The `nCopula` package

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Abstract

The *nCopula* package aims to simplify the construction process and usage of hiearchical Archimedean copulas through compound distributions in R. Is is possible to build structures with clear representations, obtain expressions for Archimedean copulas, wether they are hiearchical or not, as well as other important functions (i.e. Laplace Stieltjes Transform, pgf, etc.), given a certain path and structure. Furthermore, the generation of random vectors is possible from any given structure.

1 Introduction

Copulas are now well known tools used for dependence modeling purposes in many research topics. A d-dimensional copula is a d-variate probability distribution function for which the marginals are uniformly distributed on $(0,1)$, with $d \geq 2$. One important class of copulas is the Archimedean copula family, popular for its simple construction procedure and multivariate generalization. A d-dimensional copula C is said to be an *Archimedean copula* if

$$C(u_1,\dots,u_d) = \psi\left(\psi^{-1}(u_1)+\ldots+\psi^{-1}(u_d)\right), \text{ for } (u_1,\dots,u_d) \in [0,1]^d. \tag{1}$$

The continuous and strictly decreasing function ψ is called the generator of the copula, where $\psi : [0,\infty) \to [0,1]$, $\psi(0) = 1$ and $\lim_{t\to\infty} \psi(t) = 0$. In the same manner, $\psi^{-1} : [0,1] \to [0,\infty)$, for which $\psi^{-1}(0) = \inf\{t : \psi(t) = 0\}$, where ψ^{-1} is the inverse of the generator ψ. The set of all such functions is denoted by Ψ_∞. In fact, from Kimberling's and Bernstein's theorems, see e.g. [Kimberling, 1974], [Feller, 1971], and [Hofert, 2010], representation (1) leads to a proper copula for all $d \geq 2$ if and only if ψ is the Laplace-Stieltjes Transform (LST) of a strictly positive random variable (rv) Θ with cumulative distribution function (cdf) F_Θ, where the LST of the rv Θ is given by

$$\mathscr{L}_\Theta(t) = \int_0^\infty \mathrm{e}^{-tx}\,\mathrm{d}F_\Theta(x) = E\left[\mathrm{e}^{-t\Theta}\right]. \tag{2}$$

The use of multivariate Archimedean copulas in high dimension can be restrictive due to their exchangeability property. One can resort to another interesting class of copulas, namely vine copulas (see e.g. [Bedford and Cooke, 2002] and [Joe, 1997]). They are pair copula constructions allowing a cascade decomposition of a multivariate distribution into the product of bivariate copulas. Vine copulas force the use of $\frac{d(d-1)}{2}$ bivariate copulas which requires a high number of parameters when d, the dimension of the copula, increases.

Hierarchical Archimedean copulas provide an interesting alternative to allow asymmetries. The first approach to construct hierarchical Archimedean copulas was proposed by [Joe, 1997] who introduced the so-called nested Archimedean copulas in three and four dimensions. They are obtained by nesting into each other Archimedean copulas. They are able to capture different dependence relations between and within different groups of risks with a relatively small number of parameters (see e.g. [Górecki et al., 2016]). They were further studied by e.g. [McNeil, 2008], [Hofert, 2012], and [Hofert, 2011], in a general setting. For an Archimedean hierarchical structure to be a proper copula, a given nesting condition must be verified. For example, a 3-dimensional fully nested Archimedean copula can be written as

$$C(u_1,u_2,u_3) = C(u_1, C(u_2,u_3)) = \psi_0\Big(\psi_0^{-1}(u_1) + \psi_0^{-1}\circ\psi_1\big(\psi_1^{-1}(u_2) + \psi_1^{-1}(u_3)\big)\Big).$$

For the hierarchical structure to be a proper copula, $\psi_0^{-1}\circ\psi_1$ must have completely monotone derivatives, where ψ_0 and ψ_1 are generators of the parent and the child copulas respectively. If generators ψ_0 and ψ_1 belong to the same family, the verification of this sufficient nesting condition can be done without much problem, see [Hofert, 2010] for restrictions on the parameters.

The *copula* package implements serveral very effective functions to construct and use nested Archimedean copulas (see [Hofert et al.,]).

2 Hierarchical Archimedean copulas through compounding

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References

[Bedford and Cooke, 2002] Bedford, T. and Cooke, R. M. (2002). Vines: A new graphical model for dependent random variables. *Annals of Statistics*, pages 1031–1068.

[Feller, 1971] Feller, W. (1971). An introduction to probability and its applications, vol. ii. *Wiley, New York*.

[Górecki et al., 2016] Górecki, J., Hofert, M., and Holeňa, M. (2016). An approach to structure determination and estimation of hierarchical archimedean copulas and its application to bayesian classification. *Journal of Intelligent Information Systems*, 46(1):21–59.

[Hofert, 2010] Hofert, M. (2010). *Sampling nested Archimedean copulas with applications to CDO pricing*. PhD thesis, Universität Ulm.

[Hofert, 2011] Hofert, M. (2011). Efficiently sampling nested archimedean copulas. *Computational Statistics & Data Analysis*, 55(1):57–70.

[Hofert, 2012] Hofert, M. (2012). A stochastic representation and sampling algorithm for nested archimedean copulas. *Journal of Statistical Computation and Simulation*, 82(9):1239–1255.

[Hofert et al.,] Hofert, M., Mächler, M., et al. Nested archimedean copulas meet r: The nacopula package.

[Joe, 1997] Joe, H. (1997). *Multivariate models and multivariate dependence concepts*. CRC Press.

[Kimberling, 1974] Kimberling, C. H. (1974). A probabilistic interpretation of complete monotonicity. *Aequationes mathematicae*, 10(2):152–164.

[McNeil, 2008] McNeil, A. J. (2008). Sampling nested archimedean copulas. *Journal of Statistical Computation and Simulation*, 78(6):567–581.