

ONLY THE ANSWERS IN THE ANSWER SHEET WILL BE GRADED.

Part 1: Propositional & Predicate Logic (8 Points)

1. Are these sentences propositions? (Yes or No)

- 1.1. Do you wanna build a snowman
- 1.2. The sun is bigger than the moon
- 1.3. x is greater than y
- 1.4. Every integer x , $x = \sqrt{x} + y + 1$
- 1.5. Some DS-TAs think TA Potter is handsome

No: ពីរតារាជក្រោម

Yes

No: ពីរតារាខ្លួនគឺជាអ្នកស្ថាបន

No: ពីរតារាខ្លួនគឺជាអ្នកស្ថាបន

Yes

2. Given proposition below

"Roses are red and Violets are red, whenever you like red color"

Let p : Roses are red, q : Violet are red and r : you like red color

Choose the correct answer for each question.

- 2.1. PROPOSITION: $p \rightarrow q$

- a) $(p \wedge q) \rightarrow r$ b) $r \rightarrow (p \wedge q)$

- 2.2. CONVERSE: $q \rightarrow p$

- a) $(p \wedge q) \rightarrow r$ b) $r \rightarrow (p \wedge q)$

- 2.3. CONTRAPOSITIVE: $\neg q \rightarrow \neg p$

- a) $\neg(p \wedge q) \rightarrow \neg r$ b) $\neg r \rightarrow \neg(p \wedge q)$

- 2.4. INVERSE: $\neg p \rightarrow \neg q$

- a) $\neg(p \wedge q) \rightarrow \neg r$ b) $\neg r \rightarrow \neg(p \wedge q)$

- 2.5. NEGATION:

- a) $\neg((p \wedge q) \rightarrow r)$ b) $\neg(r \rightarrow (p \wedge q))$

3. Given the truth table below

- 3.1. The error occurs in row 2 column 5.

- 3.2. Determine the truth values of W, X, Y and Z.

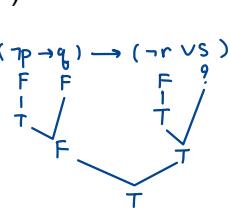
(***according to the correct truth table)

	1	2	3	4	5	6	7
p	T	T	F	F	F	F	
q		F	T		T	T	
$\neg p$			F				
$q \rightarrow \neg p$				T			
$(q \rightarrow \neg p) \wedge q$					F		
$\neg q$						F	
$((q \rightarrow \neg p) \wedge q) \leftrightarrow \neg q$							
W							T
X							F
Y							F
Z							F

4. Let p , q , r and s be the propositions. The truth values of p , q and r are F, F and F, respectively. Determine the truth value of each of these propositions.

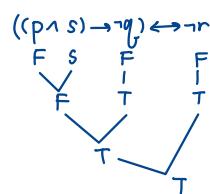
Choice: a) T b) F c) Depends on s

- 4.1. $(\neg p \rightarrow q) \rightarrow (\neg r \vee s)$



a) T
a) T

- 4.2. $((p \wedge s) \rightarrow \neg q) \leftrightarrow \neg r$



a) T
a) T

Quiz 1A (19 Oct, 11.00 - 12.00)

Propositional & Predicate Logic,
Set, Function & Relation

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$$\rightarrow (p \wedge (p \rightarrow q)) \rightarrow q \equiv \neg(p \wedge \neg(p \vee q)) \vee q \equiv (\neg p \vee (p \wedge \neg q)) \vee q \equiv ((\neg p \rightarrow p) \wedge (\neg p \vee \neg q)) \vee q \equiv \neg p \vee \neg q \vee q \equiv T \vee$$

5. Determine whether these statements

- a) Tautology b) Contradiction c) Contingency.
- 5.1. $(p \wedge (p \rightarrow q)) \rightarrow q$ (Note: This is Rule of inference ::)
- 5.2. $(p \vee q) \rightarrow \neg p \equiv \neg(p \vee q) \vee \neg p \equiv (\neg p \wedge \neg q) \vee \neg p \equiv p \wedge \neg q \rightarrow \neg p$

6. Determine whether these statements are consistent.(Yes or No) \Rightarrow The absence of contradiction.

- a) When you like chocolate croissants and buy strawberry croissants, you are not happy.
- b) If you buy strawberry croissants, you do not like chocolate croissants.
- c) You like chocolate croissants or you are happy.

Ans: Yes

$$\left. \begin{array}{l} (p \wedge q) \rightarrow \neg r \\ q \rightarrow \neg p \\ p \vee r \end{array} \right\} q \text{ is } r \text{ when } F, p \text{ when } T, q \text{ when } F$$

7. Determine the truth value of the following statements.

- 7.1. $(p \vee q) \rightarrow p \equiv (p \wedge q) \rightarrow (q \rightarrow p)$ $\left. \begin{array}{l} (p \wedge q) \rightarrow (q \rightarrow p) \equiv (\neg p \vee \neg q) \vee (\neg q \vee p) \equiv T \\ (\neg p \vee \neg q) \vee p \equiv (\neg p \wedge \neg q) \vee p \equiv (\neg p \rightarrow p) \wedge (\neg q \vee p) \equiv q \rightarrow p \end{array} \right\} \leftarrow \text{Tautology!} \leftarrow \text{Contingency}$
- 7.2. $\exists x \forall y (x^2 - y^2 = 0)$ when $x, y \in \mathbb{R}$ F
- 7.3. If $\forall x P(x)$ is true when $x \in I^+$ then $\exists x P(x)$ is always true when $x \in \mathbb{R}$ T
- 7.4. $\forall x P(x)$ is true when $x \in \emptyset$ T

8. For the arguments,

Premise:

$$\begin{aligned} &\forall x(P(x) \wedge Q(x)) \\ &\forall x(R(x) \rightarrow \neg S(x)) \\ &\forall x(\neg Q(x) \vee S(x)) \\ &\exists x \neg P(x) \end{aligned}$$

Conclusion: $\exists x \neg R(x)$

The steps:

1. $\forall x(P(x) \wedge Q(x))$ – Premise
2. $P(c) \wedge Q(c)$ – Universal Instantiation using (1)
3. $Q(c)$ – Disjunctive Syllogism using (2)
4. $\forall x(\neg Q(x) \vee S(x))$ – Premise \hookrightarrow Simplification.
5. $\neg Q(c) \vee S(c)$ for some elements c – Existential Instantiation (4)
6. $S(c)$ – Disjunctive Syllogism using (a) and (b)
7. $\forall x(R(x) \rightarrow \neg S(x))$ – Premise
8. $R(c) \rightarrow \neg S(c)$ – Universal Instantiation (7)
9. $\neg R(c)$ – Modus tollens using (6) and (8)
10. $\exists x \neg R(x)$ – Existential Generalization using (9)

8.1. Identify steps (a) and (b)

$$a = \underline{\quad 3 \quad} \quad b = \underline{\quad 5 \quad}$$

8.2. There is an error in step 3. (Only ONE error)

A

Part 2: Set (6 Points)

1. Which of the following statements is correct?

- a. $\{a, b, c, \emptyset\} \subseteq \{a, b, c, c\}$
- b. If $A \subset B$ and $A \subset C$, then $A \subset B \cap C$
- c. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
- d. $\{\emptyset\} \subset \{\{\emptyset\}, 1\}$

a) \emptyset is not in $\{a, b, c, c\}$ ∴ The statement is incorrect

b) Let $A = \{1\}$, $B = \{1, 2\}$, $C = \{1, 2\}$ which satisfy conditions
 $A \subset B$ and $A \subset C$

$B \cap C = \{1\}$ which results in $B \cap C = A$

∴ A is not a proper subset of $B \cap C$. The statement is incorrect

c) $A \subseteq B$ means all members of A must exist in B

$$B \subseteq C \quad \text{---} \quad \begin{matrix} \downarrow & & \downarrow \\ B & \subset & C \end{matrix}$$

Therefore all members of A must exist in C as well ∴ Correct

d) $\emptyset \notin \{\emptyset, 1\}$ ∵ $\{\emptyset\} \subset \{\{\emptyset\}, 1\}$

2. Which of the following statements is correct?

- a. $\{\emptyset\} \in P(\{a, b, c\})$
- b. If $A \in B$ and $B \in C$, then $A \in C$.
- c. $A \times B \times C = (A \times B) \times C$ where A, B and C are sets.
- d. Let $A = \{x \mid x \text{ is odd and } |x| < 7\}$ There are 64 possible subsets of A

a) $\emptyset \notin \{a, b, c\}$

$\{\emptyset\} \notin \{a, b, c\}$

$\{\emptyset\} \notin P(\{a, b, c\})$ ∴ The statement is incorrect

b) Let $A = 1$, $B = \{1\}$ and $C = \{1, 2\}$ # counterexample

$A \notin C$ ∴ Incorrect

c) Consider $A = \{0\}$, $B = \{1\}$, $C = \{2\}$

$A \times B \times C = \{(0, 1, 2)\}$

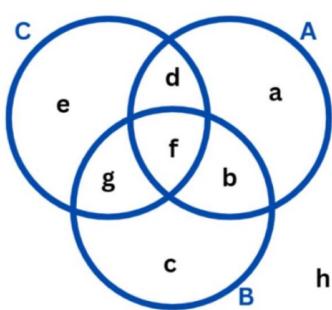
while $(A \times B) \times C = \{(0, 1), 2\}$ ∴ Incorrect

d) $A = \{-5, -3, -1, 1, 3, 5\}$

$|P(A)| = \text{Number of possible subsets of } A$

$$= 2^{|A|} = 2^6 = 64 \quad \therefore \text{Correct}$$

3. Which combination of the sets A, B, C will result in the given area



$$\begin{array}{ll}
 \text{3.1) } a, b \text{ and } d & \rightarrow f \\
 \text{a) } A' \cap (B' \cup C') & \rightarrow a, b, d \\
 \text{b) } A \cap (B' \cup C') & \rightarrow a, c, h \\
 \text{c) } C' \cap (B' \cup A') & \\
 \\
 \text{3.2) } e \text{ and } g & \rightarrow g, f \\
 \text{a) } B' \cup C' & \rightarrow e, g \\
 \text{b) } (A \cup C')' & \\
 \text{c) } (C' \cap A')' & \rightarrow e, d, a, g, f, b
 \end{array}$$

4. Let $C = \{\{\emptyset\}, \emptyset, 1, 2, \{\{\emptyset\}\}\}$

$$A = \{\{1, 2\}, \{1\}\}$$

$$\text{Find } |P(C) \cup C| + |P(A - C)| - |P(C) - A|$$

$$|P(C)| = 2^{|C|} = 2^{15} = 32$$

$$\therefore \text{Consider } C = \{\{\emptyset\}, \emptyset, 1, 2, \{\{\emptyset\}\}\}$$

$$P(C) \cap C = \{\{\emptyset\}, \{\{\emptyset\}\}, \emptyset\}$$

$$|P(C) \cup C| = |P(C)| + |C| - |P(C) \cap C| \quad \text{since } \emptyset \subset C \rightarrow \emptyset \in P(C) \text{ and } \emptyset \in C$$

$$= 32 + 5 - 3 = 34$$

$$\therefore |P(A - C)| = 2^{|A-C|} = 2^{12} = 4 \quad \text{since } A \cap C = \emptyset$$

$$\therefore |P(C) - A| = |P(C)| - |P(C) \cap A| = 32 - 2 = 30 \quad \text{since both } \{1, 2\} \text{ and } \{1\} \text{ must exist in } P(C)$$

$$\therefore |P(C) \cup C| + |P(A - C)| + |P(C) - A| = 34 + 4 - 30 = 8 //$$

5. Let $U = \{1, 2, 3, \dots, 17, 18\}$

$$A = \{x \mid x^2 \notin U\}$$

$$B = \{x \mid 2x \notin U \wedge x/3 \notin A\}$$

$$C = \{x \mid x \notin A \wedge 4x \in B\}$$

$$\text{Find } |A \cap B| + |B \cap C|$$

$$\text{Ans} = 16$$

Same as
Ques 1B

6. Consider set A, B, and C where $|A| = 7$, $|B| = 10$, $|C \cap B| = 3$, $|A \cup B| = 13$, $|C \cap A \cap B| = 2$, $|A - C| = 5$

$$6.1 \text{ Find } |(A \cup B) - C|$$

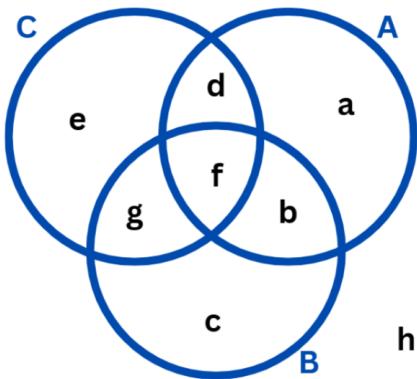
$$6.2 \text{ Find minimum cardinality of } C$$

$$6.1 \rightarrow 10$$

$$6.2 \rightarrow 3$$

- d. $\{\emptyset\} \subset \{\{\emptyset\}, 1\}$
2. Which of the following statements is correct?
- $\{\emptyset\} \in P(\{a, b, c, d\})$
 - If $A \in B$ and $B \in C$, then $A \in C$.
 - $A \times B \times C = (A \times B) \times C$ where A, B and C are sets.
 - Let $A = \{x \mid x \text{ is odd and } |x| < 7\}$. There are 64 possible subsets of A

3. Which combination of the sets A, B, C will result in the given area



- 3.1) a, b and d
 a) $A' \cap (B' \cup C')$
 b) $A \cap (B' \cup C')$
 c) $C' \cap (B' \cup A')$

- 3.2) e and g
 a) $B' \cup C'$
 b) $(A \cup C')'$
 c) $(C' \cap A')'$

4. Let $C = \{\{\emptyset\}, \emptyset, 1, 2, \{\{\emptyset\}\}\}$
 $A = \{\{1, 2\}, \{1\}\}$
 Find $|P(C) \cup C| + |P(A - C)| - |P(C) - A|$
5. Let $U = \{1, 2, 3, \dots, 17, 18\}$
 $A = \{x \mid x^2 \notin U\}$
 $B = \{x \mid 2x \notin U \wedge x/3 \in A\}$
 $C = \{x \mid x \notin A \wedge 4x \in B\}$
 Find $|A \cap B| + |B \cup C|$
6. Consider set A, B , and C where $|A| = 7$, $|B| = 10$, $|C \cap B| = 3$, $|A \cup B| = 13$, $|C \cap A \cap B| = 2$, $|A - C| = 5$
 - Find $|(A \cup B) - C|$
 - Find minimum cardinality of C

Part 3: Function & Relation (6 Points)

1. Consider the function $f(x) = \frac{2}{\sqrt{x+1}}$ from $D \rightarrow R$. Determine the values of a, b for the domain(D) and range of the function.
 $D = \underline{\hspace{2cm}}(a, \infty)\underline{\hspace{2cm}}$ $\begin{array}{l} \sqrt{x+1} > 0 \\ x > -1 \end{array} \quad \left| \begin{array}{l} \text{when } x \rightarrow \infty \\ f(x) \rightarrow 0 \end{array} \right.$ $\text{range} = \underline{\hspace{2cm}}(b, \infty)\underline{\hspace{2cm}}$ $\begin{array}{l} a = -1 \\ b = 0 \end{array} \quad \cancel{\text{orange}}$

2. Let $f(x) = 3x + 5$. Determine the value of

$$\begin{array}{ll} f(1.5) = 9.5 & \left| f(-2.5) = -2.5 \right. \\ \cancel{f(1.5)} = 9.5 & \left| \cancel{f(-2.5)} = -2.5 \right. \end{array}$$

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2.1) $[f(1.5)]$

2.2) $[f(-2.5)]$

3. Classify each function (\checkmark) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by their types of correspondences in the answer sheet.

Function	One-to-one, but not onto	Onto, but not One-to-one	One-to-one, And onto	Neither Onto, nor One-to-one
3.1) $f(x) = x^2 + 1$				\checkmark
3.2) $f(x) = x^3$	\checkmark			
3.3) $f(x) = x(x+1)(x-1)$				\checkmark
3.4) $f(x) = 3x + 5$	\checkmark			
3.5) $f(x) = 42$				\checkmark

4. Let $f(x) = 2x^2$ and $g(x) = \sqrt{x+3}$ is the function from $\mathbb{Z}^+ \rightarrow \mathbb{R}^+$. Find the value of

$$f(x): \begin{aligned} x &= 2y \\ y &= \sqrt{\frac{x}{2}} \\ f^{-1}(x) &= \sqrt{\frac{x}{2}} \end{aligned} \quad g(x): \begin{aligned} x &= \sqrt{y+3} \\ y &= x^2 - 3 \\ g^{-1}(x) &= x^2 - 3 \end{aligned} \quad \begin{aligned} f^{-1}(8) &= 2 \\ g^{-1}(f^{-1}(8)) &= g^{-1}(2) \\ &= 1 \end{aligned}$$

5. Consider set A of 3 elements. How many relations R on set A, which are symmetric?

Let $A = \{a, b, c\}$

元素	a	b	c
a	A	B	C
b	A	B	C
c	B	C	C

→ $(a, a), (b, b), (c, c)$ 3 關係
 $(a, b), (b, a)$ 1 關係
 $(a, c), (c, a)$ 2 關係
 $(b, c), (c, b)$ 3 關係
 $(a, a), (b, b), (c, c)$ 6 關係

= 2⁶ relation

6. For each of these relations on the set {1,2,3,4}, decide (\checkmark) whether it is reflexive, symmetric, antisymmetric, or transitive in the answer sheet.

Relation	Reflexive	Symmetric	Antisymmetric	Transitive
6.1) $\{(1,3),(2,2),(3,1),(4,4)\}$		\checkmark		
6.2) $\{1,2,3\} \times \{1,2,3\}$		\checkmark		\checkmark
6.3) $\{(x,y) x = y\}$	\checkmark	\checkmark	\checkmark	\checkmark
6.4) $\{(1,3),(2,3),(4,1),(4,4)\}$			\checkmark	

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Part 1: Propositional & Predicate Logic (8 Points)

1. Are these sentences propositions? (Yes or No)

- 1.1. The moon is bigger than the sun
- 1.2. Do you wanna build a sand castle
- 1.3. y is less than x
- 1.4. Some integer x , $x = \sqrt{x+1} + y + 4$
- 1.5. Every DS-TAs think TA Fah is kind

Yes
No: พิจารณาต่อไปนี้
No: y ต้องเป็น x , y ไม่เท่ากัน
No: y ต้องเป็น x
Yes

2. Given proposition below

"Jasmines are blue and Sunflower are blue, whenever you like blue color"

Let p : Jasmines are blue, q : Sunflower are blue and r : you like blue color

Choose the correct answer for each question.

- 2.1. PROPOSITION : $p \rightarrow q$

- a) $(p \wedge q) \rightarrow r$ b) $r \rightarrow (p \wedge q)$

- 2.2. CONVERSE: $q \rightarrow p$

- a) $(p \wedge q) \rightarrow r$ b) $r \rightarrow (p \wedge q)$

- 2.3. CONTRAPOSITIVE: $\neg q \rightarrow \neg p$

- a) $\neg(p \wedge q) \rightarrow \neg r$ b) $\neg r \rightarrow \neg(p \wedge q)$

- 2.4. INVERSE: $\neg p \rightarrow \neg q$

- a) $\neg(p \wedge q) \rightarrow \neg r$ b) $\neg r \rightarrow \neg(p \wedge q)$

- 2.5. NEGATION:

- a) $\neg((p \wedge q) \rightarrow r)$ b) $\neg(r \rightarrow (p \wedge q))$

3. Given the truth table below

- 3.1. The error occurs in row 2 column 5

- 3.2. Determine the truth values of W, X, Y and Z.

(***according to the correct truth table)

	1	2	3	4	5	6	7
	p	q	$\neg p$	$q \rightarrow \neg p$	$(q \rightarrow \neg p) \wedge q$	$\neg q$	$((q \rightarrow \neg p) \wedge q) \leftrightarrow \neg q$
1	T	T	F	F	F	F	W T
2	T	F	F	T	T F	T	X F
3	F	T	T	T	T	F	Y F
4	F	F	T	T	F	T	Z F

4. Let p , q , r and s be the propositions. The truth values of p , q and r are T, T and F, respectively. Determine the truth value of each of these propositions.

Choice: a) T b) F c) Depends on s

4.1. $\neg r \leftrightarrow ((p \wedge s) \rightarrow \neg q)$

c) s

4.2. $(\neg p \rightarrow q) \rightarrow (\neg r \vee s)$

a) T

$$\begin{array}{ccccc} \neg r & \leftrightarrow & ((p \wedge s) \rightarrow \neg q) \\ \begin{array}{c} F \\ | \\ T \end{array} & \begin{array}{c} T \\ | \\ ? \end{array} & \begin{array}{c} T \\ | \\ ? \end{array} & \begin{array}{c} T \\ | \\ F \end{array} & \end{array}$$

$$\begin{array}{ccccc} (\neg p \rightarrow q) \rightarrow (\neg r \vee s) \\ \begin{array}{c} T \\ | \\ F \\ / \\ T \end{array} & \begin{array}{c} T \\ | \\ T \\ / \\ T \end{array} & \begin{array}{c} F \\ | \\ T \\ / \\ T \end{array} & \end{array}$$

Quiz 1B (19 Oct, 11.00 - 12.00)

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5. Determine whether these statements are

a) Tautology b) Contradiction c) Contingency.

5.1. $(p \wedge (p \rightarrow q)) \rightarrow q \equiv \neg(p \wedge \neg p \vee q) \vee q \equiv (\neg p \vee (p \wedge q)) \vee q \equiv ((\neg p \vee p) \wedge (p \wedge q)) \vee q \equiv \top \vee q \equiv \top$

5.2. $(p \vee q) \rightarrow \neg p \equiv \neg(p \vee q) \vee \neg p \equiv (\neg p \wedge \neg q) \vee \neg p \equiv p \wedge \neg p \equiv \bot$

6. Determine whether these statements are consistent.(Yes or No) \Rightarrow The absence of contradiction.

- a) When you like chocolate P croissants and buy strawberry $\neg P$ croissants, you are not happy.
- b) You like chocolate P croissants or you are happy.
- c) If you buy strawberry $\neg P$ croissants, you do not like chocolate croissants.

Ans: Yes

$$\begin{array}{l} \neg P \wedge T \Rightarrow \bot \\ \neg P \wedge F \Rightarrow \top \\ \neg P \wedge F \Rightarrow \bot \end{array}$$

7. Determine the truth value of the following statements.

7.1. $(p \vee q) \rightarrow p \equiv (p \wedge q) \rightarrow (q \rightarrow p)$ $\begin{array}{l} (p \wedge q) \rightarrow (q \rightarrow p) \equiv (\neg p \vee \neg q) \vee (\neg q \vee p) \equiv \top \leftarrow \text{Tautology!} \\ (\neg p \vee \neg q) \rightarrow p \equiv \neg(\neg p \vee \neg q) \vee p \equiv (\neg \neg p \wedge \neg \neg q) \vee p \equiv (\neg \neg p) \wedge (\neg \neg q \vee p) \equiv q \rightarrow p \leftarrow \text{Contingency} \end{array}$

7.2. $\exists x \forall y (x^2 - y^2 = 0)$ when $x, y \in \mathbb{R}$

F

7.3. $\forall x P(x)$ is true when $x \in \emptyset$

T

7.4. If $\forall x P(x)$ is true when $x \in I^+$ then $\exists x P(x)$ is always true when $x \in \mathbb{R}$

T

8. For the arguments,

Premise:

$$\forall x(P(x) \wedge Q(x))$$

$$\forall x(R(x) \rightarrow \neg S(x))$$

$$\forall x(\neg Q(x) \vee S(x))$$

$$\exists x \neg P(x)$$

Conclusion: $\exists x \neg R(x)$

The steps:

1. $\forall x(P(x) \wedge Q(x))$ – Premise
2. $P(c) \wedge Q(c)$ – Universal Instantiation using (1)
3. $Q(c)$ – Disjunctive Syllogism using (2)
4. $\forall x(\neg Q(x) \vee S(x))$ – Premise \hookrightarrow Simplification.
5. $\neg Q(c) \vee S(c)$ for some elements c – Existential Instantiation (4)
6. $S(c)$ – Disjunctive Syllogism using (3) and (5)
7. $\forall x(R(x) \rightarrow \neg S(x))$ – Premise
8. $R(c) \rightarrow \neg S(c)$ – Universal Instantiation (7)
9. $\neg R(c)$ – Modus Tollens using (a) and (b)
10. $\exists x \neg R(x)$ – Existential Generalization using (9)

- 8.1. Identify steps (a) and (b)

a = 6

b = 8

- 8.2. There is an error in step 3. (Only ONE error)

Part 2: Set (6 Points)

1. Which of the following statements is correct?

- a. Let $A = \{x \mid x \text{ is odd and } |x| < 7\}$. There are 32 possible subsets of A
- b. $A \times B \times C = (A \times B) \times C$ where A, B and C are sets.
- c. $\{\emptyset\} \subset P(\{a, b, c, d\})$
- d. If $A \in B$ and $B \in C$, then $A \in C$.

a) $A = \{-5, -3, -1, 1, 3, 5\}$
 $|P(A)| = \text{Number of possible subsets of } A$
 $= 2^{|A|} = 2^6 = 64 \therefore \text{Incorrect}$

b) Consider $A = \{0\}, B = \{1\}, C = \{2\}$
 $A \times B \times C = \{(0, 1, 2)\}$
 while $(A \times B) \times C = \{(0, 1), 2\} \therefore \text{Incorrect}$

c) Since \emptyset is subset of any sets
 $\emptyset \in P(A)$ for all set A

Therefore $\{\emptyset\} \subset P(A) \therefore \text{Correct}$

d) Let $A = \{1\}, B = \{1\}$ and $C = \{1, 2\}$ # counterexample
 $A \notin C \therefore \text{Incorrect}$

2. Which of the following statements is correct?

- a. $\{a, b, c\} \subset \{a, b, c, c\}$
- b. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
- c. If $A \subset B$ and $A \subset C$, then $A \subset B \cap C$
- d. $\{\emptyset\} \subset \{\{\emptyset\}, 1\}$

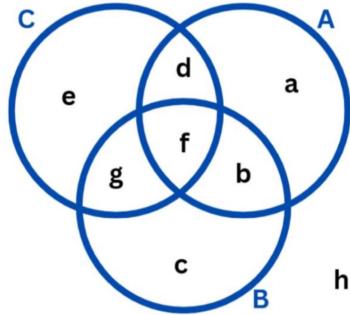
a) $\{a, b, c\} \neq \{a, b, c, c\}$
 $\{a, b, c\} \subseteq \{a, b, c\}$
 but $\{a, b, c\}$ is not a "proper" (c) subset of $\{a, b, c\}$
 $A \subset B$ "A is a proper subset of B" when $A \subseteq B$ but $A \neq B$
 $\therefore \text{Incorrect}$

b) $A \subseteq B$ means all members of A must exist in B
 $B \subseteq C$ $\begin{array}{ccc} & \nearrow & \searrow \\ & B & \end{array} \rightarrow C$
 Therefore all members of A must exist in C as well $\therefore \text{Correct}$

c) Let $A = \{1\}, B = \{1, 2\}, C = \{1, 3\}$ which satisfy conditions
 $A \subset B$ and $A \subset C$
 $B \cap C = \{1\}$ which results in $B \cap C = A$
 $\therefore A$ is not a proper subset of $B \cap C$. The statement is incorrect

d) Since $\emptyset \notin \{\emptyset, 1\}$
 $\therefore \emptyset \subset \{\emptyset, 1\}$ is incorrect

3. Which combination of the sets A, B, C will result in the given area



5.1) a, b and d
 a) $A \cap (B' \cup C')$ → a, b, d
 b) $C' \cap (B \cap A)'$ → a, c, h
 c) $A' \cap (B' \cup C')$ → f

5.2) e and g
 a) $B' \cup C' = B \cap C$ → g, f
 b) $(A \cup C')' = A' \cap C$ → e, g
 c) $(C' \cap A')' = C \cup A$ → e, d, a, g, f, b

4. Let $C = \{\{\emptyset\}, \emptyset, 4, 2, \{\{\emptyset\}\}\}$

$$A = \{\{4, 2\}, \{4\}\}$$

$$\text{Find } |P(C) \cup C| + |P(A - C)| + |P(C) - A|$$

$$|P(C)| = 2^{|C|} = 2^{15} = 32$$

Consider $C = \{\{\emptyset\}, \emptyset, 4, 2, \{\{\emptyset\}\}\}$

$$P(C) \cap C = \{\{\emptyset\}, \{\{\emptyset\}\}, \emptyset\}$$

$$|P(C) \cup C| = |P(C)| + |C| - |P(C) \cap C|$$

$$= 32 + 5 - 3 = 34$$

$|P(A - C)| = 2^{|A-C|} = 2^{12} = 4$ since $A \cap C = \emptyset$

$|P(C) - A| = |P(C)| - |P(C) \cap A| = 32 - 2 = 30$ since both $\{4, 2\}$ and $\{4\}$ are subsets of C , $\{4, 2\}$ and $\{4\}$ must exist in $P(C)$

$$\therefore |P(C) \cup C| + |P(A - C)| + |P(C) - A| = 34 + 4 + 30 = 68 //$$

5. Let $U = \{1, 2, 3, \dots, 17, 18\}$

$$A = \{x \mid x^2 \notin U\}$$

$$B = \{x \mid 2x \notin U \wedge x/3 \in A\}$$

$$C = \{x \mid x \notin A \wedge 4x \in B\}$$

$$\text{Find } |A \cap B| + |B \cup C|$$

$$A = \{5, 6, 7, \dots, 17, 18\}$$

$$B = \{10, 11, 12, \dots, 17, 18\} - \{x \mid x/3 \in A\} = \{10, 11, \dots, 17, 18\} - \{15, 18\}$$

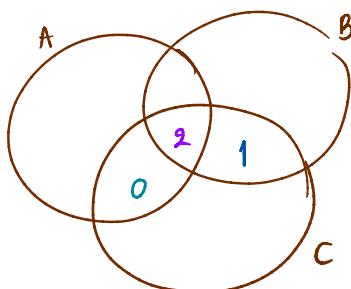
$$C = \{3, 4\}$$

$$|A \cap B| + |B \cup C| = 7 + 9 = 16 //$$

6. Consider set A, B, and C where $|A| = 7$, $|B| = 10$, $|C \cap B| = 3$, $|A \cup B| = 13$, $|C \cap A| = 2$, $|A - C| = 5$

6.1 Find minimum cardinality of C

6.2 Find $|(A \cup B) - C|$



① from $|A \cap B \cap C| = 2$

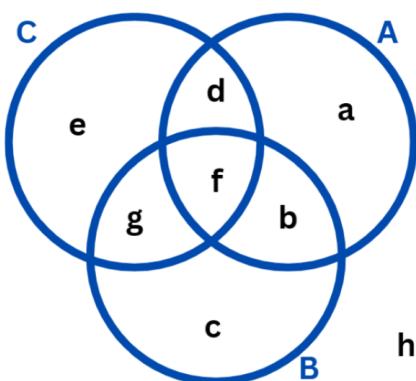
② From $|C \cap B| = 3$
 $\therefore |(C \cap B) - A| = 1$

③ From $|A - C| = 5$ and $|A| = 7 \therefore (A \cap C) - B = \emptyset$

No more information about C $\therefore |C| \geq 2+1 \quad (3) // 6.1$

$$|(A \cup B) - C| = |A \cup B| - |(A \cup B) \cap C| \\ = 13 - 3 = 10 // 6.2$$

- d. If $A \in B$ and $B \in C$, then $A \in C$.
2. Which of the following statements is correct?
- $\{a, b, c\} \subset \{a, b, c, c\}$
 - If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
 - If $A \subset B$ and $A \subset C$, then $A \subset B \cap C$
 - $\{\emptyset\} \subset \{\{\emptyset\}, 1\}$
3. Which combination of the sets A,B,C will result in the given area



- 5.1) a,b and d
 a) $A \cap (B' \cup C')$
 b) $C' \cap (B \cap A)'$
 c) $A' \cap (B' \cup C')$

- 5.2) e and g
 a) $B' \cup C'$
 b) $(A \cup C)'$
 c) $(C' \cap A)'$

4. Let $C = \{\{\emptyset\}, \emptyset, 4, 2, \{\{\emptyset\}\}\}$
 $A = \{\{4, 2\}, \{4\}\}$
 Find $|P(C) \cup C| + |P(A - C)| + |P(C) - A|$
5. Let $U = \{1, 2, 3, \dots, 17, 18\}$
 $A = \{x \mid x^2 \notin U\}$
 $B = \{x \mid 2x \notin U \wedge x/3 \in A\}$
 $C = \{x \mid x \notin A \wedge 4x \in B\}$
 Find $|A \cap B| + |B \cup C|$
6. Consider set A,B, and C where $|A| = 7$, $|B| = 10$, $|C \cap B| = 3$, $|A \cup B| = 13$, $|C \cap A \cap B| = 2$, $|A - C| = 5$
 6.1 Find minimum cardinality of C
 6.2 Find $|(A \cup B) - C|$

Part 3: Function & Relation (6 Points)

1. Consider the function $f(x) = \frac{3}{\sqrt{x-1}}$ from $D \rightarrow R$. Determine the values of a, b for the domain(D) and range of the function.
- | | | |
|---|--|---------|
| $x > 1$ | $\left \begin{array}{l} \text{if } x \rightarrow \infty \\ f(x) \rightarrow \infty \end{array} \right.$ | $a = 1$ |
| $D = \underline{\hspace{2cm}}(a, \infty)\underline{\hspace{2cm}}$ | $range = \underline{\hspace{2cm}}(b, \infty)\underline{\hspace{2cm}}$ | $b = 0$ |

2. Let $f(x) = 1.3x + 2$. Determine the value of

$$f(1) = 3.3 \quad f(-3) = -1.9$$

$$f(2) = 3 \quad f(-3) = -1$$

Quiz 1B (19 Oct, 11.00 - 12.00)

Propositional & Predicate Logic,
Set, Function & Relation

Name _____

ID _____ No. _____

2.1) $|f(1)|$

2.2) $|f(-3)|$

3. Classify each function (\checkmark) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by their types of correspondences in the answer sheet.

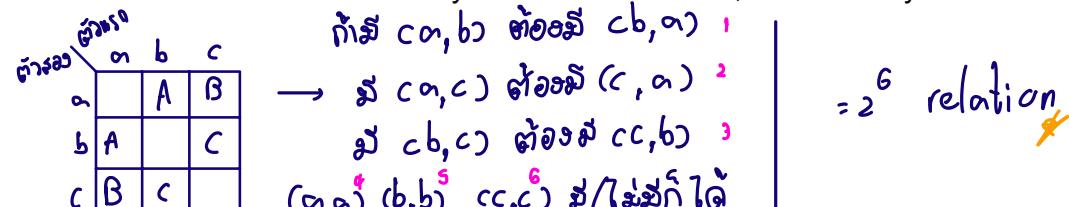
Function	One-to-one, but not onto	Onto, but not One-to-one	One-to-one, And onto	Neither Onto, nor One-to-one
3. 1) $f(x) = 99$				\checkmark
3. 2) $f(x) = 2x^2 + 3$				\checkmark
3. 3) $f(x) = 2x + 8$	\checkmark			
3. 4) $f(x) = x(x + 2)(x - 2)$				\checkmark
3. 5) $f(x) = 2x^3$	\checkmark			

4. Let $f(x) = \frac{x^2}{2}$ and $g(x) = \sqrt{2x + 4}$ is the function from $\mathbb{Z}^+ \rightarrow \mathbb{R}^+$. Find the value of

$$\begin{array}{l}
 f(x): x = \frac{y^2}{2} \quad g(x): x = \sqrt{2y+4} \quad | \quad f^{-1}(8) = 4 \\
 y = \sqrt{2x} \quad y = \frac{x^2-4}{2} \quad | \quad g^{-1}(f^{-1}(8)) = g^{-1}(4) \\
 f^{-1}(x) = \sqrt{2x} \quad g^{-1}(x) = \frac{x^2-4}{2} \quad | \quad = 6
 \end{array}$$

5. Consider set A of 3 elements. How many relations R on set A, which are symmetric?

Let $A = \{a, b, c\}$



6. For each of these relations on the set $\{1, 2, 3, 4\}$, decide (\checkmark) whether it is reflexive, symmetric, antisymmetric, or transitive in the answer sheet.

Relation	Reflexive	Symmetric	Antisymmetric	Transitive
6.1) $\{(x,y) \mid x = y\}$	\checkmark	\checkmark	\checkmark	\checkmark
6.2) $\{(1,3), (2,2), (3,1), (4,4)\}$		\checkmark		
6.3) $\{(1,3), (2,3), (4,1), (4,4)\}$			\checkmark	
6.4) $\{1,2,3\} \times \{1,2,3\}$		\checkmark		\checkmark