EMCH 792 Project 4- Advecting a Passive Droplet in a Lid-Driven Cavity using the Eulerian Implicit VOF method

Spencer Schwartz

1 Time Step computation

The time step size constraints are

$$\Delta t < \frac{\Delta x}{\mathrm{CFL}|u_{max}|} \tag{1}$$

and

$$\Delta t < \frac{(\Delta x)^2}{4\nu}. \tag{2}$$

Eq. (1) and (2) refer to the constraints caused by the convective and diffusive terms, respectively. After considering the preceding equations, we use a conservative $\Delta t = 0.0005$ for our 256x256 Cartesian grid to maximize stability and reduce the iterations of the pressure solver. For the following cases, simulations were run until the velocity difference between the time steps reached a threshold value, 1E-8, or the time reached the maximum time, $U_0 t_{max}/L = 10$.

2 Initial Conditions

The domain is a 1x1 square using no slip boundary conditions for **u** and symmetric boundary conditions for the color function, C. We obtain the initial C field from the level set function of a circle

$$R - \sqrt{(x-0.75)^2 + (y-0.75)^2} = 0 \tag{3}$$

and is illustrated in Figure 1. The top wall, to simulate a moving lid, has a tangential velocity

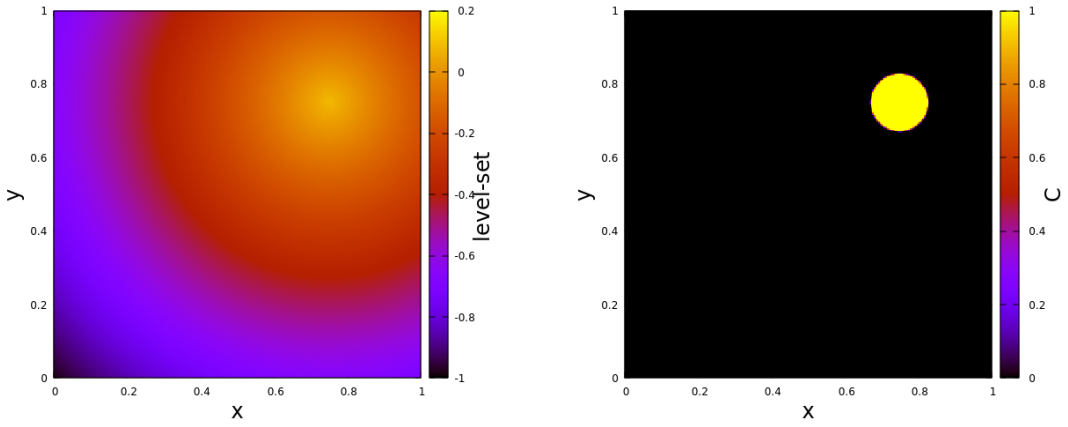

Figure 1: Initial level-set field (left) and the color function field, C, (right) at t = 0

boundary condition of $U_0 = 1$. The domain and B.C's are summarized in Figure 2.

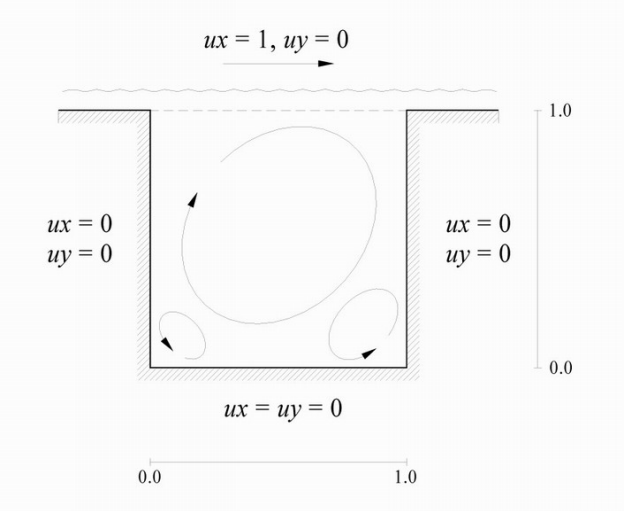

Figure 2: Diagram summarizing boundary conditions and dimensions for the domain.

3 Methods and Parameters

We use $Re = 400$ for all cases. Additionally, all cases will use a 2nd order central method for solving the advection and diffusion terms in the N.S. equations. Here we test two methods: the basic Eulerian Implicit (EI) method and the EI method used in Basilisk **?**. For this project, the problem was solved using a custom C++ program available on GitHub.

The two EI methods only differ in how they express the VOF advection equation

$$\partial_t C + \mathbf{u} \cdot \nabla C = 0. \tag{4}$$

Expanding and rearranging (4) yields

$$\partial_t C = -\nabla \cdot (\mathbf{u}C) + C\nabla \cdot \mathbf{u}. \tag{5}$$

Since these methods are split, we discretize (5) considering one dimension, i.e.

$$C^{n+1} = C^n - \Delta t(\nabla_x \cdot (u_x C^n) - C\nabla_x \cdot u_x). \tag{6}$$

What differentiates between the two methods is how they set C in the divergence term. In the basic EI approach, C=C^{n+1}, hence the implicit name, while the approach done by Basilisk is $C = 1$ if $C^n > 0.5$, otherwise $C = 0$.

Using the basic EI approach, (6) is written as

$$C^{n+1} = \frac{C^n + F_L - F_R}{1 + u^*_{x,L} - u^*_{x,R}} \tag{7}$$

where $F_i = u^*_{x,i} C_i$ denotes the flux on the left (L) or right (R) side and $u^*_{x,i} = \frac{u_x \Delta t}{h}$, a.k.a the CFL number.

Additionally, we examine the effect of normal calculation methods on the solution. Specifically, we look at the Youngs Finite Difference method (YFD) and the Centered-Column method (CCM), which uses height functions. The Mixed-Youngs-Centered (MYC) method was also tested but differed little from the CCM results, so we neglect it in the next section.

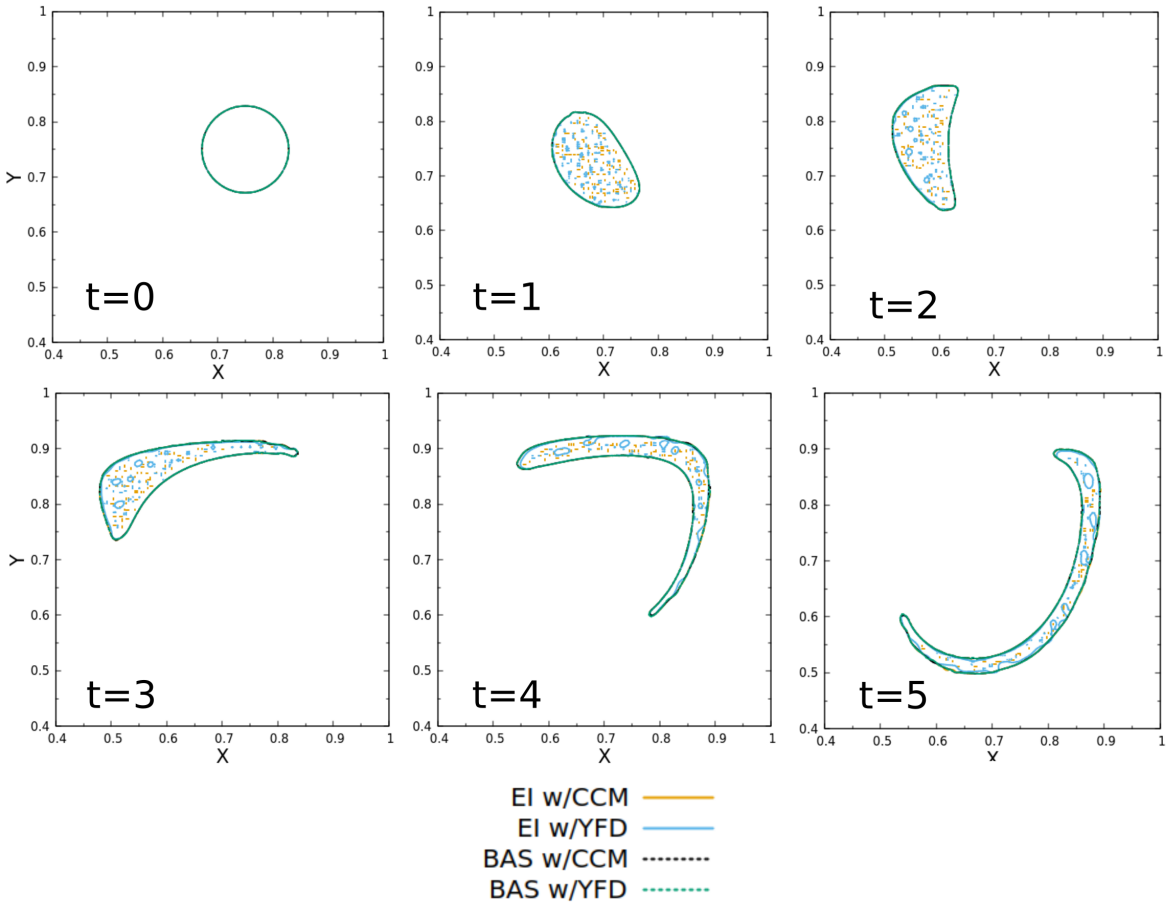

Figure 3: Droplet shape for different methods from t = 0 to 5.

Lastly, we alternate the order of the unidimensional advection every timestep to reduce the phase error.

It should be noted that, as seen in *src/vof.cpp*, an implementation of the Lagrangian explicit method was attempted but has bugs related to the out-of-cell mapping. Therefore, results using the method are not shown here.

4 Results

Each simulation is ran from t = 0 to 10. The droplet shape for each combination of method is shown in Fig. 3 and 4. The EI and Basilisk (BAS) method using the centered column normal approach (CCM) are identical except for the very tiny fragments of interface within the droplet present in the EI method's results. These "bubbles" are likely cause by rounding errors and the different treatment of C in equation (6) which does not impact the overall solution in this case. Figure 6 show that EI w/CCM does not have noticeable holes in the volume fraction field, confirming the previous assertion. Furthermore, Figure 5 shows that the volume loss from EI w/CCM and BAS w/CCM are identical.

It is observed that using the Youngs-Finite-Difference (YFD) method with the basic EI procedure produces significant holes/bubbles in the droplet, observable in Figure 3, 4, and 7. On the other

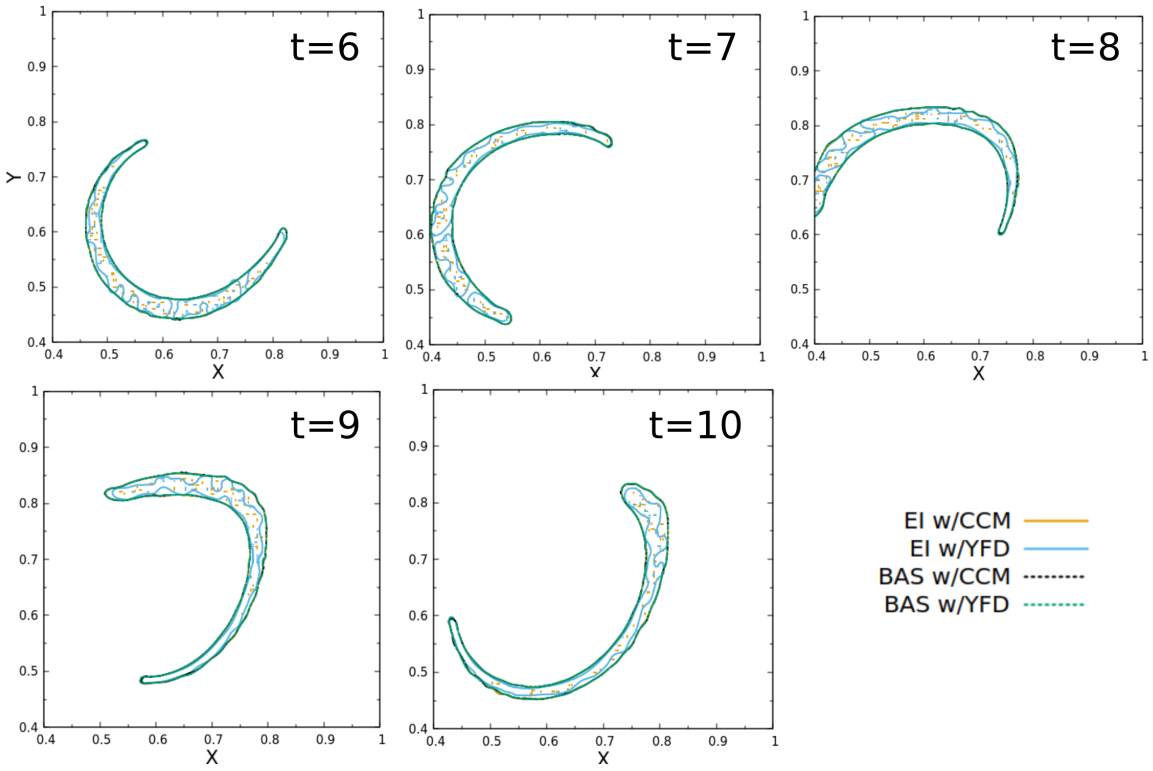

Figure 4: Droplet shape for different methods from t = 6 to 10.

hand, using YFD with Basilisk's approach only slightly impacts the final solution while keep the droplet intact. The reason for this is likely the same as why we see tiny fragments generated in the droplet for the EI w/CCM case, but now amplified due to the relatively rough accuracy of the YFD method. Consequently, the volume loss of EI w/YFD is great, as shown in Figure 5, with an almost 50% mass loss, while the other three cases remain above 4%.

5 Summary

Simulations of a passive scalar field using the Volme-of-Fluid (VOF) method in a lid driven cavity are run by solving the Navier-Stokes equation using the projection method. The simulation is solved in a custom C++ program available on GitHub. The four cases were run using a 256x256 Cartesian grid. Results show good agreement between the Eulerian Implicit (EI) advection method using the centered column method (CCM) and both cases using the Basilisk's approach. It is observed that using Youngs-Finite-Difference (YFD) method with the basic EI scheme produces nonphysical bubbles within the droplet. Compared with results using the level-set-method from project 3, the volume conservation improvement with VOF is literally ∞ since the droplet's volume goes to 0 using the level set method.

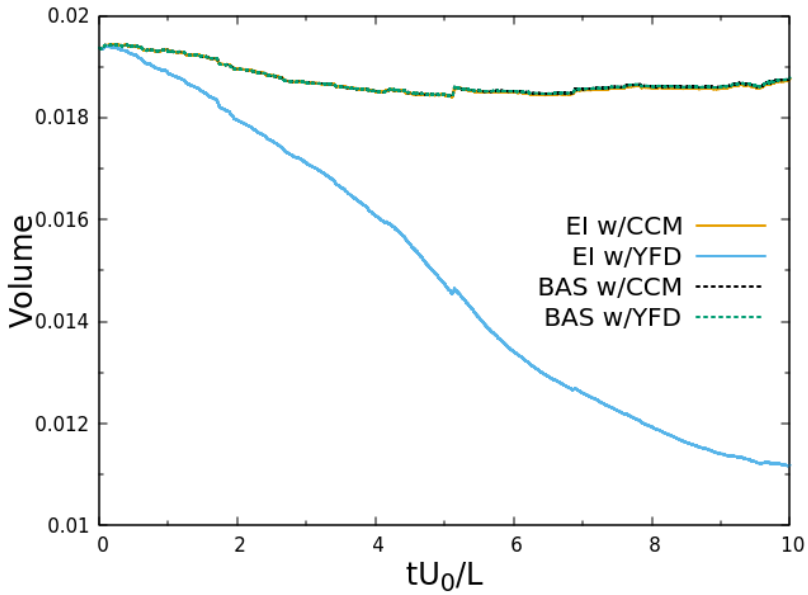

Figure 5: Volume loss over time for the two advection methods (Eularian Implict and Basilisk's version of EI) and for the two normal calculation methods (Youngs-Finite-Difference and Centered-Column-Method).

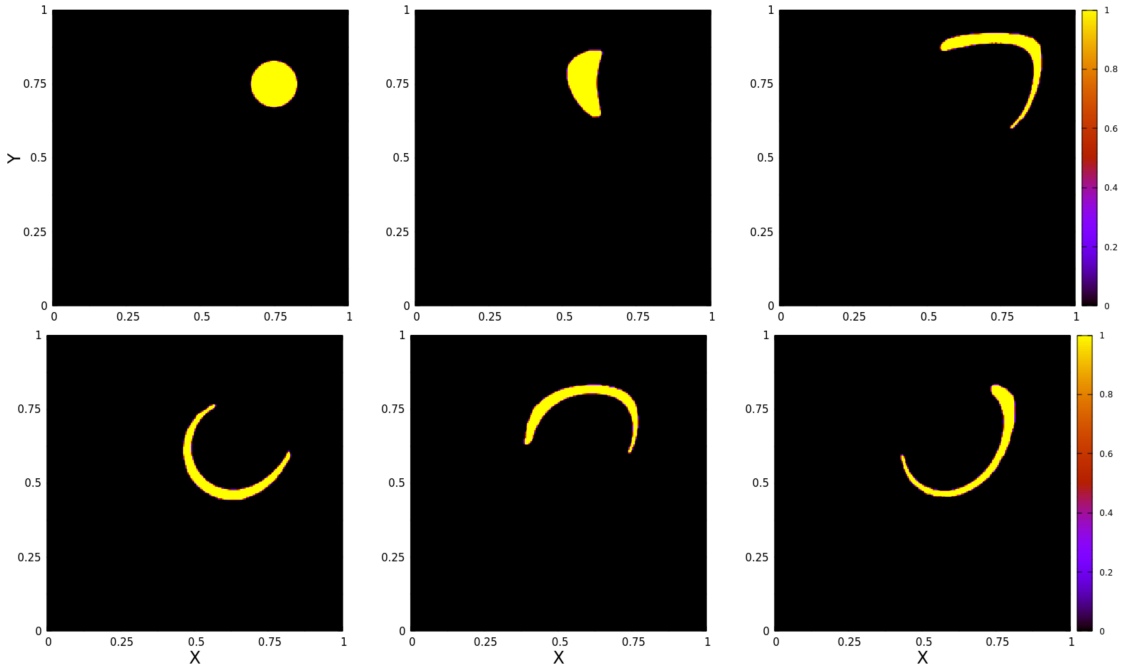

Figure 6: Volume fraction field C for t = 0, 2, 4, 6, 8, and 10 using the EI w/MYC method.

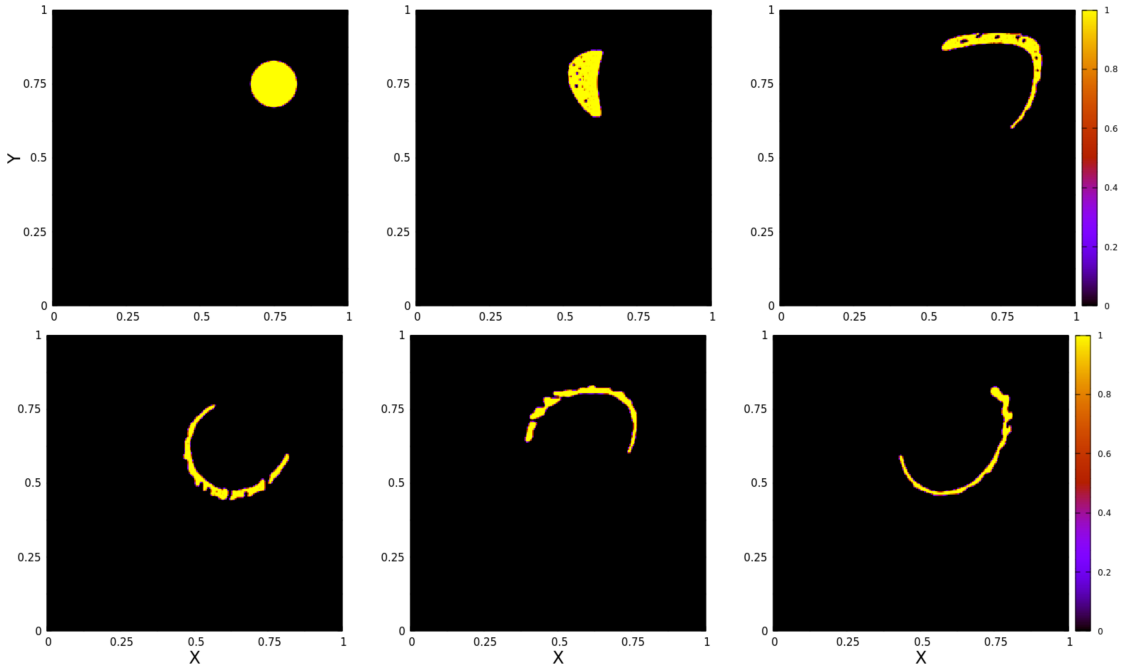

Figure 7: Volume fraction field C for t = 0, 2, 4, 6, 8, and 10 using the EI w/YFD method.

6 Resources

All code for this project can be found here:
https://github.com/SpencerSchwart/multiphase-cfd/tree/main/project4