ONLY THE ANSWERS IN THE ANSWER SHEET WILL BE GRADED.

Part 1: Propositional & Predicate Logic (8 Points)

1.   Are these sentences propositions? (Yes or No)
    1.1.   Do you wanna build a snowman   _____
    1.2.   The sun is bigger than the moon   _____
    1.3.   x is greater than y   _____
    1.4.   Every integer x, x = sqrt(x) + y + 1   _____
    1.5.   Some DS-TAs think TA Potter is handsome   _____

2.   Given proposition below
      "Roses are red and Violets are red, whenever you like red color"
    Let p: Roses are red, q: Violet are red and r: you like red color
    Choose the correct answer for each question.
    2.1.   PROPOSITION:
        a) $(p \land q) \to r$ b) $r \to (p \land q)$
    2.2.   CONVERSE:
        a) $(p \land q) \to r$ b) $r \to (p \land q)$
    2.3.   CONTRAPOSITIVE:
        a) $\neg(p \land q) \to \neg r$ b) $\neg r \to \neg(p \land q)$
    2.4.   INVERSE:
        a) $\neg(p \land q) \to \neg r$ b) $\neg r \to \neg(p \land q)$
    2.5.   NEGATION:
        a) $\neg((p \land q) \to r)$ b) $\neg(r \to (p \land q))$

3.   Given the truth table below
    3.1.   The error occurs in row _____ column _____.
    3.2.   Determine the truth values of W, X, Y and Z.
        (***according to the correct truth table)

	1	2	3	4	5	6	7
	p	q	$\neg p$	$q \to \neg p$	$(q \to \neg p) \land q$	$\neg q$	$((q \to \neg p) \land q) \leftrightarrow \neg q$
1	T	T	F	F	F	F	**W**
2	T	F	F	T	T	T	**X**
3	F	T	T	T	T	F	**Y**
4	F	F	T	T	F	T	**Z**

4.   Let p, q, r and s be the propositions. The truth values of p, q and r are F, F and F, respectively. Determine the truth value of each of these propositions.
      Choice: a) T   b)F   c) Depends on s
    4.1. $(\neg p \to q) \to (\neg r \lor s)$ _____
    4.2. $((p \land s) \to \neg q) \leftrightarrow \neg r$ _____

5.   Determine whether these statements
            a) Tautology   b) Contradiction   c) Contingency.

    5.1.   (p ∧ (p→q))→q   _____

    5.2.   (p ∨ q) → ¬p   _____

6.   Determine whether these statements are consistent.(Yes or No)
    a) When you like chocolate croissants and buy strawberry croissants, you are not happy.
    b) If you buy strawberry croissants, you do not like chocolate croissants.
    c) You like chocolate croissants or you are happy.
            Ans: _____

7.   Determine the truth value of the following statements.
    7.1.   (p ∨ q) → p ≡ (p ∧ q)→ (q →p)

            _____

    7.2. $\exists x \forall y(x^2 - y^2 = 0)$ when x, y ∈ R⁻   _____
    7.3.   If ∀xP(x) is true when x ∈ I⁺ then ∃xP(x) is always true when x∈ R   _____
    7.4.   ∀xP(x) is true when x ∈ ∅   _____

8.   For the arguments,
    Premise:
            ∀x(P(x) ∧ Q(x))
            ∀x(R(x)→¬S(x))
            ∀x(¬Q(x) ∨ S(x))
            ∃x¬P(x)
    Conclusion: ∃x¬R(x)
        The steps:
    1. ∀x(P(x) ∧ Q(x))   – Premise
    2. P(c) ∧ Q(c)   – Universal Instantiation using (1)
    3. Q(c)   – Disjunctive syllogism using (2)
    4. ∀x(¬Q(x) ∨ S(x))   – Premise
    5. ¬Q(c) ∨ S(c) for some elements c   – Existential Instantiation (4)
    6. S(c)   – Disjunctive Syllogism using _(a)_ and _(b)_
    7. ∀x(R(x)→¬S(x))   – Premise
    8. R(c)→¬S(c)   – Universal Instantiation (7)
    9. ¬R(c)   – Modus tollens using (6) and (8)
    10. ∃x¬R(x)   – Existential Generalization using (9)
    8.1.   Identify steps (a) and (b)
            a = _____   b = _____
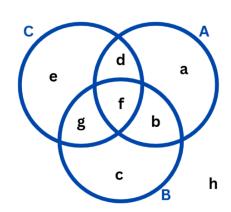   8.2.   There is an error in step _____.(Only ONE error)

Part 2: Set (6 Points)

    1. Which of the following statements is correct?
            a. $\{a, b, c, \emptyset\} \subseteq \{a, b, c, c\}$
            b. If A ⊂ B and A ⊂ C, then A ⊂ B ∩ C
            c. If A ⊆ B and B ⊆ C, then A ⊆ C

d. $\{\varnothing\} \subset \{\{\varnothing\},1\}$

2. Which of the following statements is correct?
   a. $\{\emptyset\} \in P(\{a, b, c, c\})$
   b. If A ∈ B and B ∈ C, then A ∈ C.
   c. A × B × C = (A × B) × C where A,B and C are sets.
   d. Let A = {x | x is odd and |x| < 7 } There are 64 possible subsets of A

3. Which combination of the sets A,B,C will result in the given area

3.1) a,b and d
a) A' ∩ (B' ∪ C')
b) A ∩ (B' ∪ C')
c) C' ∩ (B' ∪ A')

3.2) e and g
a) B' ∪ C'
b) (A ∪ C')'
c) (C' ∩ A')'

4. Let C = {{∅},∅,1,2,{{∅}}}
   A = {{1,2},{1}}
   Find |P(C) ∪ C| + |P(A - C)| - | P(C) - A|

5. Let U = {1,2,3,...,17,18}
   A = {x | x ² ∉ U}
   B = {x | 2x ∉ U ∧ x/3 ∉ A}
   C = {x | x ∉ A ∧ 4x ∈ B}
   Find |A ∩ B| + |B ∪ C|

6. Consider set A,B, and C where |A| = 7, |B| = 10 , |C ∩ B| = 3 , |A ∪ B| = 13 , |C ∩ A ∩ B| = 2 , |A-C| = 5
      6.1 Find |(A ∪ B) - C |
      6.2 Find minimum cardinality of C

Part 3: Function & Relation (6 Points)

1. Consider the function $f(x) = \dfrac{2}{\sqrt{x+1}}$ from $D \rightarrow R$. Determine the values of a, b for the domain(D) and range of the function.

   $D = $ _____(a, ∞)_____ $range = $ _____(b, ∞)_____

2. Let $f(x) = 3x + 5$.Determine the value of

2.1) $\lfloor f(1.5) \rfloor$ 2.2) $\lceil f(-2.5) \rceil$

3. Classify each function (✓) $f: \mathbb{Z} \longrightarrow \mathbb{Z}$ by their types of correspondences **in the answer sheet.**

Function	One-to-one, but not onto	Onto, but not One-to-one	One-to-on, And onto	Neither Onto, nor One-to-one
3.1) $f(x) = x^2 + 1$				
3.2) $f(x) = x^3$				
3.3) $f(x) = x(x + 1)(x - 1)$				
3.4) $f(x) = 3x + 5$				
3.5) $f(x) = 42$				

4. Let $f(x) = 2x^2$ and $g(x) = \sqrt{x + 3}$ is the function from $\mathbb{Z}^+ \longrightarrow R^+$. Find the value of $g^{-1}(f^{-1}(8))$

5. Consider set A of 3 elements. How many relations R on set A, which are symmetric?

6. For each of these relations on the set {1,2,3,4}, decide (✓) whether it is reflexive, symmetric, antisymmetric, or transitive **in the answer sheet.**

Relation	Reflexive	Symmetric	Antisymmetric	Transitive
6.1) {(1,3),(2,2),(3,1),(4,4)}				
6.2) {1,2,3} x {1,2,3}				
6.3) {(x,y) | x = y}				
6.4) {(1,3),(2,3),(4,1),(4,4)}				

Fill in the bubble or fill the number in the box that corresponds to your answer for each question on the exam.
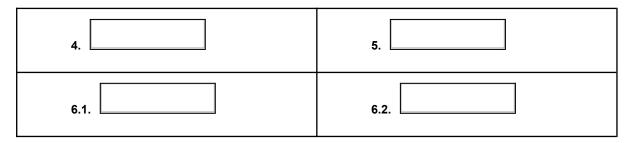
Part 1: Propositional & Predicate Logic

1.	2.
1.1. (Yes) (No) 1.2. (Yes) (No) 1.3. (Yes) (No) 1.4. (Yes) (No) 1.5. (Yes) (No)	2.1. (A) (B) 2.2. (A) (B) 2.3. (A) (B) 2.4. (A) (B) 2.5. (A) (B)

3.

3.1. row = [   ]

column= [   ]

3.2. W (True) (False)

X (True) (False)

Y (True) (False)

Z (True) (False)

4.

4.1. (A) (B) (C)

4.2. (A) (B) (C)

5.

5.1. (A) (B) (C)

5.2. (A) (B) (C)

6. (Yes) (No)

7.

7.1. (True) (False)

7.2. (True) (False)

7.3. (True) (False)

7.4. (True) (False)

8.

8.1. a = [   ]

b = [   ]

8.2. [   ]

Part 2: Set

1. (A) (B) (C) (D)	2. (A) (B) (C) (D)

3.1	(A) (B) (C)	3.2	(A) (B) (C)

Part 2: Set (Cont)

4.		5.	
6.1.		6.2.	

Part 3: Function & Relation

1	a =	b =		2.1		2.2	

3	One-to-one, but not onto	Onto, but not One-to-one	One-to-on, And onto	Neither Onto, nor One-to-one
3.1	⬭	⬭	⬭	⬭
3.2	⬭	⬭	⬭	⬭
3.3	⬭	⬭	⬭	⬭
3.4	⬭	⬭	⬭	⬭
3.5	⬭	⬭	⬭	⬭

4		5	

Choose "ALL" that apply.

6	Reflexive	Symmetric	Antisymmetric	Transitive
6.1	⬭	⬭	⬭	⬭
6.2	⬭	⬭	⬭	⬭

6.3	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$
6.4	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$