ME 565: Battery Systems and Control, University of Michigan, Winter 2024

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Homework 3

Assigned: Wednesday, January 31.
Due: Sunday, February 10 at 11:59pm Eastern Time.

Submission Instructions

Submit your assignment through Gradescope as a single PDF. During submission, Gradescope will ask you to assign page numbers to each problem. Present your solutions in a coherent, sequential flow. For questions requiring numerical results or explanations, highlight or put a box around your final answers. Include figures and code along with your final answers where applicable.

Learning Objectives

Problem 1 will help you test your ability to read and translate the battery equivalent circuit model into a set of differential equations that can be programmed and evaluated in Matlab Simulink. Problem 2 will walk you through the steps to perform parameter identification of the equivalent circuit resistance and capacitance values from experimental data. This is another important step in programming models for use in state estimation and battery management algorithms. The goal of Problem 3 is to practice implementing the ODEs from Problem 1 which describe the equivalent circuit battery model in Simulink. Finally, you will evaluate the parameter estimation strategy from Problem 2 using your own Simulink model with known parameters to see how well the estimation scheme works when the estimation model structure is correct.

Problem 1: OCV-R-RC Equations

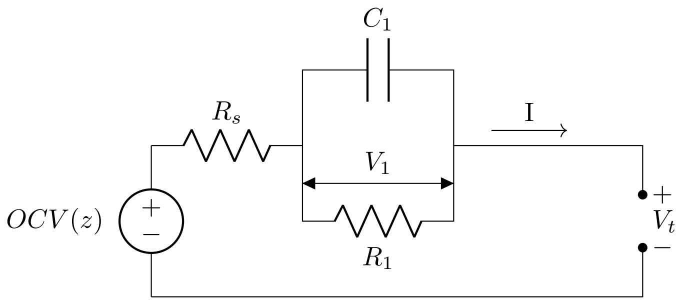

Figure 1: Equivalent OCV-R-RC model

(a) Assume R_s $(\Omega), R_1$ $(\Omega), C_1$ (F), and Q (Ah) capacity of the battery are known. Furthermore, the current is constant and equal to I (A). Write down the differential equations governing the evolution of z and V_1 in time. Assume $I > 0$ for discharging.

(b) Solve the above differential equations assuming the initial condition $[z(0), V_1(0)] = [z_0, V_0]$, and write out the resulting equations in the following form,

$$z(t) = f(t, z_0) \tag{1}$$

$$V_1(t) = g(t, V_0) \tag{2}$$

(c) Assuming that you have the function, $OCV(z(t))$. Write down the terminal voltage as a function of time, $V_t(t)$ with respect to the same initial condition as above.

Problem 2: OCV-R-RC Model Parameter Estimation

Last time in HW 2, we fitted the OCV and estimated the R parameter for the OCV-R model. For this problem you are going to identify the parameters of the OCV-R-RC model from "experimental" data, i.e. measured terminal voltage in discrete time ($V[k] = V(k\Delta t)$) using a least squares method. Download the Simulink model file `HW3_virtual_testbed` from Canvas, which has the same virtual battery from HW 2. This time in addition to being able to set the battery's initial temperature with the variable `T0a`, you are also able to set the initial state of charge with the variable `SOC_0`. Also, download the data files `polynomial.mat` which contains the polynomial fit of the OCV curve from Canvas. Note the order of coefficients in alpha is increasing $\alpha = [\alpha_0, \alpha_1, ..., \alpha_N]^T$, where $V_{ocv}(z) = OCV(z) = \alpha_0 + \alpha_1 z + \alpha_2 z^2 + ... + \alpha_N z^N$.

(a) Consider the OCV-R-RC equivalent circuit model in discrete-time:

$$\begin{aligned} z[k+1] &= z[k] - \frac{\Delta t}{Q} I[k] && (3) \\ V_1[k+1] &= \left(1 - \frac{\Delta t}{R_1 C_1}\right) V_1[k] + \frac{\Delta t}{C_1} I[k] && (4) \\ V[k] &= OCV(z[k]) - V_1[k] - R_s I[k], && (5) \end{aligned}$$

with $I > 0$ for discharging. Our goal is to identify the parameters R_s, R_1, and C_1. We will assume we know $Q = 5$Ah, and the open-circuit voltage function is described by a polynomial,

$$OCV(z) = \sum_{i=0}^{n} \alpha_i z^i. \tag{6}$$

where values for coefficients α_i and order of polynomial are provided in `polynomial`. Consider the virtual output signal

$$y[k] = OCV(z[k]) - V[k] = V_1[k] + R_s I[k], \tag{7}$$

which represents the voltage difference between the terminal voltage and the open circuit voltage. The value of y can be computed based on measurements and known data. Specifically, we can integrate current, scale the integral to SOC using $Q = 5$Ah, calculate the corresponding OCV based on (6), and then subtract the measured voltage.

As you have learned in class, an ARX (Auto-Regression with eXogenous inputs) formula,

$$y[k+1] + a_0 y[k] = b_1 I[k+1] + b_0 I[k], \tag{8}$$

can be derived for y through time shift and algebraic manipulations. A parametric form can then be obtained as

$$y[k] = \phi[k]\theta, \tag{9}$$

where

$$\begin{aligned} y[k] &= OCV(z[k]) - V[k] && (10) \\ \theta &= [b_1, b_0, a_0]^T && (11) \\ \phi[k] &= [I[k], I[k-1], -y[k-1]]. && (12) \end{aligned}$$

Write down expressions for a_0, b_1, b_0 in (8) in terms of the unknown parameters R_s, R_1, C_1.

(b) First, you need to implement a simple "SOC estimator" subsystem in the `HW3_virtual_testbed`. To do this, you will program a coulomb counter (a discrete time integrator), and initialize the integrator with the same variable `SOC_0`. Furthermore, use the variables `Delta_t` and `Tspan` to set the data acquisition rate to 1 s and the total simulation time to 600 s. **Confirm that the SOC estimator is working by initializing the SOC to 0.5, apply a charge current of $I = -2.5\ A$, and plot the evolution of SOC, voltage, current, and temperature for 10 minutes**. The final SOC should be 0.583. Note that you still need to set the workspace variable `T0a` to 25 and that the voltage protection logic is already implemented in the Simulink file.

(c) Recreate the input current profile shown in Fig. 2. This can be done using a series of `Step` blocks in Simulink or a pre-defined, 2-column array of [time, current] in the workspace brought in with the `From workspace` block. Set the initial SOC or $z(0)$ to 0.5 and start the simulation. Furthermore, use the variables `Delta_t` and `Tspan` to set the data acquisition rate to 1 s and the total simulation time to 1500 s. Use the given polynomial fit of the battery OCV function (with coefficients from `polynomial.mat`) and state-of-charge $z(k)$ to calculate the output signal $y(k)$. **Submit one figure with four subplots showing: input current, terminal voltage, calculated SOC, and calculated y versus time.** (Hint, Review the matlab documentation of the *polyval* function with regard to the order of the coefficients.)

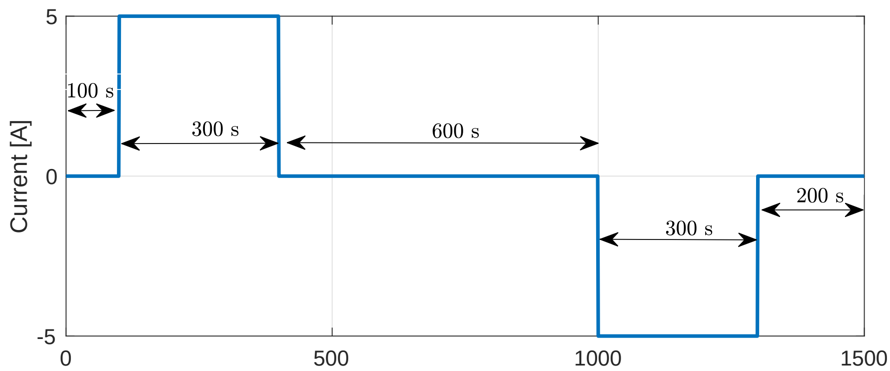

Figure 2: The input current profile.

(d) The parameters can be determined by solving the least square problem:

$$\hat{\theta} = \left(\Phi^T\Phi\right)^{-1}\left(\Phi^T Y\right) = \Phi\backslash Y, \tag{13}$$

where

$$\Phi = \begin{bmatrix} \phi(1) \\ \phi(2) \\ \cdots \\ \phi(N) \end{bmatrix}, \quad Y = \begin{bmatrix} y(1) \\ y(2) \\ \cdots \\ y(N) \end{bmatrix}. \tag{14}$$

Use the input profile in Fig. 2 to generate data at 0.25, 0.5, and 0.75 SOC. Use Matlab to compute the least squares estimate $\hat{\theta}$ for each SOC separately and provide the a_0, b_0, b_1 results for each SOC in your homework solution. The initial SOC of the battery plant model can be set using the variable SOC_0.

(e) Now solve for R_s, R_1, C_1 from the values of $\hat{\theta}$ you found. **Provide the expressions for R_s, R_1, C_1 from $a_0, b_0, b_1, \Delta T$ as well as values in your homework solution at each SOC.**

Problem 3: OCV-R-RC Model Implementation

Your final task is to implement the integrator-based, OCV-R-RC model from Problem 1 in Matlab/Simulink, using $R_s = 0.04\ \Omega$, $R_1 = 0.1\ \Omega$, $C = 300\ F$, $Q = 5\ Ah$, and the following OCV function

$$OCV(z) = 3.1264 + 3.0532z - 5.2313z^2 + 3.2152z^3. \tag{15}$$

(a) In Simulink, create an OCV-R-RC battery model. Submit a screenshot of your Simulink model with your name typed on top of the integrator block.

(b) Assume that the parameters R_s, R_1, C_1 are unknown for now and estimate the R_s, R_1, C_1 using the same method from Problem 2 collect data at 50% SOC. Generate the experimental data by designing a current signal $I(k)$, $k = 1, ..., N$ that you can use to estimate the unknown parameters. For example, consider if a constant current profile be enough, you may want to try a few different profiles. Similar to before, use a fixed sample time $\Delta t \leq 1$ sec. **Plot your current, voltage, y, and SOC profiles versus time.**

(c) Compare the true values of the parameters R_s, R_1, C_1 with their estimates in a table.

Grading Rubric

Problem		Criteria	Pts
1	a	Correct differential equations for z and V_1 versus time.	5
	b	Correct analytical solution is provided for both z and V_1.	10
	c	Correct expression for V_t.	5
2	a	Correct expressions for a_0, b_1, and b_0 in terms of the unknown parameters.	7.5
	b	A plot is provided showing the evolution of SOC, voltage, current, and temperature, for 10 minutes. The final SOC is 0.583.	7.5
	c	A figure is submitted which includes the four subplots: input current, terminal voltage, calculated SOC, and calculated y versus time.	10
	d	Correct estimations of a_0, b_0 and b_1 for each SOC are provided.	10
	e	Correct expressions for R_s, R_1, and C_1 are for each SOC are provided.	10
3	a	A screenshot of the Simulink model is correct. The model is correct.	10
	b	The experimental current, voltage, y, and SOC are provided. The plot appears correct.	10
	c	A comparison is provided of the true versus estimated parameters for R_s, R_1, and C_1.	15
		Total Points: 100	