Initial Post

by Shengdong Li

20 April 2020

1 Intro

My last name starts with an L (L) so I am solving the volume for the region bound by the equations $y = \sqrt{x}$ and $y = x^2$, rotated around the y-axis.

2 Solution

First I decided to graph the problem on Desmos, to better visualize it.

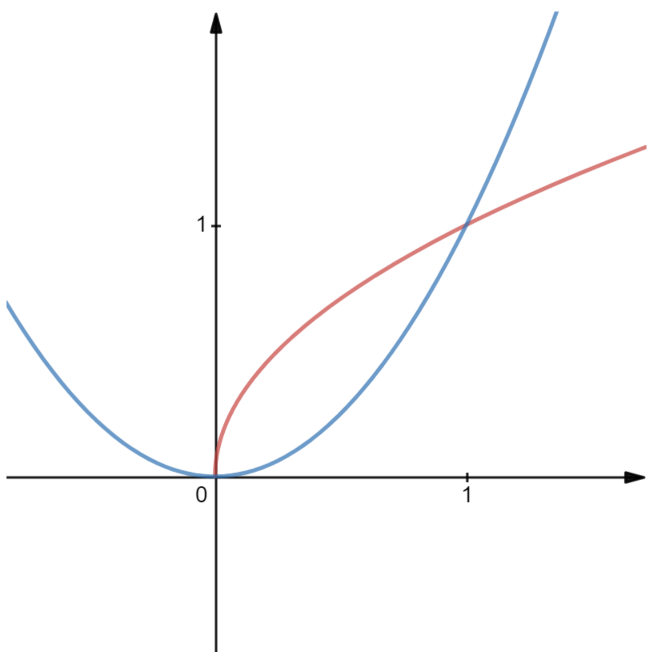

Figure 1: *Graph of $y = \sqrt{x}$ and $y = x^2$.* Desmos link here, also see embed below.

We know that the formula for finding the volume using shells is

$$V = 2\pi \int_a^b (x \cdot f(x))\, dx \tag{1}$$

Now lets try to create a formula finding the volume of an area bound by 2 curves.

$$V = 2\pi \int_a^b (x \cdot f(x))\, dx - 2\pi \int_a^b (x \cdot g(x))\, dx \tag{2}$$

This can be combined

$$V = 2\pi \int_a^b (x \cdot f(x) - x \cdot g(x))\, dx \tag{3}$$

The x can be factored out

$$V = 2\pi \int_a^b x\,(f(x) - g(x))\, dx \tag{4}$$

Now find the difference between the top function and the bottom function to find the height of the cylinder. Looking at the graph,

$$\text{Upper} = y = \sqrt{x} \tag{5}$$
$$\text{Lower} = y = x^2 \tag{6}$$

Looking at the graph, the left bound is clearly 0 and 1

$$a = \boxed{0} \tag{7}$$
$$b = \boxed{1} \tag{8}$$

Now plugin to the equation that we just made and integrate.

$$
\begin{aligned}
2\pi \int_a^b x\left(f\left(x\right) - g\left(x\right)\right) dx &= 2\pi \int_0^1 x\left(\left(\sqrt{x}\right) - \left(x^2\right)\right) dx && (9)\\
&= 2\pi \int_0^1 x\left(x^{\frac{1}{2}} - x^2\right) dx && (10)\\
&= 2\pi \int_0^1 \left(x^{\frac{3}{2}} - x^3\right) dx && (11)\\
&= 2\pi \left(\frac{2x^{\frac{5}{2}}}{5} - \frac{x^4}{4}\right)\Big|_0^1 && (12)\\
&= 2\pi \left(\frac{2\left(1\right)^{\frac{5}{2}}}{5} - \frac{\left(1\right)^4}{4} - \left(\frac{2\left(0\right)^{\frac{5}{2}}}{5} - \frac{\left(0\right)^4}{4}\right)\right) && (13)\\
&= 2\pi \left(\frac{2\left(1\right)^{\frac{5}{2}}}{5} - \frac{\left(1\right)^4}{4}\right) && (14)\\
&= 2\pi \left(\frac{2}{5} - \frac{1}{4}\right) && (15)\\
&= 2\pi \left(\frac{3}{20}\right) && (16)\\
&= \frac{3\pi}{10} && (17)\\
&\approx \boxed{0.942} && (18)
\end{aligned}
$$