

# CEDT – Digital Logic 2023

## Day 2

### Two Level Simplification

### Logic Realization

### Non-gate logics

# Working with combinational logic

- Simplification
  - two-level simplification
  - exploiting don't cares
  - algorithm for simplification
- Logic realization
  - two-level logic and canonical forms realized with NANDs and NORs
  - multi-level logic, converting between ANDs and ORs
- Time behavior
- Hardware description languages

# Simplification of two-level combinational logic

- Finding a minimal sum of products or product of sums realization
  - exploit don't care information in the process
- Algebraic simplification
  - not an algorithmic/systematic procedure
  - how do you know when the minimum realization has been found?
- Computer-aided design tools
  - precise solutions require very long computation times, especially for functions with many inputs ( $> 10$ )
  - heuristic methods employed – "educated guesses" to reduce amount of computation and yield good if not best solutions
- Hand methods still relevant
  - to understand automatic tools and their strengths and weaknesses
  - ability to check results (on small examples)

# The uniting theorem

- Key tool to simplification:  $A(B' + B) = A$  \*
- Essence of simplification of two-level logic
  - find two element subsets of the ON-set where only one variable changes its value – this single varying variable can be eliminated and a single product term used to represent both elements

$$F = A'B' + AB' = (A' + A)B' = B'$$

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

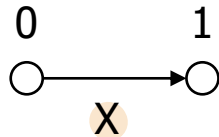
B has the same value in both on-set rows  
– B remains

A has a different value in the two rows  
– A is eliminated – ไม่สำคัญ

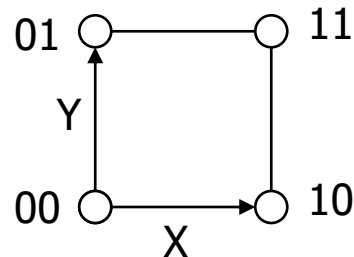
# Boolean cubes

- Visual technique for indentifying when the uniting theorem can be applied
- **n input variables = n-dimensional "cube"**

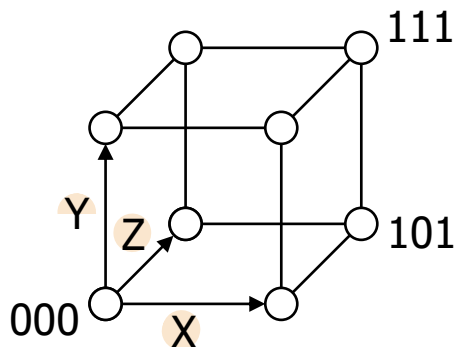
1-cube



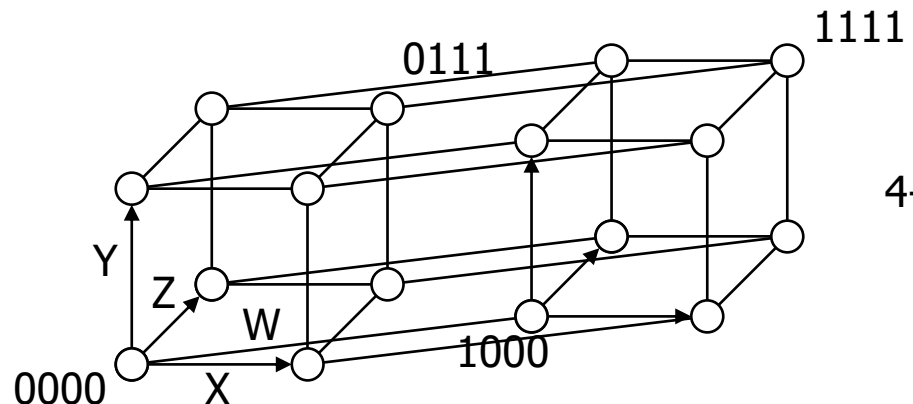
2-cube



3-cube



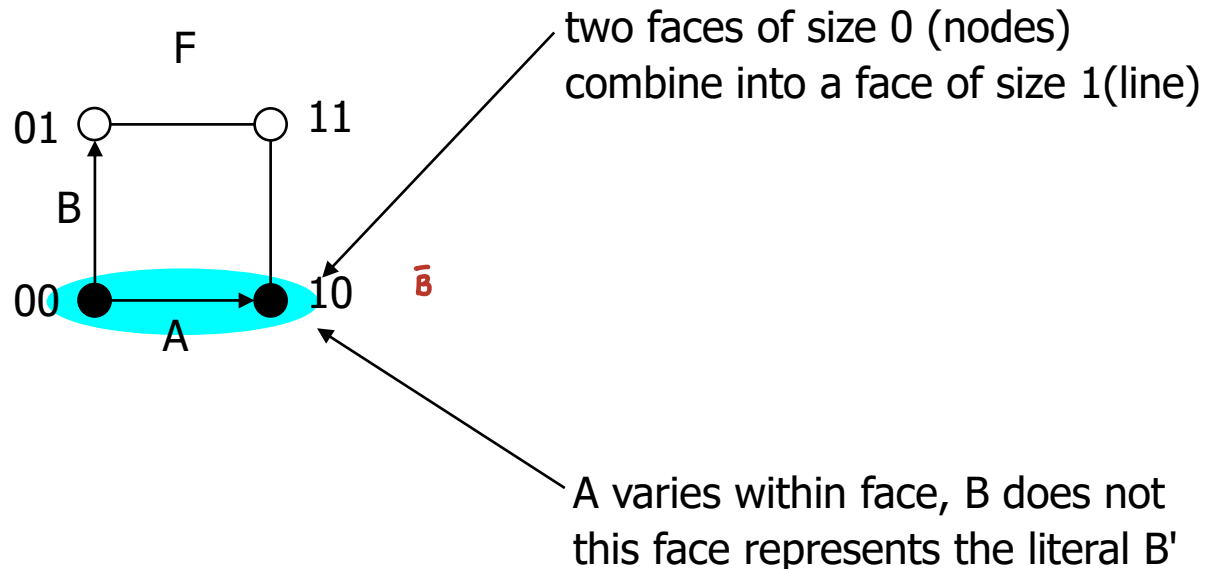
4-cube



# Mapping truth tables onto Boolean cubes

- Uniting theorem combines two "faces" of a cube into a larger "face"
- Example:

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0



ON-set = solid nodes

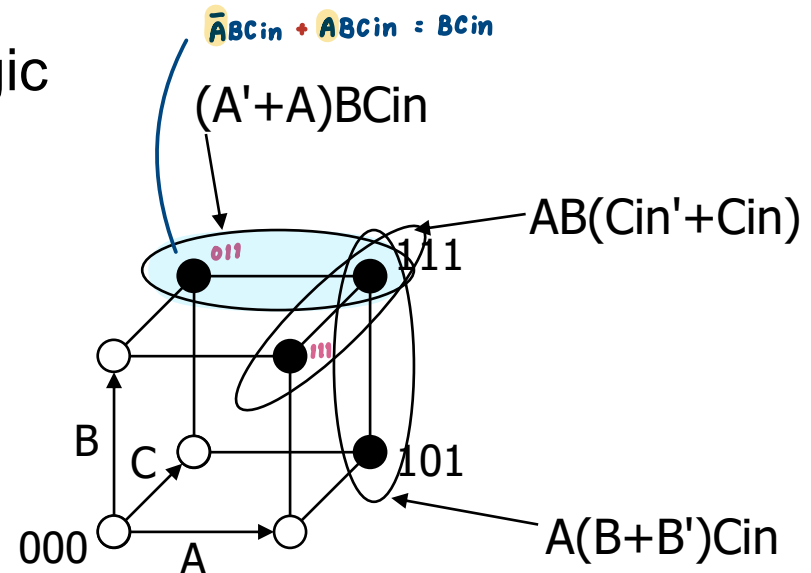
OFF-set = empty nodes

DC-set =  $\times$ 'd nodes

# Three variable example

## ■ Binary full-adder carry-out logic

A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

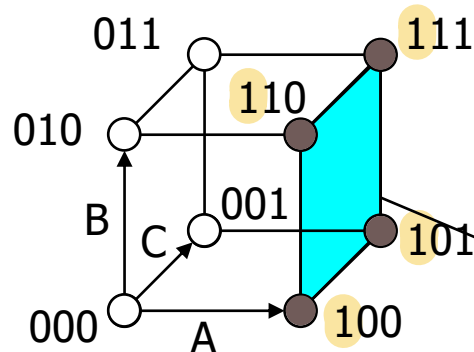


the on-set is completely covered by the combination (OR) of the subcubes of lower dimensionality - note that "111" is covered three times

$$Cout = BCin + AB + ACin \quad \text{2 literal}$$

# Higher dimensional cubes

- Sub-cubes of higher dimension than 2



$$F(A,B,C) = \sum m(4,5,6,7)$$

on-set forms a square  
i.e., a cube of dimension 2

*represents an expression in one variable  
i.e., 3 dimensions – 2 dimensions*

A is asserted (true) and unchanged  
B and C vary

This subcube represents the  
literal A



# m-dimensional cubes in a n-dimensional Boolean space

- In a 3-cube (three variables):
  - a 0-cube, i.e., a single node, yields a term in 3 literals
  - a 1-cube, i.e., a line of two nodes, yields a term in 2 literals
  - a 2-cube, i.e., a plane of four nodes, yields a term in 1 literal
  - a 3-cube, i.e., a cube of eight nodes, yields a constant term "1"
- In general,
  - an m-subcube within an n-cube ( $m < n$ ) yields a term with  $n - m$  literals

# Karnaugh maps

- Flat map of Boolean cube
  - ❑ wrap-around at edges
  - ❑ hard to draw and visualize for more than 4 dimensions
  - ❑ virtually impossible for more than 6 dimensions
- Alternative to truth-tables to help visualize adjacencies
  - ❑ guide to applying the uniting theorem
  - ❑ on-set elements with only one variable changing value are adjacent unlike the situation in a linear truth-table

		A	
		0	1
B	0	0 1	2 1
	1	1 0	3 0

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

# Karnaugh maps (cont'd)

- Numbering scheme based on Gray-code
  - e.g., 00, 01, 11, 10
  - only a single bit changes in code for adjacent map cells

AB		A			
		00	01	11	10
C	0	0	2	6	4
	1	1	3	7	5

B

		A			
		0	2	6	4
C	0	0	2	6	4
	1	1	3	7	5

B

		A			
		0	4	12	8
C	0	0	4	12	8
	1	1	5	13	9
C	2	3	7	15	11
	3	2	6	14	10

B

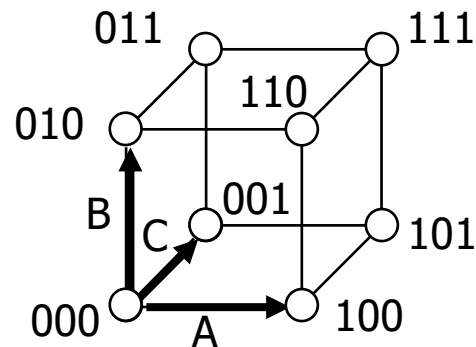
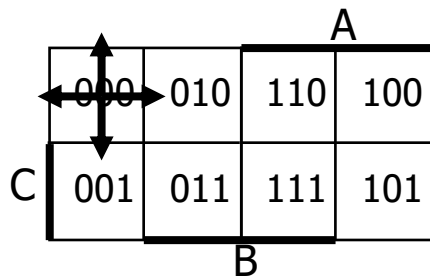
D

$$13 = 1101 = ABC'D$$

# Adjacencies in Karnaugh maps

- Wrap from first to last column
- Wrap top row to bottom row

อยู่ข้างกันใน K-map = ข้างกันใน Boolean Cube



# Karnaugh map examples

■  $F =$

	A	
B	0	1
	1	1
B	0	1
	0	0

$B'$

■  $C_{out} =$

	A			
Cin	0	0	1	0
	0	1	1	1
B	0	1	1	1

$AB + AC_{in} + BC_{in}$

$ABC_{in} + AB'C_{in} = AC_{in}$

■  $f(A,B,C) = \Sigma m(0,4,5,7)$

	A			
C	0	0	1	1
	1	0	0	1
B	0	1	1	1
	0	0	1	1

$AC + B'C' + AB'$

obtain the complement of the function by covering 0s with subcubes

# More Karnaugh map examples

			A
	0	0	1
	0	0	1
C	0	0	1
		B	

$$G(A,B,C) = A$$

			A
	1	0	0
	0	0	1
C	0	0	1
		B	

$$F(A,B,C) = \sum m(0,4,5,7) = AC + B'C'$$

			A
	0	1	1
	1	1	0
C	1	1	0
		B	

$\bar{F}$

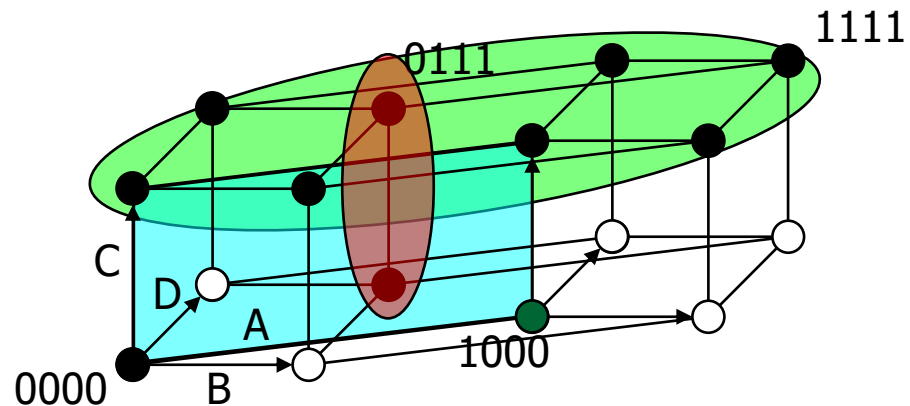
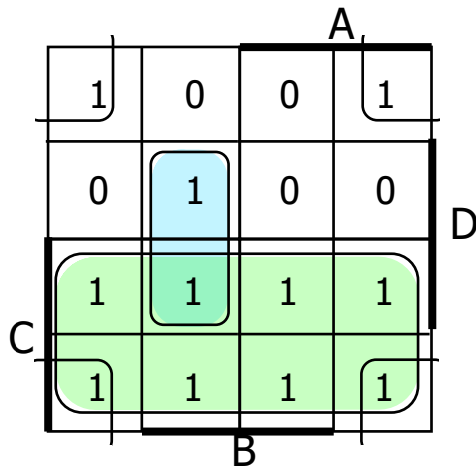
$$F' \text{ simply replace 1's with 0's and vice versa}$$

$$F'(A,B,C) = \sum m(1,2,3,6) = BC' + A'C$$

# Karnaugh map: 4-variable example

■  $F(A,B,C,D) = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$

$$F = C + A'BD + B'D'$$



find the smallest number of the largest possible subcubes to cover the ON-set  
(fewer terms with fewer inputs per term)

# Karnaugh maps: don't cares

- $f(A,B,C,D) = \Sigma m(1,3,5,7,9) + d(6,12,13)$ 
  - without don't cares
    - $f = A'D + B'C'D$

A			
0	0	X	0
1	1	X	1
1	1	0	0
0	X	0	0
B			
C			
D			



# Karnaugh maps: don't cares (cont'd)

- $f(A,B,C,D) = \Sigma m(1,3,5,7,9) + d(6,12,13)$ 
  - $f = A'D + B'C'D$  without don't cares
  - $f = A'D + C'D$  with don't cares

		A		
	0	0	X	0
	1	1	X	1
	1	1	0	0
C	0	X	0	0
		B		

by using **don't care** as a "1"  
a 2-cube can be formed  
rather than a 1-cube to cover  
this node

don't cares can be treated as  
1s or 0s  
depending on which is more  
advantageous

# Activity

- Minimize the function  $F = \Sigma m(0, 2, 7, 8, 14, 15) + d(3, 6, 9, 12, 13)$

			A	
	1	0	X	1
	0	0	X	X
C	X	1	1	0
	1	X	1	0
			B	

$$F = \cancel{AC'} + \cancel{A'C} + \cancel{BC} + \cancel{AB} + A'B'D' + B'C'D'$$

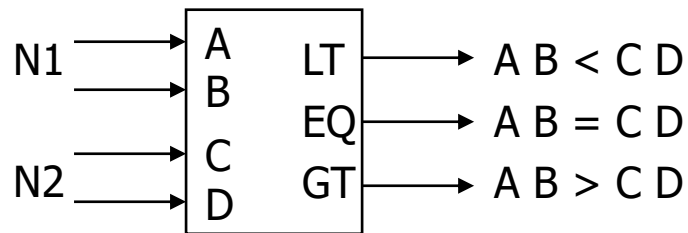
$$F = BC + \overset{3 \text{ literals}}{A'B'D'} + B'C'D'$$

$$\text{ง่ายกว่า} \quad F = A'C + AB + \overset{3 \text{ literals}}{B'C'D'}$$

			A	
	1	0	X	1
	0	0	X	X
	X	1	1	0
C	1	X	1	0
			B	

			A	
	1	0	X	1
	0	0	X	X
	X	1	1	0
C	1	X	1	0
			B	

# Design example: two-bit comparator

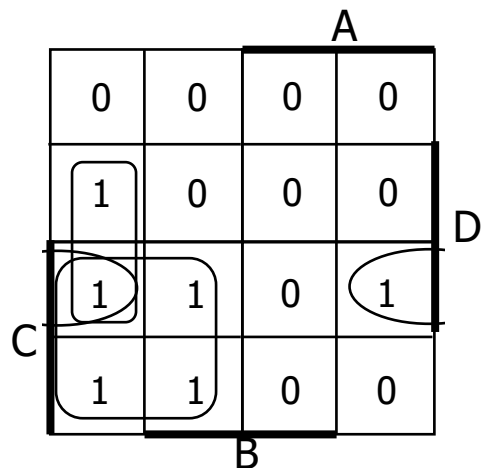


block diagram  
and  
truth table

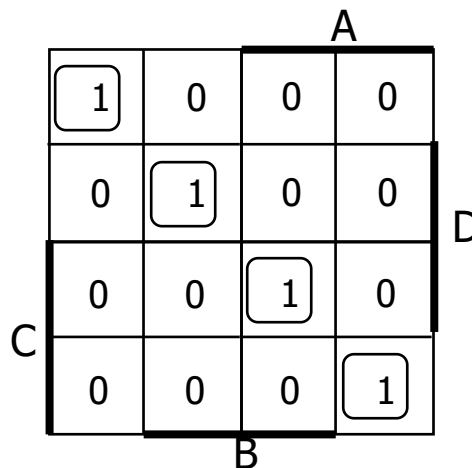
A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	1	0	0
0	1	0	0	0	0	1
		0	1	0	1	0
		1	0	1	0	0
		1	1	1	0	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	0	1	0
		1	1	1	0	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	1	0

we'll need a 4-variable Karnaugh map  
for each of the 3 output functions

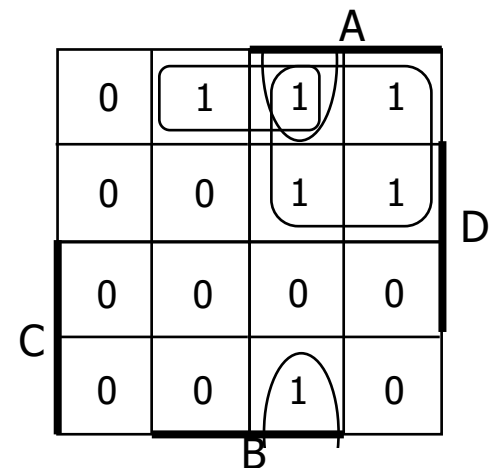
# Design example: two-bit comparator (cont'd)



K-map for LT



K-map for EQ



K-map for GT

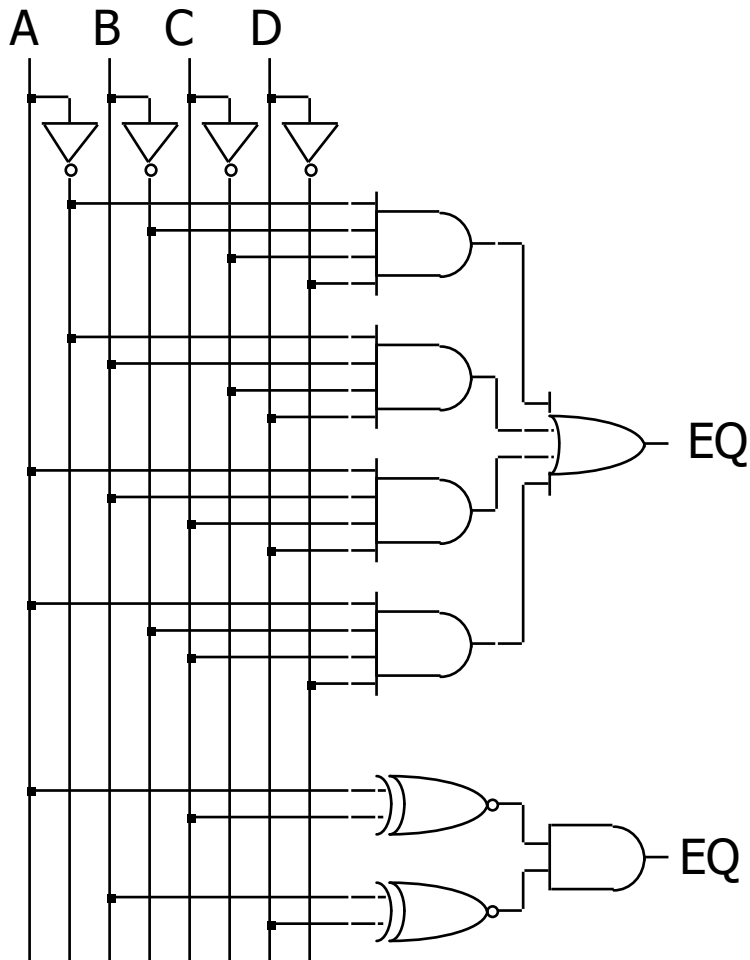
$$LT = A' B' D + A' C + B' C D$$

$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{ xnor } C) \bullet (B \text{ xnor } D)$$

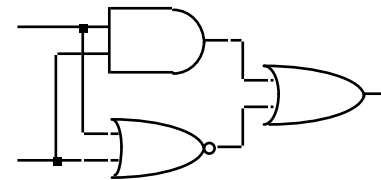
$$GT = B C' D' + A C' + A B D'$$

LT and GT are similar (flip A/C and B/D)

# Design example: two-bit comparator (cont'd)

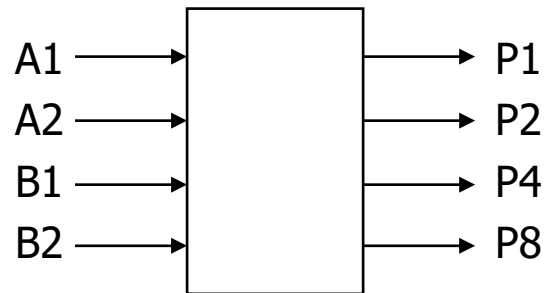


two alternative  
implementations of EQ  
with and without XOR



XNOR is implemented with  
at least 3 simple gates

# Design example: 2x2-bit multiplier

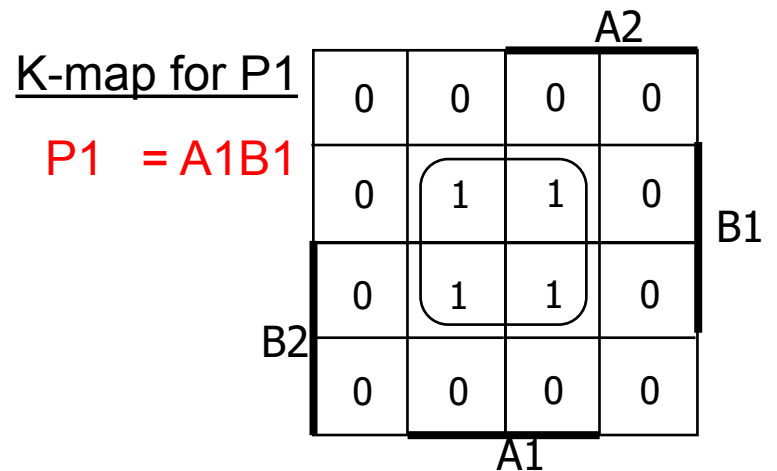
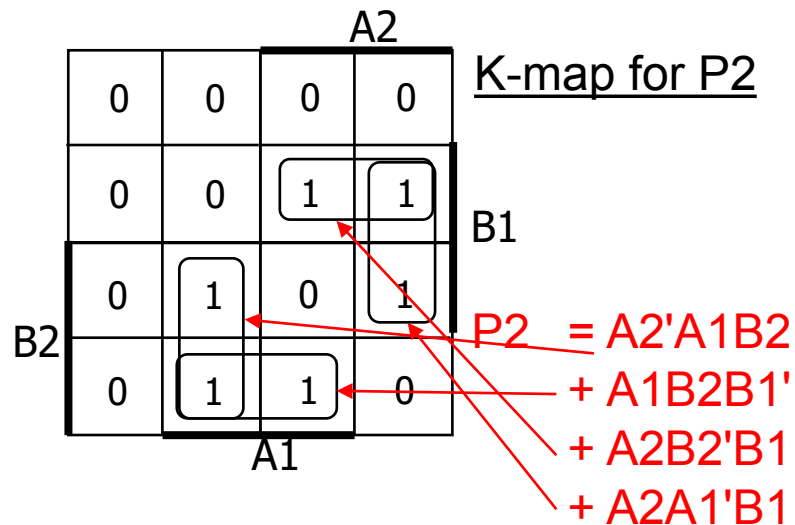
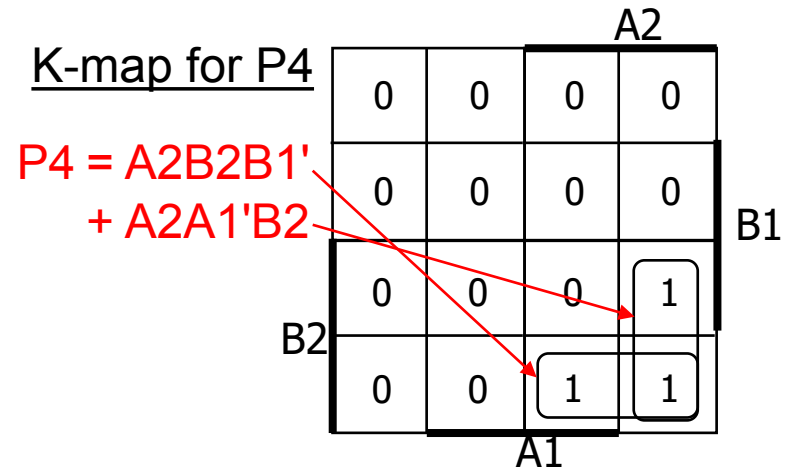
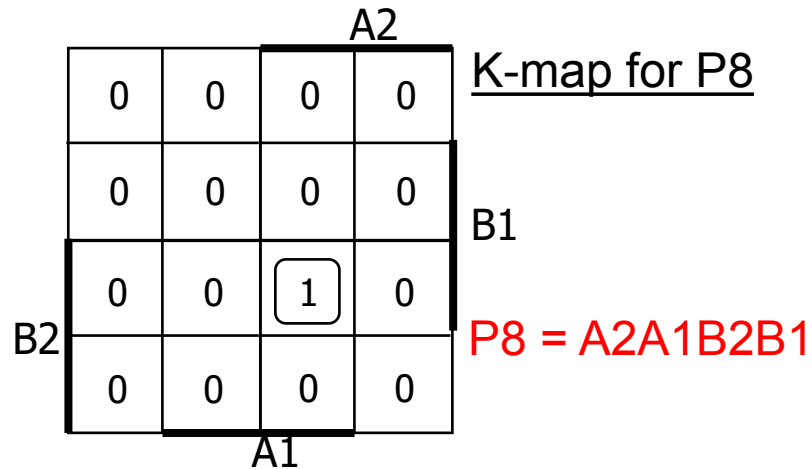


block diagram  
and  
truth table

A2	A1	B2	B1	P8	P4	P2	P1
0	0	0	0	0	0	0	0
		0	1	0	0	0	0
		1	0	0	0	0	0
		1	1	0	0	0	0
0	1	0	0	0	0	0	0
		0	1	0	0	0	1
		1	0	0	0	1	0
		1	1	0	0	1	1
1	0	0	0	0	0	0	0
		0	1	0	0	1	0
		1	0	0	1	0	0
		1	1	0	1	1	0
1	1	0	0	0	0	0	0
		0	1	0	0	1	1
		1	0	0	1	1	0
		1	1	1	0	0	1

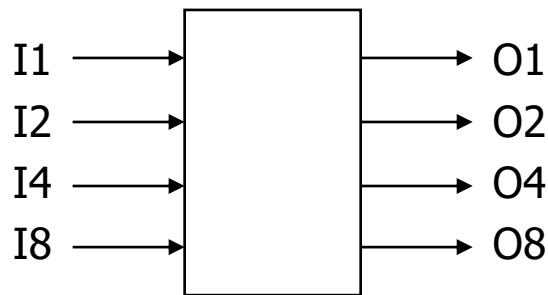
4-variable K-map  
for each of the 4  
output functions

# Design example: 2x2-bit multiplier (cont'd)



# Design example: BCD increment by 1

( Binary - coded decimal )



block diagram  
and  
truth table

I8	I4	I2	I1	O8	O4	O2	O1
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

0-9 ที่เหลือ don't care

4-variable K-map for each of  
the 4 output functions



# Design example: BCD increment by 1 (cont'd)

				I8	
	0	0	X	1	<u>O8</u>
	0	0	X	0	
	0	1	X	X	I1
I2	0	0	X	X	
					I4

$$O8 = I4 I2 I1 + I8 I1'$$

$$O4 = I4 I2' + I4 I1' + I4' I2 I1$$

$$O2 = I8' I2' I1 + I2 I1'$$

$$O1 = I1'$$

				I8	
	0	0	X	0	<u>O2</u>
	1	1	X	0	
	0	0	X	X	I1
I2	1	1	X	X	
				I4	

		I8		I1
<u>O4</u>				
I2	0	1	X	
	0	1	X	
	1	0	X	
	0	1	X	
		I4		

		I8		I1
<u>O1</u>				
I2	1	1	X	
	0	0	X	
	0	0	X	
	1	1	X	
		I4		

# Definition of terms for two-level simplification

- **Implicant**
  - single element of ON-set or DC-set or any group of these elements that can be combined to form a subcube
- **Prime implicant** \*
- implicant that can't be combined with another to form a larger subcube
- **Essential prime implicant**
  - prime implicant is essential if it alone covers an element of ON-set
  - will participate in ALL possible covers of the ON-set
  - DC-set used to form prime implicants but not to make implicant essential
- **Objective:**
  - grow implicant into prime implicants (minimize literals per term)
  - cover the ON-set with as few prime implicants as possible (minimize number of product terms)

# Examples to illustrate terms

	A			
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			

6 prime implicants:

$A'B'D$ ,  $BC'$ ,  $AC$ ,  $A'C'D$ ,  $AB$ ,  $B'CD$

essential

minimum cover:  $AC + BC' + A'B'D$

5 prime implicants:

$BD$ ,  $ABC'$ ,  $ACD$ ,  $A'BC$ ,  $A'C'D$

essential

minimum cover: 4 essential implicants

	A			
	0	0	1	0
	1	1	1	0
C	0	1	1	1
	0	1	0	0
	B			

# Algorithm for two-level simplification

- Algorithm: minimum sum-of-products expression from a Karnaugh map
  - Step 1: choose an element of the ON-set
  - Step 2: find "maximal" groupings of 1s and Xs adjacent to that element
    - consider top/bottom row, left/right column, and corner adjacencies
    - this forms prime implicants (number of elements always a power of 2)
  - Repeat Steps 1 and 2 to find all prime implicants
  - Step 3: revisit the 1s in the K-map
    - if covered by single prime implicant, it is essential, and participates in final cover
    - 1s covered by essential prime implicant do not need to be revisited
  - Step 4: if there remain 1s not covered by essential prime implicants
    - select the smallest number of prime implicants that cover the remaining 1s

# Algorithm for two-level simplification (example)

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

**2 primes around  $A'BC'D'$**

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

**2 primes around  $ABC'D$**

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

**3 primes around  $AB'C'D'$**

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

**2 essential primes**

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

**minimum cover (3 primes)**

# Activity

- List all prime implicants for the following K-map:

		A		
	X	0	X	0
	0	1	X	1
	0	X	X	0
C	X	1	1	1
	B		D	

$\checkmark$   
 $CD'$      $BC$      $\checkmark$   
 $BD$      $AB$      $\checkmark$   
 $AC'D$

- Which are essential prime implicants?  $CD'$      $BD$      $AC'D$
- What is the minimum cover?  $CD'$      $BD$      $AC'D$

# Quine-McCluskey Method \* ไม่ออกสอบ ทาลาย โอกาสทำผิดสูง

## Tabular method to systematically find all prime implicants

$$g(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + \sum d(0,7,15)$$

## Stage 1: Find all prime implicants

**Step 1: Fill Column 1 with ON-set and DC-set minterm indices. Group by number of 1's.**

Implication Table	
Column I	
0000 0	
0100 4	
1000 8	
0101 5	
0110 6	
1001 9	
1010 10	
0111 7	
1101 13	
1111 15	

# Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$$f(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + \sum d(0,7,15)$$

Stage 1: Find all prime implicants

Step 1: Fill Column 1 with ON-set and DC-set minterm indices. Group by number of 1's.

Step 2: Apply Uniting Theorem—  
Compare elements of group w/  
N 1's against those with N+1 1's.  
Differ by one bit implies adjacent.  
Eliminate variable and place in  
next column.

E.g., 0000 vs. 0100 yields 0-00  
0000 vs. 1000 yields -000

When used in a combination,  
mark with a check. If cannot be  
combined, mark with a star. These  
are the prime implicants.

Repeat until no further combinations can be made.

Implication Table		
Column I	Column II	
0000	0-00 -000	
0100		
1000	010- 01-0	
0101	100- 10-0	
0110		
1001	01-1 -101	
1010	011- 1-01	
0111		
1101	-111 11-1	
1111		



# Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$$f(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + \sum d(0,7,15)$$

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Differ by one bit implies adjacent.  
Eliminate variable and place in  
next column.

E.g., 0000 vs. 0100 yields 0-00  
0000 vs. 1000 yields -000

When used in a combination,  
mark with a check. If cannot be  
combined, mark with a star. These  
are the prime implicants.

Repeat until no further combinations can be made.

Implication Table		
Column I	Column II	Column III
0000	0-00 *	01-- *
	-000 *	
0100		-1-1 *
1000	010-	
	01-0	
0101	100- *	
0110	10-0 *	
1001	01--	01--
1010	01-1	
	-101	
0111	011-	
1101	1-01 *	
	-1-1	-1-1
1111	-111	
	11-1	

# Quine-McCluskey Method

AB \ CD		A			
		00	01	11	10
C	00	X	1	0	1
	01	0	1	1	1
	11	0	X	X	0
	10	0	1	0	1

Diagram showing the Karnaugh map with groupings for prime implicants:

- Group A: Columns 11 and 10 (top row)
- Group B: Columns 01 and 11 (bottom row)
- Group C: Rows 11 and 10 (left column)
- Group D: Rows 01 and 11 (right column)

## Prime Implicants:

$$0-00 = A' C' D'$$

$$100- = A B' C'$$

$$1-01 = A C' D$$

$$-1-1 = B D$$

$$-000 = B' C' D'$$

$$10-0 = A B' D'$$

$$01-- = A' B$$

# Quine-McCluskey Method Continued

		A			
AB		00	01	11	10
CD	00	X	1	0	1
	01	0	1	1	1
	11	0	X	X	0
	10	0	1	0	1
C		D			
		B			

**Prime Implicants:**

$$0-00 = A' C' D'$$

$$-000 = B' C' D'$$

$$100- = A B' C'$$

$$10-0 = A B' D'$$

$$1-01 = A C' D$$

$$01-- = A' B$$

$$-1-1 = B D$$

**Stage 2: find smallest set of prime implicants that cover the ON-set**  
**recall that essential prime implicants must be in all covers**  
**another tabular method– the prime implicant chart \***

# Prime Implicant Chart

	4	5	6	8	9	10	13
0,4 (0-00)	X						
0,8 (-000)				X			
8,9 (100-)				X	X		
8,10 (10-0)				X		X	
9,13 (1-01)					X		X
4,5,6,7 (01-)	X	X	X				
5,7,13,15 (-1-1)		X					X

**rows = prime implicants**  
**columns = ON-set elements**  
**place an "X" if ON-set element is**  
**covered by the prime implicant**

# Prime Implicant Chart

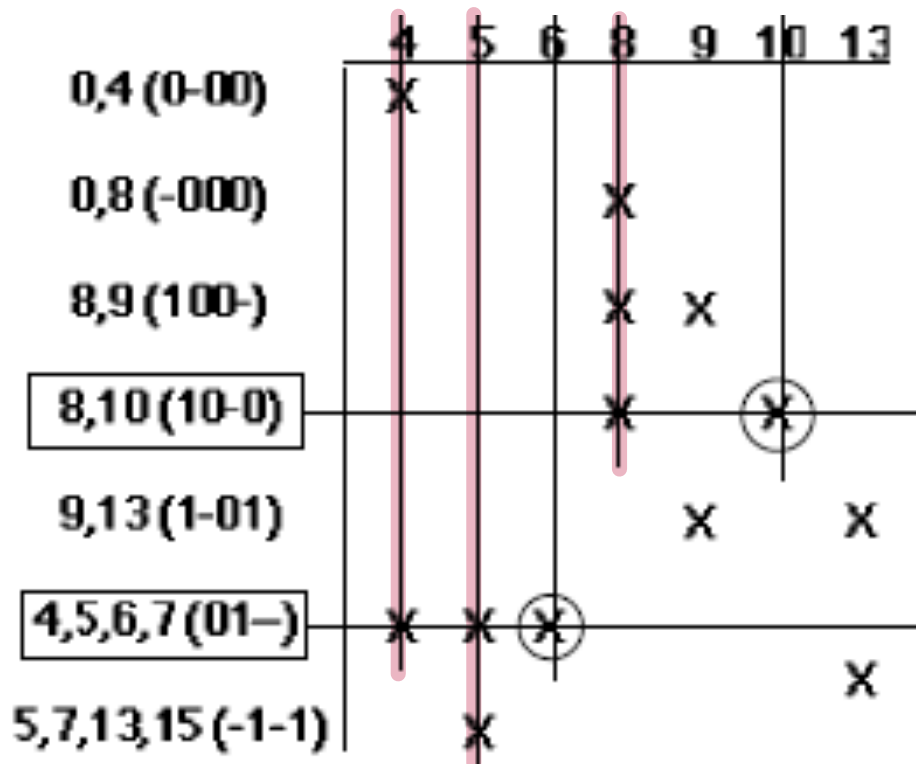
	4	5	6	8	9	10	13
0,4 (0-00)	X						
0,8 (-000)				X			
8,9 (100-)				X	X		
8,10 (10-0)				X		X	
9,13 (1-01)					X		X
4,5,6,7 (01-)	X	X	X				
5,7,13,15 (-1-1)		X					X

	4	5	6	8	9	10	13
0,4 (0-00)	X						
0,8 (-000)				X			
8,9 (100-)				X	X		
8,10 (10-0)				X		X	
9,13 (1-01)					X		X
4,5,6,7 (01-)	X	X	X				
5,7,13,15 (-1-1)		X					X

rows = prime implicants  
columns = ON-set elements  
place an "X" if ON-set element is  
covered by the prime implicant

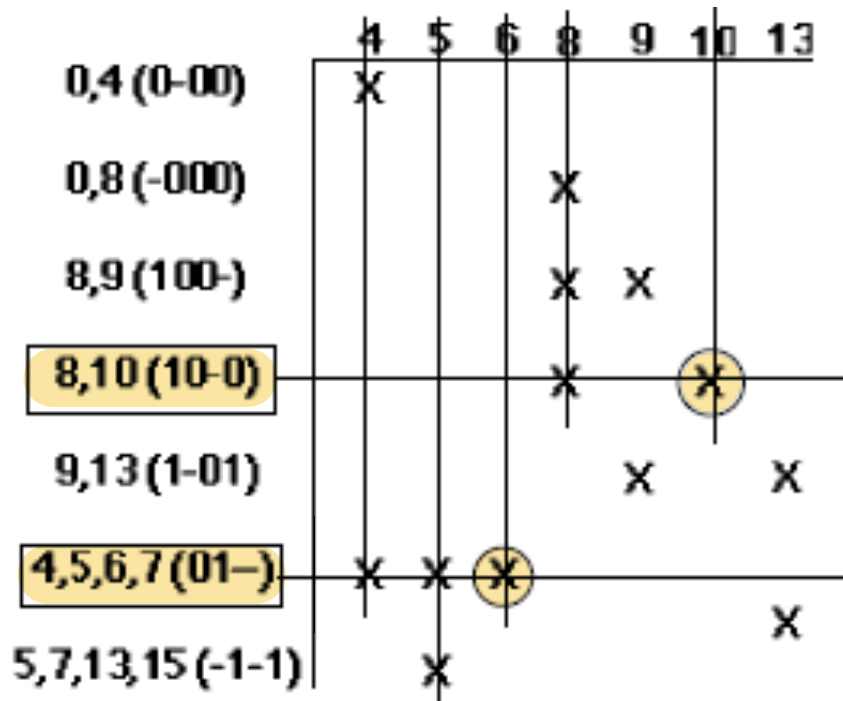
If column has a **single X**, then the  
implicant associated with the row  
is **essential**. It must appear in  
minimum cover

# Prime Implicant Chart

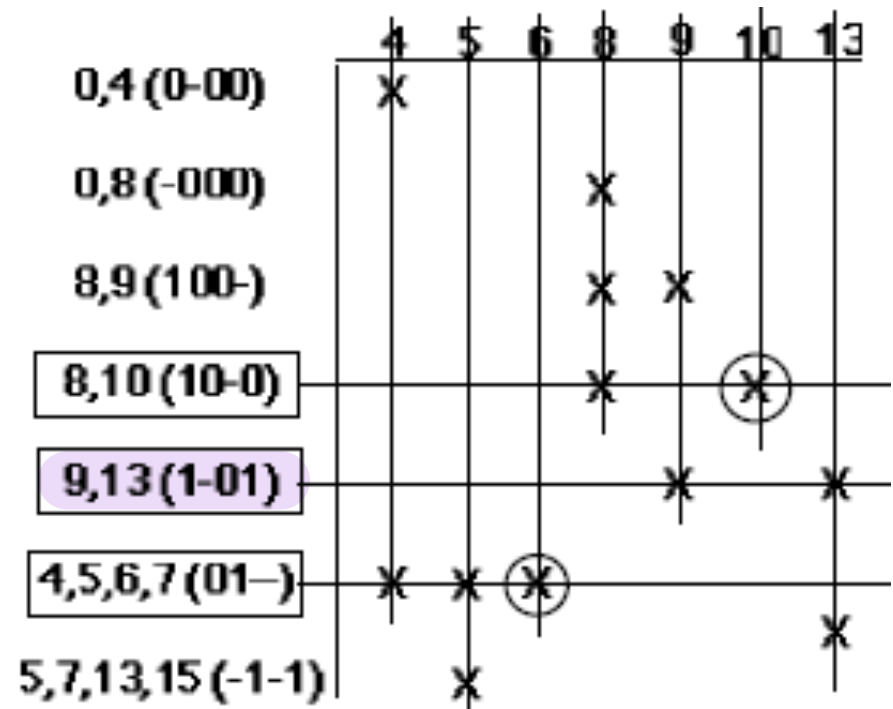


Eliminate all columns covered by essential primes

# Prime Implicant Chart



Eliminate all columns covered by essential primes



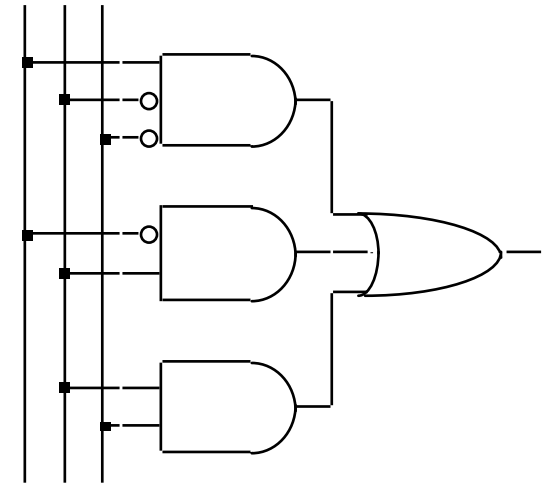
Find minimum set of rows that cover the remaining columns

$$f = A B' D' + A C' D + A' B$$

# Implementations of two-level logic

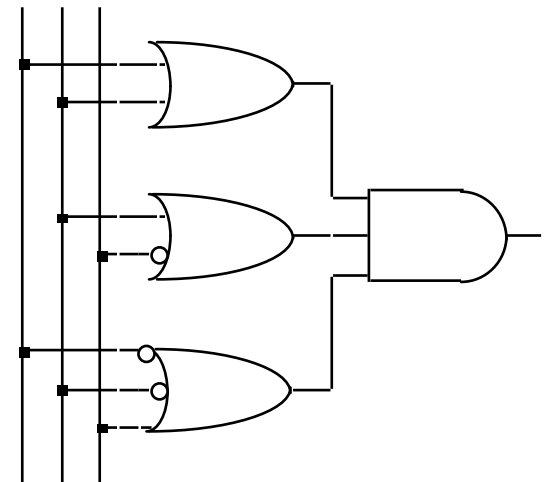
## ■ Sum-of-products

- AND gates to form product terms (minterms)
- OR gate to form sum



## ■ Product-of-sums

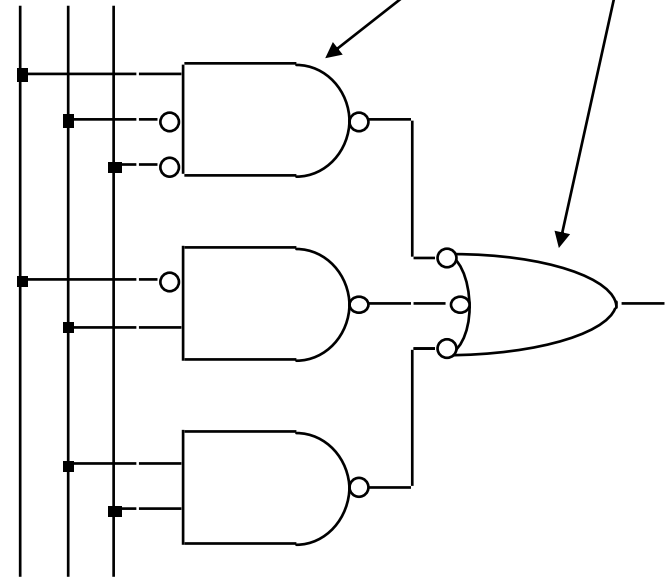
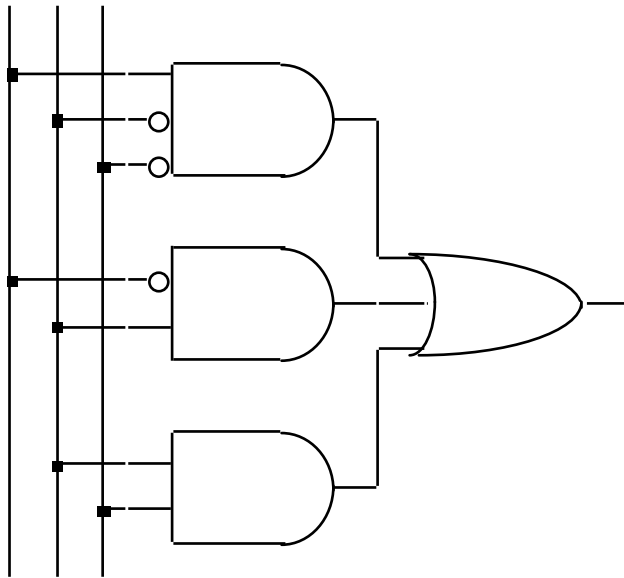
- OR gates to form sum terms (maxterms)
- AND gates to form product





# Two-level logic using NAND gates

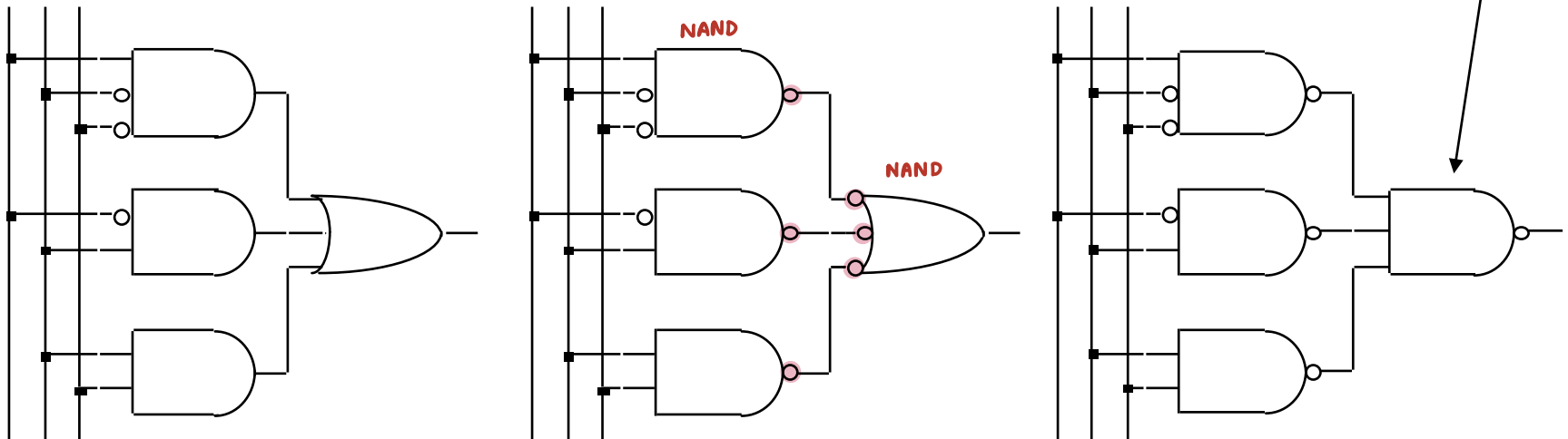
- Replace minterm **AND** gates with **NAND** gates
- Place compensating inversion at inputs of OR gate



# Two-level logic using NAND gates (cont'd)

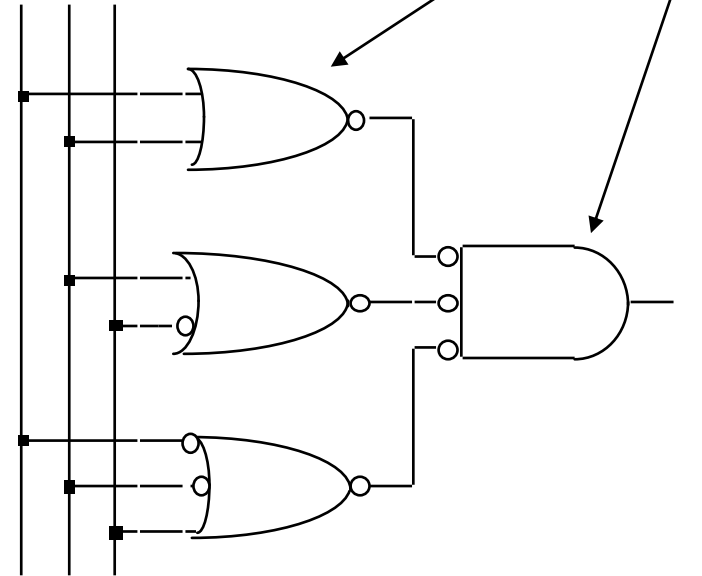
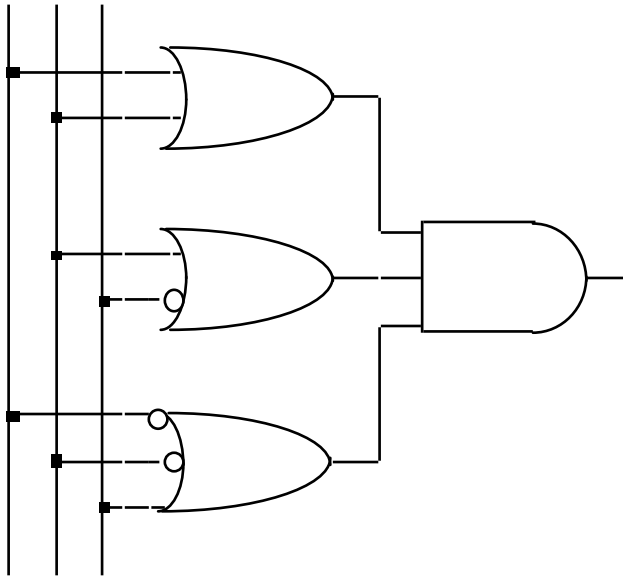
(SoP)

- OR gate with inverted inputs is a NAND gate
  - de Morgan's:  $A' + B' = (A \cdot B)'$
- Two-level NAND-NAND network
  - inverted inputs are not counted
  - in a typical circuit, inversion is done once and signal distributed



# Two-level logic using NOR gates

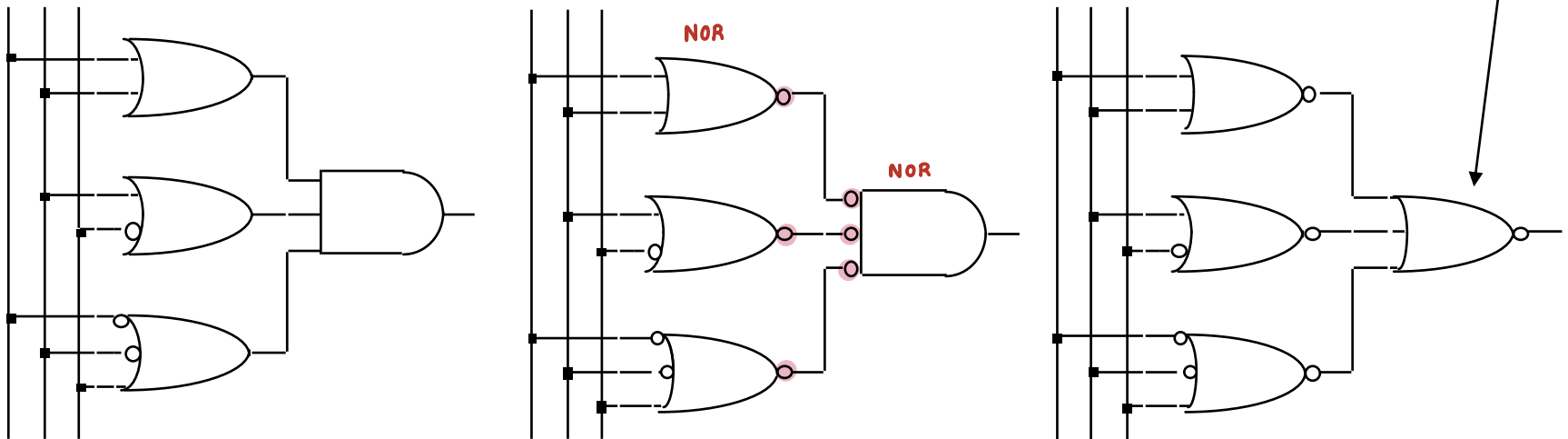
- Replace maxterm OR gates with NOR gates
- Place compensating inversion at inputs of AND gate



# Two-level logic using NOR gates (cont'd)

(PoS)

- AND gate with inverted inputs is a NOR gate
  - de Morgan's:  $A' \cdot B' = (A + B)'$
- Two-level NOR-NOR network
  - inverted inputs are not counted
  - in a typical circuit, inversion is done once and signal distributed



# Two-level logic using NAND and NOR gates

## ■ NAND-NAND and NOR-NOR networks

- de Morgan's law:  $(A + B)' = A' \cdot B'$  ①       $(A \cdot B)' = A' + B'$  ②
- written differently:  $A + B = (A' \cdot B')'$  ③       $(A \cdot B) = (A' + B')'$  ④

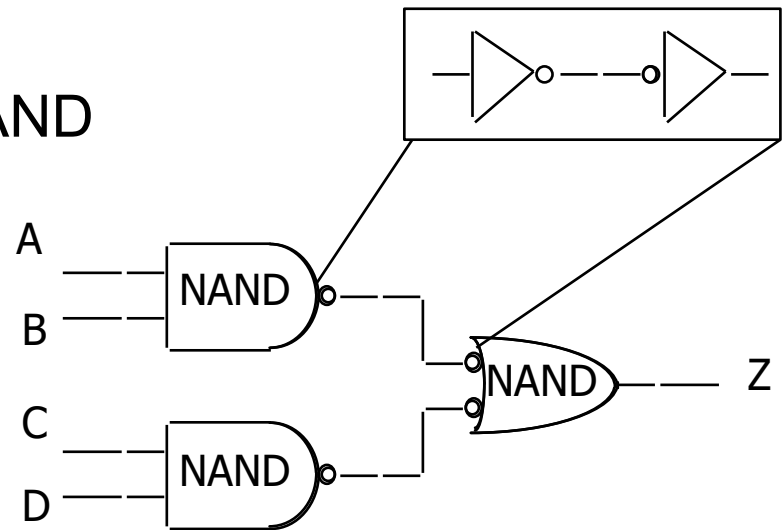
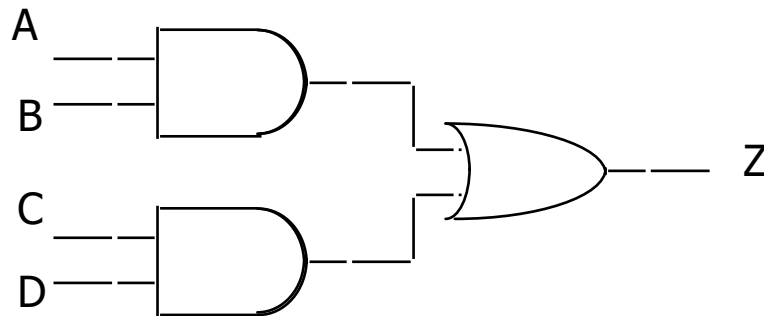
## ■ In other words —

- OR is the same as NAND with complemented inputs
- AND is the same as NOR with complemented inputs
- NAND is the same as OR with complemented inputs
- NOR is the same as AND with complemented inputs



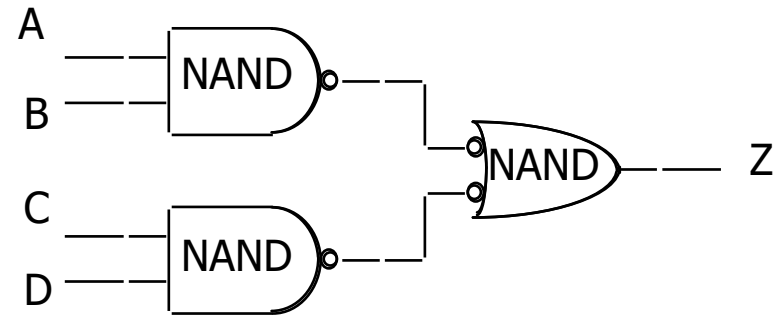
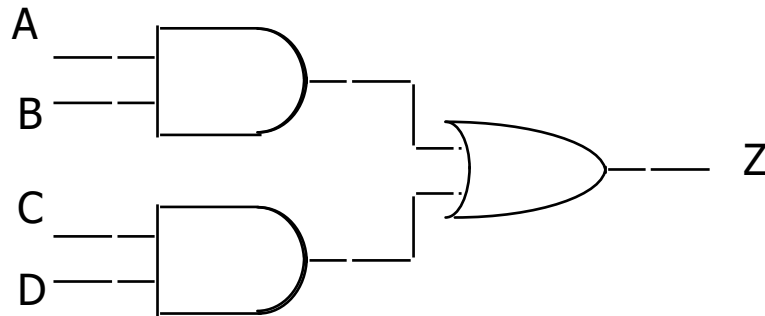
# Conversion between forms

- Convert from networks of ANDs and ORs to networks of NANDs and NORs
  - introduce appropriate inversions ("bubbles")
- Each introduced "bubble" must be matched by a corresponding "bubble"
  - conservation of inversions
  - do not alter logic function
- Example: AND/OR to NAND/NAND



# Conversion between forms (cont'd)

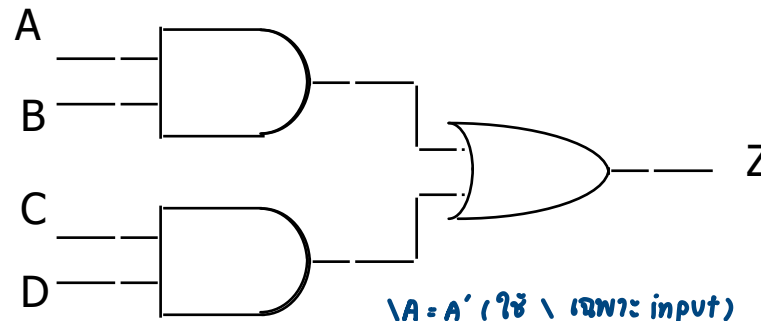
- Example: verify equivalence of two forms



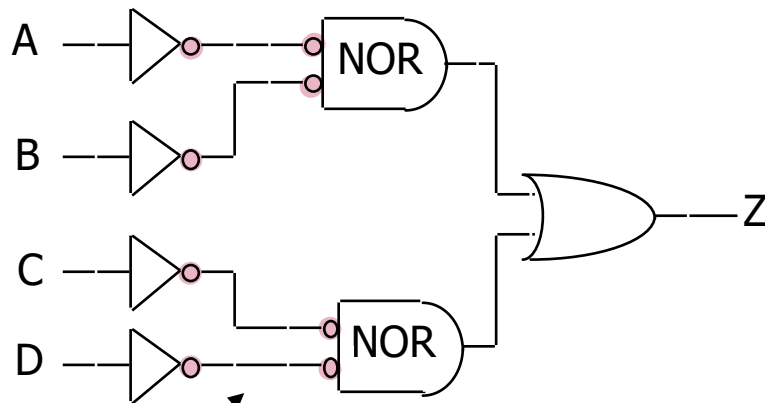
$$\begin{aligned} Z &= [ (A \cdot B)' \cdot (C \cdot D)' ]' \\ &= [ (A' + B') \cdot (C' + D') ]' \\ &= [ (A' + B')' + (C' + D')' ] \\ &= (A \cdot B) + (C \cdot D) \quad \checkmark \end{aligned}$$

# Conversion between forms (cont'd)

- Example: map AND/OR network to NOR/NOR network

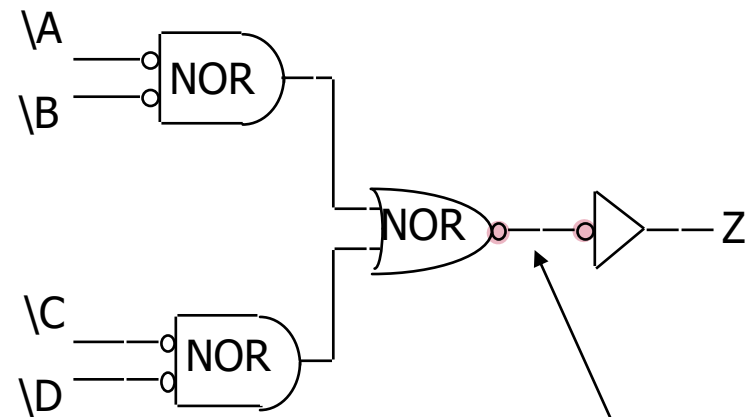


$\forall A = A' \text{ (if \textbackslash input)}$



Step 1

conserve  
"bubbles"



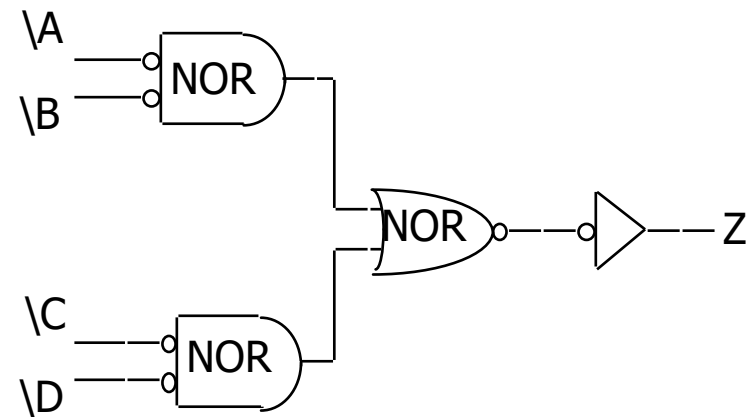
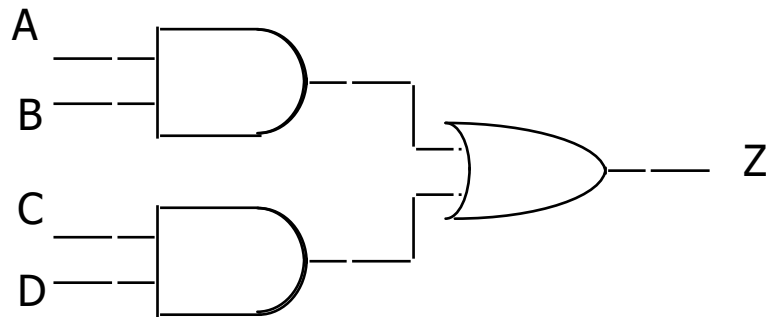
Step 2

conserve  
"bubbles"



# Conversion between forms (cont'd)

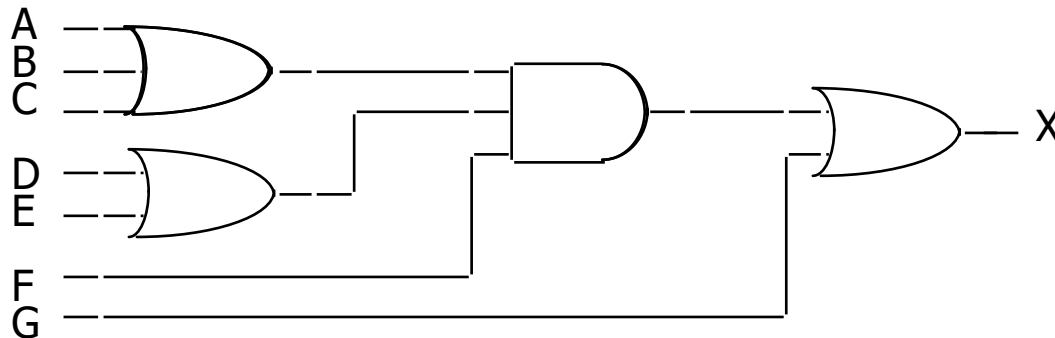
- Example: verify equivalence of two forms



$$\begin{aligned} Z &= \{ [ (A' + B')' + (C' + D')' ]' \}' \\ &= \{ (A' + B') \cdot (C' + D') \}' \\ &= (A' + B')' + (C' + D')' \\ &= (A \cdot B) + (C \cdot D) \quad \checkmark \end{aligned}$$

# Multi-level logic

- $x = A D F + A E F + B D F + B E F + C D F + C E F + G$ 
  - reduced sum-of-products form – already simplified
  - 6 x 3-input AND gates + 1 x 7-input OR gate (that may not even exist!)
  - 25 wires (19 literals plus 6 internal wires)
- $x = (A + B + C) (D + E) F + G$ 
  - factored form – not written as two-level S-o-P
  - 1 x 3-input OR gate, 2 x 2-input OR gates, 1 x 3-input AND gate
  - 10 wires (7 literals plus 3 internal wires)



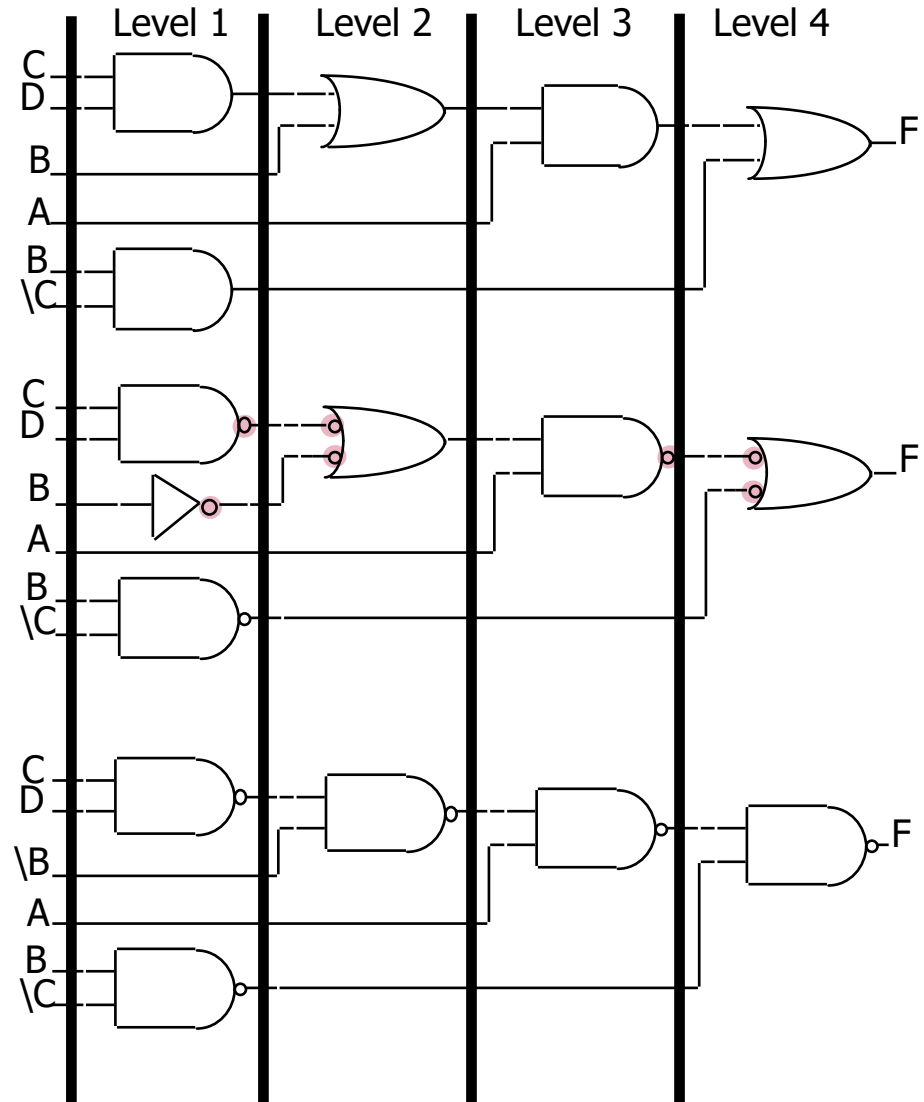
# Conversion of multi-level logic to **NAND** gates

■  $F = A(B + CD) + B C'$

original  
AND-OR  
network

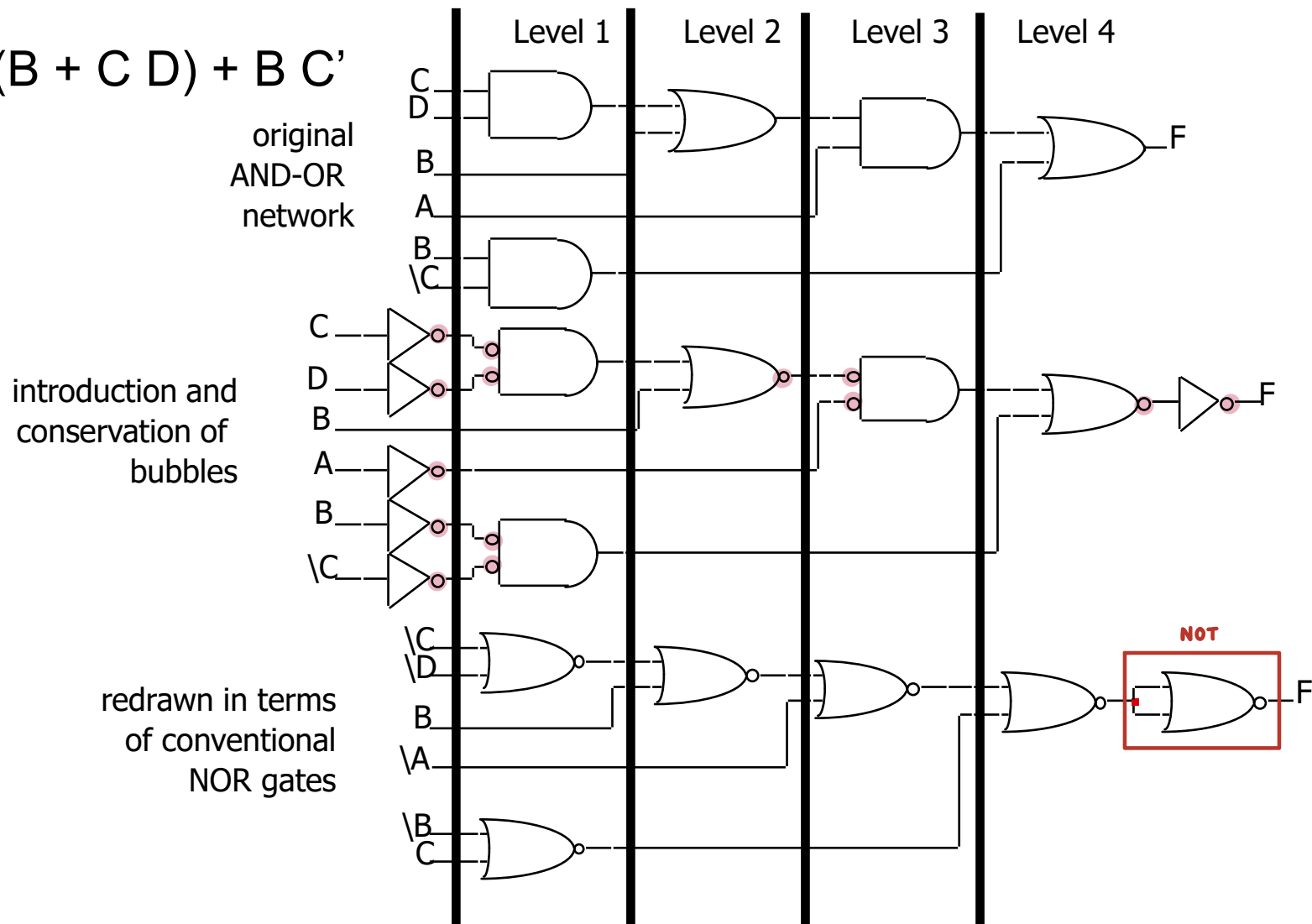
introduction and  
conservation of  
bubbles

redrawn in terms  
of conventional  
NAND gates



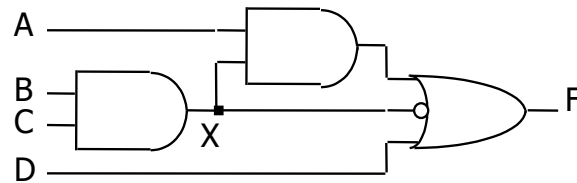
# Conversion of multi-level logic to **NORs**

■  $F = A(B + CD) + BC'$

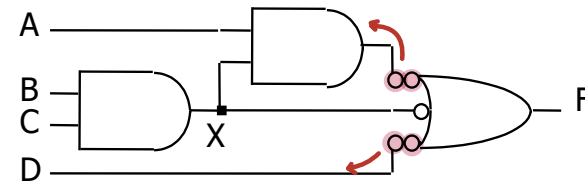


# Conversion between forms

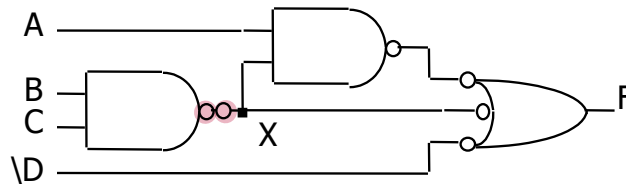
## ■ Example



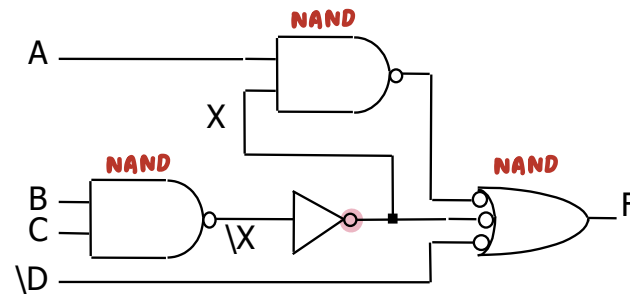
original circuit



add double bubbles to  
invert all inputs of OR gate



add double bubbles to  
invert output of AND gate

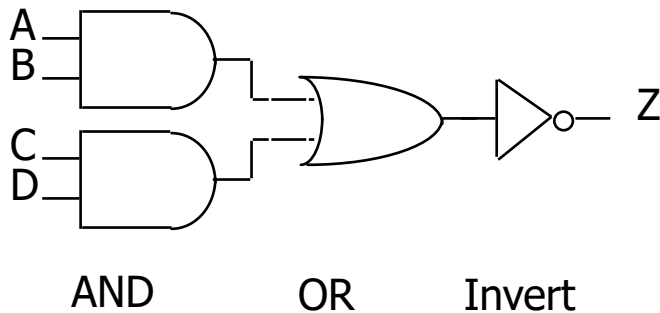


insert inverters to eliminate  
double bubbles on a wire

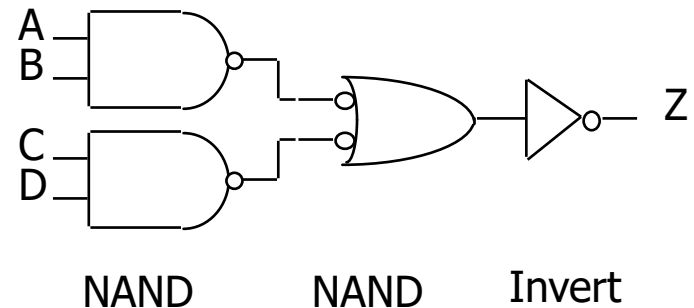
# AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
  - multiple gates "packaged" as a single circuit block

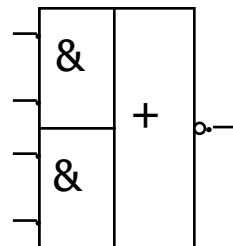
logical concept



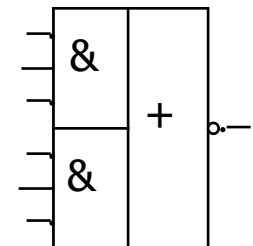
possible implementation



2x2 AOI gate symbol

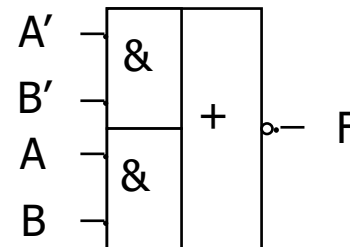
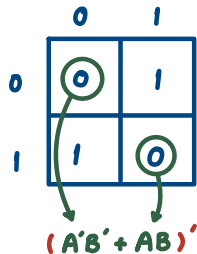


3x2 AOI gate symbol



# Conversion to AOI forms

- General procedure to place in AOI form
  - compute the complement of the function in sum-of-products form
  - by grouping the 0s in the Karnaugh map
- Example: XOR implementation
  - $A \text{ xor } B = A' B + A B'$
  - AOI form:
    - $F = (A' B' + A B)'$



# Examples of using AOI gates

## ■ Example:

- $F = AB + AC' + BC'$
- $F = (A'B' + A'C + B'C)'$
- Implemented by 2-input 3-stack AOI gate
- $F = (A + B)(A + C')(B + C')$
- $F = [(A' + B')(A' + C)(B' + C)]'$
- Implemented by 2-input 3-stack OAI gate

## ■ Example: 4-bit equality function

- $Z = (A_0 B_0 + A_0' B_0')(A_1 B_1 + A_1' B_1')(A_2 B_2 + A_2' B_2')(A_3 B_3 + A_3' B_3')$

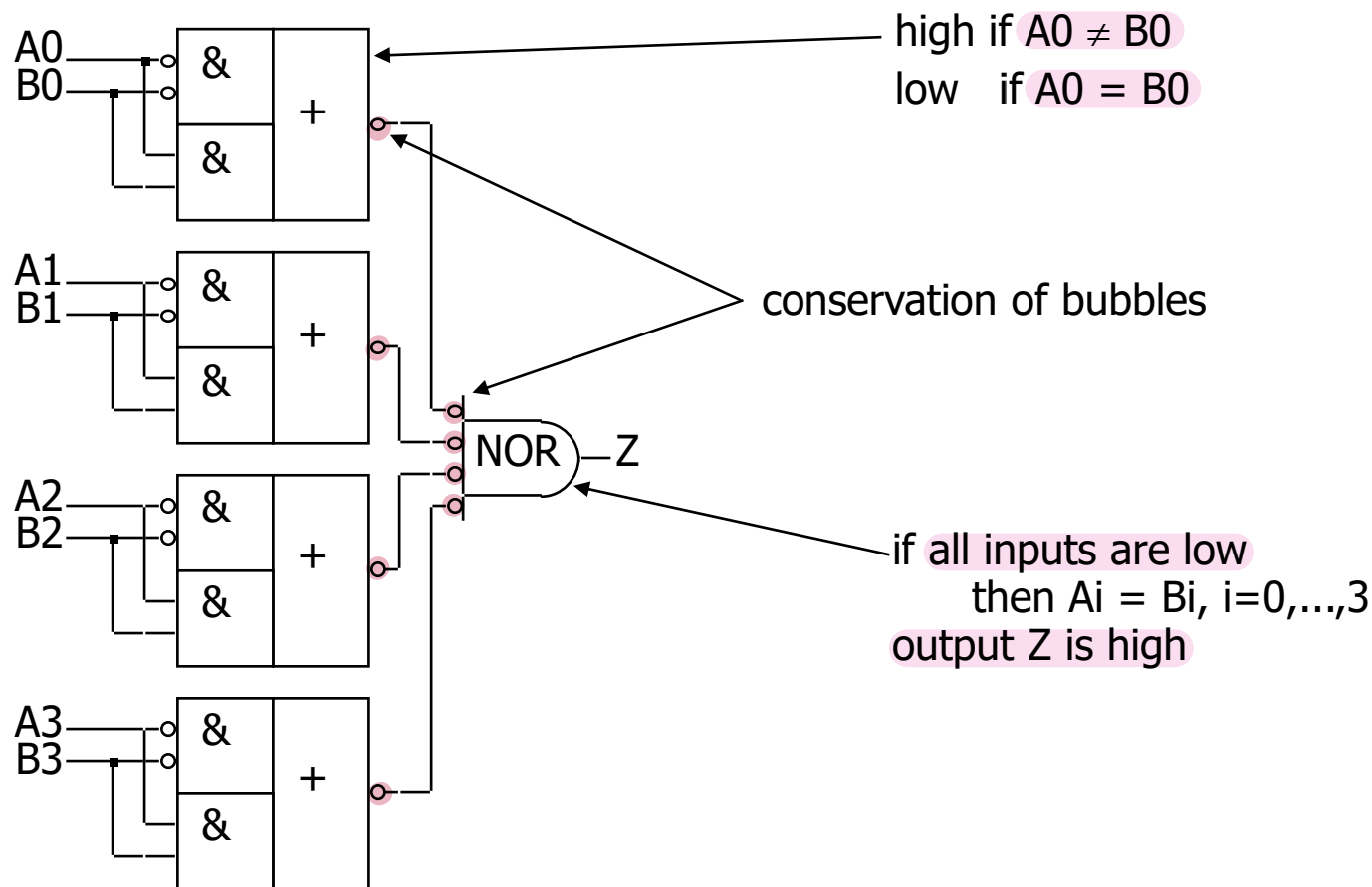
each implemented in a single 2x2 AOI gate



# Examples of using AOI gates (cont'd)

$A_3A_2A_1A_0 = B_3B_2B_1B_0$  เท่ากัน เมื่อทุก bit เท่ากัน

## ■ Example: AOI implementation of 4-bit equality function



# Summary for multi-level logic

## ■ Advantages

- circuits may be smaller
- gates have smaller fan-in
- circuits may be faster

## ■ Disadvantages

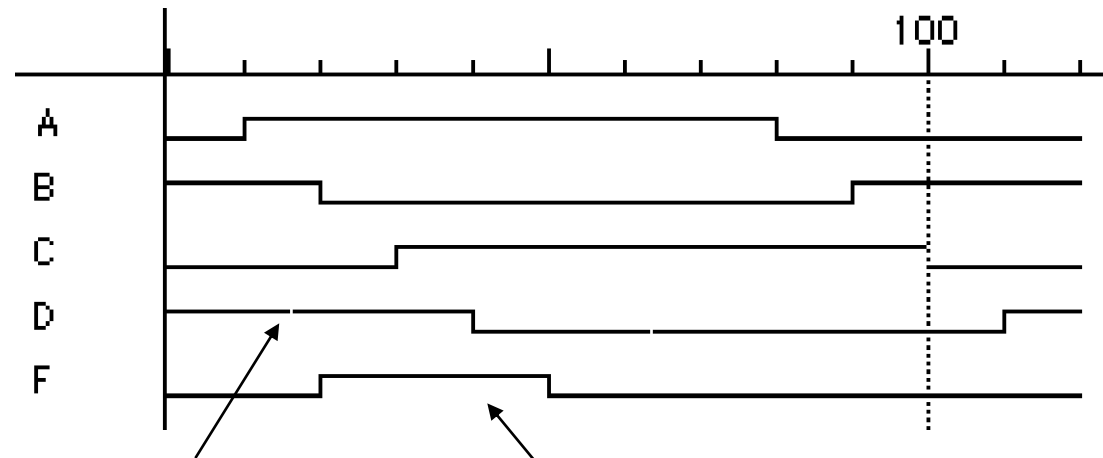
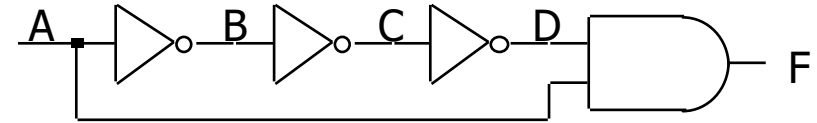
- more difficult to design
- tools for optimization are not as good as for two-level
- analysis is more complex

# Time behavior of combinational networks

- Waveforms
  - visualization of values carried on signal wires over time
  - useful in explaining sequences of events (changes in value)
- Simulation tools are used to create these waveforms
  - input to the simulator includes gates and their connections
  - input stimulus, that is, input signal waveforms
- Some terms
  - gate delay — time for change at input to cause change at output
    - min delay – typical/nominal delay – max delay
    - careful designers design for the worst case
  - rise time — time for output to transition from low to high voltage
  - fall time — time for output to transition from high to low voltage
  - pulse width — time that an output stays high or stays low between changes

# Momentary changes in outputs

- Can be useful — pulse shaping circuits
- Can be a problem — incorrect circuit operation (glitches/hazards)
- Example: pulse shaping circuit
  - $A' \cdot A = 0$
  - delays matter

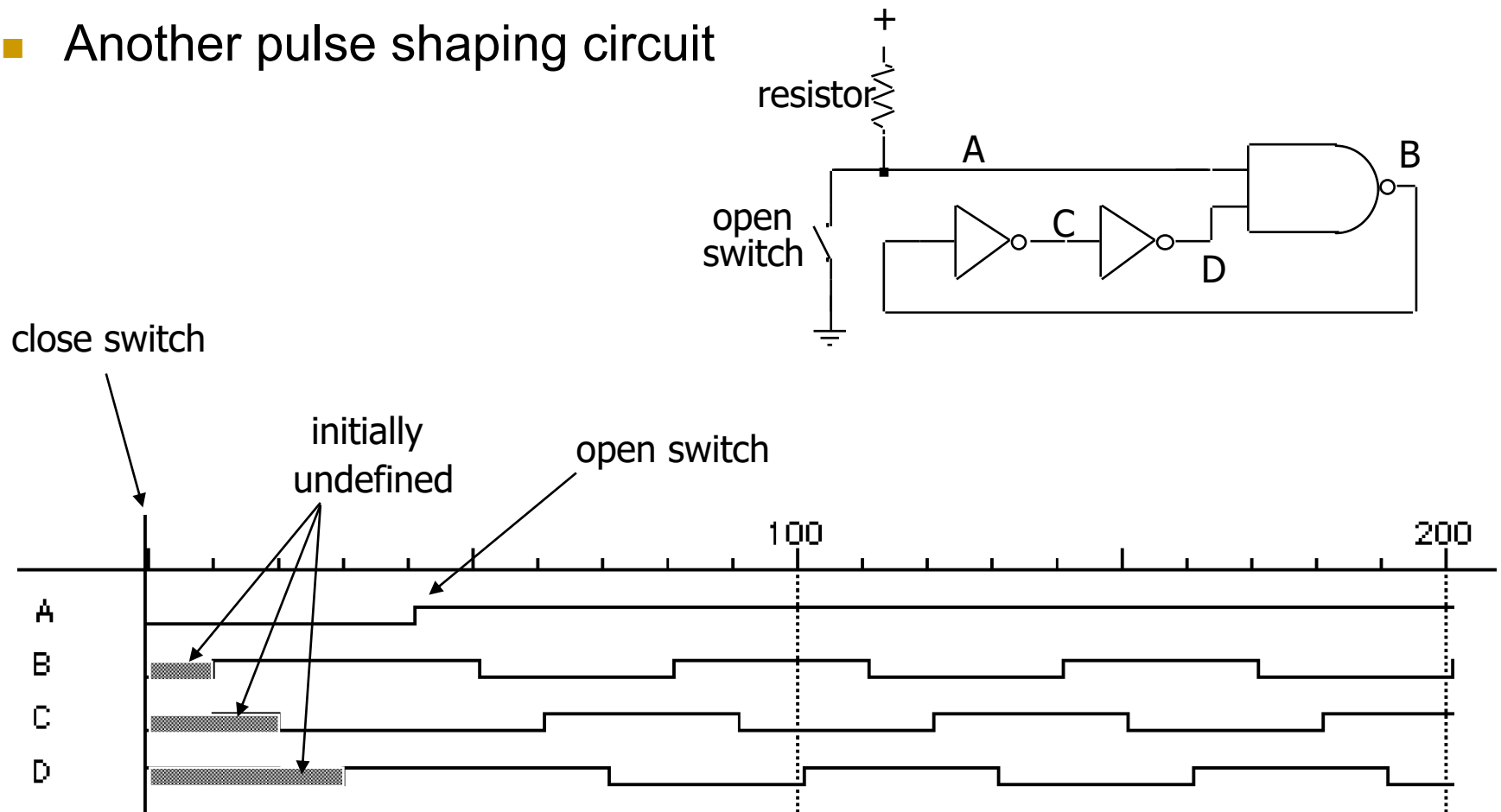


D remains high for  
three gate delays after  
A changes from low to high

F is not always 0  
pulse 3 gate-delays wide

# Oscillatory behavior

## ■ Another pulse shaping circuit



# Hardware description languages

- Describe hardware at varying levels of abstraction
- Structural description
  - textual replacement for schematic
  - hierarchical composition of modules from primitives
- Behavioral/functional description
  - describe what module does, not how
  - synthesis generates circuit for module
- Simulation semantics

# HDLs

- Abel (circa 1983) - developed by Data-I/O
  - targeted to programmable logic devices
  - not good for much more than state machines
- ISP (circa 1977) - research project at CMU
  - simulation, but no synthesis
- Verilog (circa 1985) - developed by Gateway (absorbed by Cadence)
  - similar to Pascal and C
  - delays is only interaction with simulator
  - fairly efficient and easy to write
  - IEEE standard
- VHDL (circa 1987) - DoD sponsored standard
  - similar to Ada (emphasis on re-use and maintainability)
  - simulation semantics visible
  - very general but verbose
  - IEEE standard

# Verilog

- Supports structural and behavioral descriptions
- Structural
  - explicit structure of the circuit
  - e.g., each logic gate instantiated and connected to others
- Behavioral
  - program describes input/output behavior of circuit
  - many structural implementations could have same behavior
  - e.g., different implementation of one Boolean function
- We'll mostly be using behavioral Verilog in Aldec ActiveHDL
  - rely on schematic when we want structural descriptions



# Hardware description languages vs. programming languages

- Program structure
  - instantiation of multiple components of the same type
  - specify interconnections between modules via schematic
  - hierarchy of modules (only leaves can be HDL in Aldec ActiveHDL)
- Assignment
  - continuous assignment (logic always computes)
  - propagation delay (computation takes time)
  - timing of signals is important (when does computation have its effect)
- Data structures
  - size explicitly spelled out - no dynamic structures
  - no pointers
- Parallelism
  - hardware is naturally parallel (must support multiple threads)
  - assignments can occur in parallel (not just sequentially)

# Hardware description languages and combinational logic

- Modules - specification of inputs, outputs, bidirectional, and internal signals
- Continuous assignment - a gate's output is a function of its inputs at all times (doesn't need to wait to be "called")
- Propagation delay- concept of time and delay in input affecting gate output
- Composition - connecting modules together with wires
- Hierarchy - modules encapsulate functional blocks

# Working with combinational logic summary

- Design problems
  - filling in truth tables
  - incompletely specified functions
  - simplifying two-level logic
- Realizing two-level logic
  - NAND and NOR networks
  - networks of Boolean functions and their time behavior
- Time behavior
- Hardware description languages
- Later
  - combinational logic technologies
  - more design case studies