that this idea is not feasible.

Example

```
while ( $x > 0 \vee y > 0$ )  
{if ( $y > 0$ ) :  $y' = y - 1$ ;  $x' = 5$ ;  
  else :  $x' = x - 1$ ;  $y = 5$ ; }
```

The loop is not terminating but if we using the method to conjunct the negation, we may wrongly conclude that the loop is terminating with a multiphase ranking function $\langle y, x \rangle$.

Idea2: Attempt to Merge the Blocks

Idea: if we find the jump-in-and-out situations of the execution between two blocks, we consider to merge them into one. Besides, we can unwind the loop for several times to split a big block into several small one.

\wedge	$y > 1$	$y > 0$	$y \leq 0$	
$x > 1$				
$x > 0$			I2	
	I1			
$x \leq 0$				

Intuition: in order to accurately find the ranking function for one phase, the worst case is that we need to unwind the loop for infinity times and consider every possible executing branch.

Idea2: Attempt to Merge the Blocks

Problem: One cannot tell it is the problem of the inability of multiphase ranking function for this loop, or it is really that these two blocks should be merged.

Current Thoughts

Consider some subclass of the loop with disjunction as [1] did, where they restrain there disjunction area to be a convex polyhedron in the sense of max-plus algebra.

[1] Using Dynamic Analysis to Generate Disjunctive Invariants

Max-Plus Polyhedron

We can express some transition space using convex polyhedron of max-plus algebra and