Module 3 : Methods of Proof (?q=onlinecourse/course/43512)

Method of Proof III

- **วิชชาภัทร จินดานาถ** previously submitted answers to this quiz/test on 23-Oct-2023 @ 09:06:01 and obtained **5** correct answers out of **5** .
- This test/quiz can be taken many times.
- Correct answers will NOT be revealed after submission.

Select the most appropriate choice for each problem.

1 Suppose we have the following proof.

From previous attempt

Prove that $7^{3m} - 1$ is divisible by 19 for all positive integers m.

Proof by using Mathematical Induction:

Let P(n): $19|7^{3m} - 1$

Basis Step: $P(1) : 19|342 \rightarrow P(1) \equiv T$
Inductive Step: Proof that $P(k) \rightarrow P(k+1) \equiv T$, for any positive integer k.

Let $P(k) \equiv T$ by assumption, for any positive integer k. $[1]$

We must prove that: $7^{3(k+1)} - 1$ is divisible by 19 $[2]$

$k + 1$ can be written as a positive integer r.

Therefore, $[2]$ becomes $P(r)$.

Now we can conclude that $P(r) \equiv T$, as $[1]$ states that $P(k) \equiv T$ for any positive integer k.

This proves the statement above.

Which statement is correct?

This proof is valid.

This proof is invalid.

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2 Which step is wrong in this proof by induction?

From previous attempt

• Theorem: All Americans are the same age.
• Let $S(n)$: In any group of n Americans, everyone in that group is the same age.
• Basis Step: Since everyone in a group of one American has the same age, $S(1)$ is true.

• Inductive Step:
Assume $S(n)$ is true for some n. We prove $S(n+1)$.

[1] Let G be an arbitrary set of $n+1$ Americans. We show that everyone in G has the same age by showing that any two members of G have the same age.

[2] Let $a, b \in G, a \neq b, G_a = G - \{a\}$ and $G_b = G - \{b\}$.
Since G_a has n members, b (which is in G_a) has the same age as any other person in G_a . Similarly, a has the same age as any other person in G_b.

[3] Now, let c be any person in G other than a and b. Then, $c \in G_a$ and $c \in G_b$. So, a and b both have the same age as c.
Hence, a and b have the same age.

This proves $S(n+1)$.

[1]

[2]

[3]

None of the above.

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3 Suppose we want to proof that $\sum_{i=1}^{n} i^4$ is equal to $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$, by mathematical induction

From previous attempt

Which pair of expression should be proved equal by the end of the proof:

[1] $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} + (n+1)^4$

[2] $\frac{(n+1)(n+2)(2n+3)(3(n+1)^2+3n+2)}{30}$

[3] $\sum_{i=1}^{n} i^4$

[1] and [2]

[1] and [3]

[2] and [3]

None of the above.

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4 Use strong induction to prove or disprove the statement:

From previous attempt

Every positive integer greater than or equal to 6 can be expressed as the sum of non-negative multiples of 2 and 3.

Which choice below is correct:

The statement is not true for all positive integers greater than or equal to 6.

The statement is only true for numbers that are divisible by 6.

The statement is true.

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5 What is wrong with this proof by strong induction?

From previous attempt

Theorem: For every nonnegative integer n, $5n = 0$.
Basis Step: $5 \cdot 0 = 0$.
Inductive Step: Suppose that $5j = 0$ for all nonnegative integers j with $0 \leq j \leq k$. $[1]$
Write $k+1 = i+j$, where i and j are nonnegative integers less than $k+1$. $[2]$
By the inductive hypothesis, $5(k+1) = 5(i+j) = 5i + 5j = 0 + 0 = 0$

The basis step cannot be conducted like this.

The assumption [1] is incorrect in the context of strong induction method.

The expression "k+1" in [2] cannot expand to all nonnegative integers.

The proof was miscalculated.

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