Asymmetric Aircraft Responses to Atmospheric Turbulence

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Asymmetric Aircraft Responses to Atmospheric Turbulence

For this lecture the following material was used:

- Chapter 8 of Lecture notes *Aircraft Responses to Atmospheric Turbulence*.

Contents of this lecture

Introduction

2-dim. auto-covariance functions + PSD functions

Elementary two-dimensional fields of flow

Asymmetric forces and moments due to gust velocities
- longitudinal turbulence
- lateral turbulence
- vertical turbulence

Approximation of effective one-dimensional PSD

Asymmetric equations of motions

Introduction

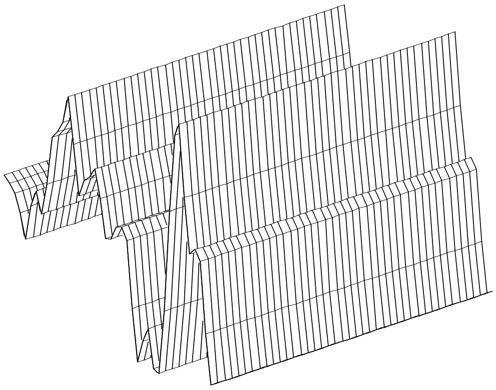
Preceding lecture: symmetric EOM in atmospheric turbulence.
- gust velocity: variations along longitudinal axis only
- aircraft motions: only in plane of symmetry

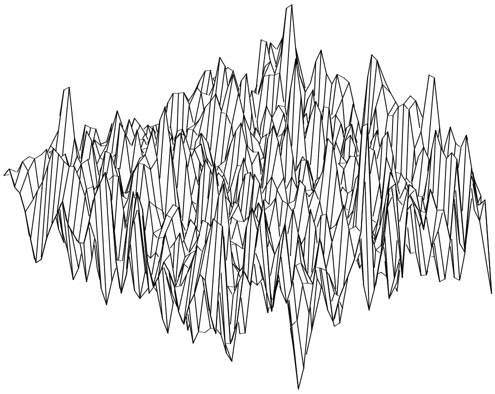
This lecture: asymmetric EOM in atmospheric turbulence.
- gust velocity: variations along longitudinal and lateral axis
- aircraft motions: asymmetric motions

→ Atmospheric turbulence (this lecture): **two-dimensional** process.

Introduction

Symmetric vs asymmetric turbulence (1-dimensional vs 2-dimensional):





2. auto-covariance functions + PSD functions

Assumption: atmospheric turbulence is:

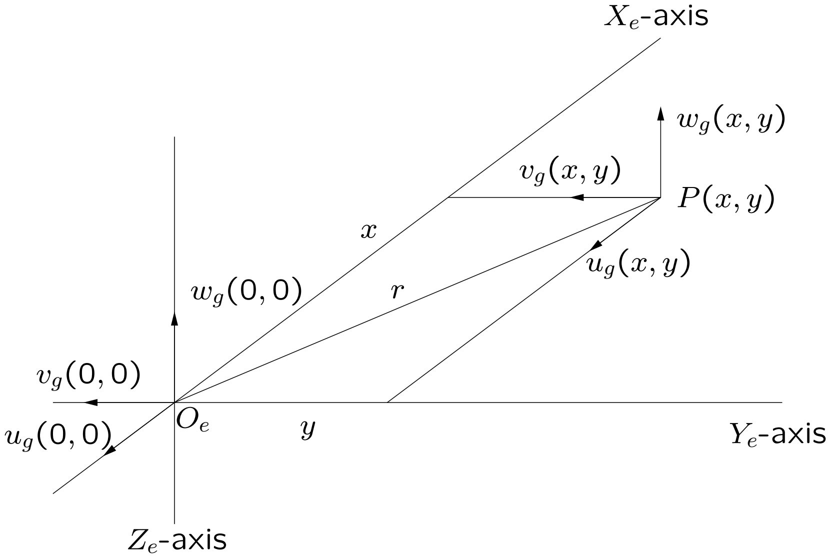
- stationary and frozen
- homogeneous
- isentropic

Atmospheric turbulence field is two-dimensional stochastic process:

$$\begin{aligned} u_g &= u_g(x,y) \\ v_g &= v_g(x,y) \\ w_g &= w_g(x,y) \end{aligned} \tag{8.1}$$

2-dim. auto-covariance functions + PSD functions

Auto-cov. func. = average relations between velocities in 2 points:



2-dim. auto-covariance functions + PSD functions

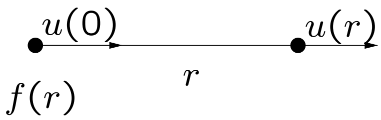
Auto-cov. funct.: between the origin O_e(0,0) and point P(x,y):

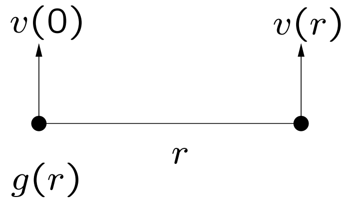
$$
\begin{aligned}
C_{u_g u_g} &= E\{u_g(0,0)\, u_g(x,y)\} \\
C_{v_g v_g} &= E\{v_g(0,0)\, v_g(x,y)\} \\
C_{w_g w_g} &= E\{w_g(0,0)\, w_g(x,y)\}
\end{aligned}
\tag{8.2}
$$

u_g, v_g and w_g are mutually independent → **cross-corr. func.** are 0.

2-dim. auto-covariance functions + PSD functions

Basic **one-dimensional** auto-covariance functions:

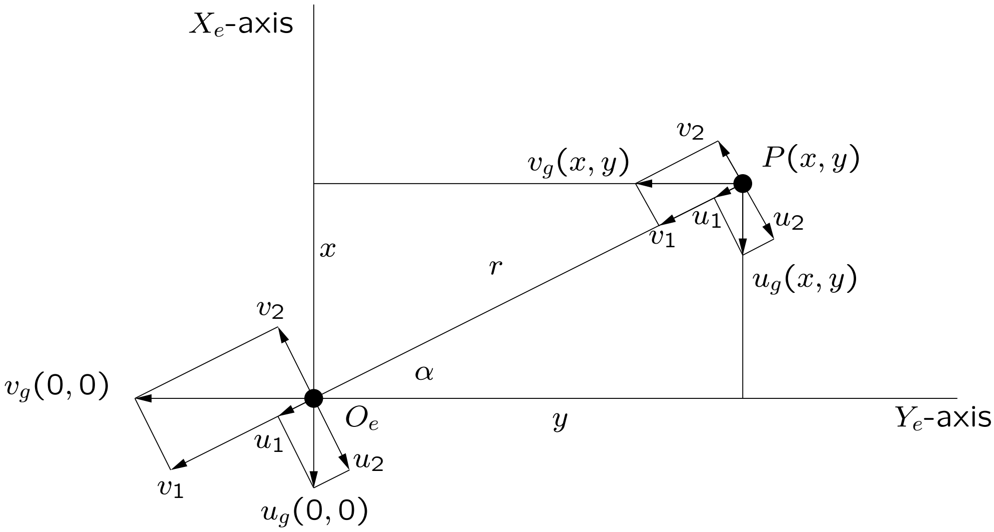




$$f(r) = \frac{\overline{u(0) \cdot u(r)}}{\bar{u}^2} \qquad g(r) = \frac{\overline{v(0) \cdot v(r)}}{\bar{v}^2}$$

2-dim. auto-covariance functions + PSD functions

One 2-dim. field $\rightarrow$ two 1-dim. fields:



2-dim. auto-covariance functions + PSD functions

When decomposed into components along and perp. to O_eP:

$$u_g = u_1 \sin\alpha + u_2 \cos\alpha = u_1 \frac{x}{r} + u_2 \frac{y}{r} \tag{8.3}$$

$$v_g = v_1 \cos\alpha + v_2 \sin\alpha = v_1 \frac{y}{r} + v_2 \frac{x}{r} \tag{8.4}$$

The expression for $C_{u_g u_g}$ then reads,

$$\begin{aligned} C_{u_g u_g} &= E\{u_g(0,0)\, u_g(x,y)\} \qquad (8.5) \\ &= E\left\{(u_1(0,0)\,\frac{x}{r} + u_2(0,0)\,\frac{y}{r})\ \ (u_1(x,y)\,\frac{x}{r} + u_2(x,y)\,\frac{y}{r})\right\} \\ &= E\left\{u_1(0,0)\,u_1(x,y)\ \left(\frac{x}{r}\right)^2 + u_2(0,0)\,u_2(x,y)\ \left(\frac{y}{r}\right)^2\right. \\ &+ \left. u_1(0,0)\,u_2(x,y)\ \left(\frac{xy}{r^2}\right) + u_2(0,0)\,u_1(x,y)\ \left(\frac{xy}{r^2}\right)\right\} \end{aligned}$$

2-dim. auto-covariance functions + PSD functions

Since,

$$E\{u_1(0,0)\,u_1(x,y)\} = \sigma_{u_g}^2 f(r) \tag{8.6}$$

$$E\{u_2(0,0)\,u_2(x,y)\} = \sigma_{u_g}^2 g(r) \tag{8.7}$$

$$E\{u_1(0,0)\,u_2(x,y)\} = E\{u_2(0,0)\,u_1(x,y)\} = 0 \tag{8.8}$$

it follows,

$$C_{u_g u_g}(x,y) = \sigma_{u_g}^2 \left\{f(r)\left(\frac{x}{r}\right)^2 + g(r)\left(\frac{y}{r}\right)^2\right\} \tag{8.9}$$

Similarly,

$$C_{v_g v_g}(x,y) = \sigma_{v_g}^2 \left\{f(r)\left(\frac{y}{r}\right)^2 + g(r)\left(\frac{x}{r}\right)^2\right\} \tag{8.10}$$

and,

$$C_{w_g w_g}(x,y) = \sigma_{w_g}^2 g(r) \tag{8.11}$$

2-dim. auto-covariance functions + PSD functions

With the **Dryden** covariance functions for $f(r)$ and $g(r)$ (6.28):

$$\begin{aligned} f(r) &= e^{-\frac{r}{L_g}} \\ g(r) &= e^{-\frac{r}{L_g}} \left(1 - \frac{1}{2}\frac{r}{L_g}\right) \end{aligned}$$

the **auto-cov. funct.** result in:

$$\boxed{\begin{aligned} C_{u_g u_g}\left(\frac{x}{L_g}, \frac{y}{L_g}\right) &= \sigma_{u_g}^2 \left\{ e^{-\frac{r}{L_g}} \left(\frac{x}{r}\right)^2 + e^{-\frac{r}{L_g}} \left(1 - \frac{1}{2}\frac{r}{L_g}\right) \left(\frac{y}{r}\right)^2 \right\} \quad (8.12) \\ C_{v_g v_g}\left(\frac{x}{L_g}, \frac{y}{L_g}\right) &= \sigma_{v_g}^2 \left\{ e^{-\frac{r}{L_g}} \left(\frac{y}{r}\right)^2 + e^{-\frac{r}{L_g}} \left(1 - \frac{1}{2}\frac{r}{L_g}\right) \left(\frac{x}{r}\right)^2 \right\} \quad (8.13) \\ C_{w_g w_g}\left(\frac{x}{L_g}, \frac{y}{L_g}\right) &= \sigma_{w_g}^2 e^{-\frac{r}{L_g}} \left(1 - \frac{1}{2}\frac{r}{L_g}\right) \quad (8.14) \end{aligned}}$$

2-dim. auto-covariance functions + PSD functions

Time-domain (cov.-func.) $\rightarrow$ freq.-domain (PSD func.)

$$\begin{aligned} &S_{u_g u_g}(\Omega_x L_g, \Omega_y L_g) \\ &S_{v_g v_g}(\Omega_x L_g, \Omega_y L_g) \\ &S_{w_g w_g}(\Omega_x L_g, \Omega_y L_g) \end{aligned} \quad (8.15)$$

with the use of the **2-dim. fourier-transform**.

$$S(\Omega_x L_g, \Omega_y L_g) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C\left(\frac{x}{L_g}, \frac{y}{L_g}\right) e^{-j(\Omega_x x + \Omega_y y)} \, d\frac{x}{L_g} d\frac{y}{L_g} \quad (8.16)$$

2-dim. auto-covariance functions + PSD functions

Two-dimensional power spectral densities:

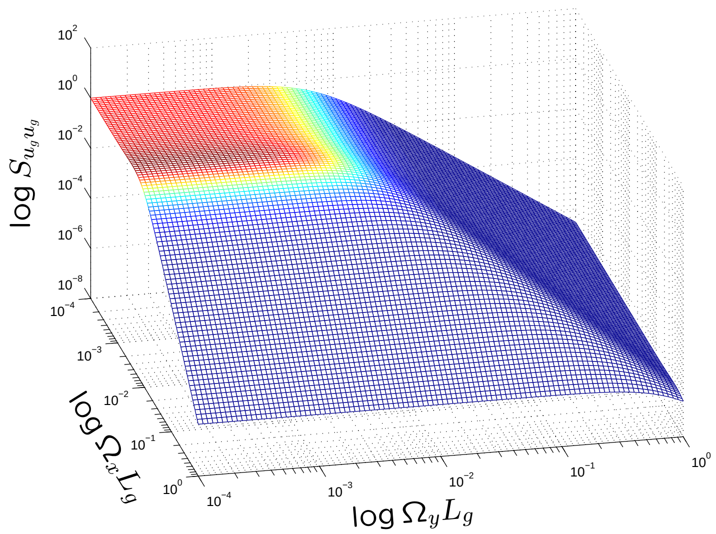
$$S_{u_g u_g}(\Omega_x L_g, \Omega_y L_g) = \pi \sigma_{u_g}^2 \frac{1 + \Omega_x^2 L_g^2 + 4\Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \tag{8.20}$$

$$S_{v_g v_g}(\Omega_x L_g, \Omega_y L_g) = \pi \sigma_{v_g}^2 \frac{1 + 4\Omega_x^2 L_g^2 + \Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \tag{8.21}$$

$$S_{w_g w_g}(\Omega_x L_g, \Omega_y L_g) = 3\pi \sigma_{w_g}^2 \frac{\Omega_x^2 L_g^2 + \Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \tag{8.22}$$

2-dim. auto-covariance functions + PSD functions

Graphical representation of $S_{u_g u_g}(\Omega_x L_g, \Omega_y L_g)$:



2-dim. auto-covariance functions + PSD functions

Relation between **2-dim.** spectrum and **1-dim.** spectrum:

$$S'_{u_g u_g}(\Omega_x L_g) = \frac{1}{\pi} \int_0^{+\infty} S_{u_g u_g}(\Omega_x L_g, \Omega_y L_g)\, d(\Omega_y L_g) \tag{8.23}$$

The one-dimensional spectrum is the sum (integral) of distributions of the 2-dim. spectrum at all lateral positions.

When elaborated,

$$S'_{u_g u_g}(\Omega_x L_g) = 2\sigma^2_{u_g} \frac{1}{1 + \Omega_x^2 L_g^2} \tag{8.24}$$

which is identical to the **one-dimensional Dryden power spectrum**.

Elementary two-dimensional fields of flow

Turbulence field = **superposition** of ∞ **elementary** fields of flow.

$$\begin{aligned} u_g &= u_{g_{max}} Re\left\{e^{j(\Omega_x x + \Omega_y y)}\right\} \\ v_g &= v_{g_{max}} Re\left\{e^{j(\Omega_x x + \Omega_y y)}\right\} \\ w_g &= w_{g_{max}} Re\left\{e^{j(\Omega_x x + \Omega_y y)}\right\} \end{aligned} \tag{8.25}$$

with Ω : spatial frequency
ω : circular frequency

The wavelengths are given by:

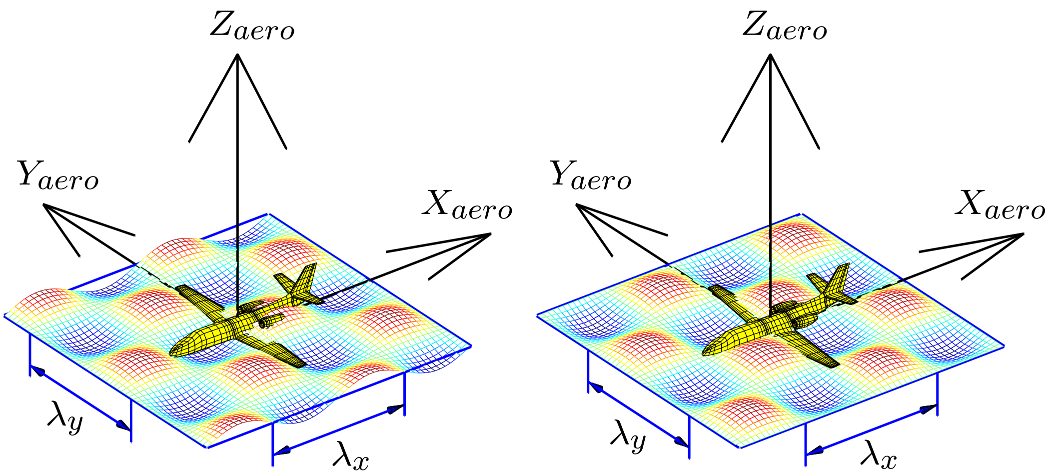
$$\lambda_x = \frac{2\pi}{\Omega_x},\ \lambda_y = \frac{2\pi}{\Omega_y}$$

Previous lecture: **1-dim. fields** $\leftrightarrow$ this lecture: **2-dim. fields**.

Note: obviously the 1-dim. fields are the 2-dim. fields for $\Omega_y = 0$.

Elementary two-dimensional fields of flow

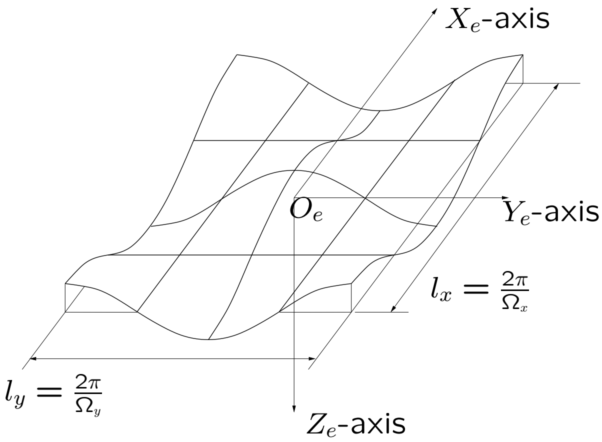
Symmetric vs **antisymmetric** 2-dim. elementary fields of flow:



a) symmetric b) antisymmetric

Elementary two-dimensional fields of flow

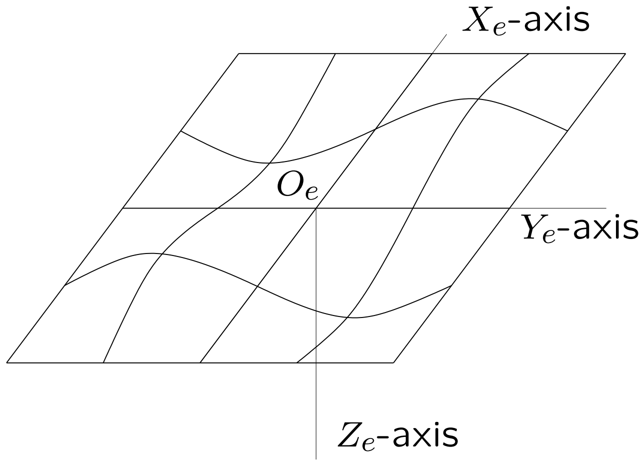
Symmetric 2-dim. elementary fields of flow:



Two-dimensional elementary flowfield, symmetric with respect to the $O_eX_eZ_e$-plane, in which the components of the gust velocity u_g, v_g or w_g change sinusoidally in the X_e- as well as in the Y_e-direction.

Elementary two-dimensional fields of flow

Antisymmetric 2-dim. elementary fields of flow:



Two-dimensional elementary flowfield, antisymmetric with respect to the $O_eX_eZ_e$-plane.

Elementary two-dimensional fields of flow

When written as:

$$\begin{aligned}
u_g &= u_{g_{max}} Re\left\{e^{j(\Omega_x x+\Omega_y y)}\right\} \\
&= u_{g_{max}} Re\left\{(\cos\Omega_x x + j\ \sin\Omega_x x)\,(\cos\Omega_y y + j\ \sin\Omega_y y)\right\} \\
&= u_{g_{max}}\,(\cos\Omega_x x\ \cos\Omega_y y - \sin\Omega_x x\ \sin\Omega_y y) \\
&= u_{g_1}(x,y) - u_{g_2}(x,y) \qquad (8.26)
\end{aligned}$$

each turbulence field can be written as **symmetric + antisymmetric** velocity-field (u_g=u_{g_1}-u_{g_2}), where

$$\begin{aligned}
u_{g_1}(x,y) &= u_{g_{max}}\ \cos\Omega_x x\ \cos\Omega_y y \quad \text{(symmetric)} \\
u_{g_2}(x,y) &= u_{g_{max}}\ \sin\Omega_x x\ \sin\Omega_y y \quad \text{(antisymmetic)}
\end{aligned} \qquad (8.27)$$

Elementary two-dimensional fields of flow

Similarly, the v_g- and w_g-fields can be separated in **symmetric** and **antisymmetric** parts:

$$\begin{array}{rcll} v_{g_1}(x,y) & = & v_{g_{max}} \cos\Omega_x x \, \cos\Omega_y y & \text{(antisymmetric)} \\ v_{g_2}(x,y) & = & v_{g_{max}} \sin\Omega_x x \, \sin\Omega_y y & \text{(symmetic)} \end{array} \qquad (8.28)$$

$$\begin{array}{rcll} w_{g_1}(x,y) & = & w_{g_{max}} \cos\Omega_x x \, \cos\Omega_y y & \text{(symmetric)} \\ w_{g_2}(x,y) & = & w_{g_{max}} \sin\Omega_x x \, \sin\Omega_y y & \text{(antisymmetic)} \end{array} \qquad (8.29)$$

Elementary two-dimensional fields of flow

When **aircraft plane of symmetry** coincides with $O_eX_eZ_e$-**plane**:

Symmetric velocity fields (u_{g_1}, v_{g_2} and w_{g_1})
→ symmetric aircraft deviations from steady flight.
→ in previous lecture: Ω_y=0, hence $v_{g_2}(x,y)$ omitted !!

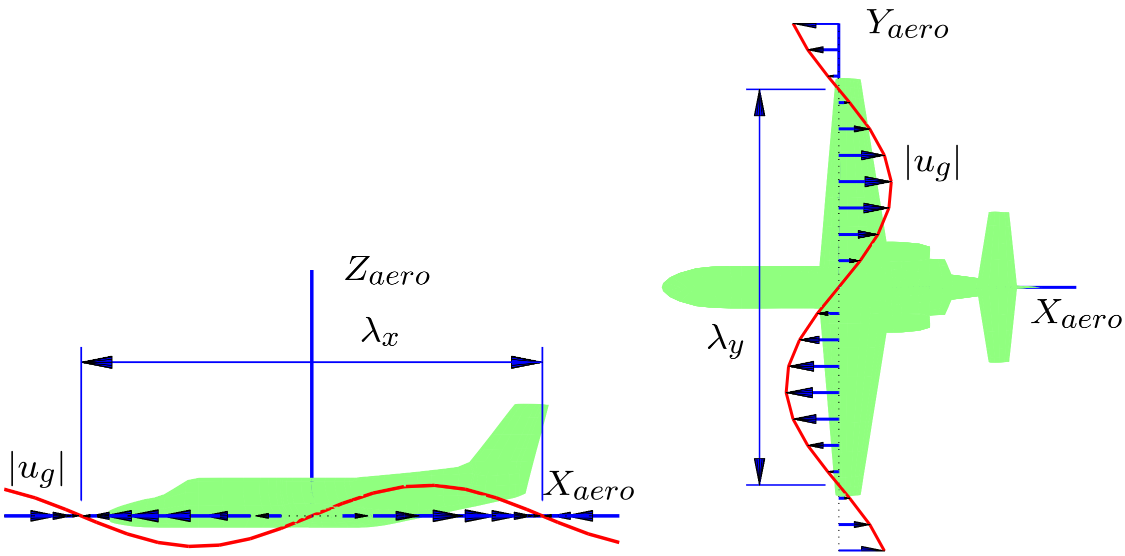
Antisymmetric velocity fields (u_{g_2}, v_{g_1} and w_{g_2})
→ asymmetric aircraft deviations from steady flight.

⇒ only antisymmetric elementary fields of flow are considered !

→ next: forces and moments due to these elementary velocity fields.
(longitudinal, lateral and vertical turbulence)

Forces and moments: longitudinal turbulence

Longitudinal turbulence:



Forces and moments: longitudinal turbulence

Antisymmetric part of the elementary u_g-field

$$u_{g_2}(x,y) = u_{g_{max}} \sin \Omega_x x \, \sin \Omega_y y$$

With,

$$u_g = u_{g_{max}} \sin \Omega_x x \tag{8.30}$$

u_{g_2} can be written as (index 2 is omitted):

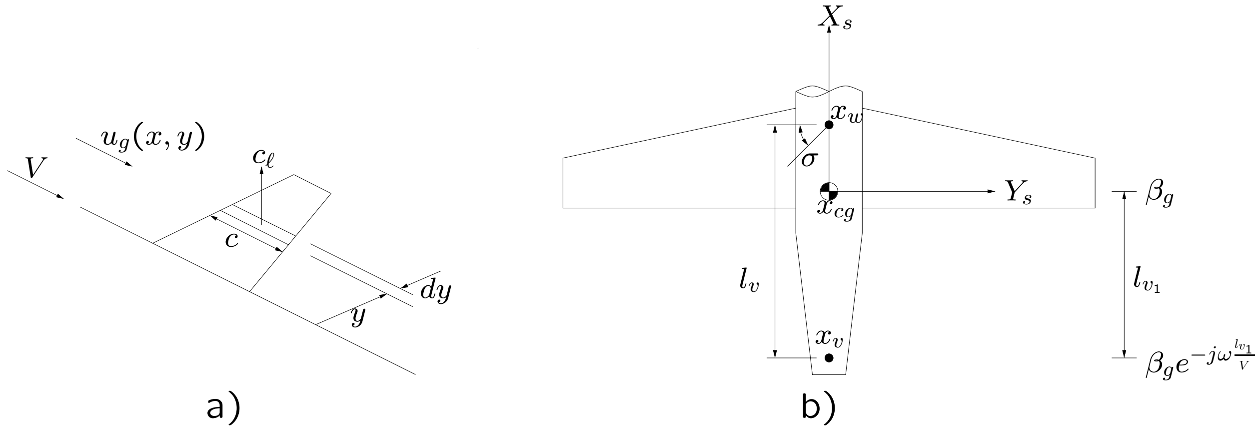
$$u_g(x,y) = u_g \sin \Omega_y y \tag{8.31}$$

Variations of u_g in the Y_e-direction $\rightarrow$ **rolling** and **yawing** moment.

(Sideforces are neglected)

Forces and moments: longitudinal turbulence

Strip-theory: wing is divided in strips: turbulence $\rightarrow$ **additional lift**



The contribution to the rolling moment by a chordwise strip of the wing of width dy at a distance y from the plane of symmetry (a), and description of the gust penetration effect for asymmetric aircraft motions (b).

Forces and moments: longitudinal turbulence

<u>**Rolling moment**</u>, due to

- (a) longitudinal turbulence $u_g(x,y)$
- (b) constant yawing velocity r

(a) $u_g(x,y)$ at chordwise **strip** $\rightarrow$ contributes to **rolling moment**:

$$dL_g = -c_l \ \frac{1}{2}\rho \left\{ [V + u_g(x,y)]^2 - V^2 \right\} \ c \ y \ dy = -\rho V \ u_{g_{max}} \ \sin\Omega_x x \ c_l c \ \sin\Omega_y y \ y \ dy \tag{8.32}$$

Total rolling moment:

$$L_g = -2\rho V \ u_g \int_0^{\frac{b}{2}} c_l c \ \sin\Omega_y y \ y \ dy \tag{8.33}$$

Forces and moments: longitudinal turbulence

$$C_{l_g} = \frac{L_g}{\frac{1}{2}\rho V^2 Sb} = -\frac{4}{Sb}\frac{u_g}{V}\int_0^{\frac{b}{2}} c_l c \,\sin\Omega_y y \; y \; dy \qquad (8.34)$$

Introducing **gust derivative** $C_{l_{u_g}}(\Omega_y \frac{b}{2})$, C_{l_g} is also written as:

$$C_{l_g} = C_{l_{u_g}}(\Omega_y \tfrac{b}{2}) \, \hat{u}_g \qquad (8.35)$$

Hence, $C_{l_{u_g}}$ can be written as,

$$\boxed{C_{l_{u_g}}(\Omega_y \tfrac{b}{2}) = -\tfrac{4}{Sb}\int_0^{\frac{b}{2}} c_l c \,\sin\Omega_y y \; y \; dy \simeq -\tfrac{4}{Sb}\int_0^{\frac{b}{2}} c_l c \; \Omega_y y^2 \; dy} \qquad (8.36)$$

(For small values of $\Omega_y \frac{b}{2}$, $u_g(x,y)$ varies approximately linearly.)

$$u_g(x,y) = u_g \sin\Omega_y y \simeq u_g \Omega_y y \qquad (8.37)$$

Forces and moments: longitudinal turbulence

(b) constant **yawing velocity** r → rolling moment:
$u_g(x,y)$ varies **linearly** instead of **sinusoidally** along wingspan and corresponds to additional velocity due to constant yawing velocity r:

$$\Delta u = -ry \qquad (8.38)$$

Total rolling moment (from strip-theory):

$$L = 2\rho V r \int_0^{\frac{b}{2}} c_l c \; y^2 \; dy = C_{l_{r_w}} \frac{rb}{2V}\frac{1}{2}\rho V^2 Sb \qquad (8.40)$$

With the contribution of the wing to C_{l_r}:

$$\boxed{C_{l_{r_w}} = \tfrac{8}{Sb^2}\int_0^{\frac{b}{2}} c_l c \; y^2 \; dy} \qquad (8.41)$$

Forces and moments: longitudinal turbulence

Thus with

(a): $$C_{l_{u_g}}(\Omega_y \frac{b}{2}) = -\frac{4}{Sb} \int_0^{\frac{b}{2}} c_l c \ \sin \Omega_y y \ y \ dy \tag{8.36}$$

(b): $$C_{l_{rw}} = \frac{8}{Sb^2} \int_0^{\frac{b}{2}} c_l c \ y^2 \ dy \tag{8.41}$$

and introduction of $h(\Omega_y \frac{b}{2})$:

$$h(\Omega_y \frac{b}{2}) = \frac{b}{2} \frac{\int_0^{\frac{b}{2}} c_l c \ \sin \Omega_y y \ y \ dy}{\int_0^{\frac{b}{2}} c_l c \ y^2 \ dy} \tag{8.43}$$

the gust derivative $C_{l_{u_g}}$ can be written as a function of $C_{l_{rw}}$.

$$\boxed{C_{l_{u_g}}(\Omega_y \frac{b}{2}) = -C_{l_{rw}} \ h(\Omega_y \frac{b}{2})} \tag{8.44}$$

Forces and moments: longitudinal turbulence

Yawing moment, due to
- longitudinal turbulence $u_g(x, y)$
- constant yawing velocity r

In an identical manner,

$$N_g = C_{n_g} \ \frac{1}{2} \rho V^2 Sb \tag{8.45}$$

where,

$$C_{n_g} = C_{n_{u_g}}(\Omega_y \frac{b}{2}) \ \hat{u}_g \tag{8.46}$$

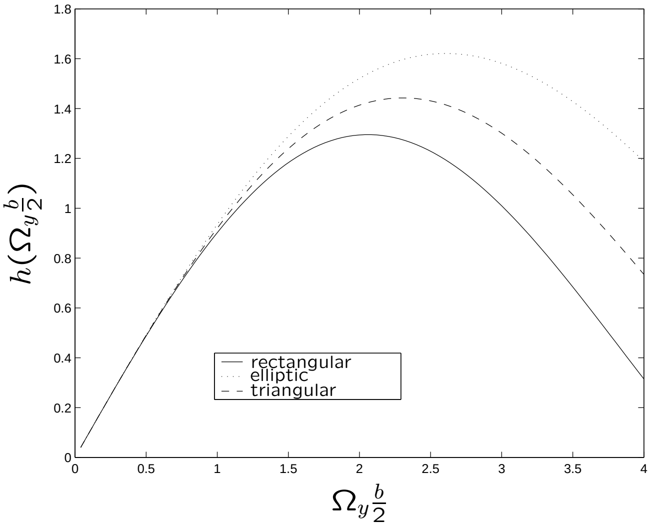
and,

$$\boxed{C_{n_{u_g}}(\Omega_y \frac{b}{2}) = -C_{n_{rw}} \ h(\Omega_y \frac{b}{2})} \tag{8.47}$$

Sideforce neglected: $C_{Y_{u_g}} = C_{Y_{rw}} = 0$ (8.48)

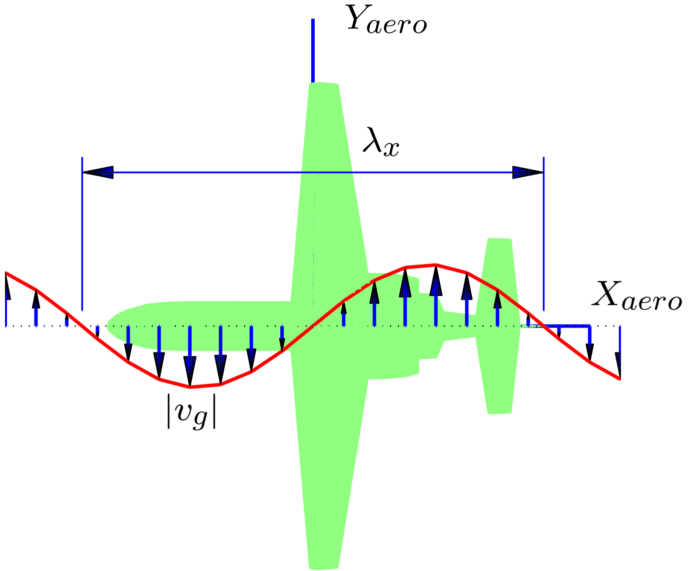
Forces and moments: longitudinal turbulence

$h(\Omega_y \frac{b}{2})$ for three different spanwise distributions of the lift.



Forces and moments: lateral turbulence

Lateral turbulence:



Forces and moments: lateral turbulence

Antisymmetric part of the elementary v_g-field

$$v_{g_1}(x,y) = v_{g_{max}}\ \cos\Omega_x x\ \cos\Omega_y y$$

Variation of v_{g_1} along wingspan neglected: $\cos\Omega_y y$=1
v_{g_1} can be written as (index 1 is omitted):

$$v_g = v_{g_{max}}\ \cos\Omega_x x \tag{8.49}$$

Definition of **gust angle of sideslip**:

$$\beta_g = \frac{v_g}{V} \tag{8.50}$$

Gust angle of sideslip → forces and moments (like α_g, previous lecture).

Forces and moments: lateral turbulence

This results in:

$$\begin{aligned} C_{Y_g} &= \left(C_{Y_{\beta_g}} + C_{Y_{\dot{\beta}_g}} D_b\right)\beta_g && (8.51)\\ C_{l_g} &= \left(C_{l_{\beta_g}} + C_{l_{\dot{\beta}_g}} D_b\right)\beta_g && (8.52)\\ C_{n_g} &= \left(C_{n_{\beta_g}} + C_{n_{\dot{\beta}_g}} D_b\right)\beta_g && (8.53)\end{aligned}$$

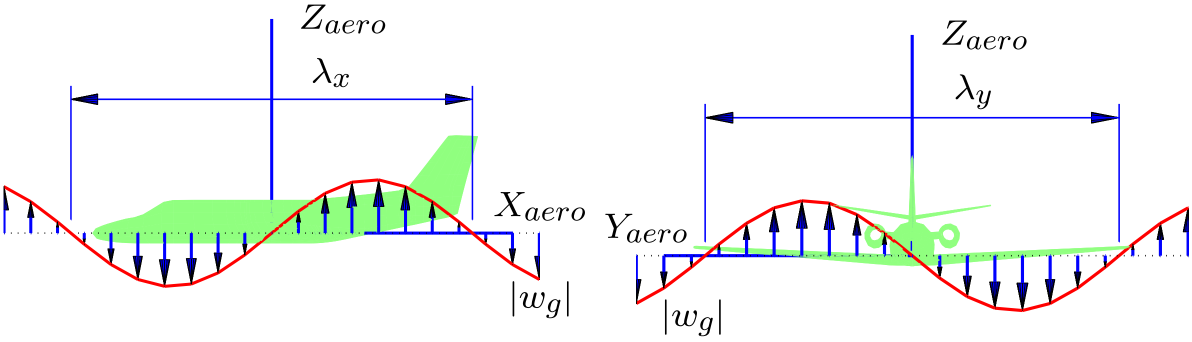
Analogue to the previous lecture,

$$\begin{array}{lllll} C_{Y_{\beta_g}} = C_{Y_\beta} & & C_{Y_{\dot{\beta}_g}} = C_{Y_{\dot{\beta}}} + \frac{1}{2}\,C_{Y_{r_{f+v}}} \\ C_{l_{\beta_g}} = C_{l_\beta} & & C_{l_{\dot{\beta}_g}} = C_{l_{\dot{\beta}}} + \frac{1}{2}\,C_{l_{r_{f+v}}} \\ C_{n_{\beta_g}} = C_{n_\beta} & & C_{n_{\dot{\beta}_g}} = C_{n_{\dot{\beta}}} + \frac{1}{2}\,C_{n_{r_{f+v}}} \end{array} \tag{8.55}$$

Straight wings and **small tailplane**: $C_{Y_{\dot{\beta}_g}} = C_{l_{\dot{\beta}_g}} = C_{n_{\dot{\beta}_g}} = 0$

Forces and moments: vertical turbulence

Vertical turbulence:



Forces and moments: vertical turbulence

Antisymmetric part of the elementary w_g-field

$$w_g(x,y) = w_{g_{max}} \sin \Omega_x x \; \sin \Omega_y y \;)$$

The **gust angle of attack**,

$$\alpha_g(x,y) = \frac{w_g(x,y)}{V} \tag{8.56}$$

varies along X_e and Y_e. With,

$$\alpha_g = \alpha_{g_{max}} \sin \Omega_x x \tag{8.57}$$

it follows:

$$\alpha_g(x,y) = \alpha_g \; \sin \Omega_y y \tag{8.58}$$

Gust angle of attack → rolling and yawing motions (like long. turb.)

Forces and moments: vertical turbulence

Analogue to longitudinal turbulence.

Rolling moment, due to
- vertical turbulence $\alpha_g(x,y)$
- constant rolling velocity p

Yawing moment, due to
- vertical turbulence $\alpha_g(x,y)$
- constant rolling velocity p

Two gust derivatives (roll and yaw):

$$C_{l_g} = C_{l_{\alpha g}}(\Omega_y \frac{b}{2})\ \alpha_g \tag{8.60}$$

$$C_{n_g} = C_{n_{\alpha g}}(\Omega_y \frac{b}{2})\ \alpha_g \tag{8.61}$$

Forces and moments: vertical turbulence

For **long wavelengths** in Y_e-direction:

$$\alpha_g(x,y) = \alpha_g\ \sin \Omega_y y \simeq \alpha_g\ \Omega_y y \tag{8.62}$$

This is an **approximation** by a **linear** distribution and corresponds to additional angle of attack due to **constant rolling velocity** p,

$$\Delta\alpha = \frac{p}{V} y \tag{8.63}$$

For small values of Ω_y,

$$\boxed{\begin{aligned} C_{l_{\alpha g}}(\Omega_y \frac{b}{2}) &= C_{l_{pw}}\ h(\Omega_y \frac{b}{2}) && (8.64) \\ C_{n_{\alpha g}}(\Omega_y \frac{b}{2}) &= C_{n_{pw}}\ h(\Omega_y \frac{b}{2}) && (8.65) \end{aligned}}$$

Sideforce neglected: $C_{Y_{\alpha g}} = C_{Y_{pw}} = 0$

Overview

For **symmetric** aircraft motions:

- forces: X, Z
- moments: m
- gust velocities: u_g, w_g
- varying along: X_e-axis (symmetric fields)

For **asymmetric** aircraft motions:

- forces: Y
- moments: l, n
- gust velocities: u_g, v_g, w_g
- varying along: X_e, Y_e-axis (only antisymmetric fields)
- exception: v_g varies only along X_e-axis

Approximation of effective one-dimensional PSD

Turbulence field:

-in X_e-direction: evolves with time: $\omega = \Omega_x V$
-in Y_e-direction: all values of Ω_y occur simultaneously.

To study the influence of Ω_y on C_{l_g} and C_{n_g}:
2-dimensional field $(X_e, Y_e) \rightarrow$ 1-dimensional "average" field (X_e).

One-dimensional spectra as function of $\Omega_x L_g$, due to u_g and w_g:
Consider C_{l_g} as function of $\Omega_x L_g$, $\Omega_y L_g$, and $B = \frac{b}{2L_g}$.

$$C_{l_g} = C_{l_{u_g}}(\Omega_y \frac{b}{2}) \hat{u}_g(\Omega_x L_g)$$

The PSD of C_{l_g} is then,

$$S_{C_{l_g}}(\Omega_x L_g, \Omega_y L_g, B) = C^2_{l_{u_g}}(\Omega_y \frac{b}{2})\; S_{\hat{u}_g}(\Omega_x L_g, \Omega_y L_g) \quad (8.66)$$

Note: duplication of indices of the PSD notation are omitted.

Approximation of effective one-dimensional PSD

When substituting $S_{\hat{u}_g}$ by,

$$S_{u_g u_g}(\Omega_x L_g, \Omega_y L_g) = \pi \sigma_{u_g}^2 \frac{1 + \Omega_x^2 L_g^2 + 4\Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \qquad (8.20)$$

and substituting $C_{l_{u_g}}$ by,

$$C_{l_{u_g}}(\Omega_y \frac{b}{2}) = -C_{l_{rw}}\ h(\Omega_y \frac{b}{2}) \qquad (8.44)$$

the PSD of C_{l_g} becomes,

$$S_{C_{l_g}}(\Omega_x L_g, \Omega_y L_g, B) \ = \ C_{l_{rw}}^2 \ \ h^2(\Omega_y \frac{b}{2}) \ \pi \ \sigma_{\hat{u}_g}^2 \frac{1 + \Omega_x^2 L_g^2 + 4\Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \qquad (8.67)$$

Approximation of effective one-dimensional PSD

The one-dimensional PSD of C_{l_g} as function of $\Omega_x Lg$ can be obtained by taking together all the contributions of Ω_y at a fixed Ω_x.

Hence, integration with respect to $\Omega_y Lg$ results,

$$S_{C_{l_g}}(\Omega_x L_g, B) \ = \ \sigma_{\hat{u}_g}^2 \ C_{l_{rw}}^2 \int_0^\infty h^2(\Omega_y \frac{b}{2}) \ \frac{1 + \Omega_x^2 L_g^2 + 4\Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \ d(\Omega_y L_g) \qquad (8.69)$$

$$S_{C_{l_g}}(\Omega_x L_g, B) \ = \ C_{l_{rw}}^2 \ I_{\hat{u}_g}(\Omega_x L_g, B) \qquad (8.70)$$

$$S_{C_{n_g}}(\Omega_x L_g, B) \ = \ C_{n_{rw}}^2 \ I_{\hat{u}_g}(\Omega_x L_g, B) \ \text{(similarly for } C_{n_g}\text{)} \qquad (8.73)$$

with effective 1-dim. PSD of $\hat{u}_g$ as a function of $\Omega_x L_g$ and B,

$$\boxed{I_{\hat{u}_g}(\Omega_x L_g, B) = \sigma_{\hat{u}_g}^2 \int_0^\infty h^2(\Omega_y \frac{b}{2}) \ \frac{1 + \Omega_x^2 L_g^2 + 4\Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \ d(\Omega_y L_g)} \qquad (8.72)$$

Approximation of effective one-dimensional PSD

The derivation can also be applied to the moments due to w_g:

$$S_{C_{l_g}}(\Omega_x L_g, B) = C_{l_{pw}}^2 \; I_{\alpha_g}(\Omega_x L_g, B) \tag{8.75}$$

$$S_{C_{n_g}}(\Omega_x L_g, B) = C_{n_{pw}}^2 \; I_{\alpha_g}(\Omega_x L_g, B) \tag{8.76}$$

With,

$$\boxed{I_{\alpha_g}(\Omega_x L_g, B) = 3\,\sigma_{\alpha_g}^2 \int_0^\infty h^2(\Omega_y \frac{b}{2}) \frac{\Omega_x^2 L_g^2 + \Omega_y^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2 + \Omega_y^2 L_g^2\right)^{5/2}} \, d(\Omega_y L_g)} \tag{8.77}$$

Approximation of effective one-dimensional PSD

When approximated:

$$I_{\hat{u}_g}(\Omega_x L_g, B) = I_{\hat{u}_g}(0, B) \frac{1 + \tau_3^2 \; \Omega_x^2 L_g^2}{\left(1 + \tau_1^2 \; \Omega_x^2 L_g^2\right)\left(1 + \tau_2^2 \; \Omega_x^2 L_g^2\right)} \tag{8.79}$$

$$I_{\alpha_g}(\Omega_x L_g, B) = I_{\alpha_g}(0, B) \frac{1 + \tau_6^2 \; \Omega_x^2 L_g^2}{\left(1 + \tau_4^2 \; \Omega_x^2 L_g^2\right)\left(1 + \tau_5^2 \; \Omega_x^2 L_g^2\right)} \tag{8.80}$$

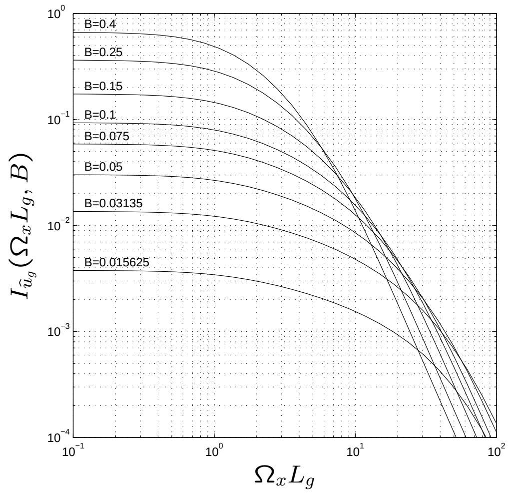
The PSD of β_g is,

$$S_{\beta_g \beta_g}(\Omega_x L_g) = \sigma_{\beta_g}^2 \frac{1 + 3\;\Omega_x^2 L_g^2}{\left(1 + \Omega_x^2 L_g^2\right)^2} \tag{8.84}$$

The values of $I_{\hat{u}_g}(0, B)$, τ_1, τ_2, τ_3 and $I_{\alpha_g}(0, B)$, τ_4, τ_5, τ_6 can be found in tables in the lecture notes.

Approximation of effective one-dimensional PSD

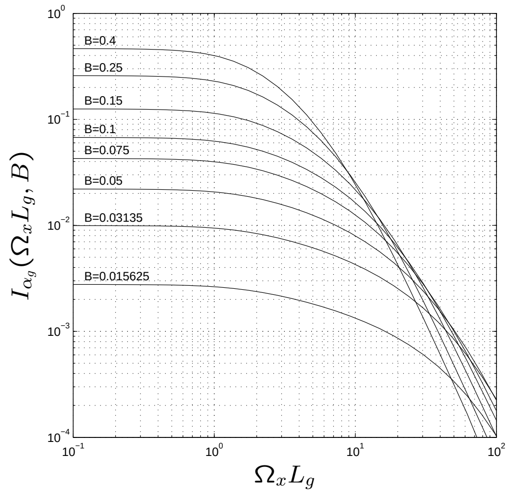
The effective one-dimensional PSD function for $\widehat{u}_g$ ($I_{\widehat{u}_g}$):



The effective (not approximated) one-dimensional power spectral density function of the horizontal gust velocity for different values of $B = \frac{b}{2L_g}$.

Approximation of effective one-dimensional PSD

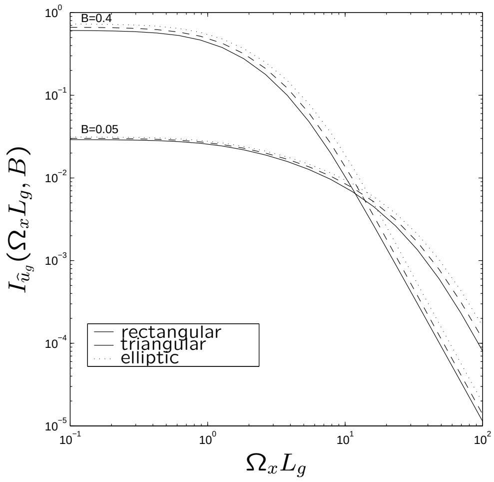
The effective one-dimensional PSD function for α_g (I_{α_g}):



The effective (not approximated) one-dimensional power spectral density function of the vertical gust velocity for different values of $B = \frac{b}{2L_g}$.

Approximation of effective one-dimensional PSD

$I_{\hat{u}_g}$ for three different lift distributions and two values for B:



The effective (not approximated) one-dimensional power spectral density function of the horizontal gust velocity for three different spanwise lift distributions at two different values for B.

Asymmetric equations of motions

For **rigid body**, small dev's from steady, symmetric and level flight:

$$\begin{bmatrix} C_{Y_\beta} - 2\mu_b D_b & C_L & C_{Y_p} & C_{Y_r} - 4\mu_b \\ 0 & -\frac{1}{2}D_b & 1 & 0 \\ C_{l_\beta} & 0 & C_{l_p} - 4\mu_b K_X^2 D_b & C_{l_r} + 4\mu_b K_{XZ} D_b \\ C_{n_\beta} & 0 & C_{n_p} + 4\mu_b K_{XZ} D_b & C_{n_r} - 4\mu_b K_Z^2 D_b \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \end{bmatrix} =$$

$$- \begin{bmatrix} 0 & C_{Y_{\delta_r}} & 0 & C_{Y_\beta} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & C_{l_{u_g}}(\Omega_y \frac{b}{2}) & C_{l_\beta} & C_{l_{\alpha_g}}(\Omega_y \frac{b}{2}) \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & C_{n_{u_g}}(\Omega_y \frac{b}{2}) & C_{n_\beta} & C_{n_{\alpha_g}}(\Omega_y \frac{b}{2}) \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ \hat{u}_g \\ \beta_g \\ \alpha_g \end{bmatrix} \qquad (8.85)$$

When used in conjunction with the random atmospheric turbulence model, and $I_{\hat{u}_g}$, I_{α_g} are used for $\hat{u}_g$ and α_g, the right-hand side can be modified.

Asymmetric equations of motions

This results in:

$$\begin{bmatrix} C_{Y_\beta} - 2\mu_b D_b & C_L & C_{Y_p} & C_{Y_r} - 4\mu_b \\ 0 & -\frac{1}{2}D_b & 1 & 0 \\ C_{l_\beta} & 0 & C_{l_p} - 4\mu_b K_X^2 D_b & C_{l_r} + 4\mu_b K_{XZ} D_b \\ C_{n_\beta} & 0 & C_{n_p} + 4\mu_b K_{XZ} D_b & C_{n_r} - 4\mu_b K_Z^2 D_b \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \end{bmatrix} =$$

$$- \begin{bmatrix} 0 & C_{Y_{\delta_r}} & 0 & C_{Y_\beta} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & -C_{l_{rw}} & C_{l_\beta} & -C_{l_{pw}} \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & -C_{n_{rw}} & C_{n_\beta} & -C_{n_{pw}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ \widehat{u}_g \\ \beta_g \\ \alpha_g \end{bmatrix} \tag{8.86}$$

Asymmetric equations of motions

This can be rewritten into **general state-space representation**:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\varphi} \\ \frac{\dot{pb}}{2V} \\ \frac{\dot{rb}}{2V} \end{bmatrix} = \begin{bmatrix} y_\beta & y_\varphi & y_p & y_r \\ 0 & 0 & 2\frac{V}{b} & 0 \\ l_\beta & 0 & l_p & l_r \\ n_\beta & 0 & n_p & n_r \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \end{bmatrix} + \begin{bmatrix} 0 & y_{\delta_r} & 0 & y_{\beta_g} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ l_{\delta_a} & l_{\delta_r} & l_{u_g} & l_{\beta_g} & l_{\alpha_g} \\ n_{\delta_a} & n_{\delta_r} & n_{u_g} & n_{\beta_g} & n_{\alpha_g} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ \widehat{u}_g \\ \beta_g \\ \alpha_g \end{bmatrix} \tag{8.87}$$

(coefficients: see table 8-4).

Turbulence field is modelled as **input** to the system, generated by turbulence filters.

Asymmetric equations of motions

Turbulence filters:

Consider the relation between the PSD of input and output signal,

$$S_{yy}(\omega) = |H(\omega)|^2 \, S_{uu}(\omega)$$

with white noise as input signal,

$$S_{uu}(\omega) = 1$$

and the turbulence velocities u_g, v_g and w_g as output signals.

Asymmetric equations of motions

Using the approximated 1-dim. PSDs $I_{\widehat{u}_g}$, I_{α_g} and $S_{\beta_g\beta_g}$ (with ω rather than Ω_x) in the relation above yields:

$$\left|H_{\widehat{u}_g w_1}(\omega)\right|^2 = \frac{L_g}{V} I_{\widehat{u}_g}(0, B) \, \frac{1 + \tau_3^2 \left(\frac{\omega L_g}{V}\right)^2}{\left(1 + \tau_1^2 \left(\frac{\omega L_g}{V}\right)^2\right)\left(1 + \tau_2^2 \left(\frac{\omega L_g}{V}\right)^2\right)} \quad (8.89)$$

$$\left|H_{\alpha_g w_3}(\omega)\right|^2 = \frac{L_g}{V} I_{\alpha_g}(0, B) \, \frac{1 + \tau_6^2 \left(\frac{\omega L_g}{V}\right)^2}{\left(1 + \tau_4^2 \left(\frac{\omega L_g}{V}\right)^2\right)\left(1 + \tau_5^2 \left(\frac{\omega L_g}{V}\right)^2\right)} \quad (8.90)$$

$$\left|H_{\beta_g w_2}(\omega)\right|^2 = \frac{L_g}{V} \sigma_{\beta_g}^2 \, \frac{1 + 3 \left(\frac{\omega L_g}{V}\right)^2}{\left(1 + \left(\frac{\omega L_g}{V}\right)^2\right)^2} \quad (8.91)$$

For example, consider $\left|H_{\widehat{u}_g w_1}(\omega)\right|^2$ for deriving the filter for $\widehat{u}_g$.

Asymmetric equations of motions

The frequency response function of the turbulence field for horizontal turbulence parallel to the longitudinal axis is given by:

$$H_{\widehat{u}_g w_1}(\omega) = \sqrt{\frac{L_g}{V} I_{\widehat{u}_g}(0,B)} \, \frac{1+\tau_3 \frac{L_g}{V} j\omega}{\left(1+\tau_1 \frac{L_g}{V} j\omega\right)\left(1+\tau_2 \frac{L_g}{V} j\omega\right)} \tag{8.92}$$

Transforming to time domain gives the differential equation:

$$\tau_1\tau_2 \left(\frac{L_g}{V}\right)^2 \ddot{\widehat{u}}_g(t) + (\tau_1+\tau_2)\frac{L_g}{V}\dot{\widehat{u}}_g(t) + \widehat{u}_g(t) =$$
$$= \sqrt{\frac{L_g}{V} I_{\widehat{u}_g}(0,B)} \, w_1(t) + \tau_3 \sqrt{\left(\frac{L_g}{V}\right)^3 I_{\widehat{u}_g}(0,B)} \, \dot{w}_1(t) \tag{8.93}$$

Asymmetric equations of motions

To obtain a state-space description, an auxiliary variable $\widehat{u}_g^*$ is introduced:

$$\widehat{u}_g^*(t) = \dot{\widehat{u}}_g(t) - \frac{\tau_3}{\tau_1\tau_2}\sqrt{\frac{V}{L_g} I_{\widehat{u}_g}(0,B)} \, w_1(t) \tag{8.94}$$

Differentiating (8.94) and substituting (8.93) and (8.94) yields:

$$\dot{\widehat{u}}_g^*(t) = \frac{1}{\tau_1\tau_2}\sqrt{\left(\frac{V}{L_g}\right)^3 I_{\widehat{u}_g}(0,B)} \, w_1(t) +$$
$$-\frac{\tau_1+\tau_2}{\tau_1\tau_2}\frac{V}{L_g}\,\widehat{u}_g^*(t) - \frac{\tau_3\,(\tau_1+\tau_2)}{(\tau_1\tau_2)^2}\sqrt{\left(\frac{V}{L_g}\right)^3 I_{\widehat{u}_g}(0,B)} \, w_1(t) - \frac{1}{\tau_1\tau_2}\left(\frac{V}{L_g}\right)^2 \widehat{u}_g(t) \tag{8.95}$$

Asymmetric equations of motions

In state-space form, using $[\widehat{u}_g \, \widehat{u}_g^*]^T$ as the state vector:

$$\begin{bmatrix} \dot{\widehat{u}}_g \\ \dot{\widehat{u}}_g^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_1\tau_2}\left(\frac{V}{L_g}\right)^2 & -\frac{\tau_1+\tau_2}{\tau_1\tau_2}\frac{V}{L_g} \end{bmatrix} \begin{bmatrix} \widehat{u}_g \\ \widehat{u}_g^* \end{bmatrix} + \begin{bmatrix} \frac{\tau_3}{\tau_1\tau_2}\sqrt{\frac{V}{L_g}I_{\widehat{u}_g}(0,B)} \\ \left(1-\frac{\tau_3(\tau_1+\tau_2)}{\tau_1\tau_2}\right)\frac{1}{\tau_1\tau_2}\sqrt{\left(\frac{V}{L_g}\right)^3 I_{\widehat{u}_g}(0,B)} \end{bmatrix} w_1 \quad (8.96)$$

Similarly, the filters for α_g and β_g can be derived:

$$\begin{bmatrix} \dot{\alpha}_g \\ \dot{\alpha}_g^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_4\tau_5}\left(\frac{V}{L_g}\right)^2 & -\frac{\tau_4+\tau_5}{\tau_4\tau_5}\frac{V}{L_g} \end{bmatrix} \begin{bmatrix} \alpha_g \\ \alpha_g^* \end{bmatrix} + \begin{bmatrix} \frac{\tau_6}{\tau_4\tau_5}\sqrt{\frac{V}{L_g}I_{\alpha_g}(0,B)} \\ \left(1-\frac{\tau_6(\tau_4+\tau_5)}{\tau_4\tau_5}\right)\frac{1}{\tau_4\tau_5}\sqrt{\left(\frac{V}{L_g}\right)^3 I_{\alpha_g}(0,B)} \end{bmatrix} w_3 \quad (8.97)$$

$$\begin{bmatrix} \dot{\beta}_g \\ \dot{\beta}_g^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\left(\frac{V}{L_g}\right)^2 & -2\frac{V}{L_g} \end{bmatrix} \begin{bmatrix} \beta_g \\ \beta_g^* \end{bmatrix} + \begin{bmatrix} \sigma_{\beta_g}\sqrt{\frac{3V}{L_g}} \\ (1-2\sqrt{3})\,\sigma_{\beta_g}\sqrt{\left(\frac{V}{L_g}\right)^3} \end{bmatrix} w_2 \quad (8.98)$$

Asymmetric equations of motions

With the use of the above given **turbulence filters** the EOM of an aircraft flying through turbulent air can be obtained:

$$\begin{bmatrix} \dot{X} \\ \dot{X}_g \end{bmatrix} = \begin{bmatrix} A_{X\dot{X}} & A_{X_g\dot{X}} \\ A_{X\dot{X}_g} & A_{X_g\dot{X}_g} \end{bmatrix} \begin{bmatrix} X \\ X_g \end{bmatrix} + \begin{bmatrix} B_{\delta\dot{X}} & B_{N\dot{X}_g} \end{bmatrix} \begin{bmatrix} \delta \\ N \end{bmatrix}$$

with δ being the control input vector and N being white noise.

Asymmetric equations of motions

$$\begin{bmatrix} A_{X\dot{X}} & A_{X_g\dot{X}} \\ A_{X\dot{X}_g} & A_{X_g\dot{X}_g} \end{bmatrix} \begin{bmatrix} X \\ X_g \end{bmatrix} =$$

$$\begin{pmatrix}
y_\beta & y_\varphi & y_p & y_r & 0 & 0 & 0 & 0 & y_{\beta_g} & 0 \\
0 & 0 & 2\frac{V}{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_\beta & 0 & l_p & l_r & l_{u_g} & 0 & l_{\alpha_g} & 0 & l_{\beta_g} & 0 \\
n_\beta & 0 & n_p & n_r & n_{u_g} & 0 & n_{\alpha_g} & 0 & n_{\beta_g} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\left(\frac{V}{L_g}\right)^2\frac{1}{\tau_1\tau_2} & -\frac{\tau_1+\tau_2}{\tau_1\tau_2}\left(\frac{V}{Lg}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\left(\frac{V}{L_g}\right)^2\frac{1}{\tau_4\tau_5} & -\frac{\tau_4+\tau_5}{\tau_4\tau_5}\left(\frac{V}{Lg}\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\left(\frac{V}{L_g}\right)^2 & -2\frac{V}{L_g}
\end{pmatrix} \cdot \begin{pmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \\ \widehat{u}_g \\ \widehat{u}_g^* \\ \alpha_g \\ \alpha_g^* \\ \beta_g \\ \beta_g^* \end{pmatrix}$$

Asymmetric equations of motions

$$\begin{bmatrix} B_{\delta\dot{X}} & B_{N\dot{X}_g} \end{bmatrix} \begin{bmatrix} \delta \\ N \end{bmatrix} =$$

$$\begin{pmatrix}
0 & y_{\delta_r} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
l_{\delta_a} & l_{\delta_r} & 0 & 0 & 0 \\
n_{\delta_a} & n_{\delta_r} & 0 & 0 & 0 \\
0 & 0 & \frac{\tau_3}{\tau_1\tau_2}\sqrt{\frac{V}{Lg}I_{\hat{u}_g}(0,B)} & 0 & 0 \\
0 & 0 & \left(1-\frac{\tau_3(\tau_1+\tau_2)}{\tau_1\tau_2}\right)\frac{1}{\tau_1\tau_2}\sqrt{\left(\frac{V}{Lg}\right)^3 I_{\hat{u}_g}(0,B)} & 0 & 0 \\
0 & 0 & 0 & \frac{\tau_6}{\tau_4\tau_5}\sqrt{\frac{V}{Lg}I_{\alpha_g}(0,B)} & 0 \\
0 & 0 & 0 & \left(1-\frac{\tau_6(\tau_4+\tau_5)}{\tau_4\tau_5}\right)\frac{1}{\tau_4\tau_5}\sqrt{\left(\frac{V}{Lg}\right)^3 I_{\alpha_g}(0,B)} & 0 \\
0 & 0 & 0 & 0 & \sigma_{\beta_g}\sqrt{\frac{3V}{Lg}} \\
0 & 0 & 0 & 0 & (1-2\sqrt{3})\sigma_{\beta_g}\sqrt{\left(\frac{3V}{Lg}\right)^3}
\end{pmatrix} \cdot \begin{pmatrix} \delta_a \\ \delta_r \\ w_1 \\ w_3 \\ w_2 \end{pmatrix}$$

Examples

Responses of the Cessna Ce-500 'Citation' to turbulence

$\sigma_{u_g} = \sigma_{v_g} = \sigma_{w_g} = 1$ m/s

Investigate the influence of:

turb. velocity comp.: long. (u_g), lat. (v_g), and vert. (w_g) gust

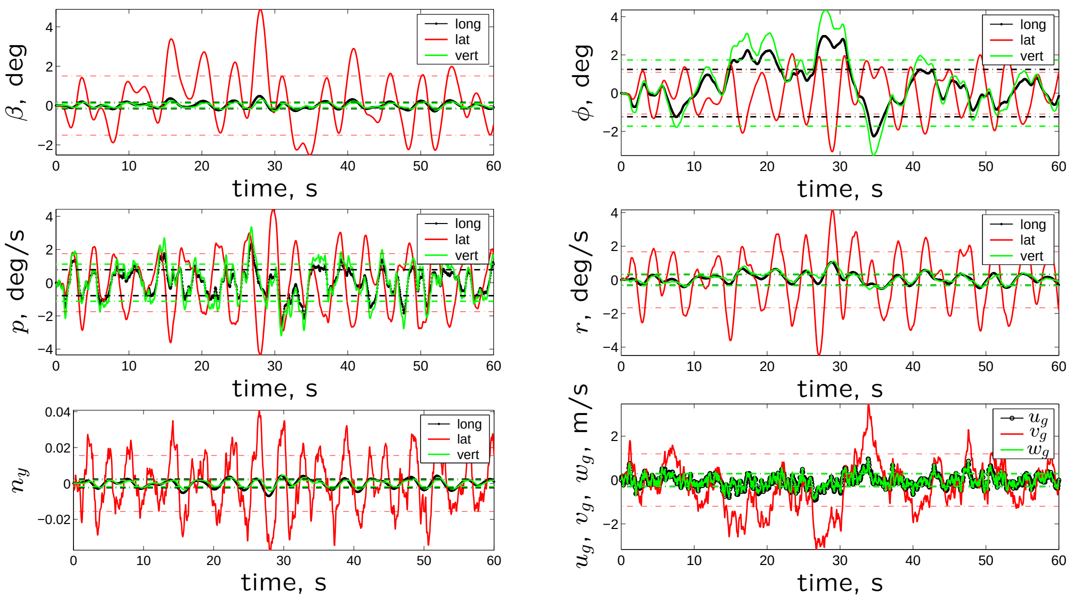
scale length: $L_g = 150$ m vs. $L_g = 1500$ m

flight condition: 'landing' $V = 59.9$ m/s vs. 'cruise' $V = 181.9$ m/s

position in the a/c: front, c.g., and rear

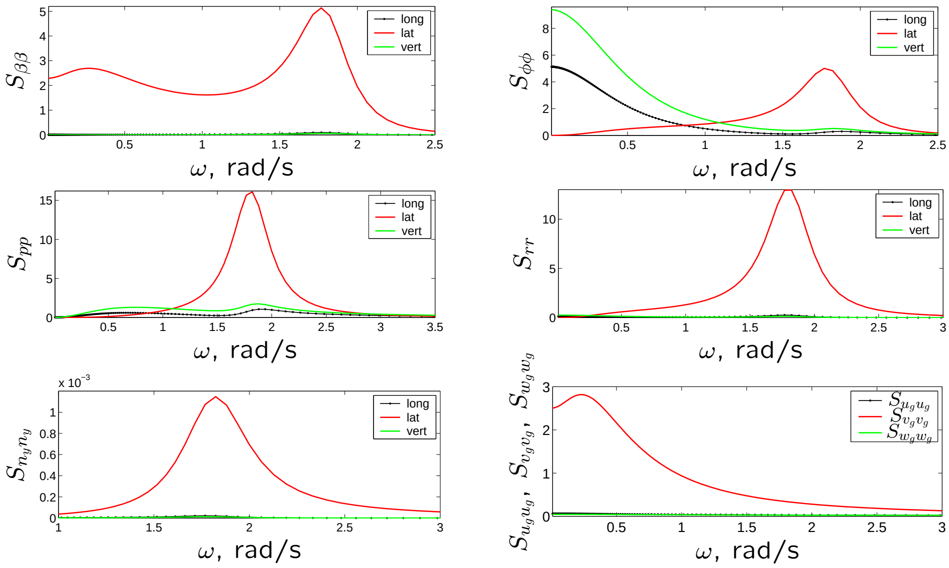
Examples

Longitudinal, lateral, and vertical turbulence



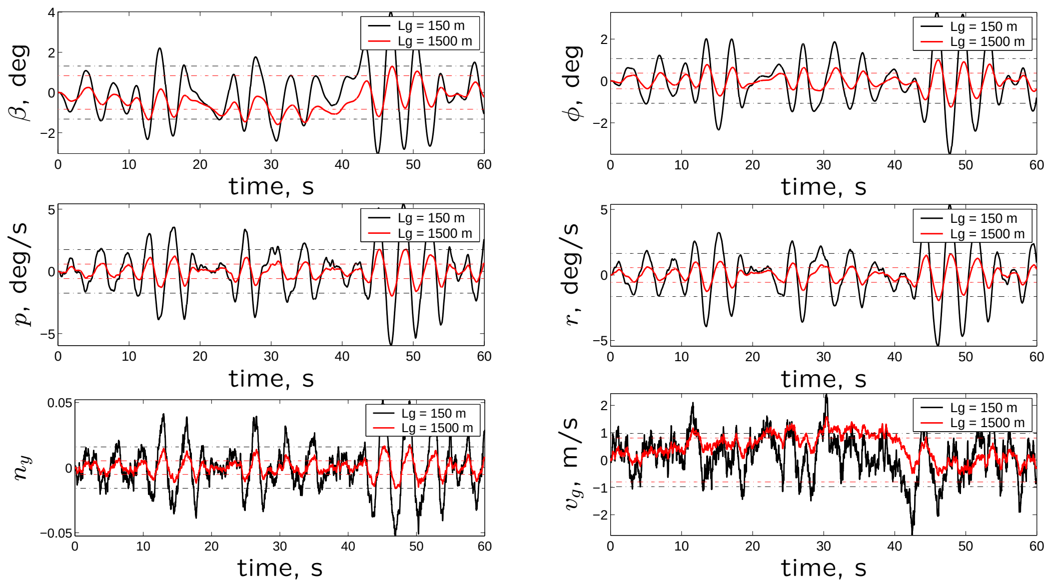
Examples

Longitudinal, lateral, and vertical turbulence (PSD)



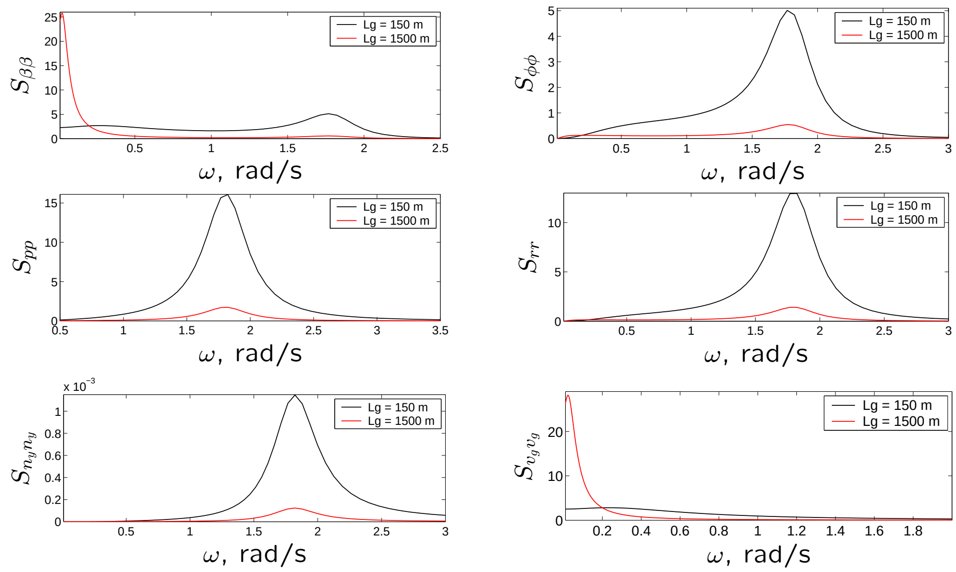
Examples

$L_g = 150$ m versus $L_g = 1500$ m with v_g



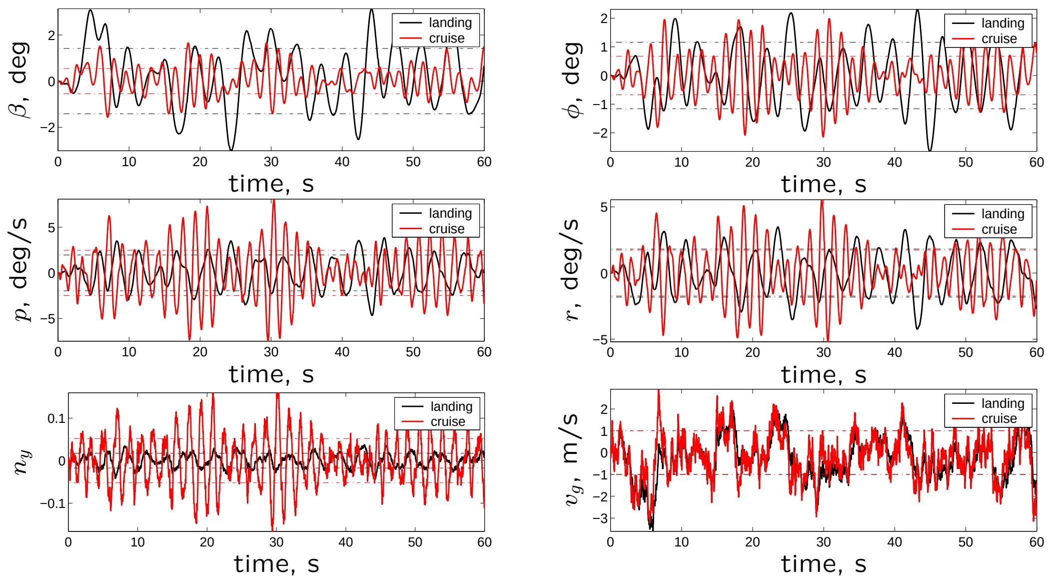
Examples

$L_g = 150$ m versus $L_g = 1500$ m with v_g (PSD)



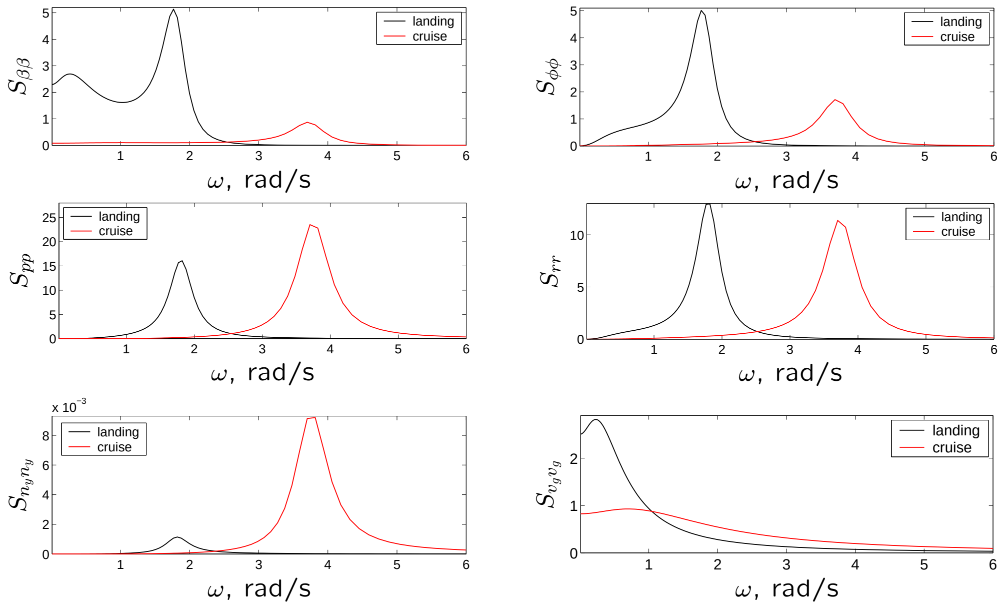
Examples

'Landing' ($V = 59.9$ m/s) vs. 'cruise' ($V = 181.9$ m/s) with v_g



Examples

'Landing' (V = 59.9 m/s) vs. 'cruise' (V = 181.9 m/s) with v_g (PSD)



Examples

A_y also depends on the position w.r.t. the c.g.