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$$\min \sum_{i,j} c_{ij} x_{ij}$$

$$\text{s. t. Flow balance constraint (we do not write it here)}$$

$$\text{Path total traversal time restriction: } \sum_{i,j} t_{ij} x_{ij} \le T$$

Designate a nonnegative scalar μ, and dualize the path total traversal time restriction to the objective function:

$$\min \sum_{i,j} c_{ij} x_{ij} + \mu \left(\sum_{i,j} t_{ij} x_{ij} - T \right) = \sum_{i,j} (c_{ij} + \mu t_{ij}) x_{ij} - \mu T$$

$$\text{s. t. Flow balance constraint}$$

In each iteration of the Lagrangian relaxation framework, the Lagrangian multiplier μ is fixed and T is a constant. As a result, we can solve a pure shortest path problem in which the link (i, j) has a modified cost $c_{ij} + \mu t_{ij}$.

Update the Lagrangian multiplier with the sub-gradient technique.

$$\mu^{k+1} = \max \left\{ \mu^k + \theta_k \left(\sum_{i,j} t_{ij} - T \right), 0 \right\}$$

$$\theta_k = 1/k$$