dE/dx calculation for P10 gas.

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① Bethe equation

z	charge number of incident particle	
Z	atomic number of absorber	
A	atomic mass of absorber	g mol^{-1}
K	$4\pi N_A r_e^2 m_e c^2$ (Coefficient for dE/dx)	0.307 075 MeV mol^{-1} cm^2
I	mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	density effect correction to ionization energy loss	

The mean rate of energy loss by moderately relativistic charged heavy particles is well-described by the "Bethe equation,"

$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] . \qquad (33.5)$$

It describes the mean rate of energy loss in the region $0.1 \lesssim \beta\gamma \lesssim 1000$ for intermediate-Z materials with an accuracy of a few percent.

For proton, 93.8 MeV≦p≦938 GeV

33.2.2. ***Maximum energy transfer in a single collision*** :

For a particle with mass M,

$$W_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} .$$

The density effect correction is usually computed using Sternheimer's parameterization [16]:

$$\delta(\beta\gamma) = \begin{cases} 2(\ln 10)x - \overline{C} & \text{if } x \geq x_1; \\ 2(\ln 10)x - \overline{C} + a(x_1 - x)^k & \text{if } x_0 \leq x < x_1; \\ 0 & \text{if } x < x_0 \text{ (nonconductors);} \\ \delta_0 10^{2(x-x_0)} & \text{if } x < x_0 \text{ (conductors)} \end{cases} \qquad (27.4)$$

P10 is almost nonconductor.

Here $x = \log_{10}\eta = \log_{10}(p/Mc)$. $\overline{C}$ (the negative of the C used in Ref. 16) is obtained by equating the high-energy case of Eq. (27.4) with the limit given in Eq. (27.3). The

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dE/dx for mixture or compound

33.2.10. ***Energy loss in mixtures and compounds*** :

A mixture or compound can be thought of as made up of thin layers of pure elements in the right proportion (Bragg additivity). In this case,

$$\left\langle \frac{dE}{dx} \right\rangle = \sum w_j \left\langle \frac{dE}{dx} \right\rangle_j , \tag{33.13}$$

where $dE/dx|_j$ is the mean rate of energy loss (in MeV g cm^{-2}) in the jth element. Eq. (33.5) can be inserted into Eq. (33.13) to find expressions for $\langle Z/A \rangle$, $\langle I \rangle$, and $\langle \delta \rangle$; for example, $\langle Z/A \rangle = \sum w_j Z_j/A_j = \sum n_j Z_j / \sum n_j A_j$. However, $\langle I \rangle$ as defined this way is an underestimate, because in a compound electrons are more tightly bound than in the free elements, and $\langle \delta \rangle$ as calculated this way has little relevance, because it is the electron density that matters. If possible, one uses the tables given in Refs. 16 and 29, that include effective excitation energies and interpolation coefficients for calculating the density effect correction for the chemical elements and nearly 200 mixtures and compounds. Otherwise, use the recipe for δ given in Ref. 5 and 17, and calculate $\langle I \rangle$ following the discussion in Ref. 10. (Note the "13%" rule!)

P10 gas: mixture of Ar:CH4 = 9:1

According to the text marked by red line, we should use methane's I and δ, without calculating them by assuming CH4 = C + 4H.

In Ref. 16, mean excitation energies and coefficients for calculating density effect for argon and methane are introduced.

From Ref.16: R.M. Sternheimer, S.M. Seltzer, and M.J. Berger, “The Density Effect for the Ionization Loss of Charged Particles in Various Substances,” Atomic Data and Nuclear Data Tables 30, 261 (1984)

※ -C is C bar, X_0 is x_0 and m is k in PDG paper.

TABLE I. Density Effect Parameters for Elemental Substances

See page 266 for Explanation of Tables

Material	Z	Z/A	I (ev)	Density, ρ_0 (g/cm^3)	$h\nu_p$ (eV)	ρ	-C	X_0	X_1	a	m	δ_0	Δ_{max}
HYDROGEN	1	0.99216	19.2	8.3748E-05	0.263	1.412	9.5835	1.8639	3.2718	0.14092	5.7273	0.0	0.024
HYDROGEN, LIQUID	1	0.99216	21.8	6.0000E-02	7.031	1.546	3.2632	0.4759	1.9215	0.13483	5.6249	0.0	0.021
HELIUM	2	0.49967	41.8	1.6632E-04	0.263	1.700	11.1393	2.2017	3.6122	0.13443	5.8347	0.0	0.024
LITHIUM	3	0.43221	40.0	5.3400E-01	13.844	1.535	3.1221	0.1304	1.6397	0.95136	2.4993	0.14	0.062
BERYLLIUM	4	0.44384	63.7	1.8480E+00	26.098	1.908	2.7847	0.0592	1.6922	0.80392	2.4339	0.14	0.029
BORON	5	0.46254	76.0	2.3700E+00	30.170	2.320	2.8477	0.0305	1.9688	0.56224	2.4512	0.14	0.024
CARBON (GRAPHITE, DENS 2.265)	6	0.49954	78.0	2.2650E+00	30.652	2.290	2.8680	-0.0178	2.3415	0.26142	2.8697	0.12	0.038
CARBON (GRAPHITE, DENS 2.0)	6	0.49954	78.0	2.0000E+00	28.803	2.376	2.9925	-0.0351	2.4860	0.20240	3.0036	0.10	0.038
CARBON (GRAPHITE, DENS 1.7)	6	0.49954	78.0	1.7000E+00	26.555	2.490	3.1550	0.0480	2.5387	0.20762	2.9532	0.14	0.038
NITROGEN	7	0.49976	82.0	1.1653E-03	0.695	1.984	10.5400	1.7378	4.1323	0.15349	3.2125	0.0	0.086
OXYGEN	8	0.50002	95.0	1.3315E-03	0.744	2.314	10.7004	1.7541	4.3213	0.11778	3.2913	0.0	0.101
FLUORINE	9	0.47372	115.0	1.5803E-03	0.788	2.450	10.9653	1.8433	4.4096	0.11083	3.2962	0.0	0.121
NEON	10	0.49556	137.0	8.3851E-04	0.587	2.577	11.9041	2.0735	4.6421	0.08064	3.5771	0.0	0.110
SODIUM	11	0.47847	149.0	9.7100E-01	19.641	2.648	5.0526	0.2880	3.1962	0.07772	3.6452	0.08	0.098
MAGNESIUM	12	0.49373	156.0	1.7400E+00	26.708	2.331	4.5297	0.1499	3.0668	0.08163	3.6166	0.08	0.073
ALUMINUM	13	0.48181	166.0	2.6989E+00	32.860	2.180	4.2395	0.1708	3.0127	0.08024	3.6345	0.12	0.061
SILICON	14	0.49848	173.0	2.3300E+00	31.055	2.103	4.4351	0.2014	2.8715	0.14921	3.2546	0.14	0.059
PHOSPHORUS	15	0.48428	173.0	2.2000E+00	29.743	2.056	4.5214	0.1696	2.7815	0.23610	2.9158	0.14	0.057
SULFUR	16	0.49906	180.0	2.0000E+00	28.789	2.131	4.6659	0.1580	2.7159	0.33992	2.6456	0.14	0.059
CHLORINE	17	0.47951	174.0	2.9947E-03	1.092	1.734	11.1421	1.5555	4.2994	0.19849	2.9702	0.0	0.041
ARGON	18	0.45059	188.0	1.6620E-03	0.789	1.753	11.9480	1.7635	4.4855	0.19714	2.9618	0.0	0.037

TABLE II. Density Effect Parameters for Compounds and Mixtures

See page 266 for Explanation of Tables

Material	Z/A	I (ev)	Density, ρ_0 (g/cm^3)	$h\nu_p$ (eV)	ρ	-C	X_0	X_1	a	m	Δ_{max}
LITHIUM TETRABORATE	0.48487	94.6	2.4400E+00	31.343	2.360	3.2093	0.0737	2.6502	0.11075	3.4389	0.048
LUNG (ICRP)	0.54965	75.3	1.0500E+00	21.891	2.184	3.4708	0.2261	2.8001	0.08588	3.5353	0.089
M3 WAX	0.55512	67.9	1.0500E+00	22.000	1.975	3.2540	0.1523	2.7529	0.07864	3.6412	0.044
MAGNESIUM CARBONATE	0.49814	118.0	2.9580E+00	34.979	2.388	3.4319	0.0860	2.7997	0.09219	3.5003	0.045
MAGNESIUM FLUORIDE	0.48153	134.3	3.0000E+00	34.634	2.330	3.7105	0.1369	2.8630	0.07934	3.6485	0.085
MAGNESIUM OXIDE	0.49622	143.8	3.5800E+00	38.407	2.412	3.6404	0.0575	2.8580	0.08313	3.5968	0.055
MAGNESIUM TETRABORATE	0.49014	108.3	2.5300E+00	32.089	2.430	3.4328	0.1147	2.7635	0.09703	3.4893	0.044
MERCURIC IODIDE	0.40933	684.5	6.3600E+00	46.494	1.892	6.3787	0.1040	3.4728	0.21513	2.7264	0.047
METHANE	0.62334	41.7	6.6715E-04	0.588	1.662	9.5243	1.6263	3.9716	0.09253	3.6257	0.112
METHANOL	0.56176	67.6	7.9140E-01	19.214	2.125	3.5160	0.2529	2.7639	0.08970	3.5477	0.080

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② Landau-Vavilov distribution

33.2.9. ***Fluctuations in energy loss*** :

For detectors of moderate thickness x (*e.g.* scintillators or LAr cells),* the energy loss probability distribution $f(\Delta; \beta\gamma, x)$ is adequately described by the highly-skewed Landau (or Landau-Vavilov) distribution [24,25]. The most probable energy loss is [26]†

$$\Delta_p = \xi \left[\ln \frac{2mc^2\beta^2\gamma^2}{I} + \ln \frac{\xi}{I} + j - \beta^2 - \delta(\beta\gamma) \right] , \tag{33.11}$$

where $\xi = (K/2)\langle Z/A \rangle z^2 (x/\beta^2)$ MeV for a detector with a thickness x in g cm^{-2}, and $j = 0.200$ [26]. ‡ While dE/dx is independent of thickness, Δ_p/x scales as $a \ln x + b$. The density correction $\delta(\beta\gamma)$ was not included in Landau's or Vavilov's work, but it was later included by Bichsel [26]. The high-energy behavior of $\delta(\beta\gamma)$ (Eq. (33.6)) is such that

$$\Delta_p \underset{\beta\gamma \gtrsim 100}{\longrightarrow} \xi \left[\ln \frac{2mc^2\xi}{(\hbar\omega_p)^2} + j \right] . \tag{33.12}$$

x: thickness in g/cm2, P10 gas density = 1.534 mg/cm3.

For our case, x~1.534*10^-3 g/cm2 (one pad ~ 10mm, track angle dependent)

※ Δ_p is energy loss, not dE/dx.

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dE/dx calculated by two methods; Bethe-Bloch and Landau-Vavilov.
And plotted as a function of particle rigidity (100~3000 MeV/c/Z).