Description of Atmospheric Turbulence

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Description of Atmospheric Turbulence

For this lecture the following material was used:

- Chapter 6 of Lecture notes *Aircraft Responses to Atmospheric Turbulence* by (Mulder, van der Vaart, & van Staveren).

Contents of this lecture

Introduction

Meteorological mechanisms of atmospheric turbulence

Different kinds of turbulence

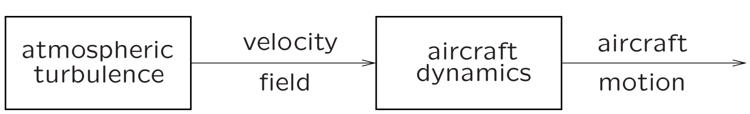
Mathematical description of atmospheric turbulence
- deterministic vs. stochastic models
- stochastic theory
- assumptions
- elementary fields
- correlation functions
- von Karman Spectra
- Dryden Spectra

Parameter selection + examples

Introduction

Purpose of this course :

Analysis of **aircraft responses** (motion) in **turbulent air**



Model of aircraft dynamics:
- input: atmospheric turbulence → given as velocity field
- output: aircraft motions

Turbulence **velocity vector** is usually **decomposed** into:
(*stability reference frame* F_s)

1. $\bar{u}_g$: longitudinal gust velocity

   $\longrightarrow$ *only of importance in ground proximity (+ low-speed)*

2. $\boxed{\bar{v}_g \text{ : side gust velocity}}$

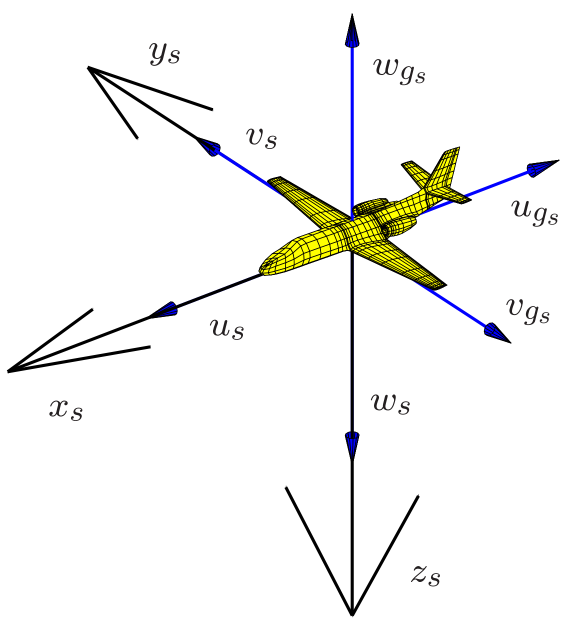
   $\longrightarrow$ *forces on vertical tail and fuselage, yawing and rolling moments*

3. $\boxed{\bar{w}_g \text{ : vertical gust velocity}}$

   $\longrightarrow$ *vertical acceleration and pitching moments*

$\Rightarrow$ **fatigue** on structure, **uncomfortable ride** for passengers and pilot

Stability reference frame F_s (s: stability, g: gust):
!! mind negative directions of gust velocity definitions !!



Meteorological mechanisms: **1. Vertical Stability**

Heat convection is one driving force for atmospheric turbulence

Geometric lapse rate λ describes the temperature profile of the air (place- & altitude- and weather-dependent).

$$\lambda = \frac{dT}{dh} \qquad \Rightarrow \qquad T = T_0 + \lambda \Delta h$$

e.g. $\lambda_{ISA} : -0.0065°/m$

Process lapse rate β describes the rate at which the temperature of an air particle decreases during adiabatic ascent (depends on relative air humidity, temperature and pressure).

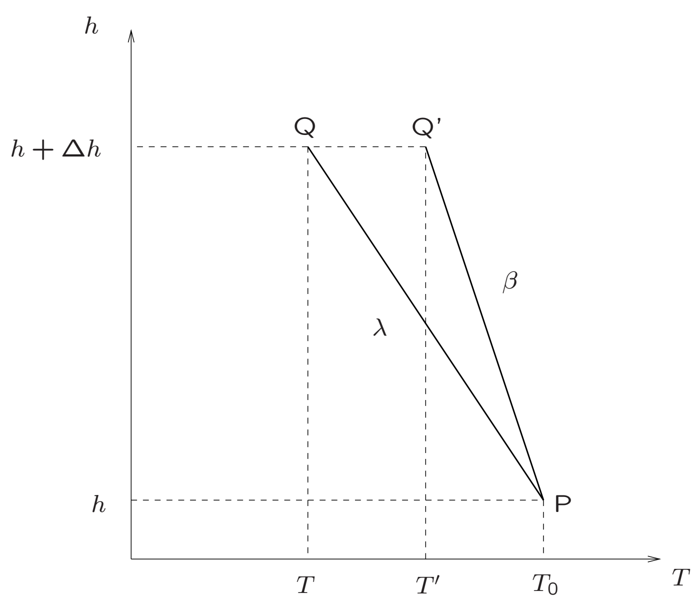
$$T' = T_0 - \beta \Delta h$$

e.g. $\beta_{dry} = 0.0098°/m$

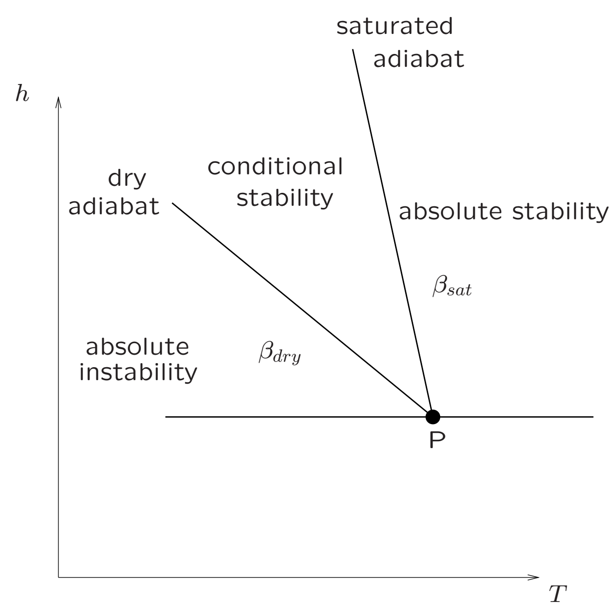
Stability:

$|\lambda| > \beta$: unstable situation
$|\lambda| < \beta$: stable situation

Example vertical instability

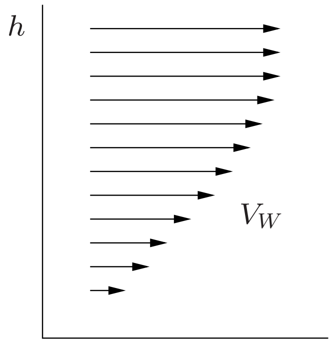


Conditions for vertical stability

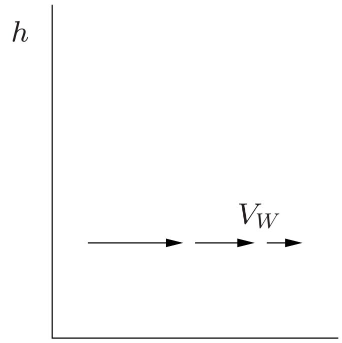


Meteorological mechanisms: **2. wind shear**

Wind shear causes turbulence due to 'mechanical friction' between layers of air.
Wind shear is primarily induced by local surface roughness.



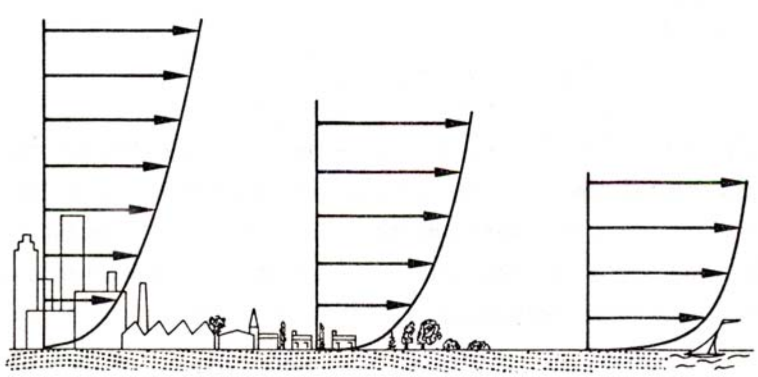
Vertical wind shear



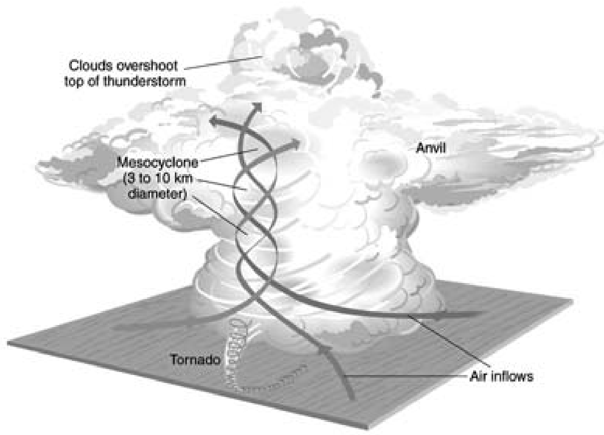
Horizontal wind shear

Different kinds of turbulence

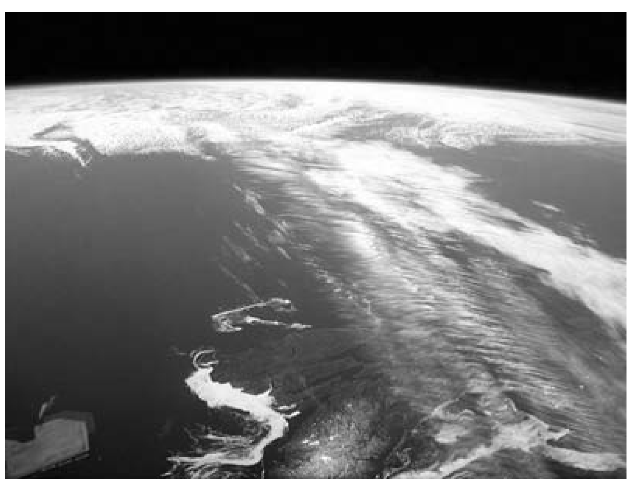
Turbulence near ground due to increased wind shear. Wind shear effect is dominant near ground. Vertical stability/heat convection has a larger contribution with increasing altitude.



Turbulence in clouds due to saturated and unstable air in cumulus clouds (sometimes severe turbulence)

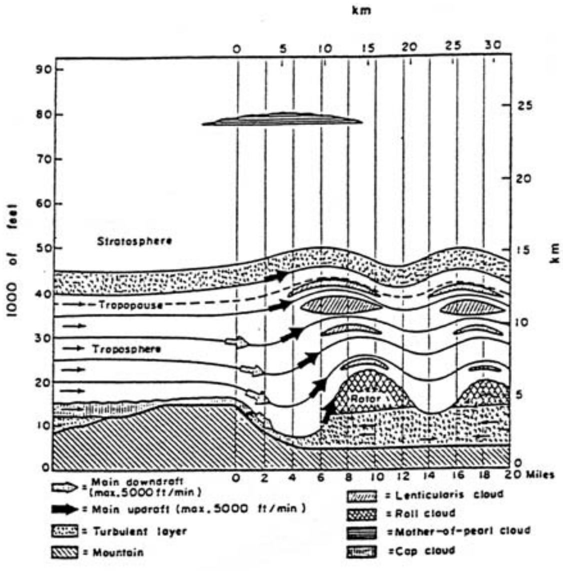


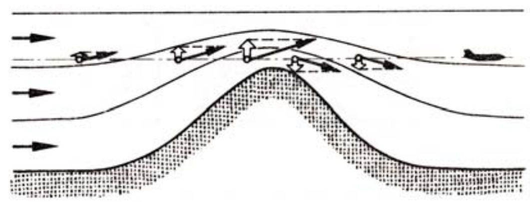
Clear-air turbulence can occur at any altitude, and can have any cause: wind shear or convective currents. e.g. in regions of jet streams (light to moderate turbulence)



jet-stream above Canada

Mountain wave turbulence when strong winds cross mountain chain: heavy vertical wind shear + severe turbulence in 'rotor' cloud





Turbulence intensity

Turbulence is divided into four degrees of intensity.

- **Light** The occupants may be required to use seat belts, but objects in the aircraft remain at rest.
- **Moderate** The occupants require seat belts and are occasionally thrown against the belt. Unsecured objects in the aircraft move about.
- **Severe** The aircraft may be momentarily out of control. Occupants are thrown violently against the belt and back into the seat. Objects not secured in the aircraft are tossed about.
- **Extreme** This is a rarely encountered condition in which the aircraft is violently tossed about and is practically impossible to control. It may cause structural damage.

Statistical description: deterministic vs. stochastic models

Navier-Stokes equations:

- Fundamental physical laws governing air motion.
- *Deterministic* set of coupled partial differential equations
- Very complex and hard to solve (only numerical)

Field of application:

- Different types of flow: around bicycle, airplane, etc.
- Turbulence in aircraft boundary layer (small scale)
- Atmospheric turbulence (large to very large scale)
- Etc.

$\rightarrow$ Differences in **scale**.

The Navier-Stokes equations are **deterministic**, but:

**the predictability of turbulent flow is limited
to very short time intervals,
no matter how well the initial conditions are known.**

$\Rightarrow$ atmospheric turbulence is 'random' (*for current application*)

$\Rightarrow$ **stochastic methods** are required.

Statistical description: stochastic theory

Stochastic variables enter classical **dynamic equations of motions** through:

$$\begin{aligned} &u = u' + \bar{u} = u' + \bar{u}_g, \\ &v = v' + \bar{v} = v' + \bar{v}_g, \\ &w = w' + \bar{w} = w' + \bar{w}_g, \\ &etc. \end{aligned} \tag{1}$$

where
- u' : mean value
- $\bar{u}$: stochastic variation
- u_g : gust component

The **mean components** of the gust velocity field :
- important for navigation and guidance
- does not affect aerodynamics.

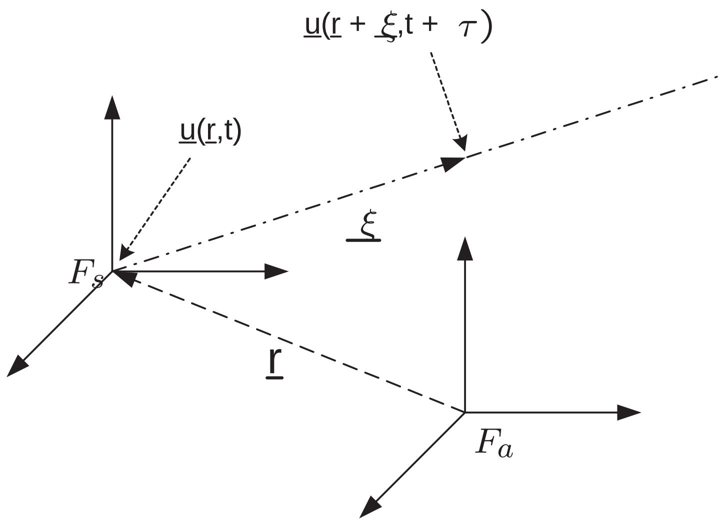
Now, interested in statistics of the random gust velocity vector $\underline{\bar{u}}_g$.

Choose an arbitrary 'atmosphere fixed frame of reference F_a', relative to which mean motion is zero ⇒ mean gust velocity eliminated

Definitions

t	absolute time	[s]
τ	time separation	[s]
λ	wave length	[m]
ω	circular frequency	[rad/s]
$\Omega = \frac{2\pi}{\lambda}$	spatial frequency	[rad/m]
$\underline{\bar{u}} = (\bar{u_1}, \bar{u_2}, \bar{u_3})^T$	stochastic (gust) wind field	[m/s]
$\underline{r} = (x_1, x_2, x_3)^T$	position vector	[m]
$\underline{\xi} = (\xi_1, \xi_2, \xi_3)^T$	separation distance vector	[m]
$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$	spatial frequency vector	[rad/m]

Statistical description: stochastic theory



Time domain:
Stochastic processes can be characterized by (ref. previous lectures) *correlation functions*:

$$C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r},t,\underline{r}+\underline{\xi},t+\tau) = E\{\underline{\bar{u}}(\underline{r},t)\cdot\underline{\bar{u}}(\underline{r}+\underline{\xi},t+\tau)\} \qquad (2)$$

Frequency domain:
When these correlation functions are transformed to the frequency domain, *spectral densities* are obtained, with the multi-dimensional Fourier-transform.

$$S_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r},t,\underline{\Omega},\omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r},t,\underline{r}+\underline{\xi},t+\tau)e^{-j(\underline{\Omega}\cdot\underline{\xi}+\omega\tau)}d\xi_1 d\xi_2 d\xi_3 d\tau \qquad (3)$$

Very generally, **atmospheric turbulence** can be described as :

$$\boxed{\underline{\bar{u}}(\underline{r}, t) = (\bar{u}_1, \bar{u}_2, \bar{u}_3)^T},$$

which is a **velocity field**, being a

3-dimensional stochastic field, depending on four variables (three space dimensions and one time dimension).

This problem is **too complex** to handle generally
→ **assumptions** have to be made.

Statistical description: assumptions

Atmospheric turbulence is . . .

1. **a random process** with Gaussian distribution. Not theoretically correct, but true for many practical situations. With this assumption, covariance matrix describes stochastic process completely !

2. **stationary + frozen.** Taylor's hypothesis. Speed of air particles through air relatively small compared to speed aircraft. Time (t) and time separation (τ) vanishes in covariance matrix:

$$\boxed{\begin{array}{lcl} C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r}, t; \underline{r}+\underline{\xi}, t+\tau) & \rightarrow & C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r}; \underline{r}+\underline{\xi}) \\ S_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r}, t; \underline{\Omega}, \omega) & \rightarrow & S_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r}; \underline{\Omega}) \end{array}}$$

Atmospheric turbulence is . . .

3. homogeneous along the flight path. For nearly horizontal flight this is a reasonable approximation.
Position ($\underline{r}$) vanishes from covariance matrix:

$$\boxed{\begin{array}{lcl} C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r};\underline{r}+\underline{\xi}) & \rightarrow & C_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{\xi}) \\ S_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{r};\underline{\Omega}) & \rightarrow & S_{\underline{\bar{u}}\underline{\bar{u}}}(\underline{\Omega}) \end{array}}$$

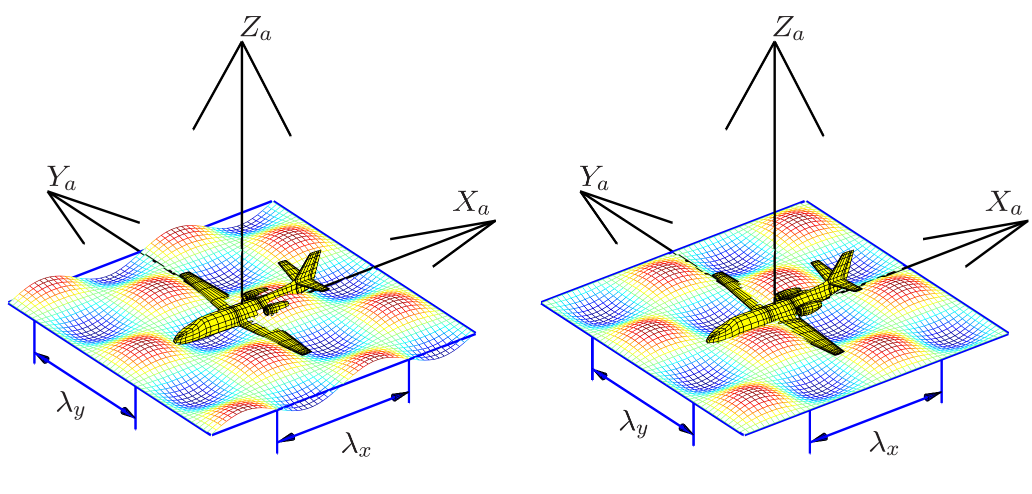
4. an isotropic process. All statistical properties are independent of the orientation of the axis. This is true for high-altitude, away from the earth's boundary layer.

$$\boxed{\sigma^2_{\bar{u}_1} = \sigma^2_{\bar{u}_2} = \sigma^2_{\bar{u}_3} = \sigma^2}$$

$\Rightarrow$ the simplest model:
Gaussian, frozen, homogeneous and isotropic turbulence.

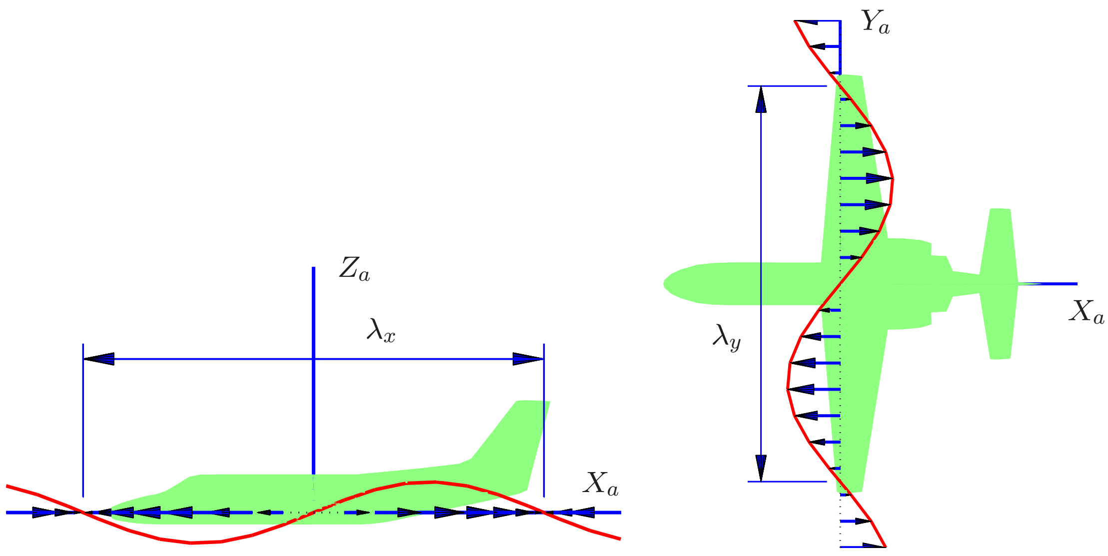
Statistical description: elementary field

An elementary turbulence field (1 spatial frequency Ω):

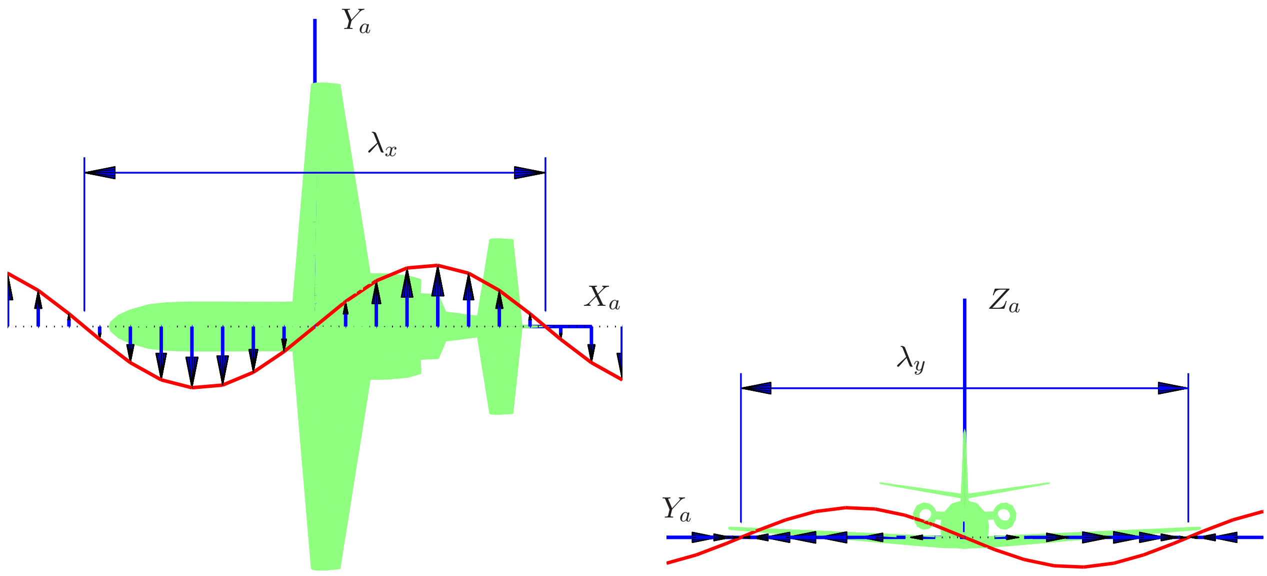


Symmetric field   Asymmetric field

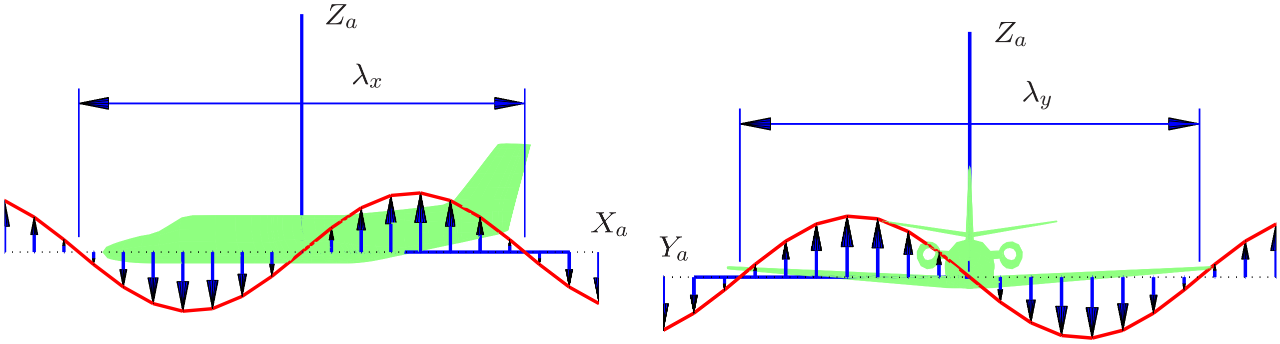
Longitudinal turbulence u_g:



Lateral turbulence v_g:



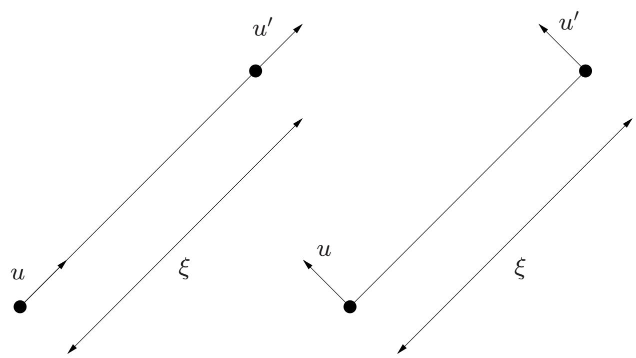
Vertical turbulence w_g:



Statistical description: correlation functions

Given the assumptions, two fundamental spatial correlation functions can be found, valid for the whole turbulence field:

$f(\xi)$	**longitudinal**	parallel to a connecting line between two points
$g(\xi)$	**lateral**	normal to a connecting line between two points



Longitudinal correlation   Lateral correlation

Based on these two correlation function, the covariance matrix of atmospheric turbulence can be written as:

$$C_{ij}(\underline{\xi}) = \sigma^2\left(\frac{f(|\underline{\xi}|) - g(|\underline{\xi}|)}{|\underline{\xi}|^2}\xi_i\xi_j + g(|\underline{\xi}|)\delta_{ij}\right), \tag{4}$$

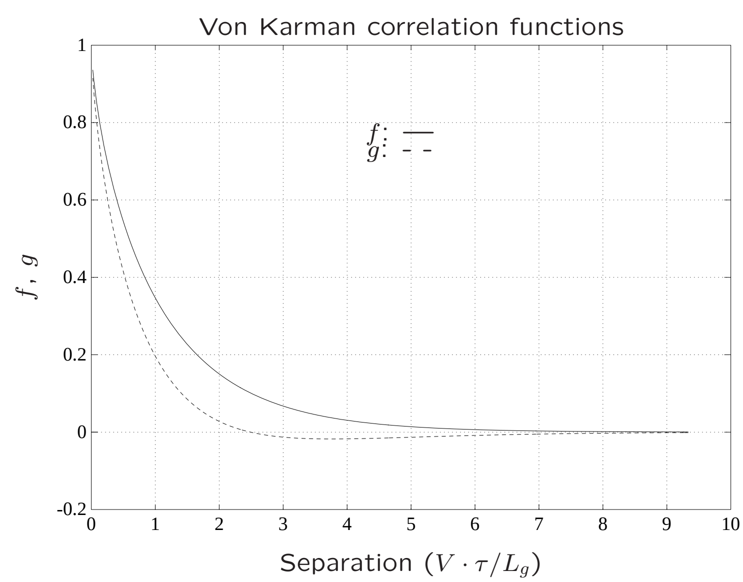
where δ_{ij} is the Kronecker delta (1 if i=j, 0 if i≠j).

Now, restrict to atmospheric turbulence that varies **along the aircraft's flight path**, so which is encountered in the aircraft's c.g.

⇒ select separation distance vector $\underline{\xi}$ along flight path (in stability reference frame)

$$\underline{\xi} = (\xi_1, \xi_2, \xi_3)^T = (V \cdot \tau, 0, 0)^T \tag{5}$$

The accompanying correlation functions are found **theoretically** (Von Karman) and have shown to be well-correlated with **wind tunnel experiments**.



Statistical description: integral scale of turbulence

Turbulence can be found at **different scales**:

- boundary layer of wing
- wake of an aircraft
- atmosphere

These differences can be quantified by the 'integral scale of turbulence' L_g:

Longitudinal scale:

$$L_g = \int_0^\infty f(\xi) d\xi$$

Lateral scale :

$$L_g' = \int_0^\infty g(\xi) d\xi$$

with $\xi = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$

Interpretation:

L_g is a measure of the **spacial extent** of **significant correlation**. It can be interpreted as "*the width of a unit-height rectangle that contains the same area as the correlation function*".

Typical value at high-altitudes:

$$\boxed{L_g = 300m}$$

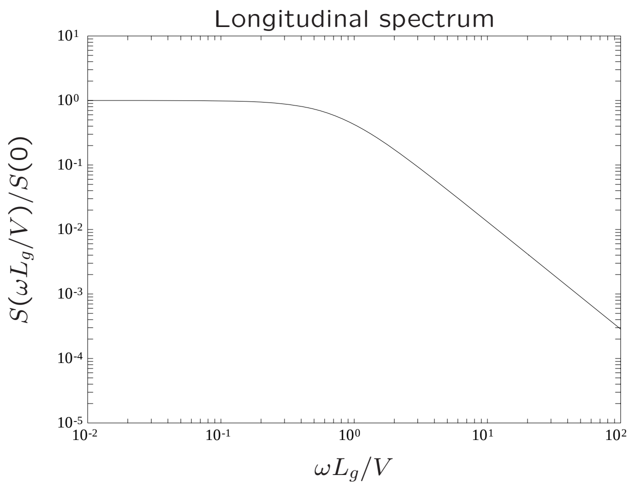
Relation:

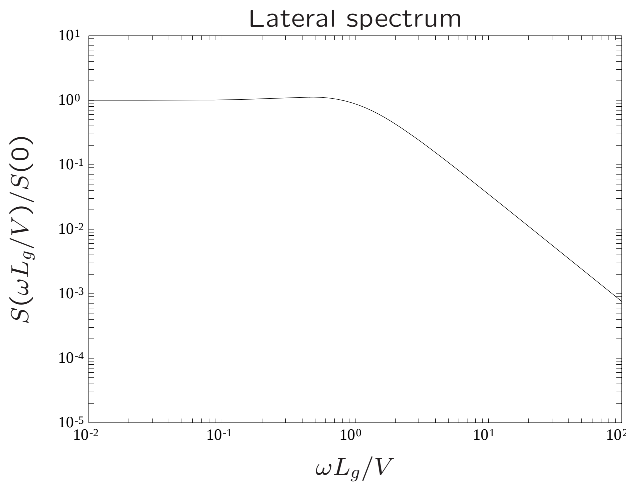
Because of the **continuity condition** for incompressible fluids, a direct relation between $f(\xi)$ and $g(\xi)$ can be found.
This is also reflected in the relation between L_g and L_g'.

$$\boxed{L_g = 2L_g'}$$

Statistical description: von Kármán spectra

The von Kármán functions $f(\xi)$ and $g(\xi)$ yield spectra that seem to **best fit the available theoretical and experimental data** on atmospheric turbulence.





With transformation from spatial (ξ,Ω) to time domain (τ,ω):

$$\xi = V \cdot \tau \qquad \text{and} \qquad \Omega = \frac{\omega}{V} \text{ (} \rightarrow \text{ from Taylor's hypothesis),}$$

the **analytic functions of von Kármán spectra** for the turbulence velocities $(\bar{u}_g, \bar{v}_g, \bar{w}_g)^T$ in F_s are:

$$S_{\bar{u}_g\bar{u}_g}(\omega) = 2\sigma^2 \frac{L_g}{V} \frac{1}{[1 + (1.339 L_g \frac{\omega}{V})^2]^{\frac{5}{6}}} \tag{6}$$

$$S_{\bar{v}_g\bar{v}_g}(\omega) = \sigma^2 \frac{L_g}{V} \frac{1 + \frac{8}{3}(1.339 L_g \frac{\omega}{V})^2}{[1 + (1.339 L_g \frac{\omega}{V})^2]^{\frac{11}{6}}} \tag{7}$$

$$S_{\bar{w}_g\bar{w}_g}(\omega) = \sigma^2 \frac{L_g}{V} \frac{1 + \frac{8}{3}(1.339 L_g \frac{\omega}{V})^2}{[1 + (1.339 L_g \frac{\omega}{V})^2]^{\frac{11}{6}}} \tag{8}$$

However, these functions are **not rational functions** and are **difficult to use** in computations (non-linear).

Rational transfer functions can be transformed to **linear differential equations**.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{k \cdot s + l}{a \cdot s^2 + b \cdot s + c} \tag{9}$$

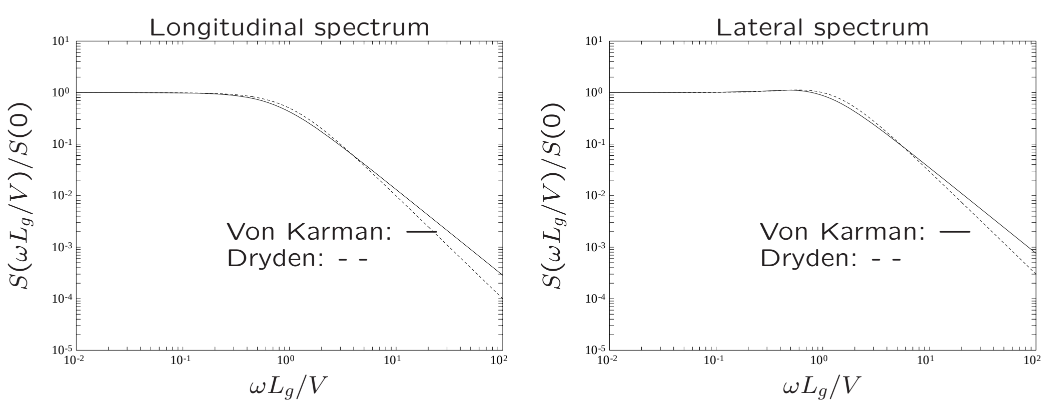
$$\Rightarrow \quad a \cdot y'' + b \cdot y' + c \cdot y = k \cdot x' + l \cdot x \tag{10}$$

$\Rightarrow$ Solution: **Dryden spectra**.

Statistical description: Dryden spectra

Difference between **von Kármán** spectra and **Dryden** spectra:

- low-frequent: same asymptote
- high-frequent: slight difference (Dryden: 2^{nd} order)
- yield **much the same results** for aircraft responses



Dryden spectral densities (**frequency domain**):

$$S_{\bar{u}_g\bar{u}_g}(\omega) = 2\sigma^2 \frac{L_g}{V} \frac{1}{1 + (L_g \frac{\omega}{V})^2} \tag{11}$$

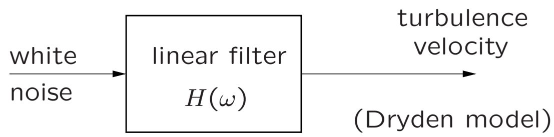
$$S_{\bar{v}_g\bar{v}_g}(\omega) = \sigma^2 \frac{L_g}{V} \frac{1 + 3(L_g \frac{\omega}{V})^2}{[1 + (L_g \frac{\omega}{V})^2]^2} \tag{12}$$

$$S_{\bar{w}_g\bar{w}_g}(\omega) = \sigma^2 \frac{L_g}{V} \frac{1 + 3(L_g \frac{\omega}{V})^2}{[1 + (L_g \frac{\omega}{V})^2]^2} \tag{13}$$

Again,
- since stationary turbulence field (Taylor's hypothesis): $\omega = \Omega \cdot V$
- since isotropic turbulence field: spectra for $\underline{\bar{v}}$ and $\underline{\bar{w}}$ are identical.

GOAL to model atmospheric turbulence, characterized by given Dryden spectra

HOW? **white noise** through a **forming filter**.



With $\bar{n}$ being white noise:

$$S_{\bar{n}\bar{n}} = 1, \tag{14}$$

$$S_{\bar{y}\bar{y}}(\omega) = |H(\omega)|^2 S_{\bar{n}\bar{n}} \tag{15}$$

$$\Rightarrow |H(\omega)|^2 = S_{\bar{y}\bar{y}}(\omega) \tag{16}$$

where $S_{\bar{y}\bar{y}}(\omega)$ is known = Dryden spectra

The variance of the gust velocities is given by σ^2. Perform check:

$$\begin{aligned}
\sigma^2_{\bar{u}_g} &= \tfrac{1}{2\pi}\int_{-\infty}^{+\infty} S_{\bar{u}_g\bar{u}_g}(\omega)d\omega \\
&= \tfrac{1}{2\pi}2\sigma^2\tfrac{L_g}{V}\int_{-\infty}^{+\infty}\tfrac{1}{1+(L_g\frac{\omega}{V})^2}d\omega \\
&= \tfrac{\sigma^2}{\pi}\tfrac{L_g}{V}\tfrac{V}{L_g}[arctan(L_g\tfrac{\omega}{V})]_{-\infty}^{+\infty} \\
&= \tfrac{\sigma^2}{\pi}[\tfrac{\pi}{2}+\tfrac{\pi}{2}] \\
&= \sigma^2 \\
&= C_{\bar{u}_g\bar{u}_g}(\tau=0)
\end{aligned}$$

Transformation: Frequency domain → Time domain

Spectral densities $\underset{1}{\rightarrow}$ transfer functions $\underset{2}{\rightarrow}$ differential equations $\underset{3}{\rightarrow}$ state-space

1. <u>Spectral densities → transfer functions:</u>

$$S_{\bar{u}_g\bar{u}_g}(\omega) = 2\sigma^2\frac{L_g}{V}\frac{1}{1+(L_g\frac{\omega}{V})^2} = |H_{\bar{u}_g w_1}(\omega)|^2 \tag{17}$$

$$S_{\bar{v}_g\bar{v}_g}(\omega) = \sigma^2\frac{L_g}{V}\frac{1+3(L_g\frac{\omega}{V})^2}{[1+(L_g\frac{\omega}{V})^2]^2} = |H_{\bar{v}_g w_2}(\omega)|^2 \tag{18}$$

$$S_{\bar{w}_g\bar{w}_g}(\omega) = \sigma^2\frac{L_g}{V}\frac{1+3(L_g\frac{\omega}{V})^2}{[1+(L_g\frac{\omega}{V})^2]^2} = |H_{\bar{w}_g w_3}(\omega)|^2 \tag{19}$$

with $w_1, w_2 \& w_3$ being white noise.

So,

$$H_{\bar{u}_g w_1}(\omega) = \frac{\bar{u}_g(\omega)}{w_1(\omega)} = \sigma\sqrt{\frac{2L_g}{V}}\frac{1}{1 \pm \frac{L_g}{V}j\omega} \tag{20}$$

$$H_{\bar{v}_g w_2}(\omega) = \frac{\bar{v}_g(\omega)}{w_2(\omega)} = \sigma\sqrt{\frac{L_g}{V}}\frac{1 \pm \sqrt{3}\frac{L_g}{V}j\omega}{(1 \pm \frac{L_g}{V}j\omega)^2} \tag{21}$$

$$H_{\bar{w}_g w_3}(\omega) = \frac{\bar{w}_g(\omega)}{w_3(\omega)} = \sigma\sqrt{\frac{L_g}{V}}\frac{1 \pm \sqrt{3}\frac{L_g}{V}j\omega}{(1 \pm \frac{L_g}{V}j\omega)^2} \tag{22}$$

Two solutions are possible (+ and -). Minus signs would lead to unstable filters and are rejected (physical reasons).

2. <u>Transfer functions → differential equations:</u> ($j\omega = s = \frac{d}{dt}$)

$$\frac{L_g}{V}\dot{u}_g(t) + u_g(t) = \sigma\sqrt{\frac{2L_g}{V}}w_1(t) \tag{23}$$

$$\frac{L_g^2}{V^2}\ddot{w}_g(t) + 2\frac{L_g}{V}\dot{w}_g(t) + w_g(t) = \sigma\sqrt{\frac{L_g}{V}}w_3(t) + \sigma\frac{L_g}{V}\sqrt{\frac{3L_g}{V}}\dot{w}_3(t) \tag{24}$$

<u>Second order equation of w_g (and v_g) → 2 first order equations:</u>

Extra parameter :

$$w_g^*(t) = \dot{w}_g(t) - \sigma\sqrt{\frac{3V}{L_g}}w_3(t) \tag{25}$$

3. **Dryden** in **state-space**: white noise → turbulence velocity field:

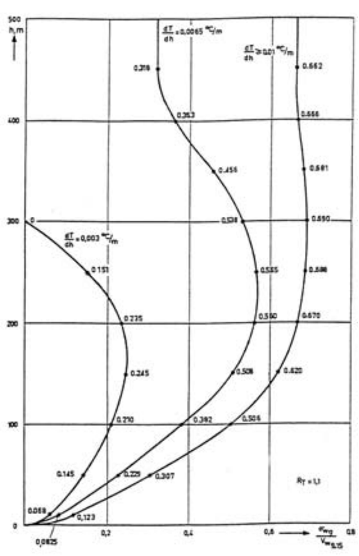
$$\dot{u_g}(t) = -\frac{V}{L_g}u_g(t) + \sigma\sqrt{\frac{2V}{L_g}}w_1(t) \tag{26}$$

$$\begin{bmatrix} \dot{v}_g(t) \\ \dot{v}_g^*(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{V^2}{L_g^2} & -2\frac{V}{L_g} \end{bmatrix} \begin{bmatrix} v_g(t) \\ v_g^*(t) \end{bmatrix} + \begin{bmatrix} \sigma\sqrt{\frac{3V}{L_g}} \\ (1-2\sqrt{3})\sigma\sqrt{(\frac{V}{L_g})^3} \end{bmatrix} w_2(t) \tag{27}$$

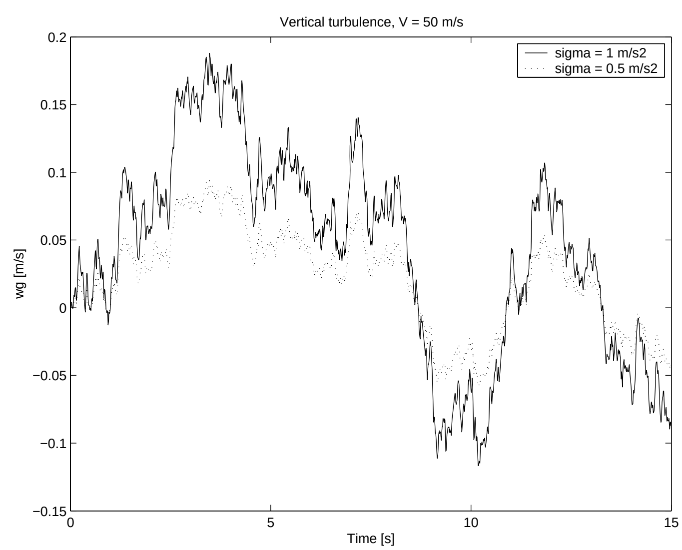
$$\begin{bmatrix} \dot{w}_g(t) \\ \dot{w}_g^*(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{V^2}{L_g^2} & -2\frac{V}{L_g} \end{bmatrix} \begin{bmatrix} w_g(t) \\ w_g^*(t) \end{bmatrix} + \begin{bmatrix} \sigma\sqrt{\frac{3V}{L_g}} \\ (1-2\sqrt{3})\sigma\sqrt{(\frac{V}{L_g})^3} \end{bmatrix} w_3(t) \tag{28}$$

Parameter selection σ and L_g

The quantitative values of σ and L_g are based on experimental data.

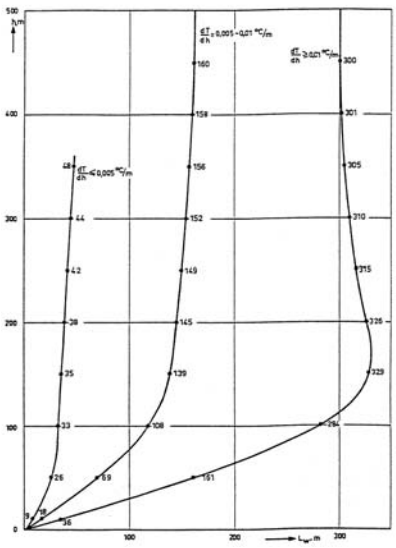


$\sigma_{w_g} = f(h, \lambda, V_{w_{9.15}})$

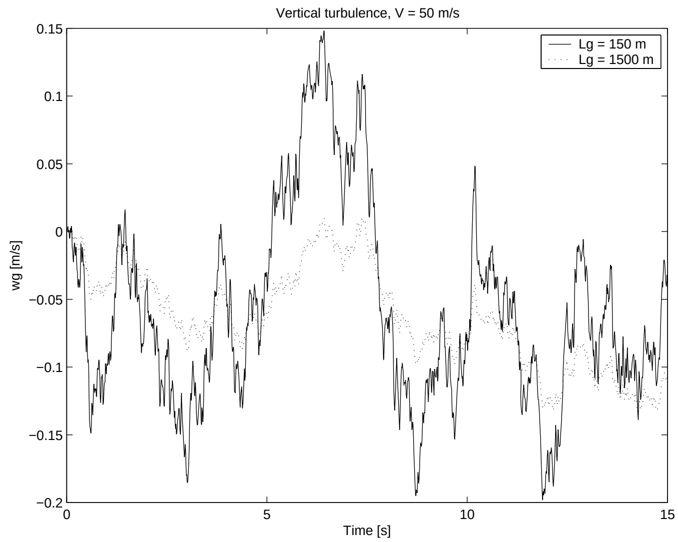


$\sigma_{w_g} = 1m/s^2$ vs $0.5m/s^2$

Parameter selection σ and L_g



scale $L_g(h) = f(h, \lambda)$



$L_g = 150m$ vs $1500m$

Turbulence at low altitude

At low altitudes, turbulence can **NO LONGER** be considered:
- **Homogeneous** (position independent) due to varying terrain roughness
- **Isotropic** (orientation independent) since w_g appears to behave differently from u_g and v_g at low altitudes.

However,
- terrain variations around most airports are relatively small
- approach and landing manoeuvres are nearly horizontal flights

$\Rightarrow$ Turbulence can be considered quasi-homogeneous

$\Rightarrow$ Dryden model is still good approximation of encountered turbulence during approach and landing of aircraft

Turbulence at low altitude

Remember, for isotropic turbulence $\sigma_{\bar{u}_g}$, $\sigma_{\bar{v}_g}$ and $\sigma_{\bar{w}_g}$ are equal.

For turbulence at low altitudes, the followings relations have been found from measured data:

$$
\begin{aligned}
\frac{\sigma_{\bar{u}_g}}{\sigma_{\bar{w}_g}} &= \frac{\sigma_{\bar{v}_g}}{\sigma_{\bar{w}_g}} = 2.5 && 0\ m \leq h < 15\ m \\
\frac{\sigma_{\bar{u}_g}}{\sigma_{\bar{w}_g}} &= \frac{\sigma_{\bar{v}_g}}{\sigma_{\bar{w}_g}} = 1.25 - 0.001\ h && 15\ m \leq h < 250\ m \qquad (29) \\
\frac{\sigma_{\bar{u}_g}}{\sigma_{\bar{w}_g}} &= \frac{\sigma_{\bar{v}_g}}{\sigma_{\bar{w}_g}} = 1 && h > 250\ m
\end{aligned}
$$

$\Rightarrow$ A complete turbulence model (Dryden) applicable at high altitude and during approach and landing.