书面作业2.2 参考解答或提示

第1部分 基础

无

第2部分 理论

无

第3部分 综合应用（T2 可选做）

T1. 形式化并证明下列自然语言推理是有效的. 个体域均为全总域.

（1）所有的玫瑰和蔷薇都是芳香带刺的，因此，所有的玫瑰都是带刺的.

（2） 三角函数都是周期函数；一些三角函数是连续函数；所以，一些周期函数是连续函数.

（3）所有图灵奖获得者都是计算机科学家,一些图灵奖获得者大学本科时主修数学专业.所以,一些计算机科学家在大学本科时主修数学专业.

（4）每个科学工作者都是勤奋的, 每个既勤奋又聪明的人在他的事业中都将获得成功, 王大志是科学工作者并且是聪明的, 所以, 王大志在他的事业中将获得成功.

（5）如果一个人长期吸烟或酗酒, 那么他身体绝不会健康; 如果一个人身体不健康，那么他就不能参加体育比赛;有人参加了体育比赛. 所以，有人不长期酗酒.

（6）小王是一年级生理科生; 每个非文科的一年级生都有辅导员; 凡小王的辅导员都是理科生; 所有的理科生都不是文科生. 所以，至少有一个不是文科生的辅导员.

(1) 设S(x):x是三角函数; T(x):x是周期函数; P(x):x是连续函数, 则推理可以形式化为:

$\forall x(S(x)\rightarrow T(x)), \exists x(S(x)\wedge P(x)) \Rightarrow \exists x(T(x)\wedge P(x))$.

证明:

(1) $\exists x(S(x)\wedge P(x))$	P
(2) $S(c)\wedge P(c)$	ES(1)
(3) $\forall x (S(x)\rightarrow T(x))$	P
(4) $S(c)\rightarrow T(c)$	US(3)
(5) $S(c)$	T,I(2)
(6) $T(c)$	T,I(4)(5)
(7) $P(c)$	T,I(2)
(8) $T(c)\wedge P(c)$	T,I(6)(7)
(9) $\exists x(T(x)\wedge P(x))$	EG(8)

(2) 设P(x):x是玫瑰; Q(x):x是蔷薇; R (x):x是芳香的; S(x):x是带刺的, 则推理可以形式化为:

$\forall x((P(x)\vee Q(x))\to(R(x)\wedge S(x))) \Rightarrow \forall x(P(x)\to S(x))$

证明：可以直接证明，也可以用CP规则，以及反证法.

直接法

(1) $\forall x((P(x)\vee Q(x))\to(R(x)\wedge S(x)))$ P

(2) $P(x)\vee Q(x)\to R(x)\wedge S(x)$ US(1)

(3) $(\neg P(x)\wedge\neg Q(x))\vee(R(x)\wedge S(x))$ T,I(2)

(4) $(\neg P(x)\vee S(x))\wedge(\neg P(x)\vee R(x))\wedge(\neg Q(x)\vee S(x))\wedge(\neg Q(x)\vee S(x))$ T,I(3)

(5) $\neg P(x)\vee S(x)$ T,I(4)

(6) $P(x)\to S(x)$ T,I(5)

$\forall x(P(x)\to S(x))$ UG,5

CP规则

(1) $\forall x((P(x)\vee Q(x))\to(R(x)\wedge S(x)))$ P

(2) $P(x)\vee Q(x)\to R(x)\wedge S(x)$ US(1)

(3) $P(x)$ P(附加，for: $P(x)\to S(x)$ ）

(4) $P(x)\vee Q(x)$ T,I(3)

(5) $R(x)\wedge S(x)$ T,I(2)(4)

(6) $S(x)$ T,I(5)

(7) $P(x)\to S(x)$ CP(3)(6)

(8) $\forall x(P(x)\to S(x))$ UG(7)

反证法(略).

（3）根据题意，论域应为全总域，现设S(x):x是图灵奖获得者; T(x):x是计算机科学家; P(x):x大学时主修数学, 则推理可以形式化为:

$\forall x(S(x)\to T(x)), \exists x(S(x)\wedge P(x)) \Rightarrow \exists x(T(x)\wedge P(x))$.

证明:

(1) $\exists x(S(x)\wedge P(x))$ P

(2) $S(c)\wedge P(c))$ ES(1)

(3) $\forall x\ (S(x)\to T(x))$ P

(4) $S(c)\to T(c)$ US(3)

(5) $S(c)$ T,I(2)

(6) $T(c)$ T,I(4)(5)

(7) $P(c)$ T,I(2)

(8) $T(c)\wedge P(c)$ T,I(6)(7)

(9) $\exists x(T(x)\wedge P(x))$ EG(8)

（4）令M(x)：x是人；K(x)：x是科学工作者；Q(x)：x勤奋；T(x)：x聪明；S(x)：x将获得成功；a：王大志，则推理可以形式化为：

$\forall x((M(x)\wedge K(x))\to Q(x)),\ \forall x((M(x)\wedge Q(x)\wedge T(x))\to S(x)),\ M(a)\wedge K(a)\wedge T(a)\Rightarrow S(a)$

证明：

(1) $M(a)\wedge K(a)\wedge T(a)$　P

(2) $\forall x((M(x)\wedge K(x))\to Q(x))$　P

(3) $(M(a)\wedge K(a))\to Q(a)$　US,(2)

(4) $M(a)\wedge K(a)$　T,I,(1)

(5) $Q(a)$　T,I,(2),(4)

(6) $M(a)\wedge T(a)$　T,I,(1)

(7) $M(a)\wedge Q(a)\wedge T(a)$　T,I,(5),(6)

(8) $\forall x((M(x)\wedge Q(x)\wedge T(x))\to S(x))$　P

(9) $(M(a)\wedge Q(a)\wedge T(a))\to S(a)$　US,(8)

10) $S(a)$　T,I,(7),(9)

（5）令M(x)：x是人；C(x)：x长期吸烟；K(x)：x长期酗酒；J(x)：x身体健康；P(x)：x能参加体育比赛，则推理可以形式化为：

$\forall x((M(x)\wedge(C(x)\vee K(x)))\to\neg J(x)),\ \forall x((M(x)\wedge\neg J(x))\to\neg P(x)),\ \exists x(M(x)\wedge P(x))\Rightarrow\exists x(M(x)\wedge\neg K(x))$

证明：

1) $\exists x(M(x)\wedge P(x))$　P

2) $M(c)\wedge P(c)$　ES,(1)

3) $\forall x((M(x)\wedge\neg J(x))\to\neg P(x))$　P

4) $(M(c)\wedge\neg J(c))\to\neg P(c)$　US,(3)

5) $P(c)$　T,I,(2)

6) $\neg(M(c)\wedge\neg J(c))$　T,I,(4),(5)

7) $\neg M(c)\vee J(c)$　R,E,(6)

8) $M(c)$　T,I,(2)

9) $J(c)$　T,I,(7),(8)

10) $\forall x((M(x)\wedge(C(x)\vee K(x)))\to\neg J(x))$　P

11) $(M(c)\wedge(C(c)\vee K(c)))\to\neg J(c)$　US,(10)

12) $\neg(M(c)\wedge(C(c)\vee K(c)))$　T,I,(9),(11)

13) $\neg M(c)\vee(\neg C(c)\wedge\neg K(c))$　R,E,(12)

14) $\neg C(c)\wedge\neg K(c)$　T,I,(8),(13)

15) $\neg K(c)$　T,I,(14)

16) $M(c)\wedge\neg K(c)$　T,I,(8),(15)

17） $\exists x(M(x)\land\neg K(x))$ EG,(16)

（6）令a：小王；S(x)：x是一年级生；L(x)：x是理科生；W(x)：x是文科生；F(x,y)：x是y的辅导员，则推理可以形式化为:

$\forall x(S(x)\land\neg W(x)\to\exists yF(y,x))$, $S(a)$, $L(a)$, $\forall x(F(x, a)\to L(x))$, $\forall x(L(x)\to\neg W(x)) \Rightarrow \exists x\exists y(\neg W(x)\land F(x, y))$.

证明:

(1) $\forall x(L(x)\to\neg W(x))$	P
(2) $L(a)\to\neg W(a)$	US,(1)
(3) $L(a)$	P
(4) $\neg W(a)$	T,I,(2),(3)
(5) $S(a)$	P
(6) $S(a)\land\neg W(a)$	T,I,(4),(5)
(7) $\forall x(S(x)\land\neg W(x)\to\exists yF(y,x))$	P
(8) $S(a)\land\neg W(a)\to\exists yF(y, a)$	US,(7)
(9) $\exists yF(y, a)$	T,I,(6),(8)
(10) $F(c, a)$	ES,(9)
(11) $\forall x(F(x, a)\to L(x))$	P
(12) $F(c, a)\to L(c)$	US,(11)
(13) $L(c)$	T,I,(10),(12)
(14) $L(c)\to\neg W(c)$	US,(1)
(15) $\neg W(c)$	T,I,(13),(14)
(16) $\neg W(c)\land F(c, a)$	T,I,(10),(15)
(17) $\exists y(\neg W(c)\land F(c, y))$	EG,(16)
(18) $\exists x\exists y(\neg W(x)\land F(x, y))$	EG,(17)

因此，该推理是有效的.

T2. Consider the following problem. We know that horses are faster than dogs and that there is a greyhound that is faster than every rabbit. We know that Harry is a horse and that Ralph is a rabbit. Our job is to derive the fact that Harry is faster than Ralph (可以应用直接证明方法与归结法. 注意要增加两个公认的事实：Greyhound是Dog，速度可以传递比较的).

Problem translated in FOPL:

$\forall x\ \forall y\ ((Horse(x) \land Dog(y)) \to Faster(x,y))$

$\exists y\ (Greyhound(y) \land \forall z\ (Rabbit(z) \to Faster(y,z)))$

Horse(Harry)

Rabbit(Ralph)

Derive the following fact:

Faster(Harry, Ralph)

Added axioms to represent commonsense knowledge:

$\forall y\ (Greyhound(y) \to Dog(y))$

$\forall x\ \forall y\ \forall z\ ((Faster(x,y) \land Faster(y,z)) \rightarrow Faster(x,z))$

Proving using Proof Theory and a set of inference rules

1.	$\forall x\ \forall y\ ((Horse(x) \land Dog(y)) \rightarrow Faster(x,y))$	Premise
2.	$\exists y\ (Greyhound(y) \land \forall z\ (Rabbit(z) \rightarrow Faster(y,z)))$	Premise
3.	$\forall y\ (Greyhound(y) \rightarrow Dog(y))$	Premise
4.	$\forall x \forall y \forall z\ (Faster(x,y) \land Faster(y,z) \rightarrow Faster(x,z))$	Premise
5.	Horse(Harry)	Premise
6.	Rabbit(Ralph)	Premise
7.	$Greyhound(Greg) \land \forall z\ (Rabbit(z) \rightarrow Faster(Greg,z))$	ES (2)
8.	Greyhound(Greg)	T,I (7)
9.	$\forall z\ (Rabbit(z) \rightarrow Faster(Greg,z))$	T,I (7)
10.	$Rabbit(Ralph) \rightarrow Faster(Greg,Ralph)$	US (9)
11.	Faster(Greg,Ralph)	T,I (6)(10)
12.	$Greyhound(Greg) \rightarrow Dog(Greg)$	US (3)
13.	Dog(Greg)	T,I (12)(8)
14.	$Horse(Harry) \land Dog(Greg) \rightarrow Faster(Harry, Greg)$	US (1)
15.	$Horse(Harry) \land Dog(Greg)$	T,I (5)(13)
16.	Faster(Harry, Greg)	T,I (14)(15)
17.	$Faster(Harry, Greg) \land Faster(Greg, Ralph) \rightarrow Faster(Harry,Ralph)$	US (4)
18.	$Faster(Harry, Greg) \land Faster(Greg, Ralph)$	T,I (11)(16)
19.	Faster(Harry,Ralph)	T,I (17)(19)

QED.