Solving some random equation

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1 Intro

Hello, Bobby, that is a very good problem. John approached the problem by using cross-sections perpendicular to the $x - axis$, making semicircles as stated in your word problem. However, I think that another viable way to approach the problem is to use cross-sections perpendicular to the $y - axis$, which would then make full circles circles.

2 Solution

I found that the equation of a circle with a radius of 10 has an equation of

$$x^2 + y^2 = 10^2 \tag{1}$$

Next, I wanted to make the equation of the circle in terms of y

$$\begin{align} x^2 + y^2 &= 10^2 \tag{2} \\ x^2 &= 10^2 - y^2 \tag{3} \\ x &= \pm\sqrt{10^2 - y^2} \tag{4} \end{align}$$

For now, since the right side of a circle yields the same value as the left, we can just simplify the $\pm$ to 2

$$x = 2\sqrt{10^2 - y^2} \tag{5}$$

Like John, I then made a desmos graph to visualize the problem.

We can define the $A(x)$ function of a circle in terms of r

$$A(r) = \pi r^2 \tag{6}$$

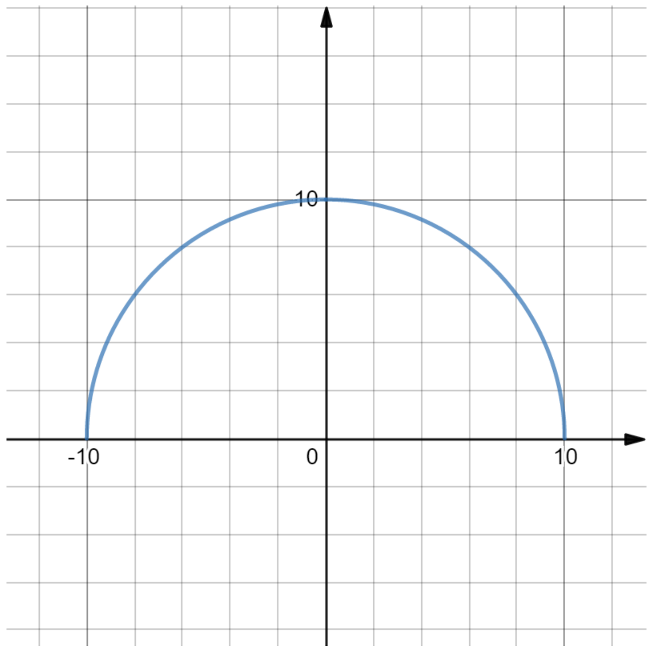

Figure 1: *Bread Container.* Desmos link here, also see embed below

Keep in mind that the radius is half the horizontal

$$r = \frac{1}{2}x \tag{7}$$

Substitute x with our equation of the circle, which simplifies things

$$= \frac{1}{2}(2\sqrt{10^2 - y^2}) \tag{8}$$
$$= \sqrt{10^2 - y^2} \tag{9}$$

Now we can substitute the r in $A(r)$ to get $A(y)$

$$A(y) = \pi(\sqrt{10^2 - y^2})^2 \tag{10}$$
$$= \boxed{\pi(10^2 - y^2)} \tag{11}$$

Since the container is a semicircle, the bottom is 0 and the top is r

$$r = 10 \tag{12}$$
$$a = \boxed{0} \tag{13}$$
$$b = r \tag{14}$$
$$= \boxed{10} \tag{15}$$

Now, we can just integrate the area function from 0 to 10 to get our volume!

$$\begin{aligned}
\int_a^b A\left(y\right)dy &= \int_0^{10} \pi(10^2 - y^2)dy && (16)\\
&= \pi \int_0^{10} (10^2 - y^2)dy && (17)\\
&= \pi\left(10^2 y - \frac{y^3}{3}\right)\Big|_0^{10} && (18)\\
&= \pi\left(10^2\left(10\right) - \frac{\left(10\right)^3}{3} - \left(10^2\left(0\right) - \frac{\left(0\right)^3}{3}\right)\right) && (19)\\
&= \pi\left(10^2\left(10\right) - \frac{\left(10\right)^3}{3}\right) && (20)\\
&\approx 666.67\pi && (21)\\
&\approx \boxed{2094.40cm^3} && (22)\\
& && (23)
\end{aligned}$$

3 Conclusion

Compared to Joe's solution, I found it very interesting how in step 16 our integrals looked nearly identical to each other, disregarding the differences between y and x. Looking at all the steps, I've made the conclusion that solving dy and dx is about the same difficulty and efficiency. However, dx could definitely take more steps if you do not recognize that $\int_{-10}^{10}$ can be rewritten as $2\int_0^{10}$. I also find that doing circular cross-sections are slightly more intuitive. What do you think?