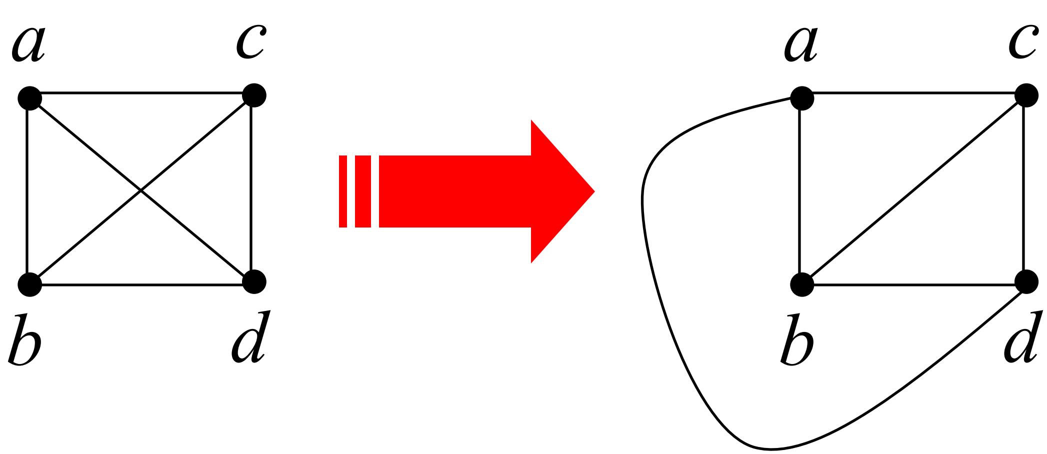
Section 10.7

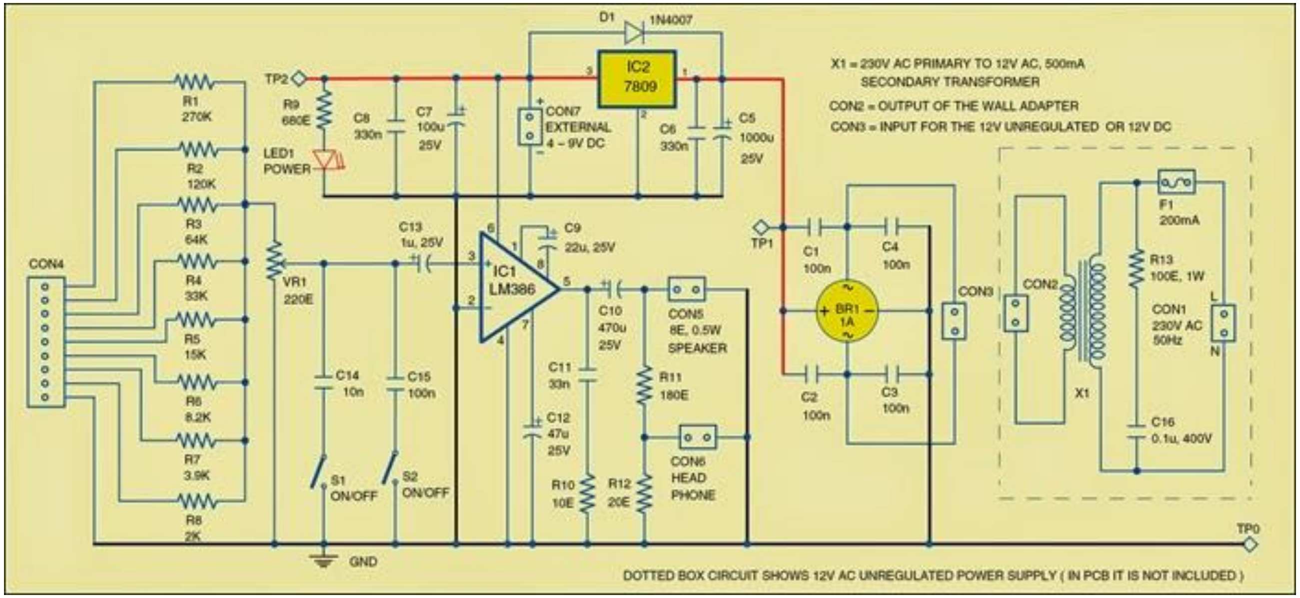
- Planar Graphs

Planar Graph

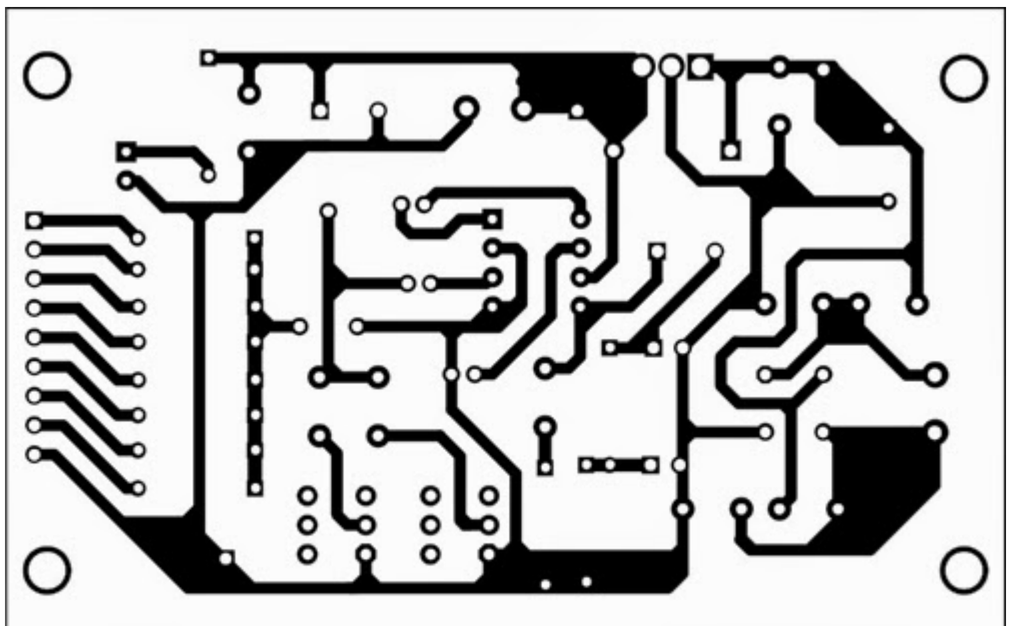
A graph is planar if it can be drawn in the plane without any edges crossing.

Planar Representation

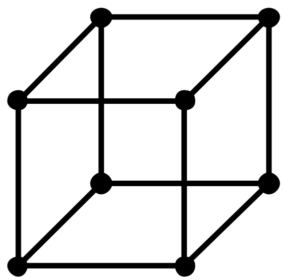




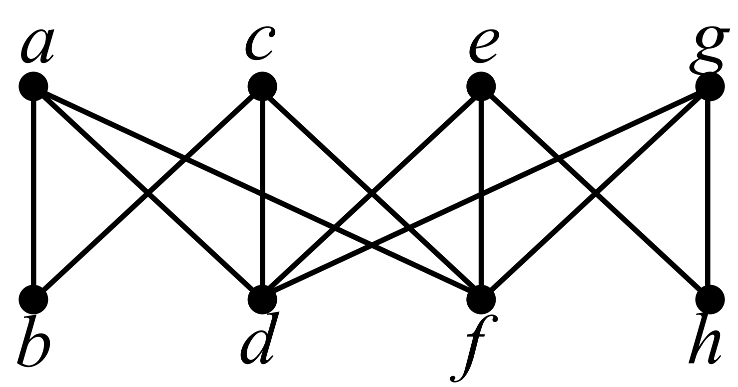
D1 1N4007
IC2 7809
X1 = 230V AC PRIMARY TO 12V AC, 500mA SECONDARY TRANSFORMER
CON2 = OUTPUT OF THE WALL ADAPTER
CON3 = INPUT FOR THE 12V UNREGULATED OR 12V DC
TP2
R9 680E
LED1 POWER
C8 330n
C7 100u 25V
CON7 EXTERNAL 4 – 9V DC
C6 330n
C5 1000u 25V
R1 270K
R2 120K
R3 64K
R4 33K
R5 15K
R6 8.2K
R7 3.9K
R8 2K
CON4
VR1 220E
C13 1u, 25V
C9 22u, 25V
IC1 LM386
C10 470u 25V
CON5 8E, 0.5W SPEAKER
C11 33n
C12 47u 25V
R11 180E
CON6 HEAD PHONE
R10 10E
R12 20E
C14 10n
C15 100n
S1 ON/OFF
S2 ON/OFF
GND
TP1
C1 100n
C4 100n
BR1 1A
C2 100n
C3 100n
CON3
CON2
X1
F1 200mA
R13 100E, 1W
CON1 230V AC 50Hz
C16 0.1u, 400V
TP0
DOTTED BOX CIRCUIT SHOWS 12V AC UNREGULATED POWER SUPPLY (IN PCB IT IS NOT INCLUDED)

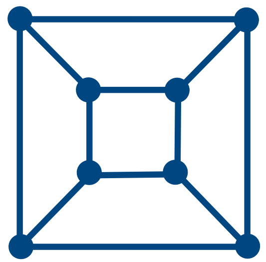


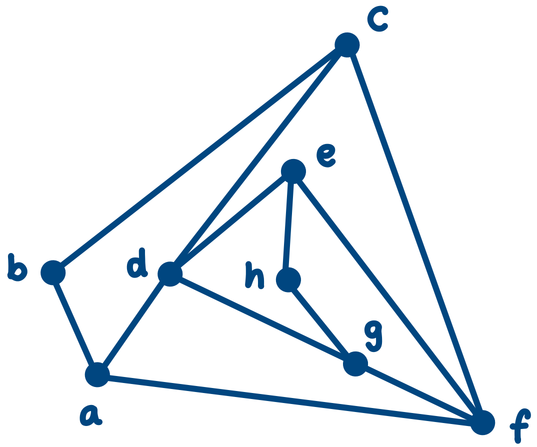
Planar or Not?



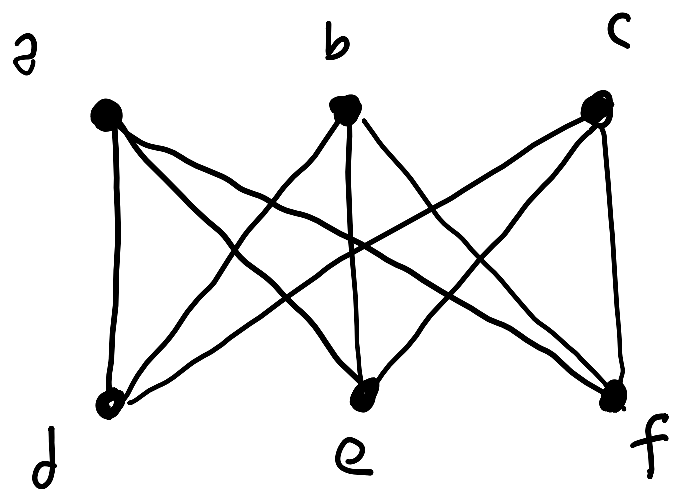
Q_3

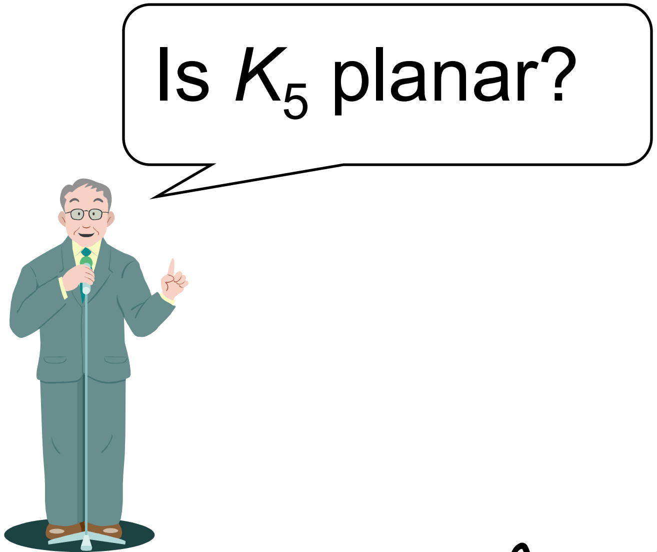


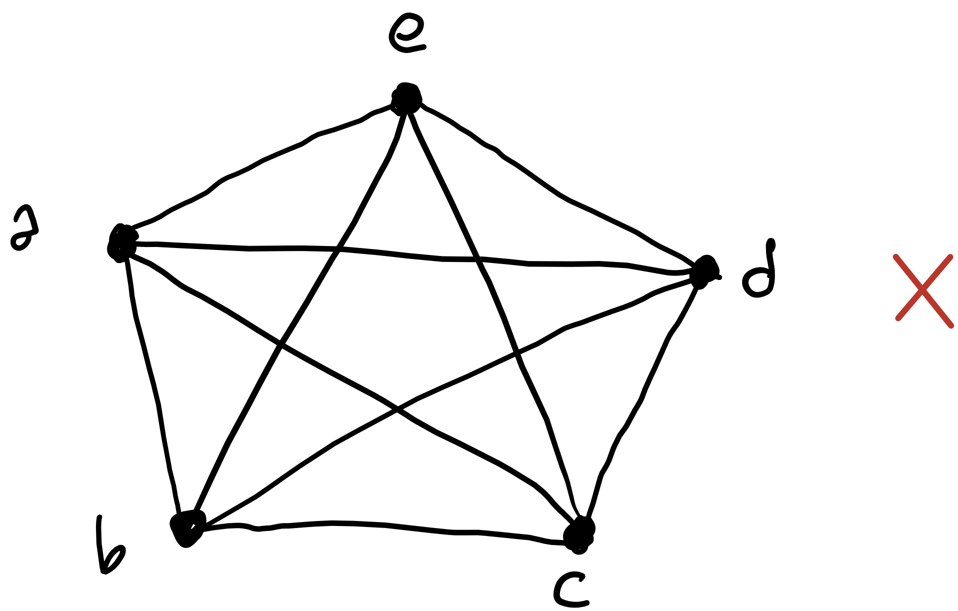




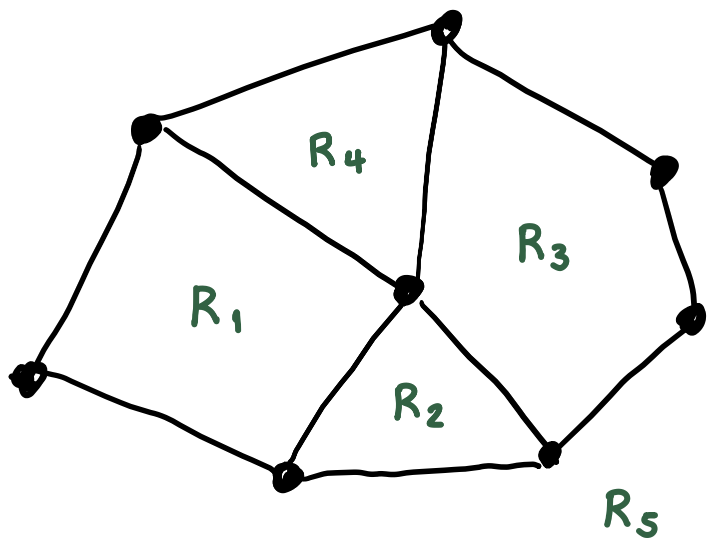
Is $K_{3,3}$ planar?




Is K_5 planar?


e
a
d
b
c

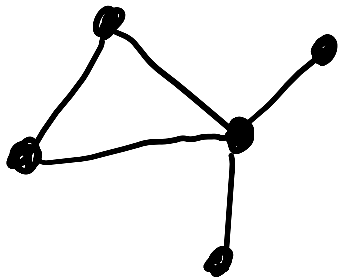
region



r = 5

e = 11

v = 8

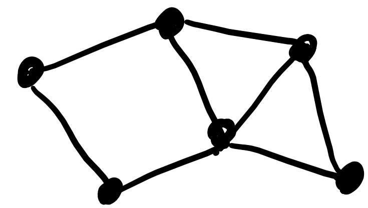


$r = 2$

$e = 5$

$v = 5$

$r - e + v = 2$

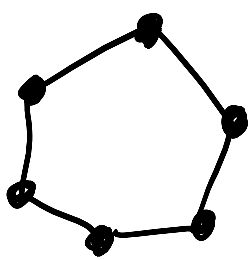


$r = 4$

$e = 8$

$v = 6$

$r - e + v = 2$

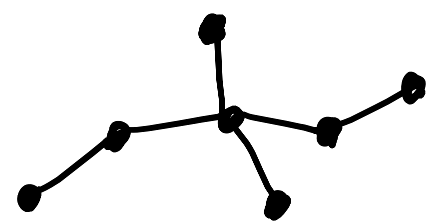


$r = 2$

$e = 6$

$v = 6$

$r - e + v = 2$



$r = 1$

$e = 6$

$v = 7$

$r - e + v = 2$

Euler's Formula

Let G be a connected planar simple graph with

e = Number of edges

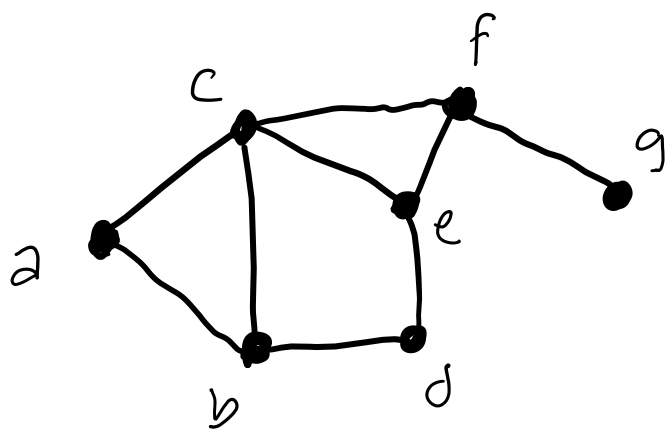
v = Number of vertices

r = Number of regions in a planar representation of G.

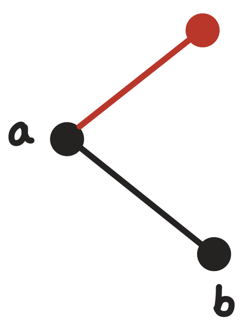
Then,

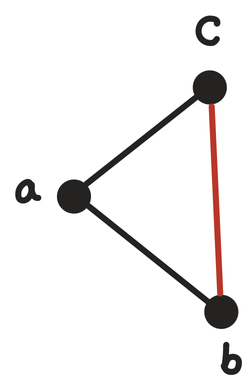
$$\boxed{r = e - v + 2}$$

Simple planar connected graph G



$r - e + v = 2$





$r = 1$

$e = 1 + 1$

$v = 2 + 1$

$\overset{+1}{r} - \overset{+1}{e} + v = 2$

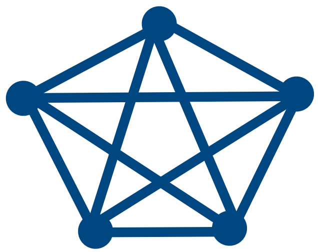
$r = 1 + 1$

$e = 2 + 1$

$v = 3$

$\overset{+1}{r} - \overset{+1}{e} + v = 2$

If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$.



$v = 5$

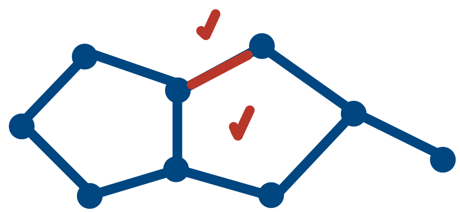
$e = 10$

$10 \leq 15 - 6$

$10 \leq 9$

$\therefore$ ✗ planar

If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$.



$r \quad e \geq \frac{3r}{2}$

$r - e + v = 2$

$r = e - v + 2$

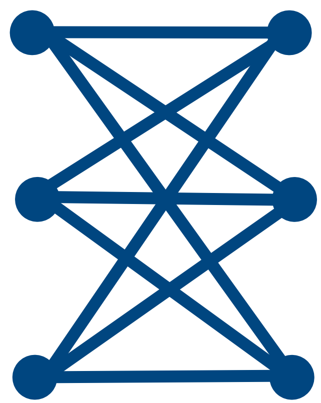
$\frac{2e}{3} \geq e - v + 2$

$0 \geq \frac{e}{3} - v + 2$

$\frac{e}{3} \leq v - 2$

$e \leq 3v - 6$

If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$.



$K_{3,3}$

$v = 6$

$e = 9$

$9 \leq 12 - 4$

$9 \leq 8$

$\therefore$ × planar

If G is a connected planar simple graph, then G has a vertex of degree not exceeding five.

ใช้ contradiction proof

โดยสมมติแย้งให้แต่ละ vertex มีดีกรีอย่างน้อย 6

$2e \geq 6v$ — ① $e \geq 3v$

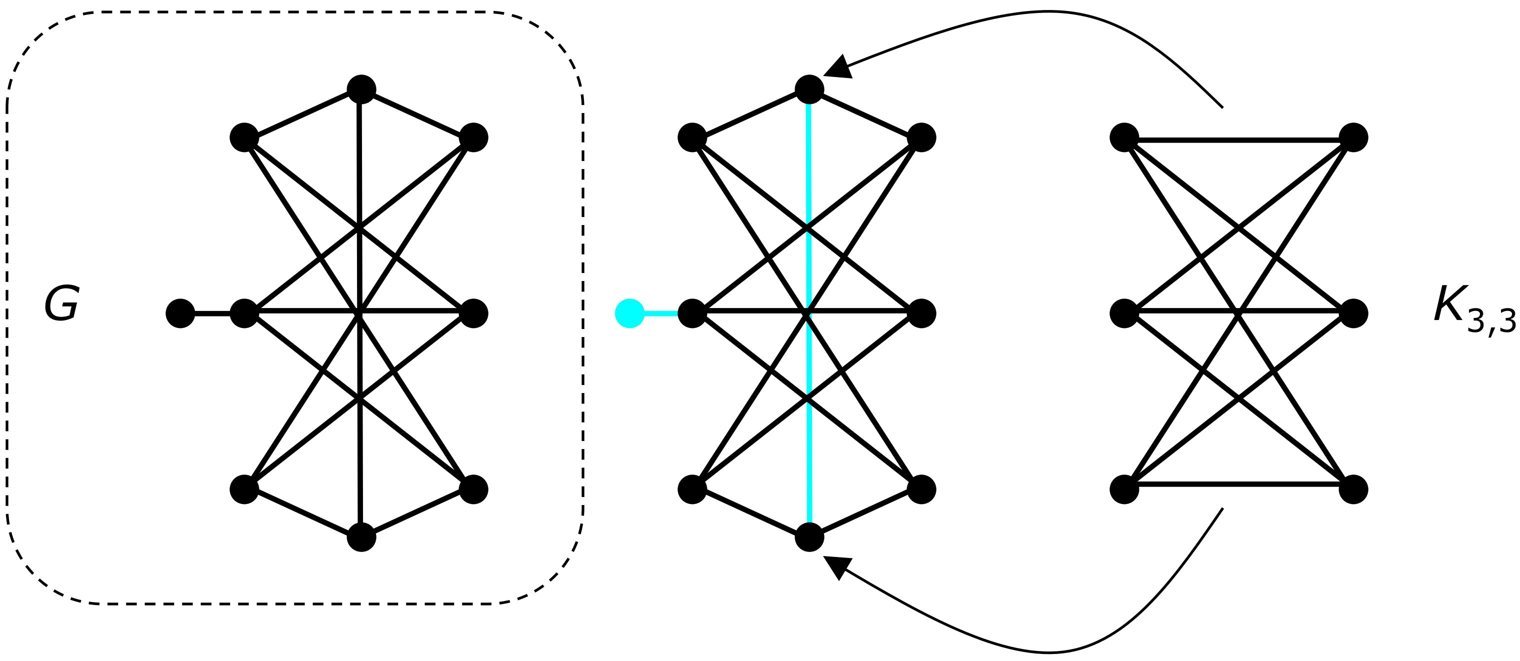
$e \leq 3v-6$ — ②

$3v \leq e \leq 3v-6$

Kuratowski's Theorem

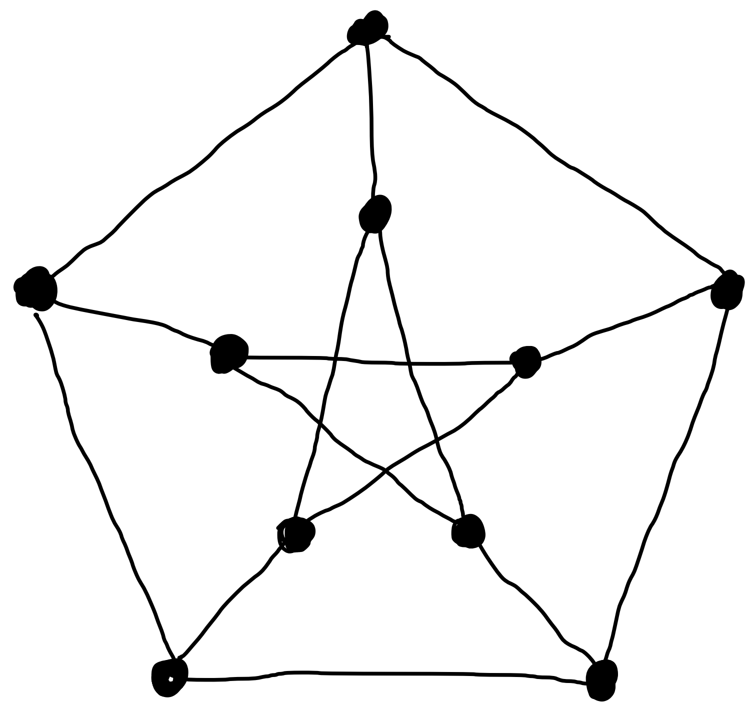
A graph is nonplanar $\leftrightarrow$ it contains a subgraph homeomorphic to $K_{3,3}$ or K_5.

คล้ายๆ



Kuratowski's Theorem

A graph is nonplanar ↔ it contains a subgraph homeomorphic to $K_{3,3}$ or K_5.



Kuratowski's Theorem

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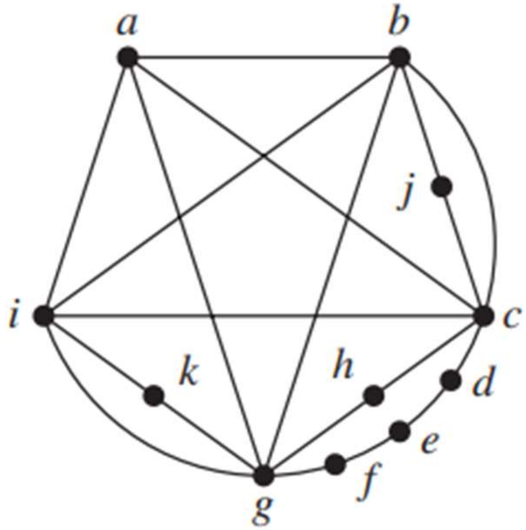
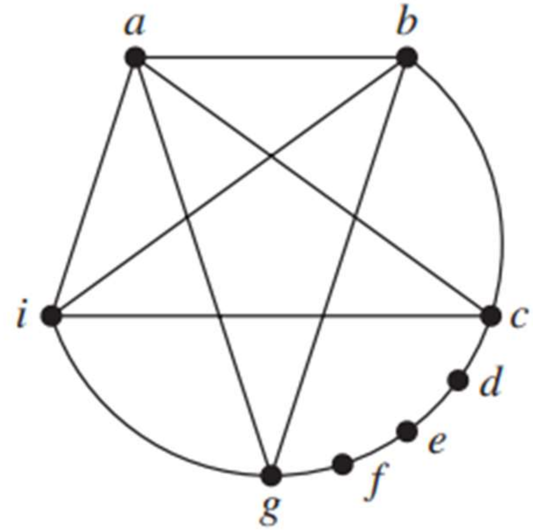
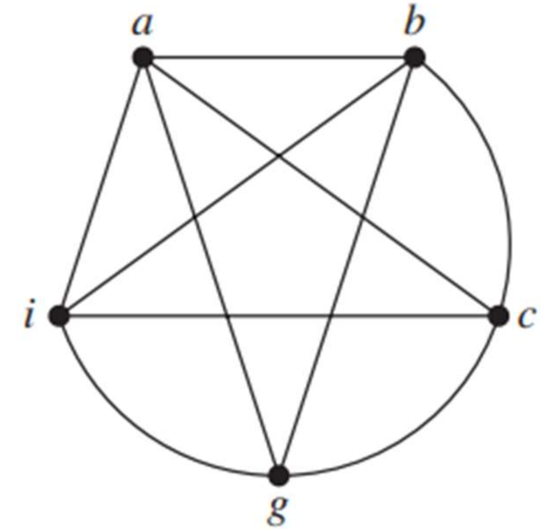


G



H



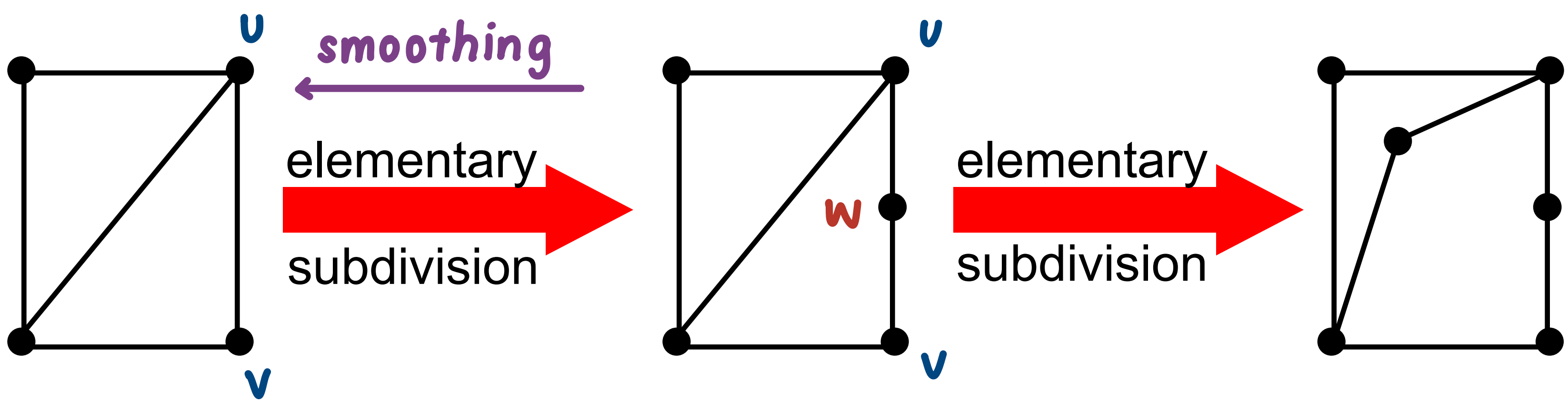
K_5

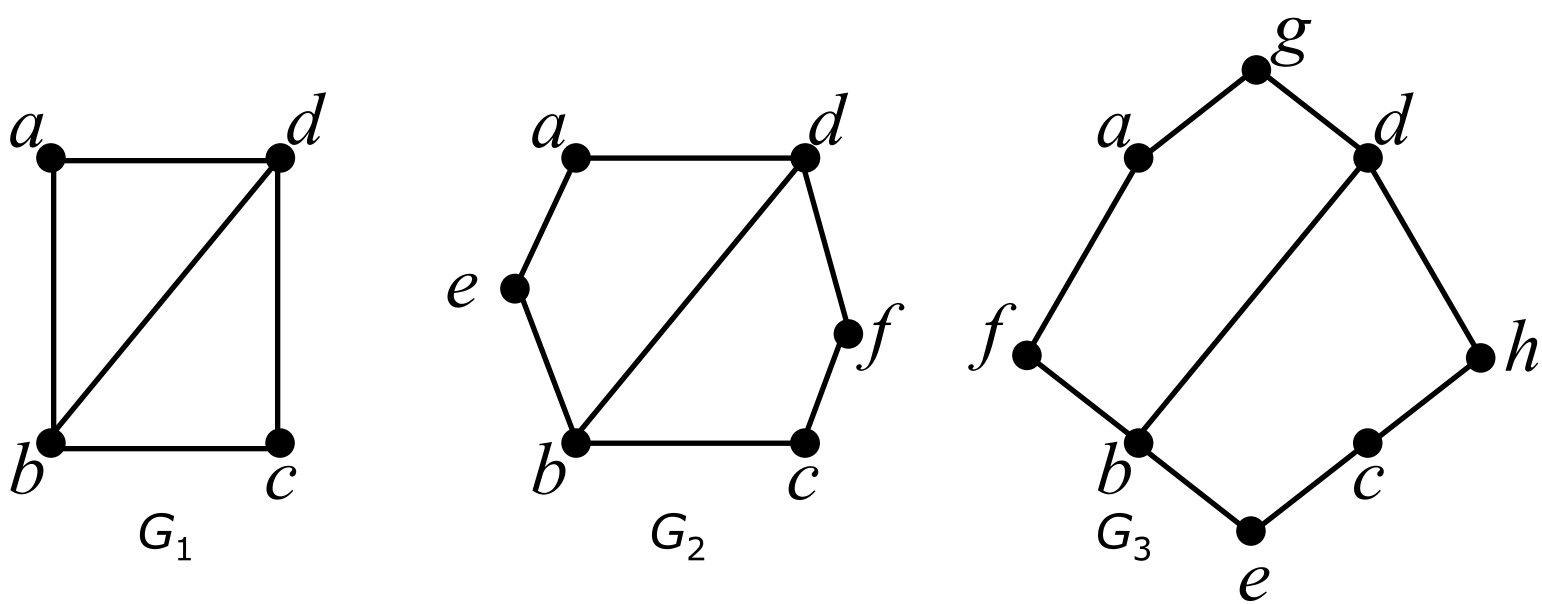
Homeomorphism

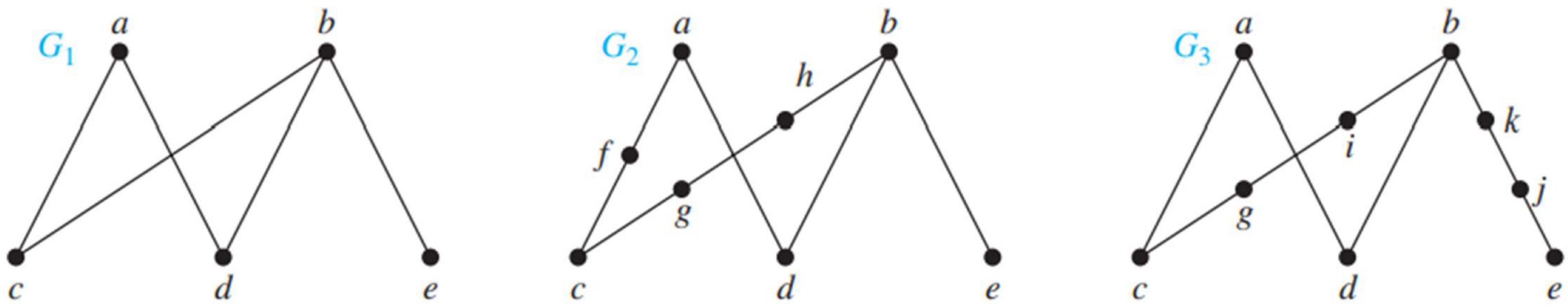
- $G=(V,E)$ and $H=(W,F)$ are **homeomorphic** if they can be obtained from the same graph by a sequence of elementary subdivisions.

Elementary Subdivision

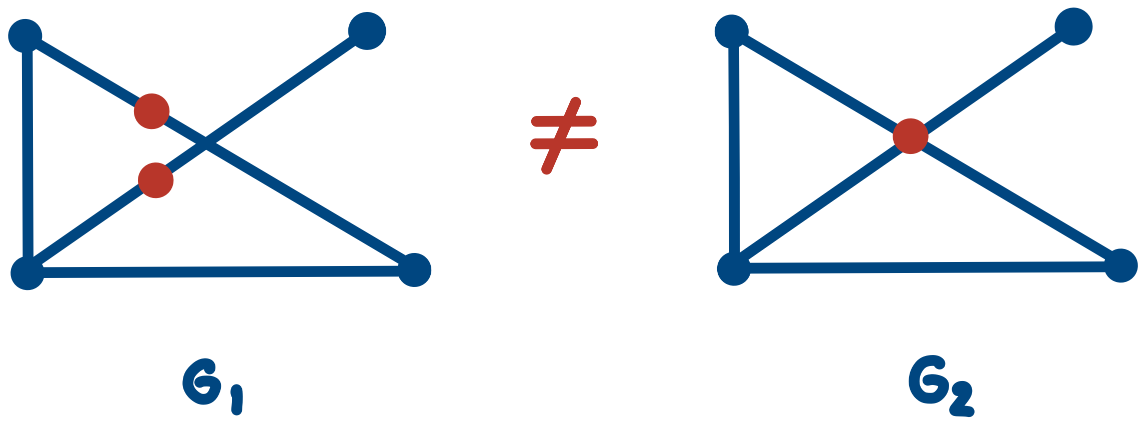
- An **Elementary Subdivision** is an operation that removes an edge $\{u,v\}$ and adding a new vertex w together with edges $\{u,w\}$ and $\{w,v\}$

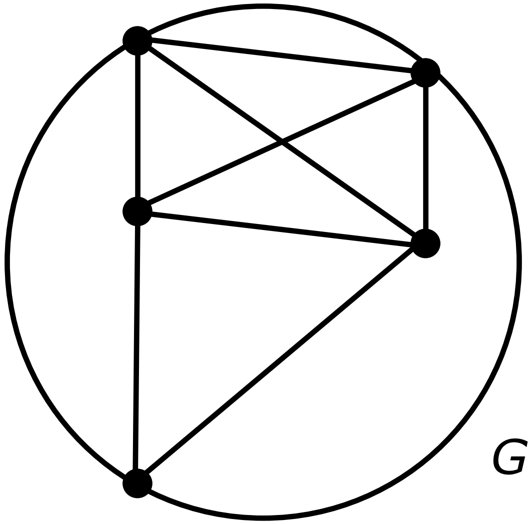



a
d
b
c
G_1
a
d
e
f
b
c
G_2
g
a
d
f
h
b
c
G_3
e



G1
a
b
c
d
e
G2
a
b
h
f
g
c
d
e
G3
a
b
k
i
j
g
c
d
e


≠
G1
G2

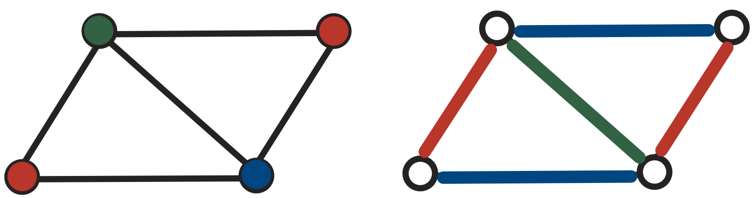


Use Kuratowski's theorem to show that G is nonplanar.

Section 10.7

Today's Topics

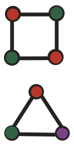
- Planar Graphs
- Graph Coloring



Graph Coloring

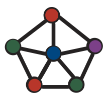
- A **coloring** of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.
- The **Chromatic Number** of *G*, $\chi(G)$, is the least number of colors needed for a coloring.

Find the chromatic number of:

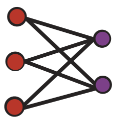


C_n v : คู่ — 2 สี

v : คี่ — 3 สี



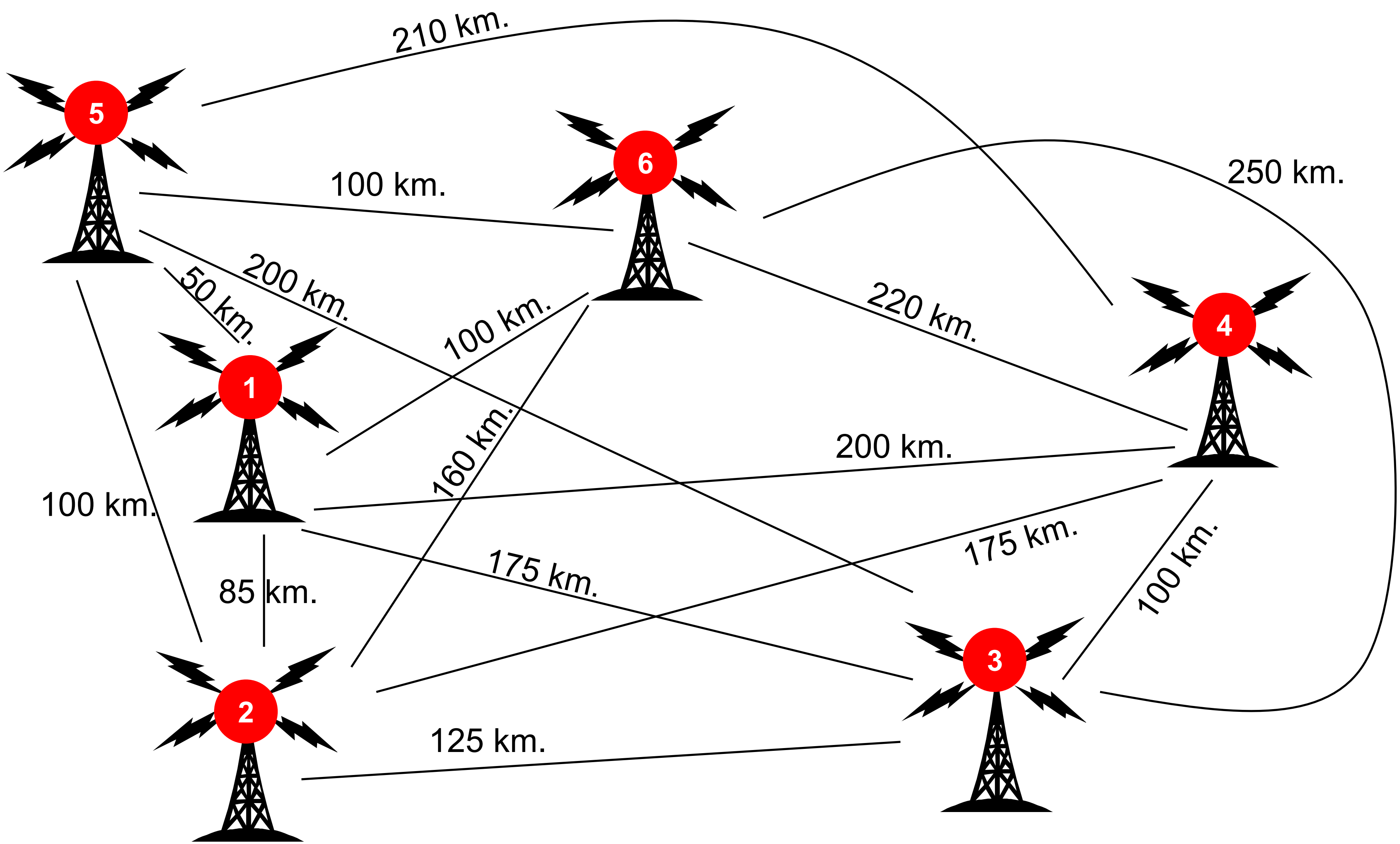
W_n จำนวนสีของ $C_n + 1$



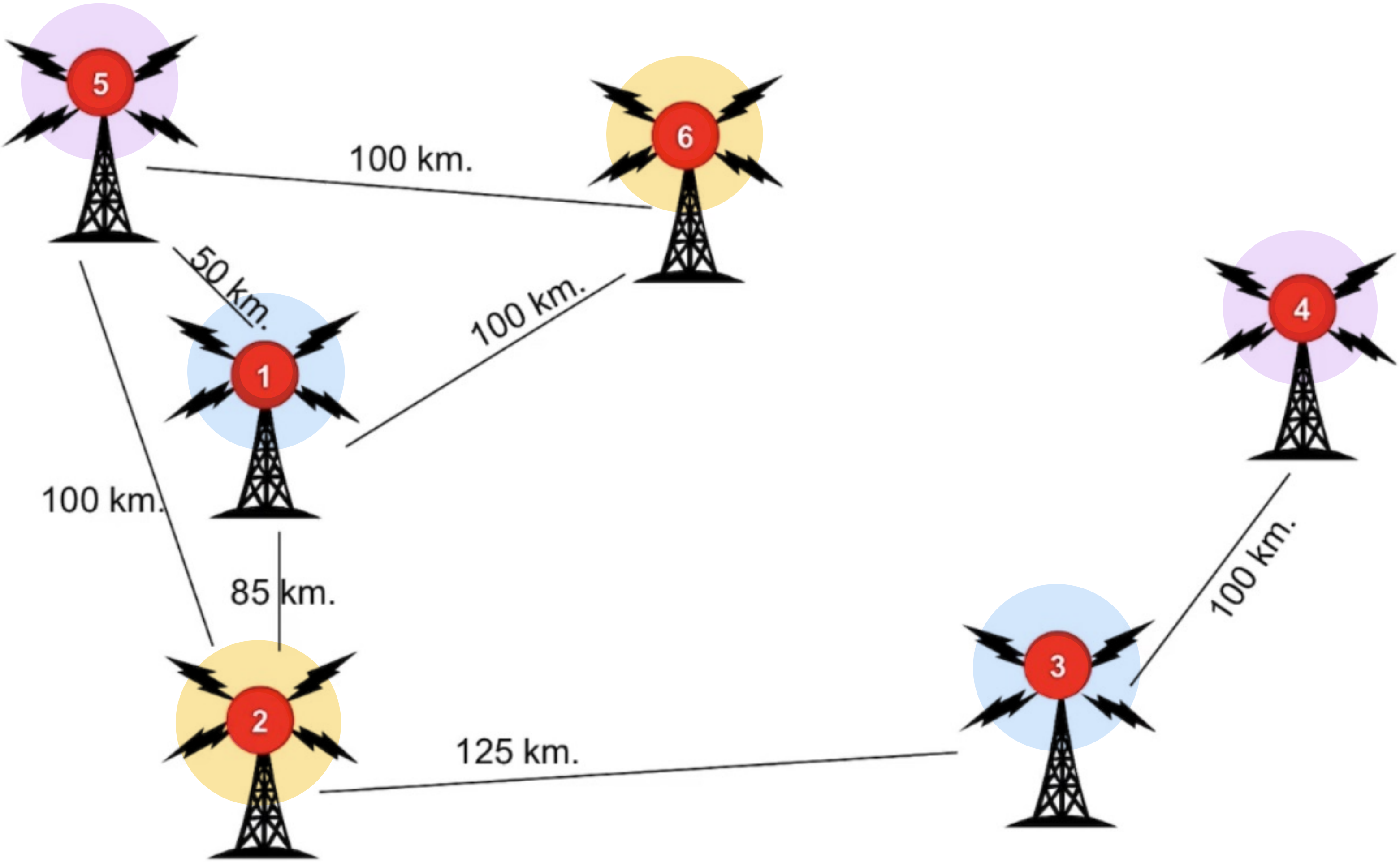
$K_{m,n}$ 2 สี (bipartite graph)

Q_n 2 สี (bipartite graph)

K_n จำนวน vertex



Two radio stations cannot use the same frequency channel when they are within 150 km. of each other. How many channels are needed?

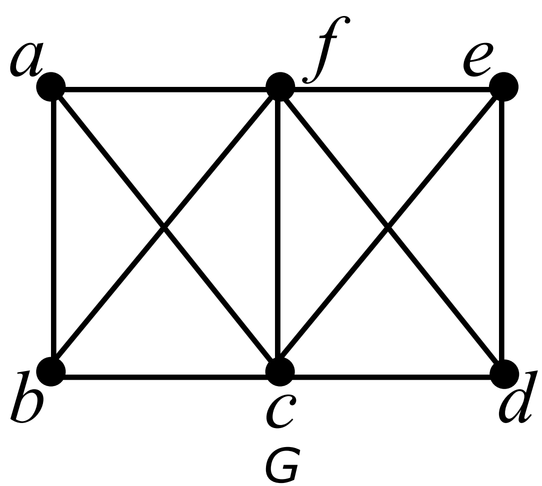


Two radio stations cannot use the same frequency channel when they are within 150 km. of each other. How many channels are needed?

The Four Color Theorem

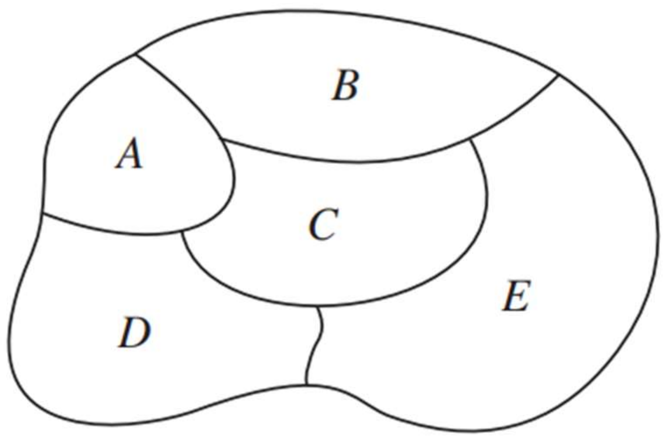
The chromatic number of a planar graph is no greater than 4.

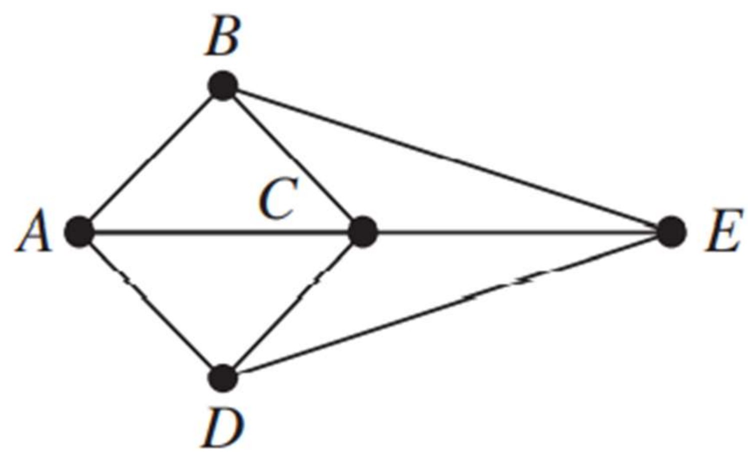
Find the chromatic number of G.



The Four Color Theorem

The chromatic number of a planar graph is no greater than 4.





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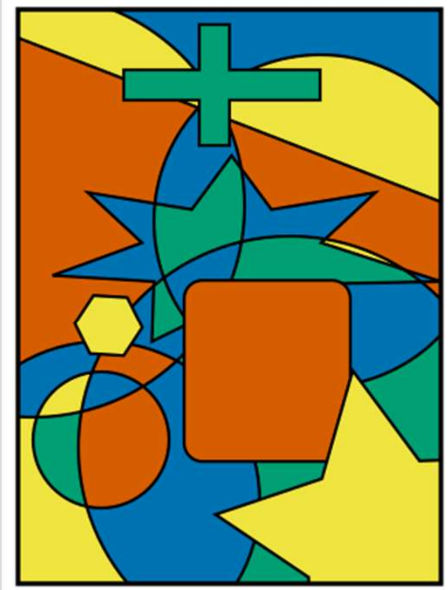
Four color theorem

From Wikipedia, the free encyclopedia

In mathematics, the **four color theorem**, or the **four color map theorem**, states that no more than four colors are required to color the regions of any map so that no two adjacent regions have the same color. *Adjacent* means that two regions share a common boundary curve segment, not merely a corner where three or more regions meet.[1] It was the first major theorem to be proved using a computer. Initially, this proof was not accepted by all mathematicians because the computer-assisted proof was infeasible for a human to check by hand.[2] The proof has gained wide acceptance since then, although some doubters remain.[3]

The four color theorem was proved in 1976 by Kenneth Appel and Wolfgang Haken after many false proofs and counterexamples (unlike the five color theorem, proved in the 1800s, which states that five colors are enough to color a map). To dispel any remaining doubts about the Appel–Haken proof, a simpler proof using the same ideas and still relying on computers was published in 1997 by Robertson, Sanders, Seymour, and Thomas. In 2005, the theorem was also proved by Georges Gonthier with general-purpose theorem-proving software.



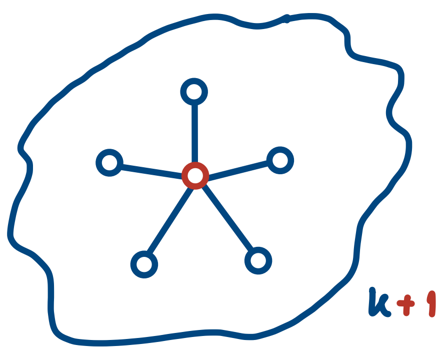
Example of a four-colored map

The chromatic number of a planar graph is no greater than 6.

S(n) : planar graph with n vextices can be color using 6 colors

Basis step : S(6)

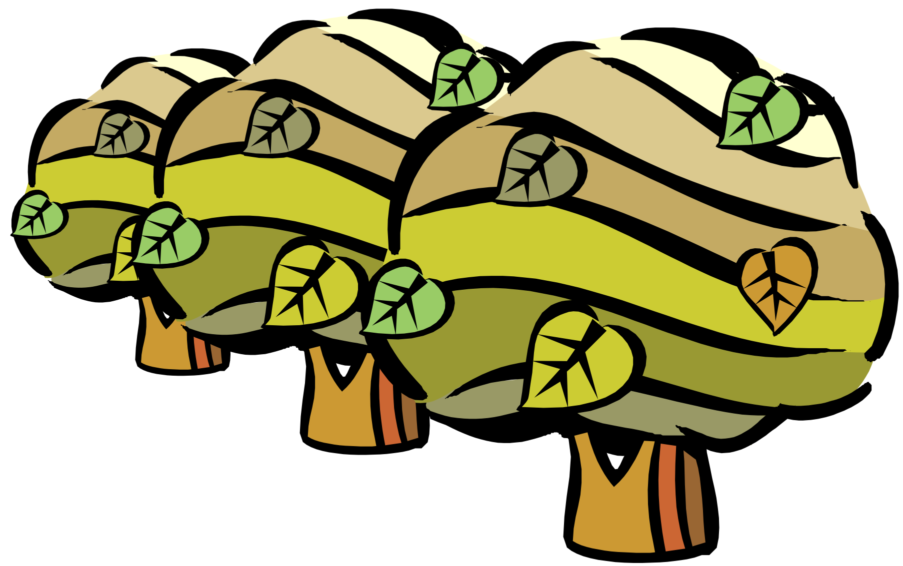
Inductive step : $S(k) \rightarrow S(k+1)$



Sections 11.1-11.2

- Trees
  - Rooted Trees
  - *m*-ary Trees
- Properties of Trees

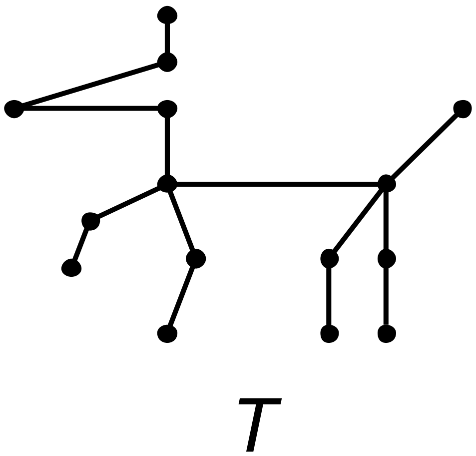
Trees

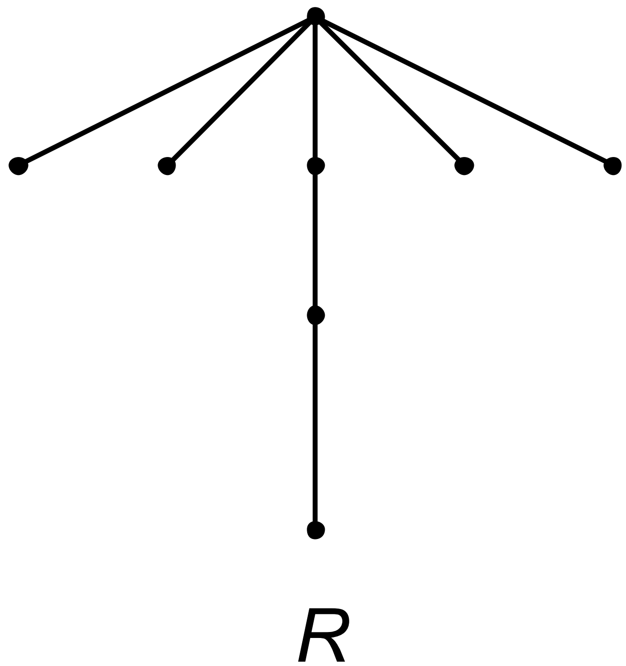


Trees

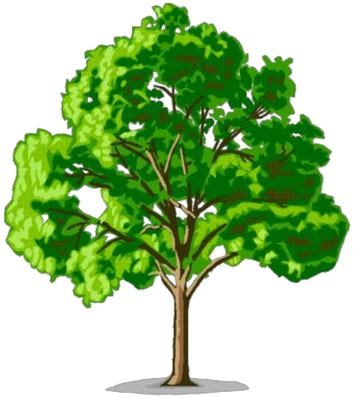
Definition:

A **tree** is a connected graph with no simple circuits





A connected graph with no simple circuits



A graph with no simple circuits but not necessarily connected

???

forest

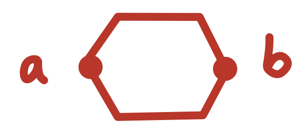
Trees

An undirected graph is a tree ↔ there is a unique path between any two of its vertices.

มีทางเดินมากกว่า 1 ทาง ระหว่างบางคู่ vertex

ดังนั้น ทำให้เกิด circuit ผ่านคู่ vertex ดังกล่าว

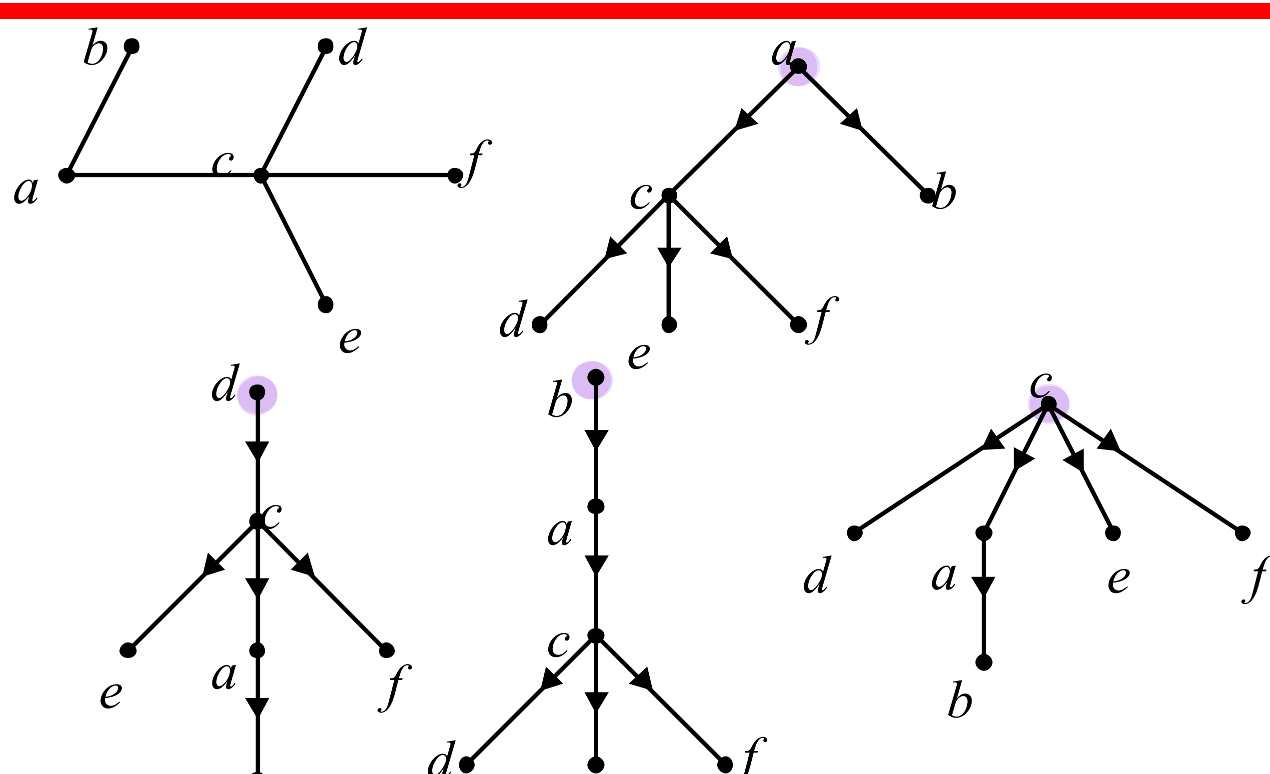
ดังนั้น กราฟไม่เป็น tree



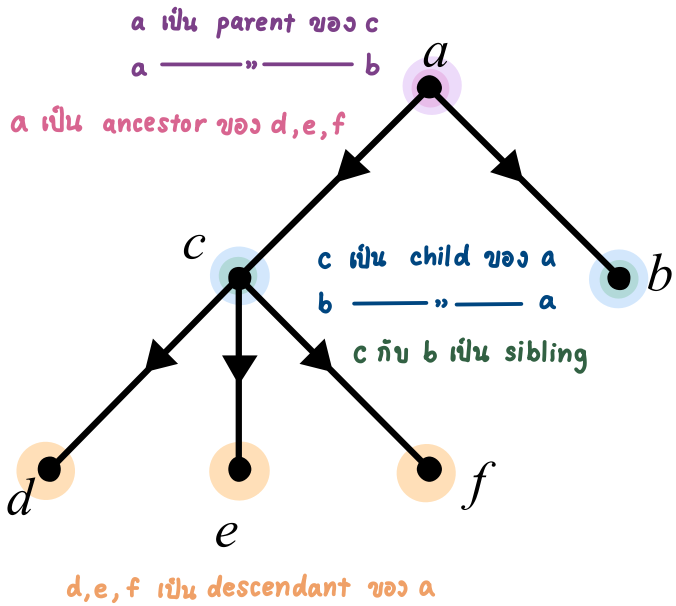
Rooted Trees

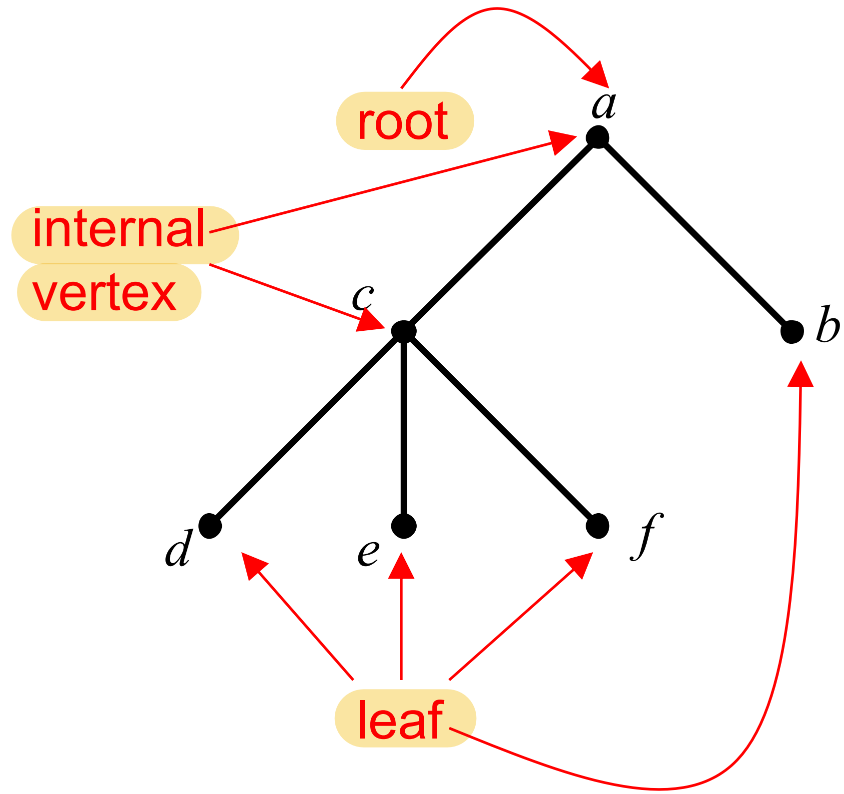
Definition:

A **rooted tree** is tree in which one vertex has been designated as the root and every edge is directed away from the root.



Rooted Trees



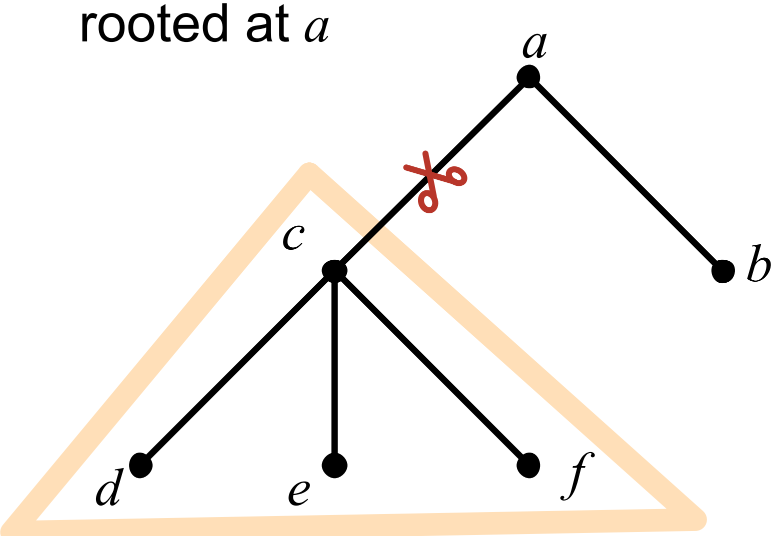


parent, child, siblings

ancestor, descendant

Subtrees

rooted at a

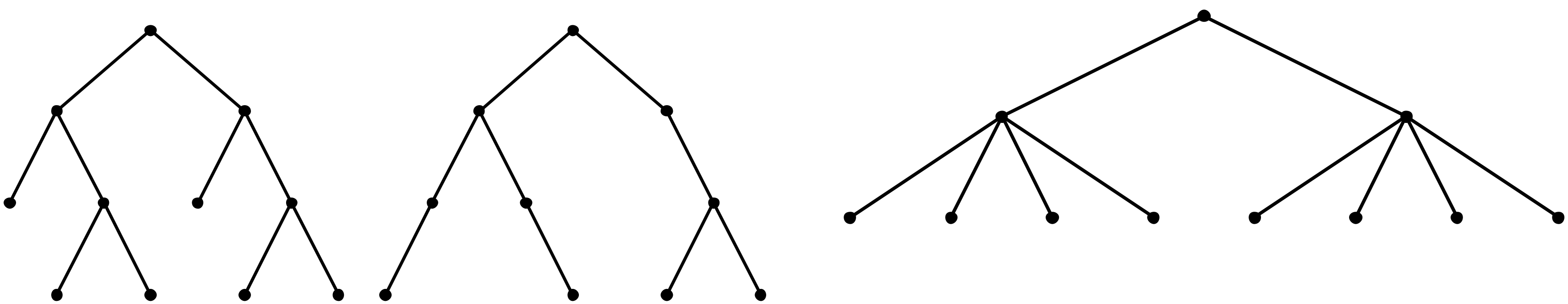


The **subtree** with v as its root is the subgraph of the tree consisting of v and its descendants and all edges incident to these descendants.

m-ary Trees

Definition:

An *m*-ary tree is a **rooted tree** whose every internal vertex has no more than *m* children



full 2-ary tree

4-ary tree

2-ary tree

When every internal vertex has exactly *m* children → **Full *m*-ary Tree**

Properties of Trees

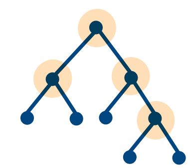
$\overset{1}{\cancel{r}} - e + v = 2$

$v = e + 1$

$e = v - 1$

A tree with *n* vertices has *n*-1 edges.

A full *m*-ary tree with *i* internal vertices contains $n = mi+1$ vertices.



$n = 4(2)+1$

$= 9$

A full 4-ary tree has 13 vertices.
How many leaves are there?

$4i + 1 = 13$

$i = (13-1)/4 = 3$

leaf $= 13 - 3 = 10$ #

A full m-ary tree with

(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,

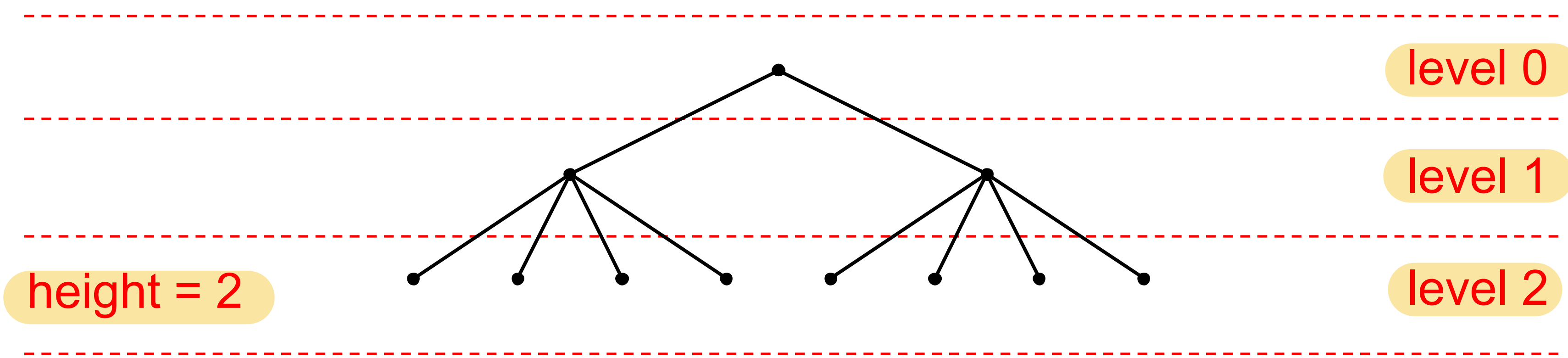
(ii) i internal vertices has $n = mi + 1$ vertices and $l = (m-1)i + 1$ leaves,

(iii) l leaves has $n = (ml - 1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

Level and Height

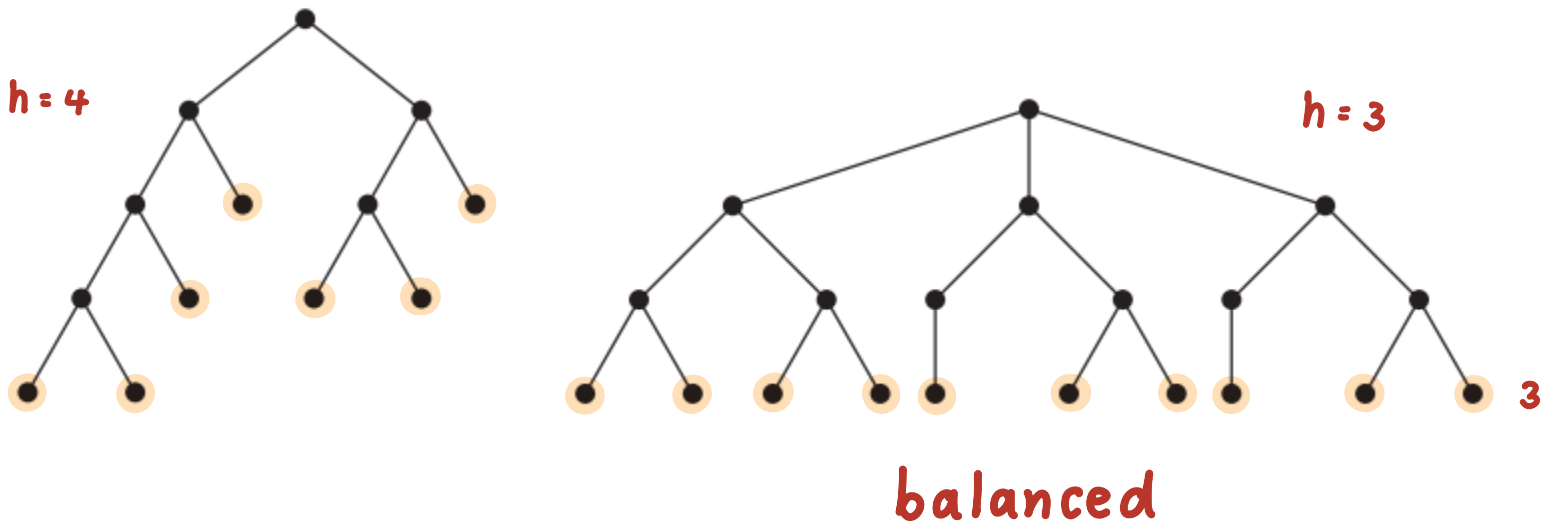
The **level** of a vertex in a rooted tree is the length of the unique path form the root to this vertex.

Height = maximum of the levels



There are at most m^h leaves in an m-ary tree of height h.

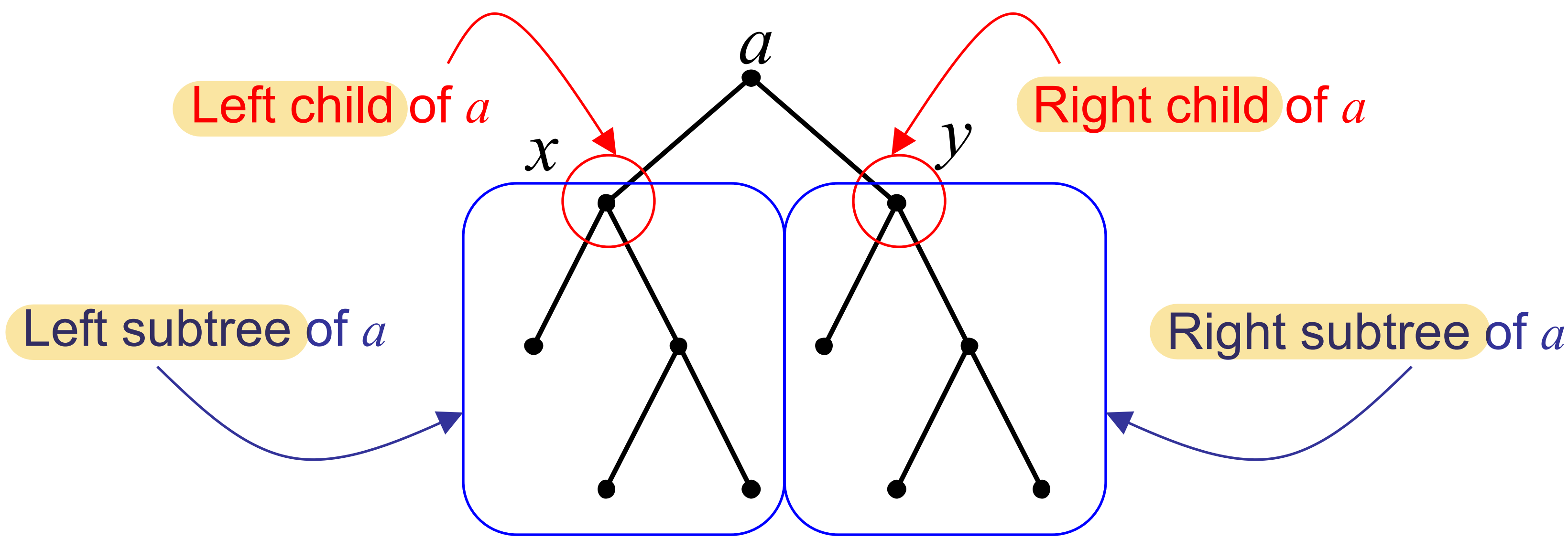
An m-ary tree of height h is balanced if all leaves are at level h or $h - 1$.


h = 4
h = 3
3
balanced

Ordered Rooted Trees

Siblings are ordered from left to right

Ordered binary tree (Binary tree)



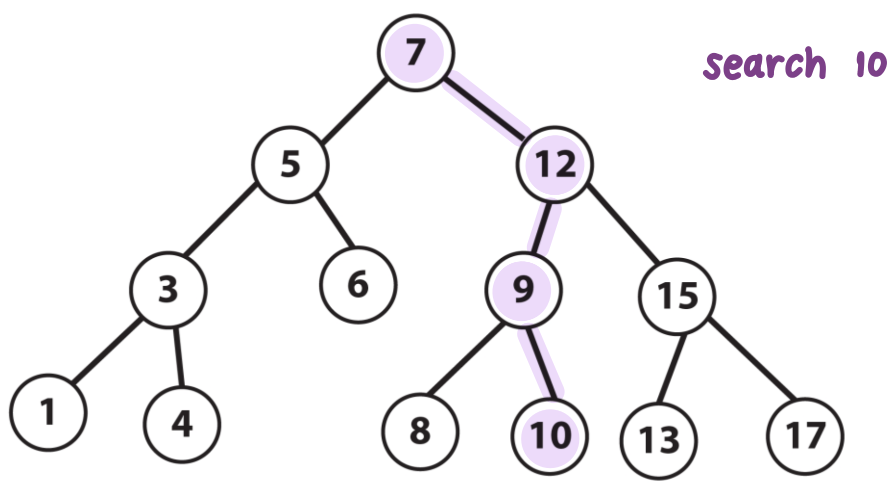
Sections 11.1-11.2

Trees

- Tree Applications
  - Binary Search Trees
  - Decision Trees
  - Prefix Codes

Binary Search Trees

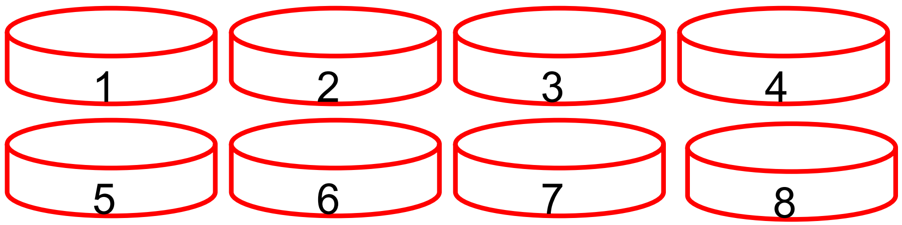
- Each vertex is labeled with a **key**.
- The key of a vertex is:
  - larger than the keys of all vertices in its left subtree.
  - smaller than the keys of all vertices in its right subtree.



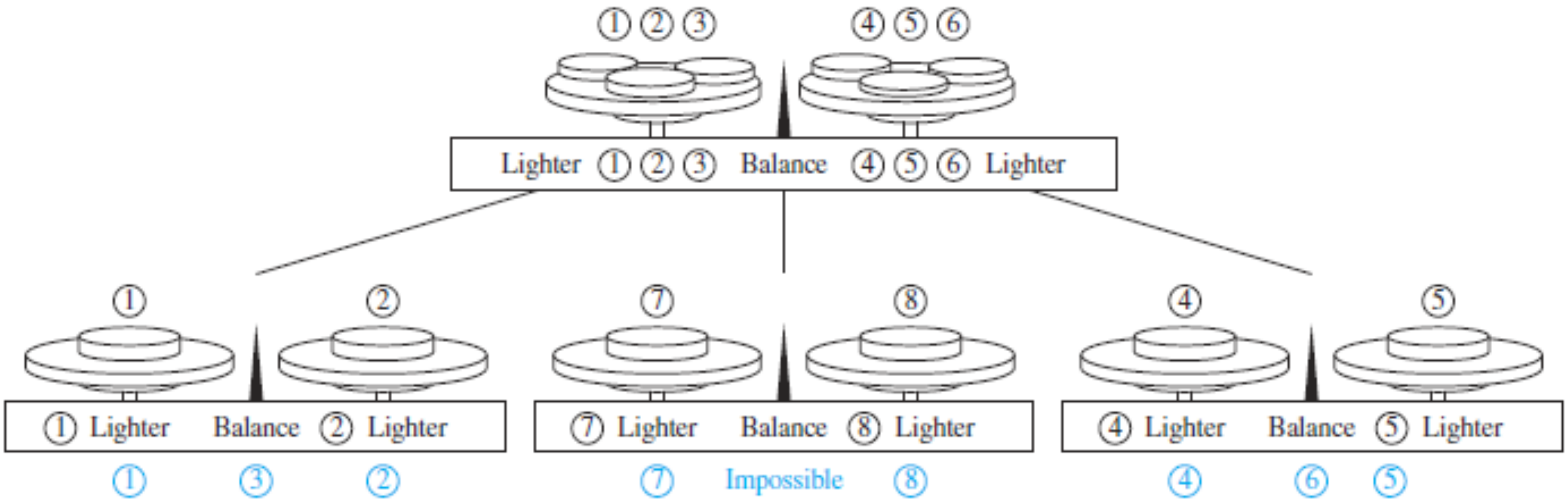
Decision Trees

How many weightings are needed to find the lighter coin?

There is one coin that is lighter than the others



Decision Trees

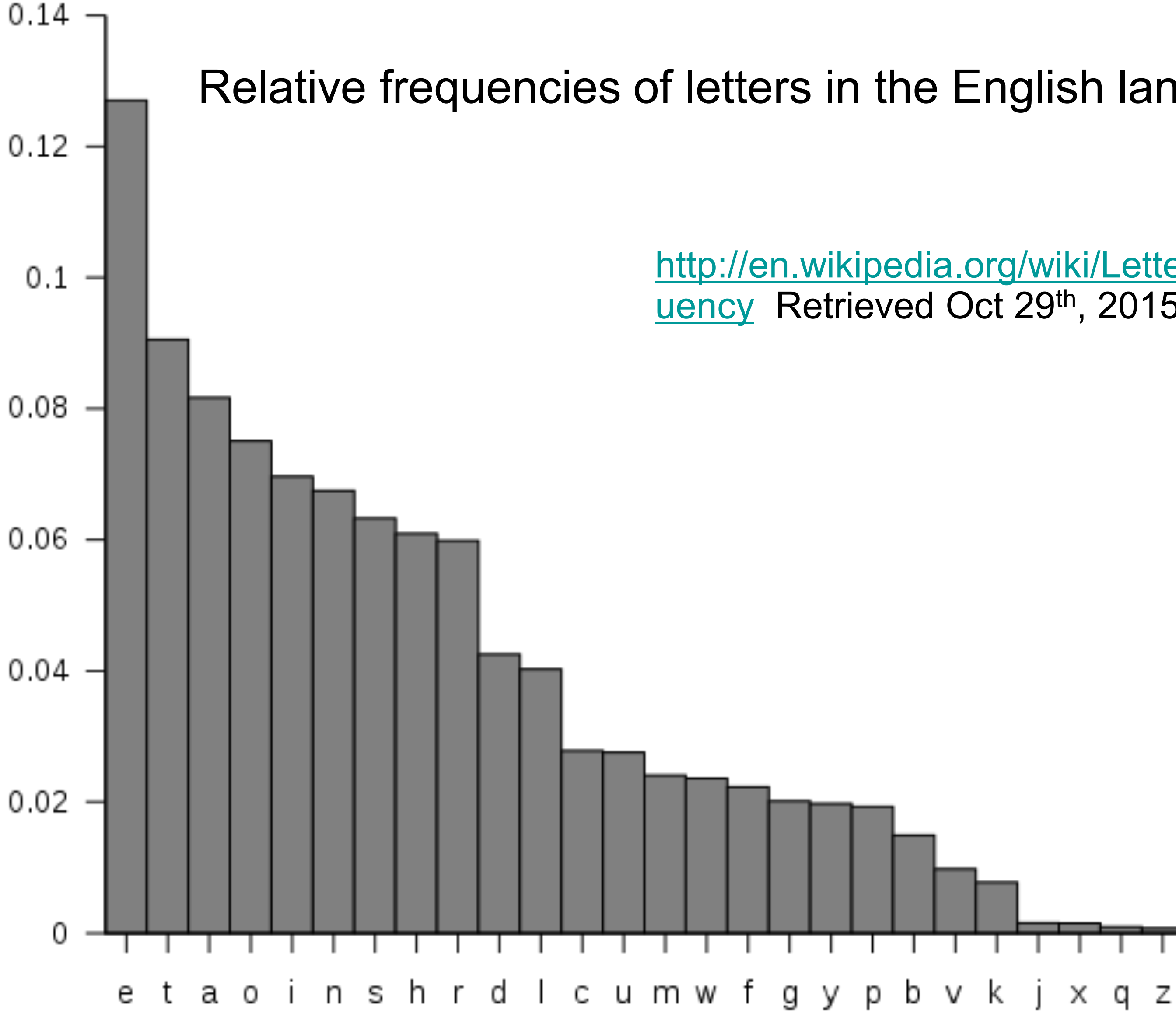


Character Encoding

Given 26 different symbols (E.g. a-z), how many bits are required to encode a symbol?

How many bits are used to encode a string of *n* characters?

Can we use less?



Relative frequencies of letters in the English language

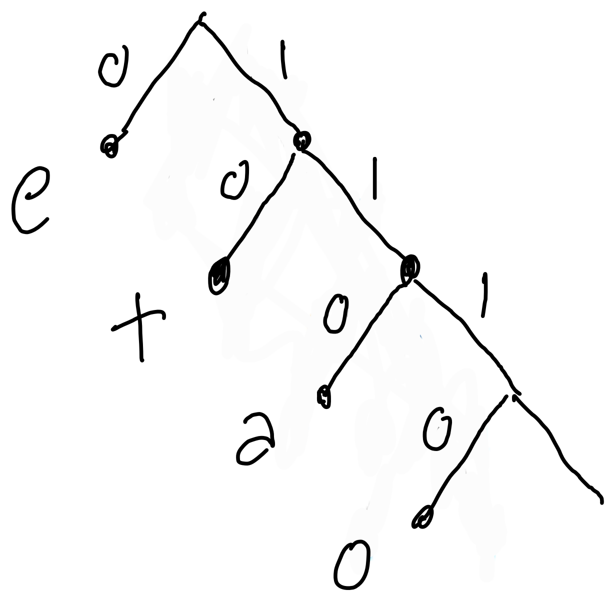
http://en.wikipedia.org/wiki/Letter_frequency Retrieved Oct 29th, 2015

Prefix Code

The bit string for a letter never occurs as the first part of the bit string for another letter

tea

10 0 110



e	0
t	10
a	110
o	1110

Section 11.3

Tree Traversal

post order traversal – ลูกถูกอ่านหมดแล้ว ถึงอ่านตัวนั้นได้

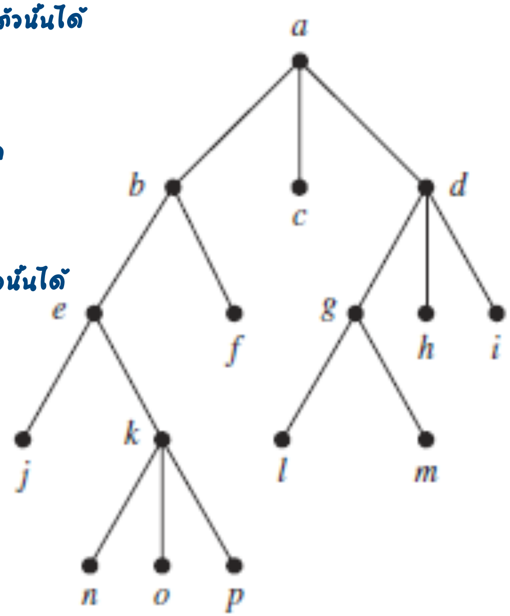
j n o p k e f b c l m g h i d a

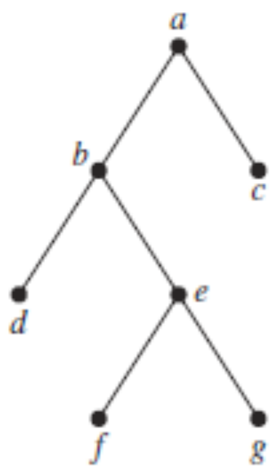
pre order traversal – อ่านตัวนั้นเองก่อน ตามด้วยลูก

a b e j k n o p f c d g l m h i

in order traversal – อ่านลูกตัวทางซ้ายก่อน ถึงอ่านตัวนั้นได้

j e n k b f a c l g m d h i

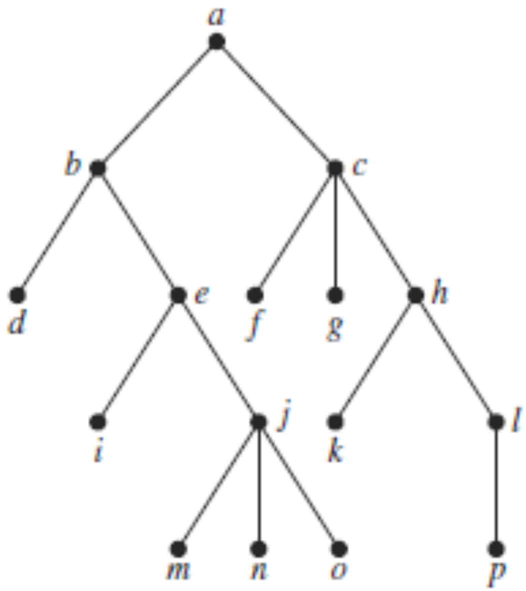




post order d f g e b c a

pre order a b d e f g c

in order d b e f g a c

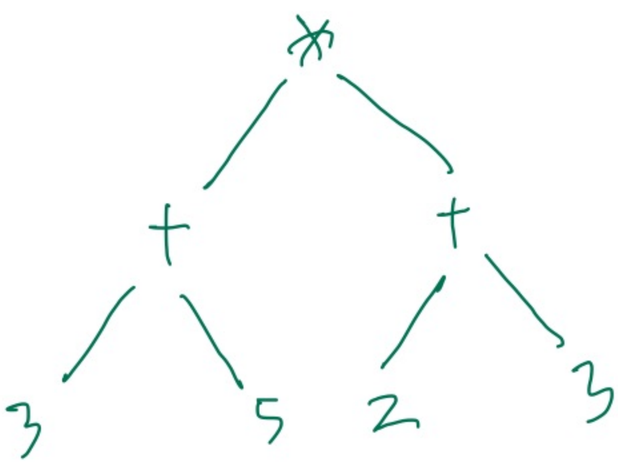


post order d i m n o j e b f g k p l h c a

pre order a b d e i j m n o c f g h k l p

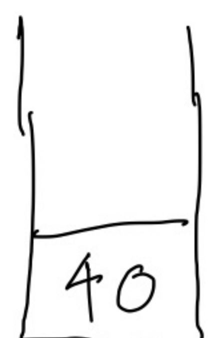
in order d b i e m j n o a f c g k h p l

Postfix Calculator



$3 \; 5 \; + \; 2 \; 3 \; + \; *$

$(3+5)*(2+3)$



Section 11.4

Spanning Trees

Definition:

Let G be a simple graph. A **spanning tree** of G is a subgraph of G that is a tree containing every vertex of G.