书面作业1.3 参考解答或提示

第1部分 基础

无

第2部分 理论

T1. 请用直接法和间接法证明 $(P\vee Q)\wedge(P\rightarrow R)\wedge(Q\rightarrow S)\wedge\neg S\Rightarrow R$.

直接证明

1) $(P\vee Q)\wedge(P\rightarrow R)\wedge(Q\rightarrow S)\wedge\neg S$	P
2) $Q\rightarrow S$	T,I, (1)
3) $\neg S$	T,I, (1)
4) $\neg Q$	T,I, (2)(3)
5) $P\vee Q$	T,I, (1)
6) P	T,I, (4)(5)
7) $P\rightarrow R$	T,I, (1)
8) R	T,I, (6)(7)

间接证明

1) $(P\vee Q)\wedge(P\rightarrow R)\wedge(Q\rightarrow S)\wedge\neg S$	P
2) $P\rightarrow R$	T,I, (1)
3) $\neg R$	P(附加)
4) $\neg P$	T,I, (2)(3)
5) $P\vee Q$	T,I, (1)
6) Q	T,I, (4)(5)
7) $Q\rightarrow S$	T,I, (1)
8) S	T,I, (6)(7)
9) $\neg S$	T,I, (1)
10) $S\wedge\neg S$	T,I, (8)(9)
11) R	IP

T2. 证明二难推理 $(\alpha\rightarrow\gamma)\wedge(\beta\rightarrow\gamma)\wedge(\alpha\vee\beta)\Rightarrow\gamma$ 与 假言三段论$(\alpha\rightarrow\beta)\wedge(\beta\rightarrow\gamma)\Rightarrow\alpha\rightarrow\gamma$

二难推理：

(1)(〈→©)∧(®→©)∧(〈∨®)	P
(2)(〈→©)∧(®→©)	T,I (1)
(3)(¬〈∨©)∧(¬®∨©)	R,E (2)
(4)(¬〈∧¬®)∨©	R,E (3)
(5)¬(〈∨®)∨©	R,E (4)
(6)〈∨®	T,I (1)
(7)©	T,I (5)(6)

假言三段论

(1)〈	P(附加，for 〈→©)

(2)(〈→®)∧(®→©)　P
(3)〈→®　T,I (2)
(4)®　T,I (1)(3)
(5)®→©　T,I (2)
(6)©　T,I (4)(5)
(7)〈→©　CP(1)–(6)

T3. 用CP规则证明:

A→(B→C)，(C∧D)→E，¬F→(D∧¬E)⇒A→(B→F).

证明:

1）A　P（附加）
2）A→(B→C)　P
3）B→C　T,I,(1),(2)
4）(C∧D)→E　P
5）C→(D→E)　R,E,(4)
6）B→(D→E)　T,I,(3),(5)
7）¬F→(D∧¬E)　P
8）(D→E)→F　R,E,(7)
9）B→F　T,I,(6),(8)
10）A→(B→F)　CP

T4. 用反证法证明:

(A→B)∧(C→D)，(B→E)∧(D→F)，¬(E∧F)，A→C⇒¬A.

证明：1）¬(¬A)　P（附加）
2）A　R,E,(1)
3）A→C　P
4）C　T,I,(2),(3)
5）(A→B)∧(C→D)　P
6）A→B　T,I,(5)
7）B　T,I,(2),(6)
8）C→D　T,I,(5)
9）D　T,I,(4),(8)
10）(B→E)∧(D→F)　P
11）B→E　T,I,(10)
12）E　T,I,(7),(11)
13）D→F　T,I,(10)
14）F　T,I,(9),(13)
15）¬E∧F)　P
16）E→¬F　R,E,(15)
17）¬F　T,I,(12),(16)
18）F∧¬F　T,I,(14),(17),矛盾
19) ¬A　反证法（IP）

T5. Prove that the following rule, called the Destructive Dilemma rule(破坏性二难), can be derived from the original and derived proof rules.

Premises: ¬ C ∨ ¬ D， A → C， B → D

Conclusion: ¬ A ∨ ¬ B.

Textbook: (假言易位) A → C ≡ ¬ C→¬ A, B → D≡ ¬ D→¬ B, ...

(1) A → C	P
(2) ¬ C→¬ A	RE(1)
(3) B → D	P
(4) ¬ D→¬ B	RE(3)
(5) ¬ C ∨ ¬ D	P
(6) ¬ A ∨ ¬ B	TI(2)(4)(5) 构造性二难

QED（表示证明结束）.

或：结论转化为蕴含式，利用 CP 来证明，

(1) A → C	P
(2) B → D	P
(3) ¬ C ∨ ¬ D	P
(4) A	P(附加, forA → ¬B)
(5) C	TI(1)(4)
(6) ¬¬C	RE(5)
(7) ¬D	TI(3)(6)
(8) ¬B	TI(2)(7)
(9) A → ¬B	CP(4)(8)
(10) ¬ A ∨ ¬ B	RE(9)

QED.

T6. Two students came up with the following different wffs to formalize the statement(命题) "If A then B else C."

(A ∧ B) ∨ (¬ A ∧ C).

(A → B) ∧ (¬ A → C).

Prove that the two wffs are equivalent by finding formal proofs for the following two statements.

a. ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)).

b. ((A → B) ∧ (¬ A → C)) → ((A ∧ B) ∨ (¬ A ∧ C)).

真值表方法（略）

推理方法：变换+CP+ IP（间接证明）？

a.（含两个蕴含式，两次 CP 规则）

(1) A	P(附加, for A → B)
(2) ¬ ¬ A	RE(1)

(3) ¬ ¬ A ∨ ¬ C TI(2)

(4) ¬ (¬ A ∧ C) RE(3)

(5) (A ∧ B) ∨ (¬ A ∧ C) P(**附加, for** ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)))

(6) A ∧ B TI(4)(5)

(7) B TI(6)

(8) A → B CP(1)-(7)

(9) ¬ A **P(附加, for** ¬ A → C)

(10)¬ A ∨ ¬ B TI(9)

(11)¬ (A ∧ B) RE(10)

(12)¬ A ∧ C TI(5)(11)

(13)C TI(12)

(14)¬ A → C CP(9)-(13)

(15) (A → B) ∧ (¬ A → C) TP(9)(14)

(16) ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)) CP(5)(15)

QED.

a.（等值变换，直接证明前件为真，后件亦为真）

(1) (A ∧ B) ∨ (¬ A ∧ C) P(**附加, for** ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)))

(2) (A ∨¬ A) ∧ (A ∨ C) ∧ (B ∨¬ A) ∧ (B ∨ C) RE(1)

(3) (¬ A ∨ B) ∧ (A ∨ C) ∧ (B ∨ C) RE(2)

(4) (¬ A ∨ B) ∧ (A ∨ C) TI(3)

(5) (A → B) ∧ (¬ A → C) RE(4)

(6) ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)) CP(1)(5)

QED.

b.（直接证明方法）

(1) (A → B) ∧ (¬ A → C) P(**附加, for** ((A ∧ B) ∨ (¬ A ∧ C)) → ((A → B) ∧ (¬ A → C)))

(2) (¬B → ¬A) ∧ (¬ A → C) RE(1)

(3) (¬B → C) TI(2)

(4) (B ∨ C) RE(3)

(5) (¬ A ∨ B) ∧ (A ∨ C) RE(1)

(6) (A ∨ ¬ A) TI(排中律)

(7) (A ∨¬ A) ∧ (¬ A ∨ B) ∧ (A ∨ C) ∧ (B ∨ C) TI(4)(5)(6)

(8) (A ∧ B) ∨ (¬ A ∧ C) RE(7)

(9) (A → B) ∧ (¬ A → C)→(A ∧ B) ∨ (¬ A ∧ C) CP(1)(9)

QED.

也可以用构造性二难推得到（4）.

对于比较复杂的结论，就本题看，上述直接证明并非最佳方法. 结论为析取式时，还可以考虑反证法.

a.（IP 方法）

(1) ¬((A ∧ B) ∨ (¬ A ∧ C)) P(附加,for ((A → B) ∧ (¬ A → C)) → ((A ∧ B) ∨ (¬ A ∧ C)))

(2) (¬A ∨ ¬B) ∧ (A ∨ ¬C) RE(1)

(3) (B → ¬A) ∧ (C → A) RE(2)

(4) B → ¬A TI(3)

(5) C → A TI(3)

(6) (A → B) ∧ (¬ A → C) P

(7) A → B TI(6)

(8) $\neg A \to C$ TI(6)

(9) $A \to \neg A$ TI(4)(7)

(10) $\neg A$ RE(9)

(11) $\neg A \to A$ TI(5)(8)

(12) A RE(11)

(13) $\neg A \wedge A$ TI(10)(12) 矛盾

(14) $((A \to B) \wedge (\neg A \to C)) \to ((A \wedge B) \vee (\neg A \wedge C))$ IP(1)-(13)

QED.

第3部分 应用（T2选做）

T1. 在某一次足球比赛中，四支球队进行了比赛，已知情况如下，问结论是否有效? 请给出形式化证明.

前提：若A队得第一，则B队或C队获亚军；若C队获亚军，则A队不能获冠军；若D队获亚军，则B队不能获亚军；A 队获第一.

结论： D队不是亚军.

符号化:

令P: A队获冠军，Q: B队获亚军，R: C队获亚军:S: D队获亚军，则

前提: $P\to(Q\vee R)$，$R\to\neg P$, $S\to\neg Q$，P

结论: S

直接证明：

(1) P P

(2) $P\to(Q\vee R)$ P

(3) $Q\vee R$ T,I,(1),(2)

(4) $R\to\neg P$ P

(5) $\neg R$ T,I,(1),(4)

(6) Q T,I,(3),(5)

(7) $S\to\neg Q$ P

(8) $\neg S$ T,I,(6),(7)

反证法略.

T2. Consider the following argument that aims to prove that Superman does not exist.

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil he would be impotent(虚弱无能); if he were unwilling to prevent evil he would be malevolent（邪恶）;Superman does not prevent evil; If superman exists he is neither malevolent nor impotent. Therefore Superman does not exist.

First, letters are employed to represent the propositions as follows:

a: Superman is able to prevent evil
w: Superman is willing to prevent evil
i: Superman is impotent
m: Superman is malevolent
p: Superman prevents evil
e: Superman exists

Then, the argument above is formalized in propositional logic as follows:

Premises	
P_1	$(a \wedge w) \rightarrow p$
P_2	$(\neg a \rightarrow i) \wedge (\neg w \rightarrow m)$
P_3	$\neg p$
P_4	$e \rightarrow \neg i \wedge \neg m$
Conclusion	$P_1 \wedge P_2 \wedge P_3 \wedge P_4 \Rightarrow \neg e$

直接证明（利用构造性二难推理）

(1) a∧w→p	P
(2) ¬p	P
(3) ¬(a ∧ w)	TI(1)(2)
(4) ¬a ∨ ¬w	RE(3)
(5) (¬a → i) ∧ (¬w → m)	P
(6) i ∨ m	TI(4)(5) 构造性二难
(7) ¬(¬i ∧ ¬m)	RE(7)
(8) e → ¬i ∧ ¬m	P
(9) ¬e	TI(7)(8)

QED.

直接证明（利用二难推理）

(1) a∧w→p	P
(2) ¬p	P
(3) ¬(a ∧ w)	TI(1)(2)
(4) ¬a ∨ ¬w	RE(3)
(5) (¬a → i) ∧ (¬w → m)	P
(6) ¬a → i	TI(5)
(7) ¬a	P(附加，for ¬a → i∨m)
(8) i	TI(6)(7)
(9) i ∨ m	TI(8)
(10) ¬a → i∨m	CP(7)(9) 注：(7)-(10)为一个子证明
(11) ¬w → m	TI(5)
(12) ¬w	P(附加，for ¬w → i∨m)
(13) m	TI(11)(12)
(14) i ∨ m	TI(13)

(15) $\neg w \rightarrow i \vee m$	CP(12)(15) 注：(12)-(14)为一个子证明
(16) $\neg a \vee \neg w \rightarrow i \vee m$	TI(10)(15) 二难推理
(17) $i \vee m$	TI(4)(16)
(18) $\neg(\neg i \wedge \neg m)$	RE(17)
(19) $e \rightarrow \neg i \wedge \neg m$	P
(20) $\neg e$	TI(18)(19)
QED.	

反证法

(1) e	P(附加)
(2) $e \rightarrow \neg i \wedge \neg m$	P
(3) $\neg i \wedge \neg m$	TI(1)(2)
(4) $\neg i$	TI(3)
(5) $\neg m$	TI(3)
(6) $(\neg a \rightarrow i) \wedge (\neg w \rightarrow m)$	P
(7) $\neg a \rightarrow i$	TI(6)
(8) $\neg w \rightarrow m$	TI(6)
(9) a	TI(4)(7)
(10) w	TI(5)(8)
(11) $a \wedge w$	TI(9)(10)
(12) $a \wedge w \rightarrow p$	P
(13) p	TI(11)(12)
(14) $\neg p$	P
(15) $p \wedge \neg p$	TI(13)(14), 矛盾
QED.	IP(1)-(15)

下面证明过程仅供参考，其给出依据中 P_n 指使用第 n 个条件，给出了具体的依据，利用了一些教材未给出的规则. 同学们可以尝试把证明依据按照教材方式简写或补充, 第二种证明过程还可以简化，如直接应用拒取式而无需变换后使用析取三段论.

Proof that Superman does not exist

1.	$a \wedge w \rightarrow p$	*Premise* 1
2.	$(\neg a \rightarrow i) \wedge (\neg w \rightarrow m)$	*Premise* 2
3.	$\neg p$	*Premise* 3
4.	$e \rightarrow (\neg i \wedge \neg m)$	*Premise* 4
5.	$\neg p \rightarrow \neg(a \wedge w)$	1, *Contrapositive*
6.	$\neg(a \wedge w)$	*3, 5 Modus Ponens*
7.	$\neg a \vee \neg w$	*6, De Morgan's Law*
8.	$\neg(\neg i \wedge \neg m) \rightarrow \neg e$	*4, Contrapositive*
9.	$i \vee m \rightarrow \neg e$	*8, De Morgan's Law*
10.	$(\neg a \rightarrow i)$	*2,* $\wedge$ *Elimination*
11.	$(\neg w \rightarrow m)$	*2,* $\wedge$ *Elimination*
12.	$\neg\neg a \vee i$	10, A→ B equivalent to ¬A∨ B
13.	$\neg\neg a \vee i \vee m$	11, ∨ Introduction
14.	$\neg\neg a \vee (i \vee m)$	
15.	$\neg a \rightarrow (i \vee m)$	14, A → B equivalent to ¬A∨ B
16.	$\neg\neg w \vee m$	11, A → B equivalent to ¬A∨ B
17.	$\neg\neg w \vee (i \vee m)$	
18.	$\neg w \rightarrow (i \vee m)$	17, A → B equivalent to ¬A∨ B
19.	$(i \vee m)$	7, 15, 18 ∨*Elimination*
20.	$\neg e$	*9, 19 Modus Ponens*

Second Proof

1.	$\neg p$	P_3
2.	$\neg(a \wedge w) \vee p$	P_1 (A → B ≡ ¬A ∨ B)
3.	$\neg(a \wedge w)$	1, 2 A ∨ B, ¬B $\vdash$ A
4.	$\neg a \vee \neg w$	3, De Morgan's Law
5.	$(\neg a \rightarrow i)$	P_2 (∧-Elimination)
6.	$\neg a \rightarrow i \vee m$	5, $x \rightarrow y \vdash x \rightarrow y \vee z$
7.	$(\neg w \rightarrow m)$	P_2 (∧-Elimination)
8.	$\neg w \rightarrow i \vee m$	7, $x \rightarrow y \vdash x \rightarrow y \vee z$
9.	$(\neg a \vee \neg w) \rightarrow (i \vee m)$	8, $x \rightarrow z$, $y \rightarrow z \vdash x \vee y \rightarrow z$
10.	$(i \vee m)$	4, 9 Modus Ponens
11.	$e \rightarrow \neg(i \vee m)$	P_4 (De Morgan's Law)
12.	$\neg e \vee \neg(i \vee m)$	11, (A → B ≡ ¬A ∨ B)
13.	$\neg e$	10, 12 A ∨ B, ¬B $\vdash$ A

Therefore, the conclusion that Superman does not exist is a valid deduction from the given premises.