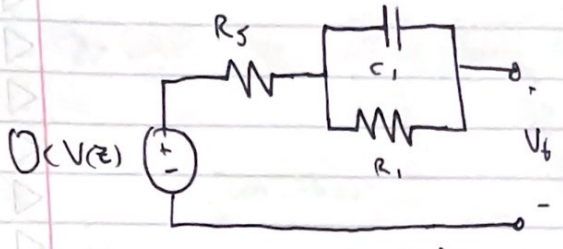
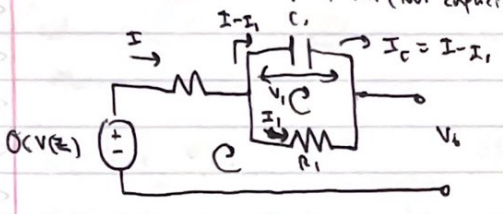
HW3 P1 A ← V_1 →



a) R_s, R_1, C_1, Q (Capacity Ah) are Known, $I = const > 0$

$$\frac{\partial z}{\partial t} = -I \cdot \frac{1}{C} \qquad C = Q \qquad \therefore \boxed{\dot{z} = \frac{-I}{Q}}$$

↑ Capacity Ah (NOT capacitance)



"ohms law for capacitors"

$$i = C \frac{\partial V}{\partial t}$$

i = Instantaneous current

C = capacitance in Farads

$\frac{\partial V}{\partial t}$ = Instantaneous rate of voltage change (V/s)

$$V_1 = I_1 R_1 \rightarrow I_1 = \frac{V_1}{R_1}$$

$$\frac{\partial V_1}{\partial t} = \frac{i_c}{C_1} \Rightarrow \frac{I - I_1}{C_1} \Rightarrow \frac{I - \frac{V_1}{R_1}}{C_1} \Rightarrow \frac{I}{C_1} - \frac{V_1}{R_1 C_1}$$

$$\boxed{\frac{\partial V_1}{\partial t} = \frac{I}{C_1} - \frac{V_1}{R_1 C_1}}$$

HW 3 P1 B

$$\dot{z} = \frac{-I(t)}{Q} \qquad \dot{V}_1 = \frac{I(t)}{C_1} - \frac{V_1}{R_1 C_1} \qquad [z(0), V_1(0)] = [z_0, V_0]$$

$$\frac{\partial z}{\partial t} = \frac{-I(t)}{Q} \Rightarrow \frac{\partial z}{\partial t} = \frac{-I}{Q} \Rightarrow \partial z = -\frac{I}{Q}\partial t \Rightarrow \int \partial z = \int -\frac{I}{Q}\partial t$$

(const $\therefore -I(t) = -I$)

$$z(t) = -\frac{I}{Q}t + C \Rightarrow \boxed{z(t) = -\frac{I}{Q}t + z_0} \quad (C = \text{const.}) \qquad z(0) = z_0 \Rightarrow \boxed{f(t, z_0) = z_0} \quad @\ t = 0$$

$$\frac{\partial V_1}{\partial t} = \frac{I(t)}{C_1} - \frac{V_1(t)}{R_1 C_1} \Rightarrow \frac{\partial V_1}{\partial t} = \frac{I}{C_1} - \frac{V_1(t)}{R_1 C_1} \rightarrow \partial V_1 = \left(\frac{I}{C_1} - \frac{V_1(t)}{R_1 C_1}\right)\partial t$$

$$\rightarrow R_1 C_1 \partial V_1 = (I R_1 - V_1(t))\partial t \rightarrow \frac{1}{R_1 C_1} \frac{1}{-V(t) + IR_1}$$

$$\rightarrow -\frac{1}{V(t) - IR_1}\partial V_1 = \frac{1}{R_1 C_1}\partial t$$

$$\rightarrow \int -\frac{1}{V_1(t) - IR_1}\partial V_1 = \int \frac{1}{R_1 C_1}\partial t \Rightarrow -\ln(V_1(t) - IR_1) = \frac{t}{R_1 C_1} + C \quad (\text{raise to } e)$$

$$\int -\frac{1}{V_1(t) - IR_1}\partial V_1 \rightarrow -\int \frac{1}{y - y'}\partial(y - y') \text{ where } \int \frac{1}{x}\partial x = \ln(|x|)$$

$$\Rightarrow \ln\left((V_1(t) - IR_1)^{-1}\right) = \frac{t}{R_1 C_1} + C \rightarrow \frac{1}{V_1(t) - IR_1} = e^{\frac{t}{R_1 C_1} + C} \Rightarrow V_1(t) - IR_1 = e^{-\frac{t}{RC}} \cdot \left(e^{C}\right)$$

$$V_1(t) = C e^{-\frac{t}{R_1 C_1}} + IR_1 \qquad V_1(0) = V_0 = C e^{-\frac{0}{R_1 C_1}} + IR_1$$

$$V_0 = C + IR_1$$

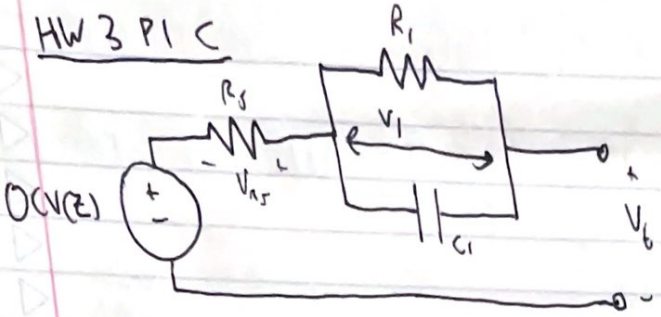
$$V_0 - IR_1 = C$$

$$\boxed{V_1(t) = (V_0 - IR_1)\, e^{-\frac{t}{R_1 C_1}} + IR_1}$$

$$V_1(t) = V_0 e^{-\frac{t}{R_1 C_1}} - IR_1 e^{-\frac{t}{R_1 C_1}} + IR_1$$

$$V_1(0) = \boxed{V_0 = g(t, V_0)} \quad @\ t = 0$$

HW 3 P1 C



$$-V_{R_s} + OCV(z^{(n)}) - V_1(t) - V_t(t) = 0$$

$$V_t(t) = OCV(z^{(n)}) - V_1(t) \quad ; \quad \text{where} \quad V_1(t) = V_0 e^{-\frac{t}{R_1C_1}} - IR_1 e^{-\frac{t}{R_1C_1}} + IR_1$$

$$-V_{R_s} \qquad z(t) = -\frac{I}{Q}t + z_0$$

$$V_t(t) = OCV\left(-\frac{I}{Q}t + z_0\right) - V_0 e^{-t/R_1C_1} + IR_1 e^{-t/R_1C_1} + IR_1 - IR_s$$

HW 3 Q2 D/E Done

$$R_5 = b_1$$

$$\frac{R_5 \Delta T}{R_1 C_1} - R_5 + \frac{\Delta T}{C_1}$$

$$R_5\left(\frac{\Delta T}{R_1 C_1} - 1\right) + \frac{\Delta T}{C_1} = b_0$$

$$\frac{\Delta T}{C_1} - R_5\left(1 - \frac{\Delta T}{R_1 C_1}\right) = b_0$$

$$\frac{\Delta T}{C_1} - R_5(a_0) = b_0$$

$$C_1 = \frac{\Delta T}{b_0 + R_5 a_0}$$

$$\boxed{C_1 = \frac{\Delta T}{b_0 + b_1 a_0}}$$

$$a_0 = 1 - \frac{\Delta T}{R_1 C_1} C_1$$

$$C_1 a_0 = C_1 - \frac{\Delta T}{R_1 C_1} C_1$$

$$C_1 a_0 - C_1 = -\frac{\Delta T}{R_1}$$

$$C_1(a_0 - 1) = \frac{-\Delta T}{R_1}$$

$$\frac{C_1(a_0 - 1)}{-\Delta T} = \frac{1}{R_1}$$

$$\boxed{R_1 = \frac{-\Delta T}{C_1(a_0 - 1)}}$$

Q2 E

2

Contents

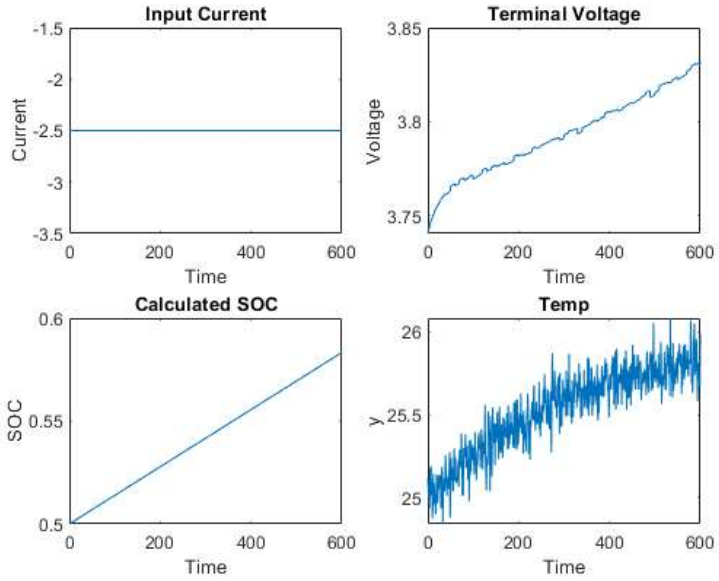
A

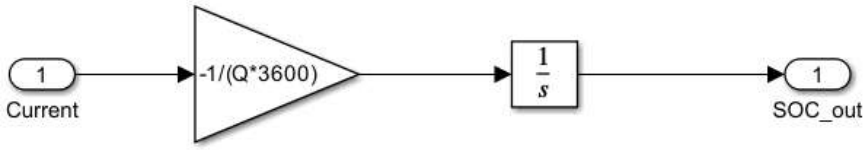
```
clc
clear


% a0 = 1 - deltat/R1C1
% b0 = Rs*Deltat/R1C1 - Rs + deltat/C1
% B1 = Rs
```

B

```
load 'polynomial.mat';
% Initialize variables
T0a = 25;
Delta_t  = 1;
Tspan = 600;
Q = 5;
Current_Input = -2.5;
SOC_0 = 0.5;
mdl = "HW3_virtual_testbed"
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out = sim(simIn);
Current = -2.5 * ones(601, 1);

figure;

subplot(2,2,1);
plot(out.tout, Current);
title('Input Current');
xlabel('Time');
ylabel('Current');

subplot(2,2,2);
plot(out.tout, out.V_out);
title('Terminal Voltage');
xlabel('Time');
ylabel('Voltage');

subplot(2,2,3);
plot(out.tout, out.SOC_out);
title('Calculated SOC');
xlabel('Time');
ylabel('SOC');

subplot(2,2,4);
plot(out.tout, out.Temp);
title('Temp');
xlabel('Time');
ylabel('y');

fprintf("The value of SOC at T=10mins is %d", out.SOC_out(end))

image_data = imread('HW3_2b_socestimator.PNG');
figure;
imshow(image_data);
```

```
mdl =

    "HW3_virtual_testbed"

The value of SOC at T=10mins is 5.833333e-01
```

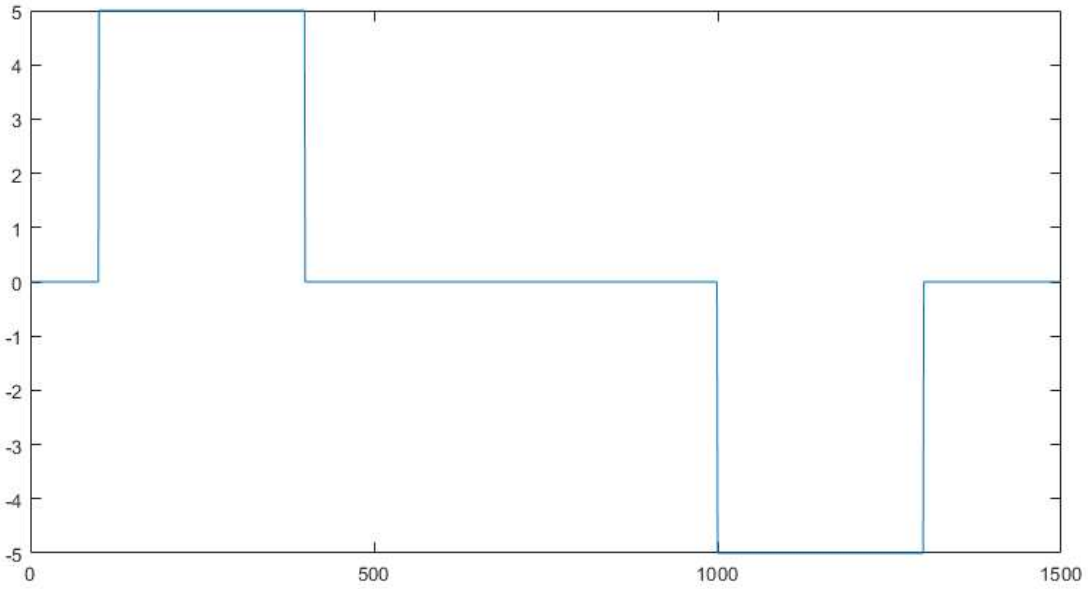


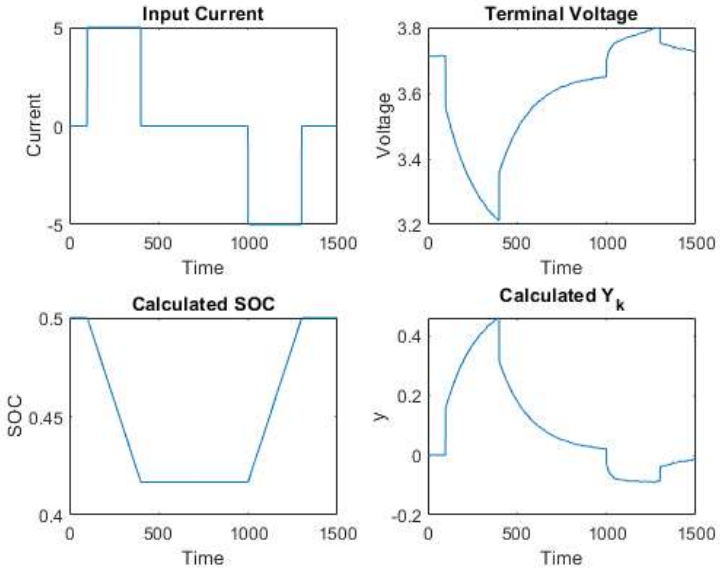


C

```
clc
clear


load 'polynomial.mat';
Step_Time = [0, 100, 400, 1000, 1300, 1500];
Step_Current = [0, 5, 0, -5, 0];
T = 0:1500;
%Preallocatie input current
Current = zeros(size(T));

for i = 1:5
    Current(T >= Step_Time(i) & T < Step_Time(i+1)) = Step_Current(i);
end
```

```
Current_Input = [0:1500;Current]';
plot(T, Current);

% Initialize variables
T0a = 25;
Delta_t  = 1;
Tspan = 1500;
Q = 5;
SOC_0 = 0.5;
alpha = flipud(alpha);

mdl = "HW3_virtual_testbed_2";
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out = sim(simIn);

% Plot of all graphs
figure;

subplot(2,2,1);
plot(out.tout, out.I_out);
title('Input Current');
xlabel('Time');
ylabel('Current');

subplot(2,2,2);
plot(out.tout, out.V_out);
title('Terminal Voltage');
xlabel('Time');
ylabel('Voltage');

subplot(2,2,3);
plot(out.tout(1:1501), out.SOC_out);
title('Calculated SOC');
xlabel('Time');
ylabel('SOC');

subplot(2,2,4);
plot(out.tout, out.Y_k);
title('Calculated Y_k');
xlabel('Time');
ylabel('y');
```





D

```
% 0.25 SOC
T0a = 25;
Delta_t  = 1;
Tspan = 1500;
Q = 5;
SOC_0 = 0.25;


mdl = "HW3_virtual_testbed_2";
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out_25 = sim(simIn);

%Preallocate big PHI_25
PHI_25 = zeros(1501, 3);
PHI_25(:, 1) = out_25.I_out;
PHI_25(2:end, 2) = out_25.I_out(1:end-1);
PHI_25(2:end, 3) = out_25.Y_k(1:end-1);

%solve least squares
Y_25 = out_25.Y_k;
Theta_25 = PHI_25\Y_25
```

```
b1_25 = Theta_25(1)
b0_25 = Theta_25(2)
a0_25 = Theta_25(3)

% 0.5 SOC
T0a = 25;
Delta_t  = 1;
Tspan = 1500;
Q = 5;
SOC_0 = 0.5;


mdl = "HW3_virtual_testbed_2";
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out_50 = sim(simIn);

%Preallocate big PHI_50
PHI_50 = zeros(1501, 3);
PHI_50(:, 1) = out_50.I_out;
PHI_50(2:end, 2) = out_50.I_out(1:end-1);
PHI_50(2:end, 3) = out_50.Y_k(1:end-1);

%solve least squares
Y_50 = out_50.Y_k;
Theta_50 = PHI_50\Y_50
b1_50 = Theta_50(1)
b0_50 = Theta_50(2)
a0_50 = Theta_50(3)

% 0.75 SOC
T0a = 25;
Delta_t  = 1;
Tspan = 1500;
Q = 5;
SOC_0 = 0.75;


mdl = "HW3_virtual_testbed_2";
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out_75 = sim(simIn);

%Preallocate big PHI_75
PHI_75 = zeros(1501, 3);
PHI_75(:, 1) = out_75.I_out;
PHI_75(2:end, 2) = out_75.I_out(1:end-1);
PHI_75(2:end, 3) = out_75.Y_k(1:end-1);

%solve least squares
Y_75 = out_75.Y_k;
Theta_75 = PHI_75\Y_75
b1_75 = Theta_75(1)
b0_75 = Theta_75(2)
a0_75 = Theta_75(3)
```

```
Theta_25 =

    0.0203
   -0.0201
    0.9979


b1_25 =

    0.0203


b0_25 =

   -0.0201


a0_25 =

    0.9979
```

```
Theta_50 =

    0.0203
   -0.0201
    0.9979


b1_50 =

    0.0203


b0_50 =

   -0.0201


a0_50 =

    0.9979


Theta_75 =

    0.0203
   -0.0201
    0.9975


b1_75 =

    0.0203


b0_75 =

   -0.0201


a0_75 =

    0.9975
```

E

```
%Espressions:
% Rs = b1
% C1 = deltat/(b0+b1a0)
% R1 = -deltat/c1(a0-1)

% 25% SOC
Rs_25 = b1_25
C1_25 = Delta_t/(b0_25+b1_25*a0_25)
R1_25 = -Delta_t/(C1_25*(a0_25-1))
% 50% SOC
Rs_50 = b1_50
C1_50 = Delta_t / (b0_50 + b1_50 * a0_50)
R1_50 = -Delta_t / (C1_50 * (a0_50 - 1))
% 75% SOC
Rs_75 = b1_75
C1_75 = Delta_t / (b0_75 + b1_75 * a0_75)
R1_75 = -Delta_t / (C1_75 * (a0_75 - 1))
```

```
Rs_25 =

    0.0203


C1_25 =

   5.2431e+03


R1_25 =
```

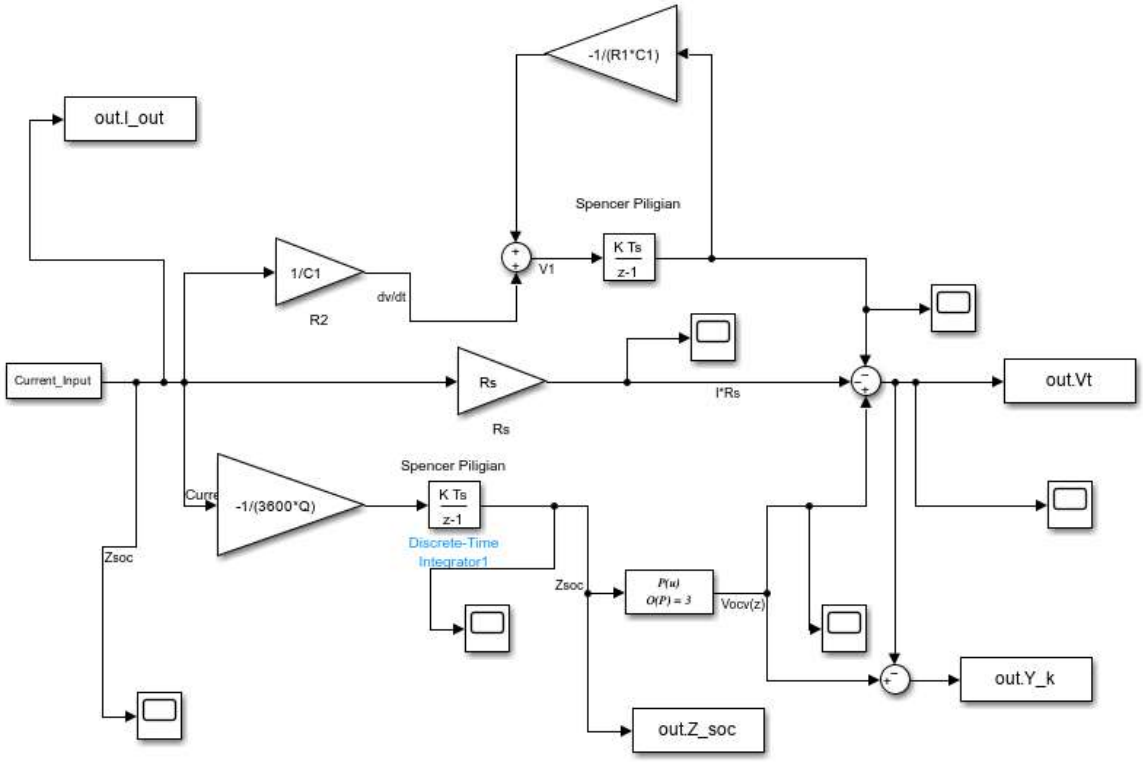
```
    0.0928

Rs_50 =

    0.0203

C1_50 =

   5.8581e+03

R1_50 =

    0.0800

Rs_75 =

    0.0203

C1_75 =

   5.5780e+03

R1_75 =

    0.0728
```

P3

A

```
OCV_Poly = [3.2152,-5.2313,3.0532,3.1264];
Delta_t  = 1;
Tspan = 1500;
Rs = 0.04;
R1 = 0.1;
C1 = 300;
Q = 5;
Soc_Init = 0.5;

mdl = "HW3_virtual_testbed_3";
set_param(mdl, 'SimulationCommand', 'update');
simIn = Simulink.SimulationInput(mdl);
simIn = setModelParameter(simIn,"StopTime",'Tspan');
out = sim(simIn);

image_data = imread('HW3_3a.PNG');

figure;
imshow(image_data);
title('3A Simulink');
```

3A Simulink

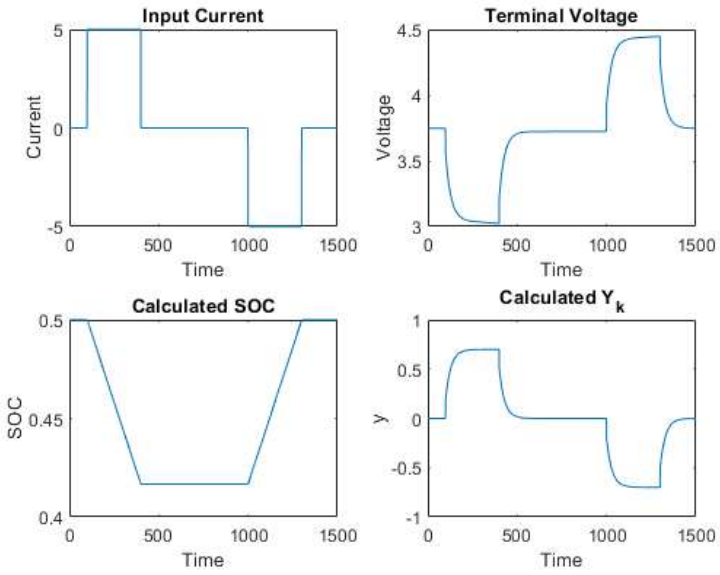


B

```
%Preallocate big phi
PHI = zeros(1501, 3);
PHI(:, 1) = out.I_out;
PHI(2:end, 2) = out.I_out(1:end-1);
PHI(2:end, 3) = out.Y_k(1:end-1);

%solve least squares
Y = out.Y_k;
Theta = PHI\Y
b1 = Theta(1)
b0 = Theta(2)
a0 = Theta(3)


Rs = b1;
C1 = Delta_t / (b0 + b1 * a0);
R1 = -Delta_t / (C1 * (a0 - 1));

figure;

subplot(2,2,1);
plot(out.tout, out.I_out);
title('Input Current');
xlabel('Time');
ylabel('Current');

subplot(2,2,2);
plot(out.tout, out.Vt);
title('Terminal Voltage');
xlabel('Time');
ylabel('Voltage');

subplot(2,2,3);
plot(out.tout, out.Z_soc);
title('Calculated SOC');
```

```
xlabel('Time');
ylabel('SOC');

subplot(2,2,4);
plot(out.tout, out.Y_k);
title('Calculated Y_k');
xlabel('Time');
ylabel('y');
```

```
Theta =

    0.0400
   -0.0353
    0.9667


b1 =

    0.0400


b0 =

   -0.0353


a0 =

    0.9667
```



C

make table

```
col = {'Actual', 'Estimated'};
titles = {'0.04', '0.01', '300'}';
Nums = double([Rs,R1,C1]);
vals =  Nums';
configTable = cell2table([titles, num2cell(vals)], 'VariableNames', col);
configTable.(1) = categorical(configTable.(1));
disp(configTable)

%Parameters calculated and estimated are the same.
```

```
    Actual    Estimated
    ______    _________

     0.04        0.04
```

```
    0.01        0.1
    300         300
```

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