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Homework 4

Assigned: Wednesday, February 07.
Due: Sunday, February 18 at 11:59pm Eastern Time.

Submission Instructions

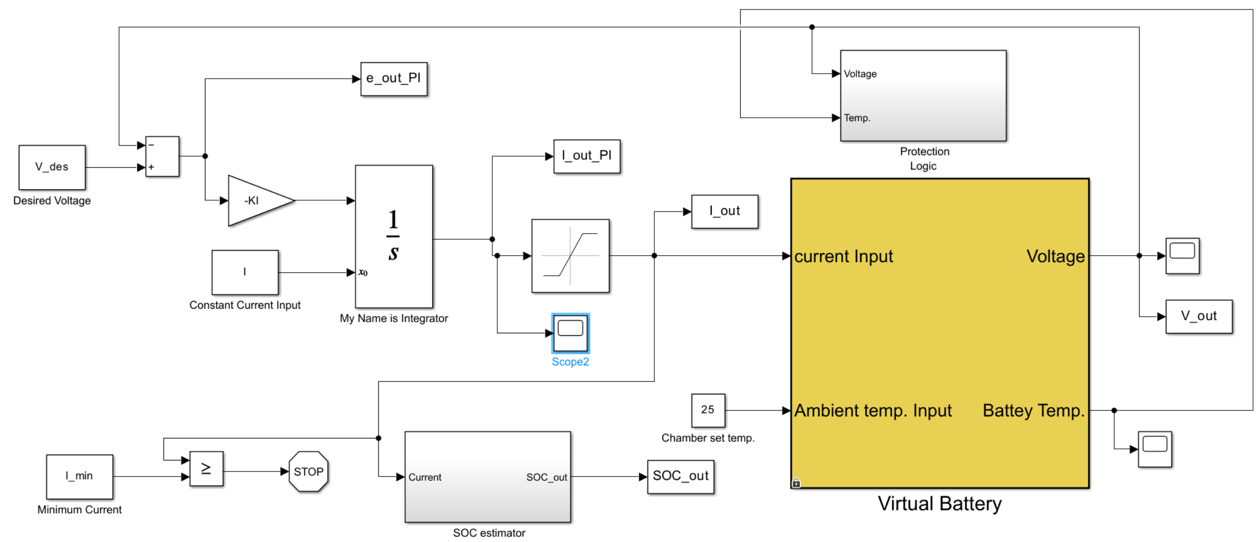
Submit your assignment through Gradescope as a single PDF. During submission, Gradescope will ask you to assign page numbers to each problem. Present your solutions in a coherent, sequential flow. For questions requiring numerical results or explanations, highlight or put a box around your final answers. Include figures and code along with your final answers where applicable.

Learning Objectives

In Problem 1, we will investigate the dynamics of battery charging using the equivalent circuit model and build a CC-CV charger controller using an integral controller with saturated output to implement the CC-phase. In Problem 2, we will graph the Nyquist plot of the equivalent circuit model and fit the RC parameters to experimental data.

Problem 1: Build a CCCV Charger Controller

In this problem, you will follow the step-by-step instructions to build a charger controller for a constant current - constant voltage (CCCV) charging protocol to avoid overcharging the battery while maximizing the stored energy. First, the saturation block will ensure the battery is charged at a constant current rate until the voltage reaches the desired voltage, V_{des}. Then, the controller will adjust the current dynamically to maintain the desired voltage by gradually decreasing the current to top off and fully charge the battery. The controller will need to use the feedback of the measured terminal voltage and address integrator anti-windup. Finally, the protocol will stop the charging session when the current decays below a preset value.



(a) In order to build the CCCV controller, first start with the `HW4_virtual_testbed` model and build an error signal between the desired voltage $\mathbf{V_{des} = 4.2\ V}$ and measured voltage as $e = V_{des} - V$. Consider the integral controller $I = -K_I \int e dt$, with initial condition I_0 = -5 A. Add a saturation block on the current signal going to the battery limit the current above -5 A. Submit a figure of your completed model that performs the CC-CV charge with the minimum current cutoff logic. Place your name in a text box on the top of the integrator. **Note: in this problem, we are using the sign convention that charging**

current is negative. **Note2: The protection logic will stop the simulation when the battery terminal voltage reaches 4.28V**

(b) Simulate a 5 A CCCV charge cycle with the integral voltage controller from part (a). Initialize the SOC to 85% (`SOC_0=0.85`) and set the variable `T0a` to 25. Use a control gain $K_I = 1$. Terminate the simulation once the absolute value of current ($|I|$) drops below 25 mA during the constant voltage phase. Submit plots of terminal voltage, state of charge, and a combined plot of the integrator state (before the saturation) and the applied current to the battery vs. time. What happens to the voltage in the CV phase? Why Doesn't the current start to decay in the CV phase as expected? (Hint: Check the integrator state).

(c) To address integrator wind-up, incorporate an anti-windup scheme. Modify the error signal to stop growing once the current saturates, such that $e = V_{des} - V + K_{aw}(I - \mathrm{sat}(I))$. Repeat the simulation from part (b) with the anti-windup scheme, using a control gain $K_I = 1$ and $K_{aw} = 1$. Submit plots of terminal voltage, SOC, and applied current vs. time.

(d) Try different control gains $K_I = (0.5, 5, 50)$ and compare total charge time, voltage overshoot beyond the maximum allowable voltage, and the achieved SOC at the end of charge. Submit a table listing controller gains, the corresponding charge times, voltage overshoots, and the estimated SOCs from coulomb counting at the end of the charge. Based on the results, which gain you will choose for the controller? Explain your reasoning.

(e) Assuming the OCV-R Equivalent circuit model (ECM) defined by

$$\dot{z} = -\frac{I}{Q_{max} \cdot 3600}$$
$$V = OCV(z) - I \cdot R$$

is accurate in modeling the dynamics of the cell voltage vs. current, answer the following questions.

Find the denominator of the closed loop transfer function from the voltage set point to the battery voltage for the following two controllers: $I = -K_P e$ (proportional control) and $I = -K_I \int e dt$ (integral control). You can ignore the anti-windup and saturation blocks for this question.

(f) (Bonus) Plot the Root-Locus diagram (rltool) for the closed loop system and discuss how the controller gain, K_I, can affect the overshoot, rise time, and stability of the voltage response. Consider the slope of the OCV curve at the SOC when the battery hits 4.2 V under a 1C rate to be $\alpha = \frac{\mathrm{d}OCV(z)}{\mathrm{d}z} = 1$, $R = 0.02\ \Omega$ and $Q_{max} = 5\ Ah$.

Problem 2: Impedance of the Battery Model

So far, you have identified the OCV-R and the OCV-R-RC models based on their response to signals in the time domain. This time, you will investigate the model dynamics in the frequency domain by looking at the impedance, Z, which is defined by

$$V(j\omega) = Z(j\omega)I(j\omega), \tag{1}$$

where ω is the frequency of the signals. The impedance reflects the response of the battery to sinusoidal current. Compare the impedance of the two models we have developed so far, which are the OCV-R and OCV-R-RC models. **Note that the sign convention for current entering the positive terminal used in EIS characterization and what we derived in Lecture 5 is opposite what we have derived in previous lectures for the OCV-R and OCV-R-RC models.** When $I > 0$ for charging, the OCV-R equations are

$$\begin{aligned} \frac{dSOC}{dt} &= \frac{I}{Q} \\ V &= OCV(SOC) + R_s I \end{aligned} \tag{2}$$

Similarly for the OCV-R-RC model, the equations when $I > 0$ for charging are

$$
\begin{aligned}
\frac{dSOC}{dt} &= \frac{I}{Q} \\
\frac{dV_c}{dt} &= -\frac{V_c}{R_1 C_1} - \frac{I}{C_1} \\
V &= OCV(SOC) - V_c + R_s I.
\end{aligned}
\tag{3}
$$

Assume that the open circuit voltage OCV(SOC) in both cases is defined as:

$$OCV(SOC) = 3.1264 + 3.0532 \times SOC - 5.2313 \times SOC^2 + 3.2152 \times SOC^3 \tag{4}$$

Also assume that the values of the parameters in the above equations, are

$$R_s = 0.03\Omega, R_1 = 0.08\Omega, C_1 = 5000F, Q = 5Ah \tag{5}$$

Use the skeleton m-file `HW4_Skeleton_Prob2.m` to complete this problem. Most EIS equipment uses a logarithmic spacing between frequency measurement points, since we are interested in the response over several orders of magnitude. The Matlab function `logspace` can be used to generate log spaced frequencies.

(a) As the first step of deriving the model impedance, you need to linearize the open circuit voltage around a certain SOC. Calculate the slope of the OCV curve at 50% SOC based on (4), and provide the equations of the linearized OCV-R and OCV-R-RC models. Show the equations with the parameter values substituted in.

(b) Derive the expression for the impedance of the OCV-R and the OCV-R-RC models. Separate the real and the imaginary parts. Show the equations with the plugged in parameter values.

(c) Show the Nyquist plot of the impedance of the OCV-R model at 50% SOC. The frequency range is $0.01mHz$ - $1kHz$. Please plot $RE(Z)$ along the x-axis, and $-IM(Z)$ along the y-axis. Label the points where $f = 0.01mHz$ and $f = 1kHz$ are located on the graph.

(d) Show the Nyquist plot of the impedance of the OCV-R-RC model at 50% SOC. Requirements are the same as in (c).

(e) (Bonus) You will use the `Nyquist_Fit_App_Skeleton` to fit the experimental data using an OCV-R-2RC model with inductance and a Warburg CPE element in series. First, in the function `calc_nyquist`, provide the equations to calculate the real and imaginary parts of the impedance as `Z_real` and `Z_imag`, respectively. (The script as provided implements the OCV-R-RC model, and only some of the sliders parameters are used in this model. You will need to use the same variable names as implemented in the GUI portion of the script). Then, run the script to build the app and fit the data using the "Fit me" button. Provide a screenshot of the plot with the fitted EIS results and a table of your parameters for R_s, R_1, τ_1, R_2, τ_2, A_w, $\frac{\alpha}{Q}$, and L. You may need to experiment with the sliders to adjust the initial conditions of the model fitting in order to get a converged solution. Try to use your understanding of how each parameter shapes the EIS curve to get as close to the data as possible before pressing the "Fit me" button.

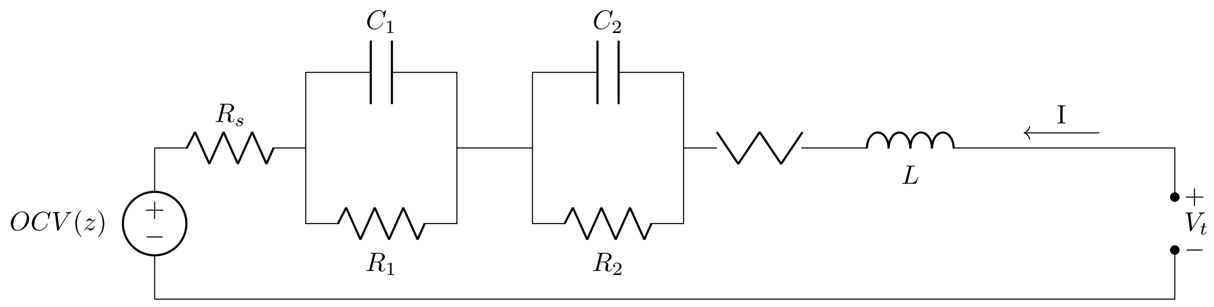


Figure 1: Equivalent OCV-R-RC-RC-W-L model

Grading Rubric

Problem		Criteria	Pts
1	a	Correct figure of the completed model that performs the CCCV charge with cutoff logic	10
	b	Plots of terminal voltage, SOC, and applied current vs. time. Correct explanation as to why current doesn't start decaying in the CV phase	5
	c	Plots of terminal voltage, SOC, and applied current vs. time	5
	d	Table listing controller gains, the corresponding charge times, voltage overshoots, and the SOC at the end of charge. The reasoning for the results	20
	e	Correct denominator of the closed loop transfer function from the voltage set point to the battery voltage for P controller and integral controller	10
	f (bonus)	Plot the Root-Locus diagram and discuss	10
2	a	Correct alpha. Slope of the OCV curve at 50% SOC	2.5
		Two analytic equations, and two with parameters plugged in Provide the equations of the linearized OCV-R model. Show the equations with the plugged in parameter values.	10
		three analytic equations and three with parameters plugged in (SOC equation didn't count for scores, it is the same with OCV-R) Provide the equations of the linearized OCV-R-RC. Show the equations with the plugged-in parameter values.	10
	b	Expression for the impedance of the OCV-R model	5
		Equations for the impedance of the OCV-R model with plugged in parameters	2.5
		Expression for the impedance of the OCV-R-RC model	5
		Equations for the impedance of the OCV-R-RC model with plugged in parameters	5
	c	Nyquist plot of the impedance of the OCV-R model at 50% SOC. Denote where are f = 0.01mHz and f = 1kHz	5
	d	Nyquist plot of the impedance of the OCV-R-RC model at 50% SOC. Denote where are f = 0.01mHz and f = 1kHz	5
	e (bonus)	Plot with the fitted EIS results	5
		Correct fitted parameters	5
		Total Points: 120	