Solutions for Questions in Chapter 6

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1 Question 6.1

1.1 Solution

As there are 2 classes (*pos* and *neg*) in this problem and the prior probabilities for each class is equal, we have:

$$P(pos) = P(neg) = 0.5$$

For the given sentence S = "I always like foreign films.", the class that the Naive bayes classifier would assign to S is computed by:

$$\begin{aligned}\hat{c} &= \text{argmax}_{c \in \{pos, neg\}} P(c) \prod_{i \in \text{positions of } S} P(w_i|c) \\ &= \text{argmax}_{c \in \{pos, neg\}} P(c) P(\text{I}|c) P(\text{always}|c) P(\text{like}|c) P(\text{foreign}|c) P(\text{films}|c)\end{aligned} \tag{1}$$

Note that we ignore the dot "." in the end of the sentence S as it does not appear in the vocabulary in this case.

For $c = pos$, we have:

$$\begin{aligned}&P(pos) P(\text{I}|pos) P(\text{always}|pos) P(\text{like}|pos) P(\text{foreign}|pos) P(\text{films}|pos) \\ &= 0.5 \times 0.09 \times 0.07 \times 0.29 \times 0.04 \times 0.08 \\ &= 2.9232 \times 10^{-06}\end{aligned} \tag{2}$$

For $c = neg$, we have:

$$\begin{aligned}&P(neg) P(\text{I}|neg) P(\text{always}|neg) P(\text{like}|neg) P(\text{foreign}|neg) P(\text{films}|neg) \\ &= 0.5 \times 0.16 \times 0.06 \times 0.06 \times 0.15 \times 0.11 \\ &= 4.752 \times 10^{-06}\end{aligned} \tag{3}$$

From equations (1), (2) and (3), we conclude that the Naive bayes classifier would assign the "*neg*" class to the given sentence.

2 Question 6.2

2.1 Solution

There are two classes for classification in this problem, i.e, "*comedy*" and "*action*" for movie reviews. In order to compute the most likely class for the new document D using the naive Bayes classifier, we

first build the vocabulary from the training documents and then compute likelihood for each word in the vocabulary (using the add-1 smoothing).

By taking the union of all the words occurring in the training documents, our vocabulary includes the 7 following words:

fun, *couple*, *love*, *fast*, *furious*, *shoot*, *fly*

From the training documents, the frequencies of the vocabulary words with respect to different classes ($count(word, class)$) are shown in the following table:

	$count(*, comedy)$	$count(*, action)$
fun	3	1
couple	2	0
love	2	1
fast	1	2
furious	0	2
shoot	0	4
fly	1	1
Total	9	11

Table 1: Word Frequencies with Respect to Different Classes

Using the add-1 smoothing, the likelihoods of the words with respect to the classes are shown below:

	$\hat{P}(*\|comedy)$	$\hat{P}(*\|action)$
fun	$\frac{3+1}{9+7} = \frac{4}{16}$	$\frac{1+1}{11+7} = \frac{2}{18}$
couple	$\frac{2+1}{9+7} = \frac{3}{16}$	$\frac{0+1}{11+7} = \frac{1}{18}$
love	$\frac{2+1}{9+7} = \frac{3}{16}$	$\frac{1+1}{11+7} = \frac{2}{18}$
fast	$\frac{1+1}{9+7} = \frac{2}{16}$	$\frac{2+1}{11+7} = \frac{3}{18}$
furious	$\frac{0+1}{9+7} = \frac{1}{16}$	$\frac{2+1}{11+7} = \frac{3}{18}$
shoot	$\frac{0+1}{9+7} = \frac{1}{16}$	$\frac{4+1}{11+7} = \frac{5}{18}$
fly	$\frac{1+1}{9+7} = \frac{2}{16}$	$\frac{1+1}{11+7} = \frac{2}{18}$

Table 2: Word Likelihoods with Respect to Different Classes

Regarding the priors, there are 3 examples of "*action*" and 2 examples of "*comedy*" in the training data, so:

$P(action) = \frac{3}{5}$ and $P(comedy) = \frac{2}{5}$

The most likely class for D = "*fast, couple, shoot, fly*" using the naive Bayes would be:

$$\begin{aligned}\hat{c} &= \text{argmax}_{c\in\{action,comedy\}} P(c) \prod_{i\in\text{positions of } D} P(w_i|c) \\ &= \text{argmax}_{c\in\{action,comedy\}} P(c)P(\text{fast}|c)P(\text{couple}|c)P(\text{shoot}|c)P(\text{fly}|c)\end{aligned} \tag{4}$$

Now, for $c = action$,

$$
\begin{aligned}
&P(action)P(\text{fast}|action)P(\text{couple}|action)P(\text{shoot}|action)P(\text{fly}|action) \\
&= \frac{3}{5} \times \frac{3}{18} \times \frac{1}{18} \times \frac{5}{18} \times \frac{2}{18} \\
&\approx 1.715 \times 10^{-4}
\end{aligned} \tag{5}
$$

For $c = comedy$, we have:

$$
\begin{aligned}
&P(comedy)P(\text{fast}|comedy)P(\text{couple}|comedy)P(\text{shoot}|comedy)P(\text{fly}|comedy) \\
&= \frac{2}{5} \times \frac{2}{16} \times \frac{3}{16} \times \frac{1}{16} \times \frac{2}{16} \\
&\approx 0.7324 \times 10^{-4}
\end{aligned} \tag{6}
$$

From equations (4), (5) and (6), we conclude that the most likely class for D using the naive Bayes classifier is "*action*.