EMCH 792 Project 3- Using the Level-Set Method to Advect a Passive Drop in a Lid-Driven Cavity

Spencer Schwartz

1 Time Step computation

The time step size constraints are

$$\Delta t < \frac{\Delta x}{\text{CFL}|u_{max}|} \tag{1}$$

and

$$\Delta t < \frac{(\Delta x)^2}{4\nu}. \tag{2}$$

Eq. (1) and (2) refer to the constraints caused by the convective and diffusive terms, respectively. After considering the preceding equations, we use a conservative $\Delta t = 0.001$ and 0.0005 for the 128x128 and 256x256 cases, respectively, to maximize stability and reduce the iterations of the pressure solver. For the following cases, simulations were run until the velocity difference between the time steps reach a threshold value, 1E-8, or the time reaches the maximum time, $U_0 t_{max}/L = 10$.

2 Initial Conditions

The domain is a 1x1 square using no slip boundary conditions for $\mathbf{u}$ and symmetric boundary conditions for the level-set function, ϕ and the initial field is defined by

$$\phi(x, y, t = 0) = R - \sqrt{(x - 0.75)^2 + (y - 0.75)^2} \tag{3}$$

and is illustrated in Figure 1. The top wall, to simulate a moving lid, has a tangential velocity

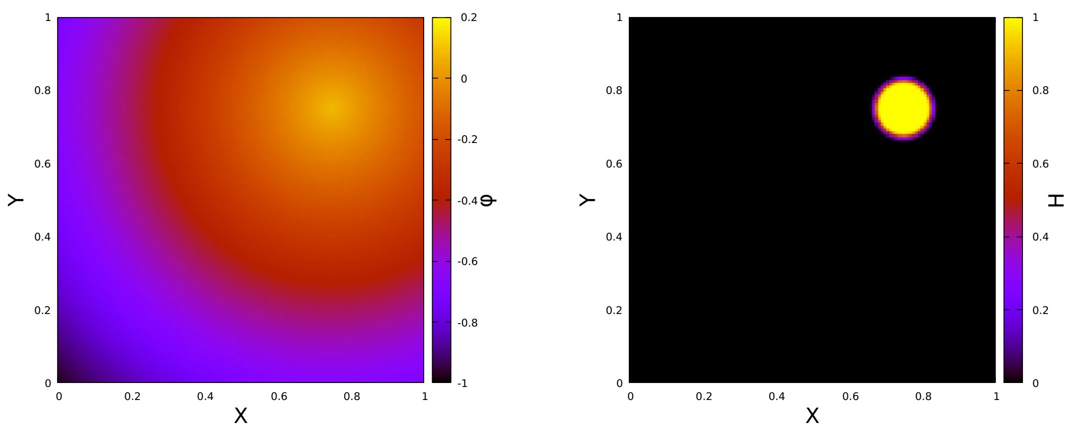

Figure 1: Initial level-set field (left) and heavy-side field (right) at t = 0, used for all cases.

boundary condition of $U_0 = 1$. The domain and B.C's are summarized in Figure 2. Two different mesh resolutions were used: 128x128 and 256x256.

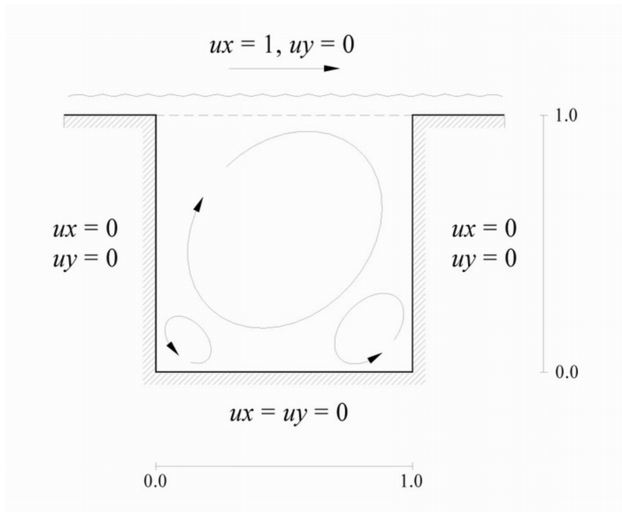

Figure 2: Diagram summarizing boundary conditions and dimensions for domain.

3 Results

We use $Re = 400$ for all cases. Additionally, all cases will use a 2nd order central method for solving the advection and diffusion terms in the N.S. equations and level-set advection equation. Furthermore, all cases use $\epsilon = 2\Delta x$, i.e. the interface thickness is two cells thick. We categorize cases based on the resolution and wether re-initialization of the levelset field is used. For this project, the problem was solved using custom C++ program.

The parameters for reinitilization are as follows:

1. $d\tau = 5 \cdot 10^{-4}, 2.5 \cdot 10^{-5}$
2. $\epsilon = 2\Delta x$
3. # of iterations = 7
4. frequency $i = 100$, 200

with the first and second number denoting the value for the coarse and fine resolution cases, resp.

First, the droplet volume over time is shown in Figure 3. It is shown that cases with reinitalization increase the droplet lifespan by $5 - 10\%$. Furthermore, increasing the resolution prolongs the lifespan by $t \approx 1$. However, cases with reinitialization generate fictitious fluid after $t = 6$, which causes the volume to suddenly increase but hits a steady state around $t = 8$. The reason for this is not known, but is likely due to boundary condition issues. Evidence of this can be found in the figures of H and ϕ at different time snapshots. Cases without reinitialization show smoothness at the boundaries for ϕ, while figures with reinitalization show a discontinuity at the wall boundaries (most visible for lvl 7). Over time, this discontinuity grows in magnitude until ϕ becomes positive, creating fake fluid. More work should be done to ensure proper treatment of ϕ near the boundaries.

Other than potentially incorrect boundary treatment, it is found that the reinitialization process is very sensitive to key parameters, such as $d\tau$ and number of iterations. The values listed above were chosen after trail and error. A more rigid method of determining better values was avoided here due to time constraints.

Figures 4-13 below showcase snapshots of ϕ and H at $t = 2, 4, 6, 8$ and 10. After $t = 4$, the droplet disappears from numerical diffusion. It is observed, though, that the droplet is better defined at $t = 4$ for the 256x256 cases (see Figure 6). At $t = 8$ and 10, fake fluid appears on the boundaries for cases with reinitilzations for reasons previously explained (see Figure 10 - 13).

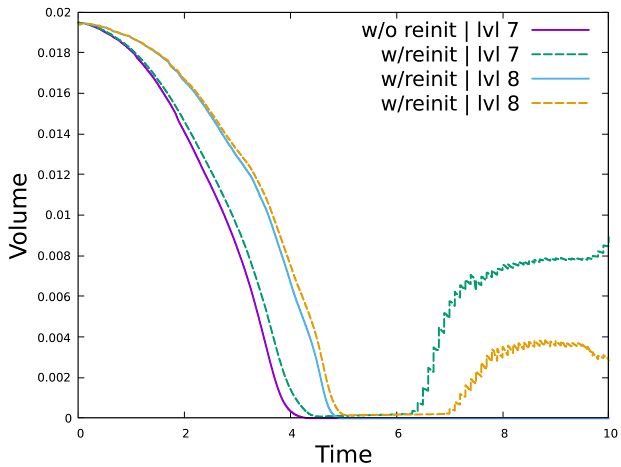

Figure 3: Volume of passive droplet over time. Results show all cases decay to zero between $t = 4$ and 5, but with fictitious fluid being created in the initialization cases after $t = 6$ or 7.

4 Summary

Simulations of a passive scalar field using the level-set method in a lid driven cavity are run by solving the Navier-Stokes equation using the projection method. The simulation is solved in a custom C++ program available on GitHub. Cases were ran at different grid resolutions, 128x128 and 256x256, and with and without reinitialization. Reinitialization is shown to help conserve mass, albeit by $5 - 10\%$ until going to zero. Additionally, the grid level of refinement is shown to impact conservation of mass/volume more significantly than the reinitialization process on a coarser grid. There are several existing bugs in the current reinitialzation code, such as the fake fluid generating on the boundaries, but they are not resolved for lack of time.

5 Resources

All code for this project can be found here:
https://github.com/SpencerSchwart/multiphase-cfd/tree/main/project3

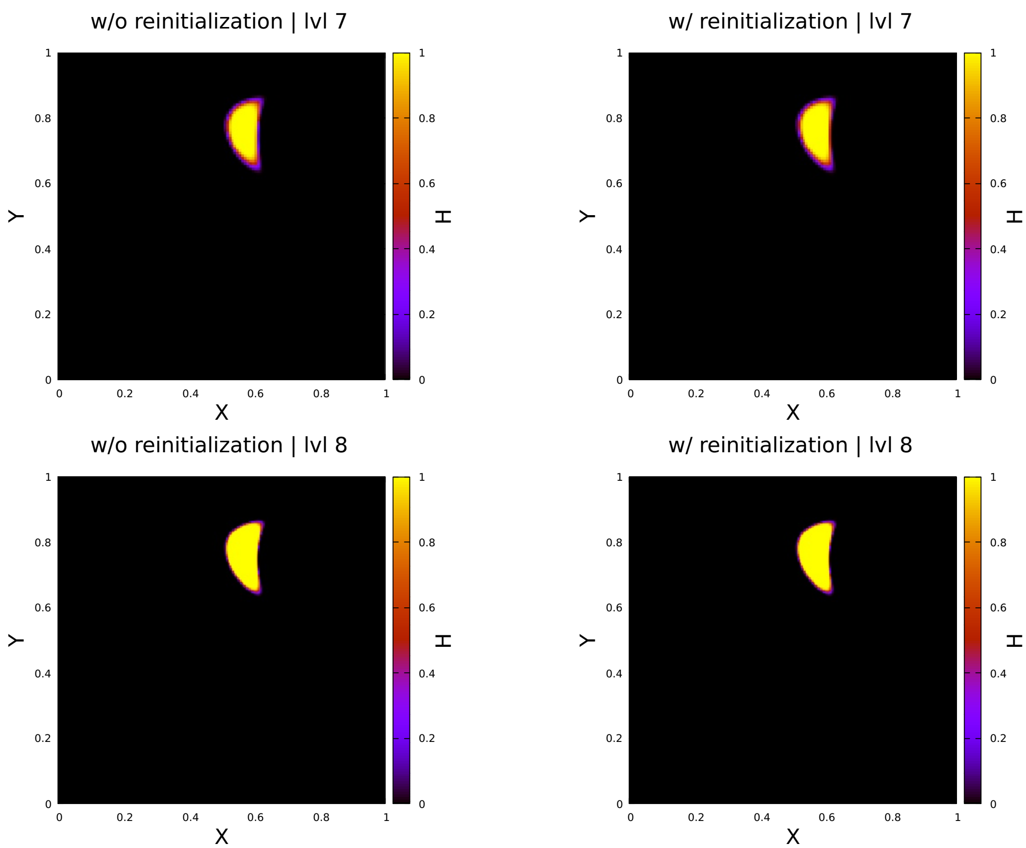

Figure 4: Snapshots of the heavyside function field for all four cases at t = 2.

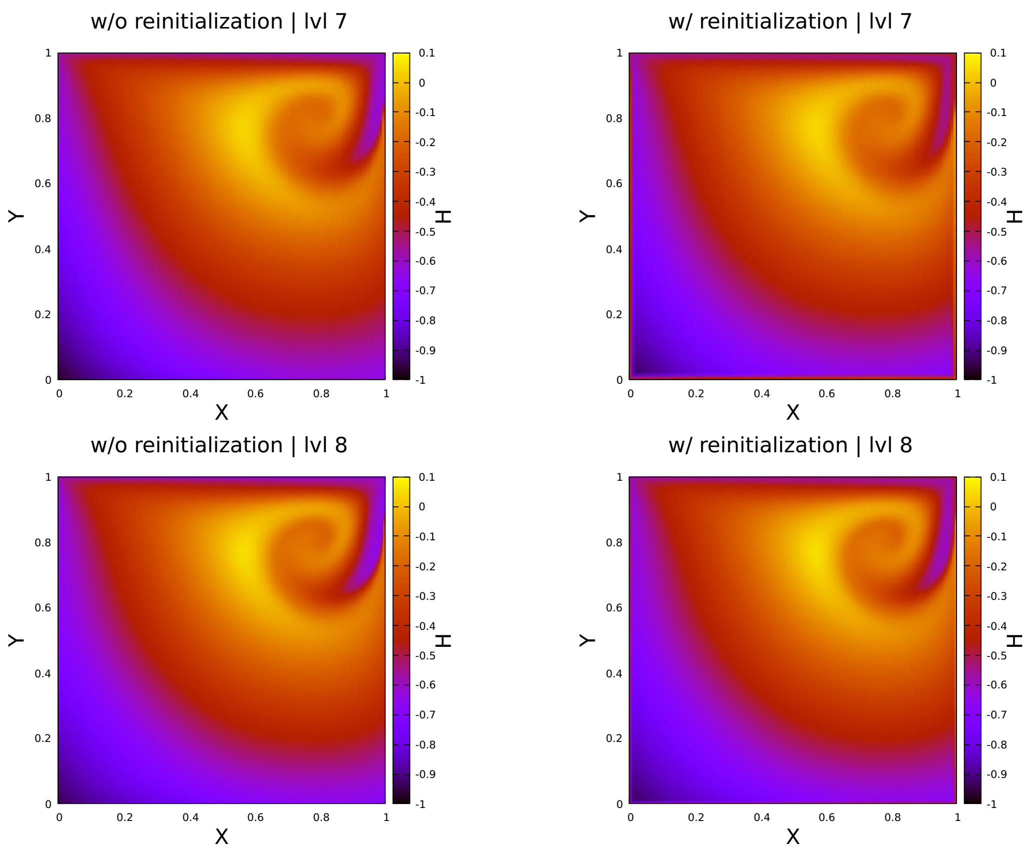

Figure 5: Snapshots of the level-set function field for all four cases at t = 2.

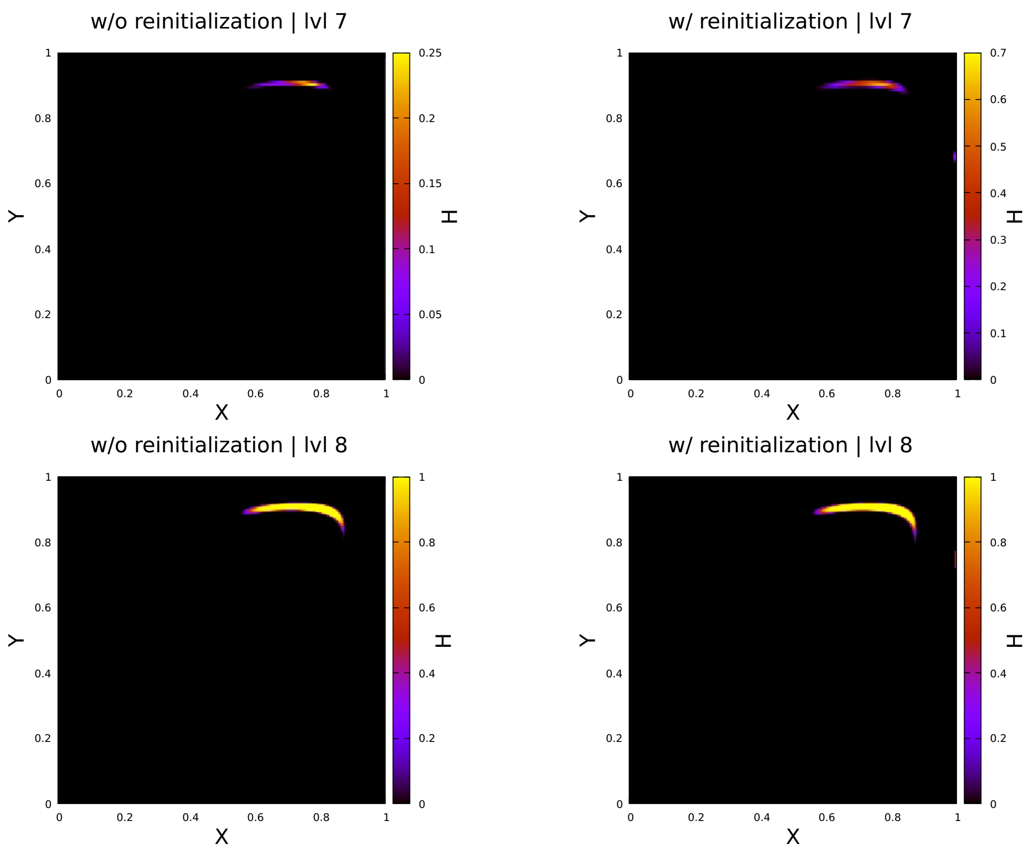

Figure 6: Snapshots of the heavyside function field for all four cases at t = 4.

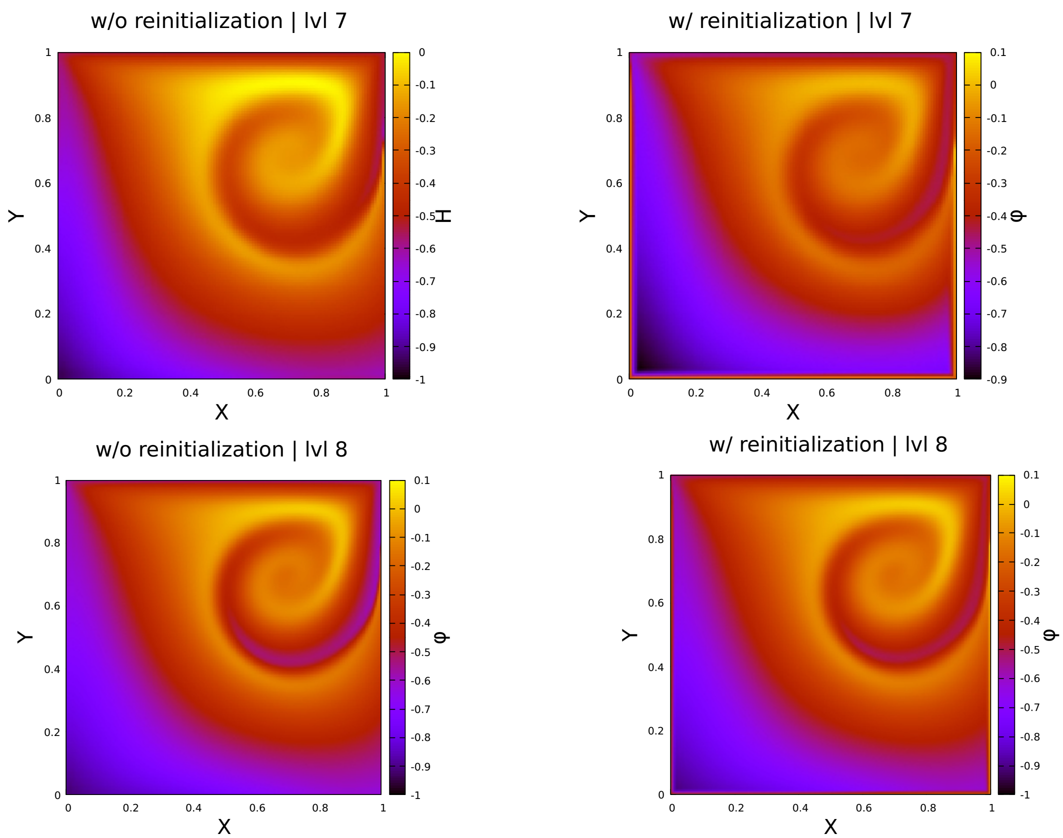

Figure 7: Snapshots of the level-set function field for all four cases at t = 4.

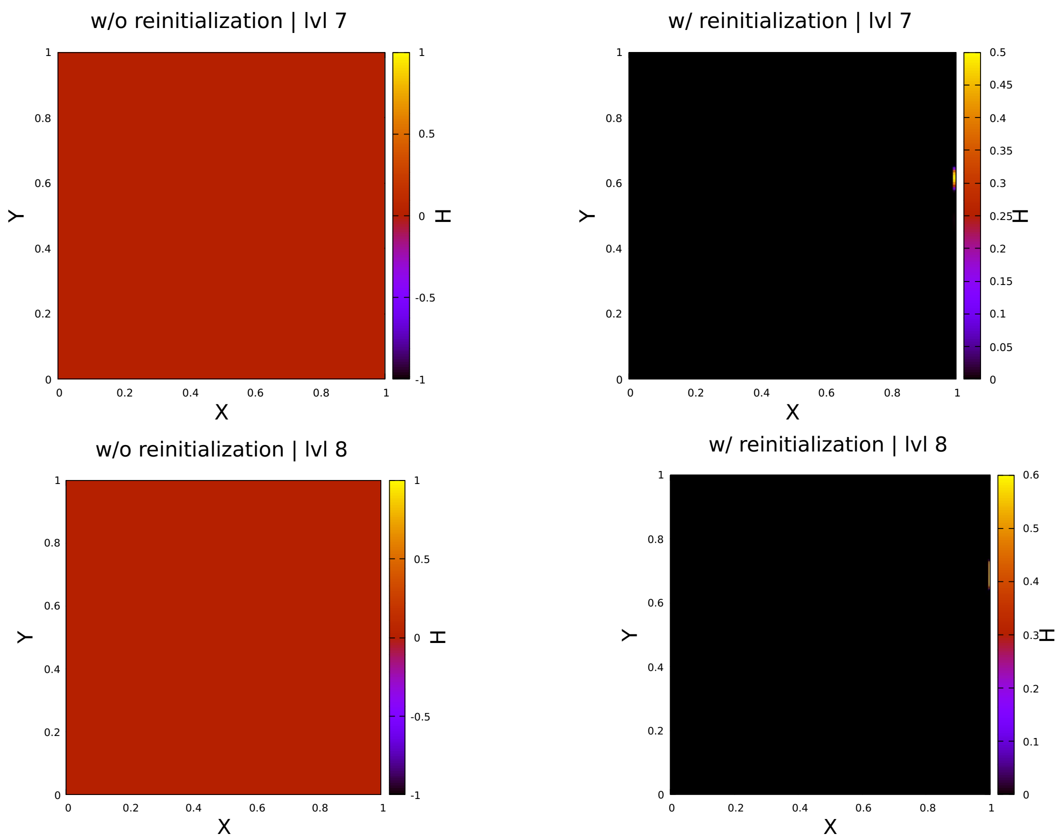

Figure 8: Snapshots of the heavyside function field for all four cases at t = 6.

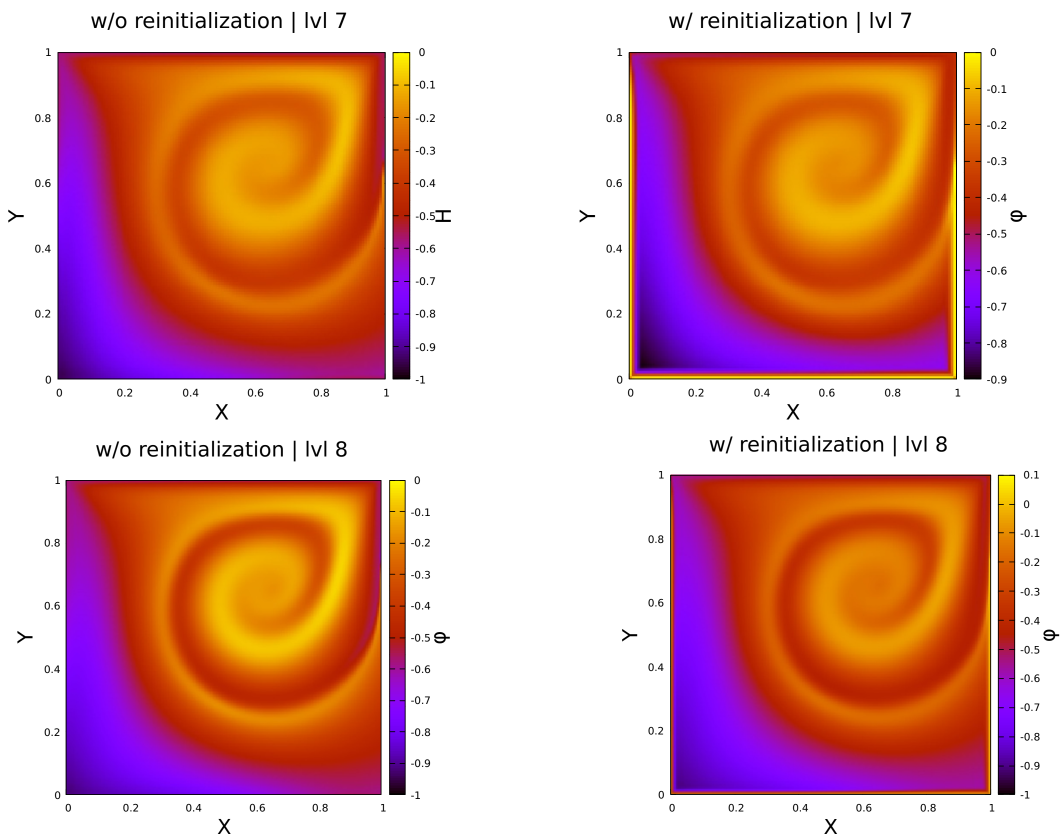

Figure 9: Snapshots of the level-set function field for all four cases at t = 6.

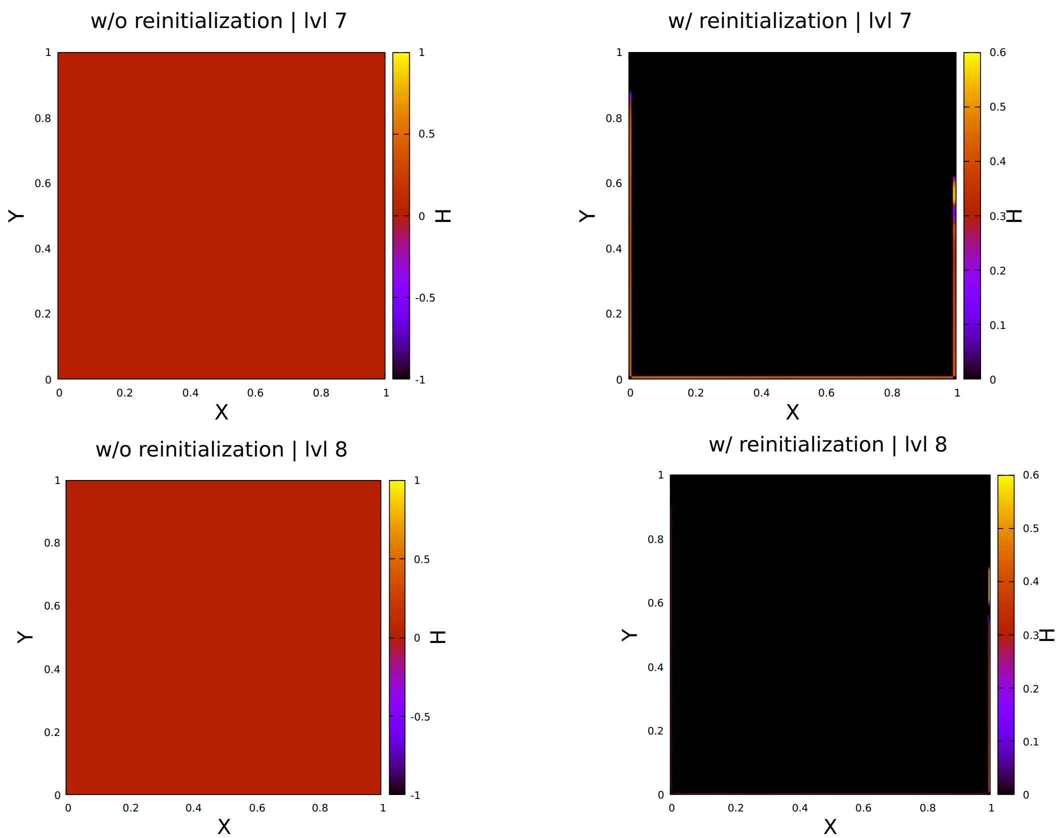

Figure 10: Snapshots of the heavyside function field for all four cases at t = 8.

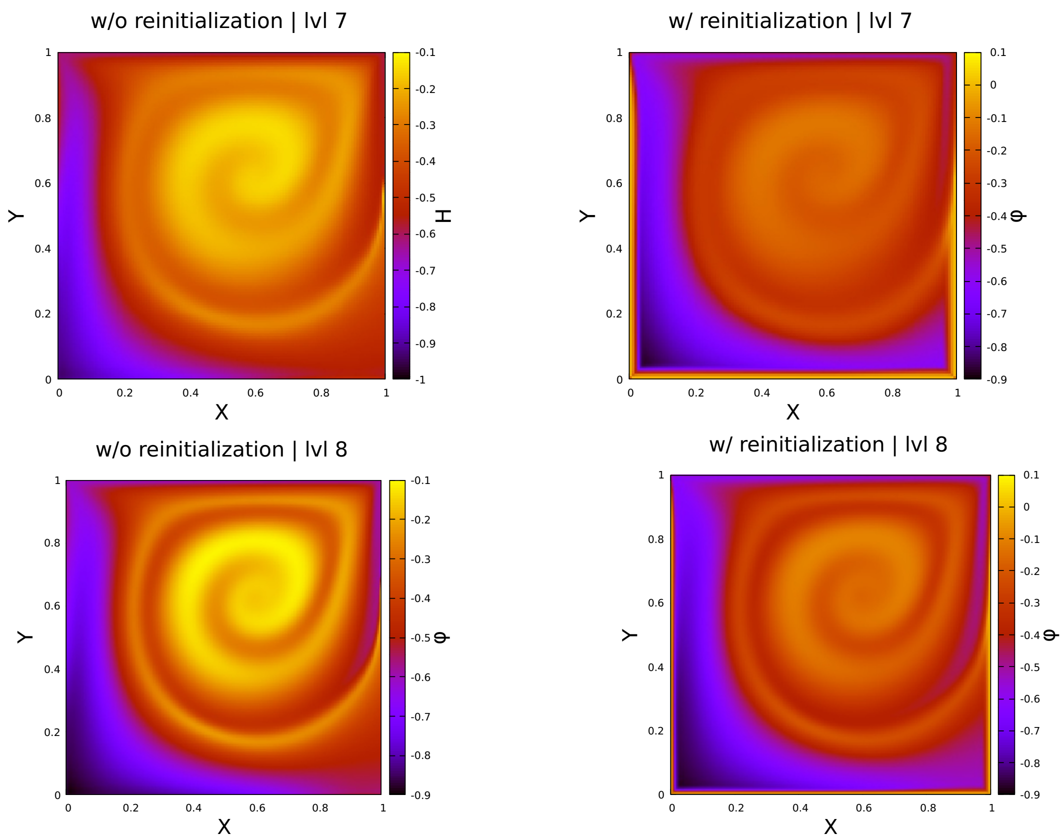

Figure 11: Snapshots of the level-set function field for all four cases at t = 8.

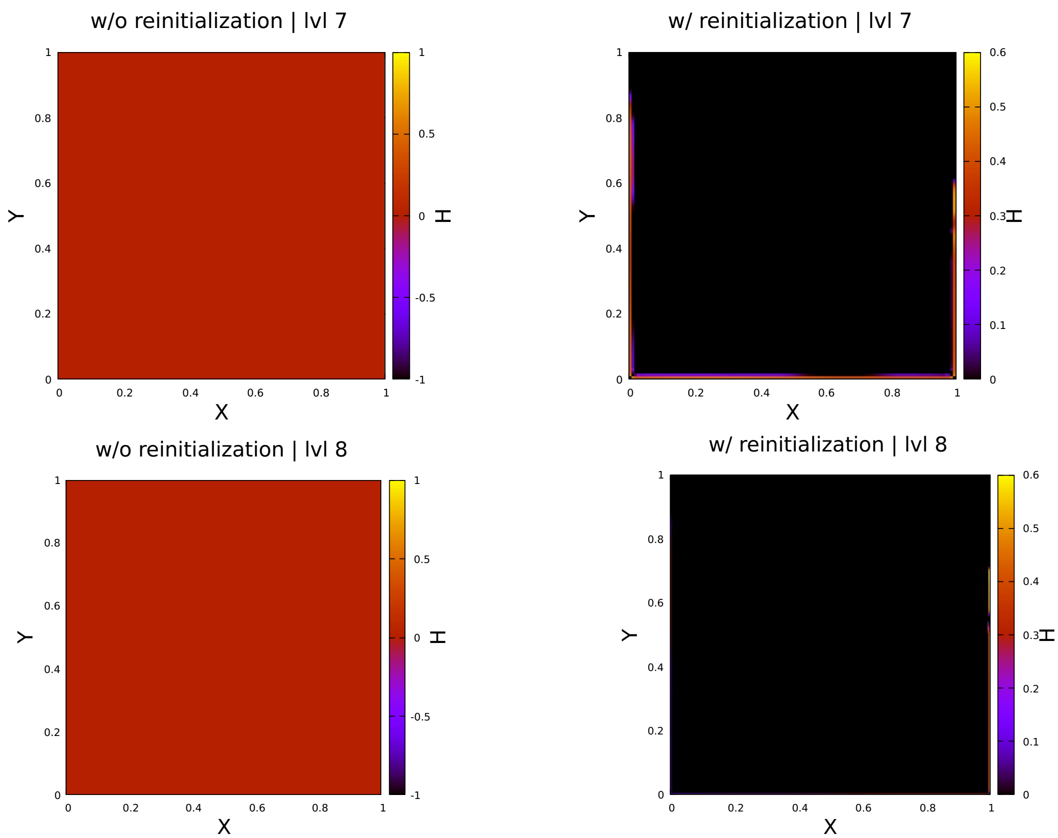

Figure 12: Snapshots of the heavyside function field for all four cases at t = 10.

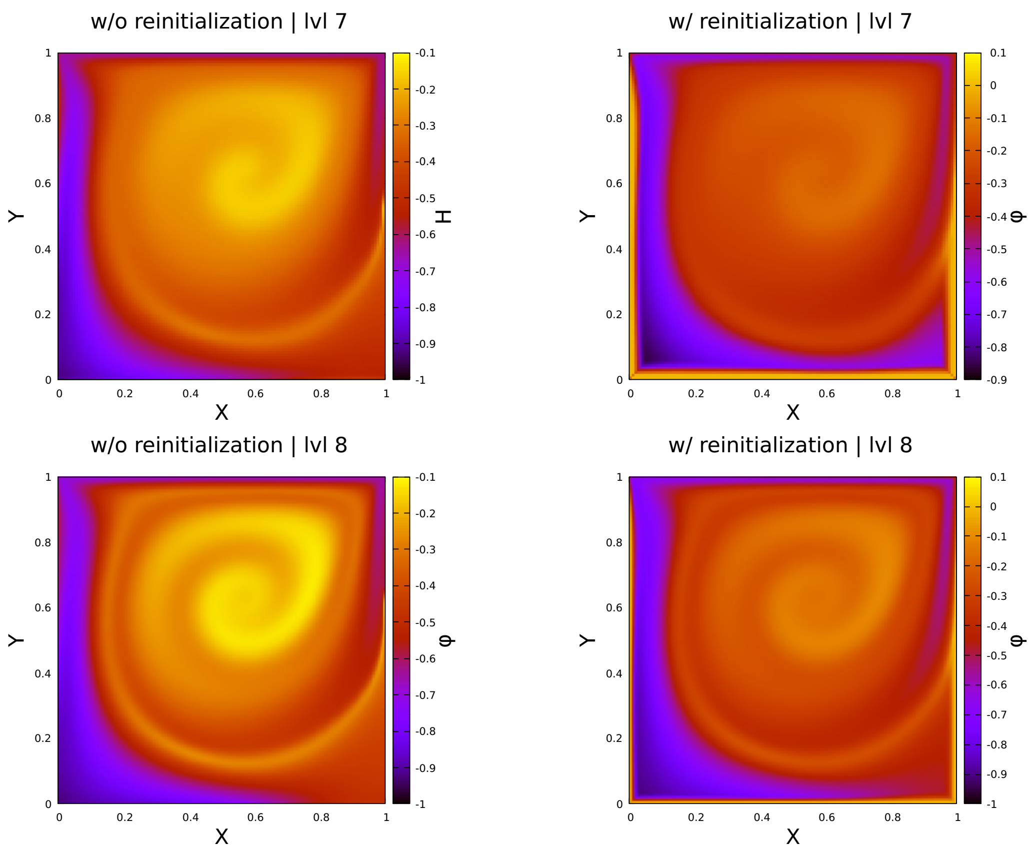

Figure 13: Snapshots of the level-set function field for all four cases at t = 10.