CHULA ΣNGINEERING
Foundation toward Innovation

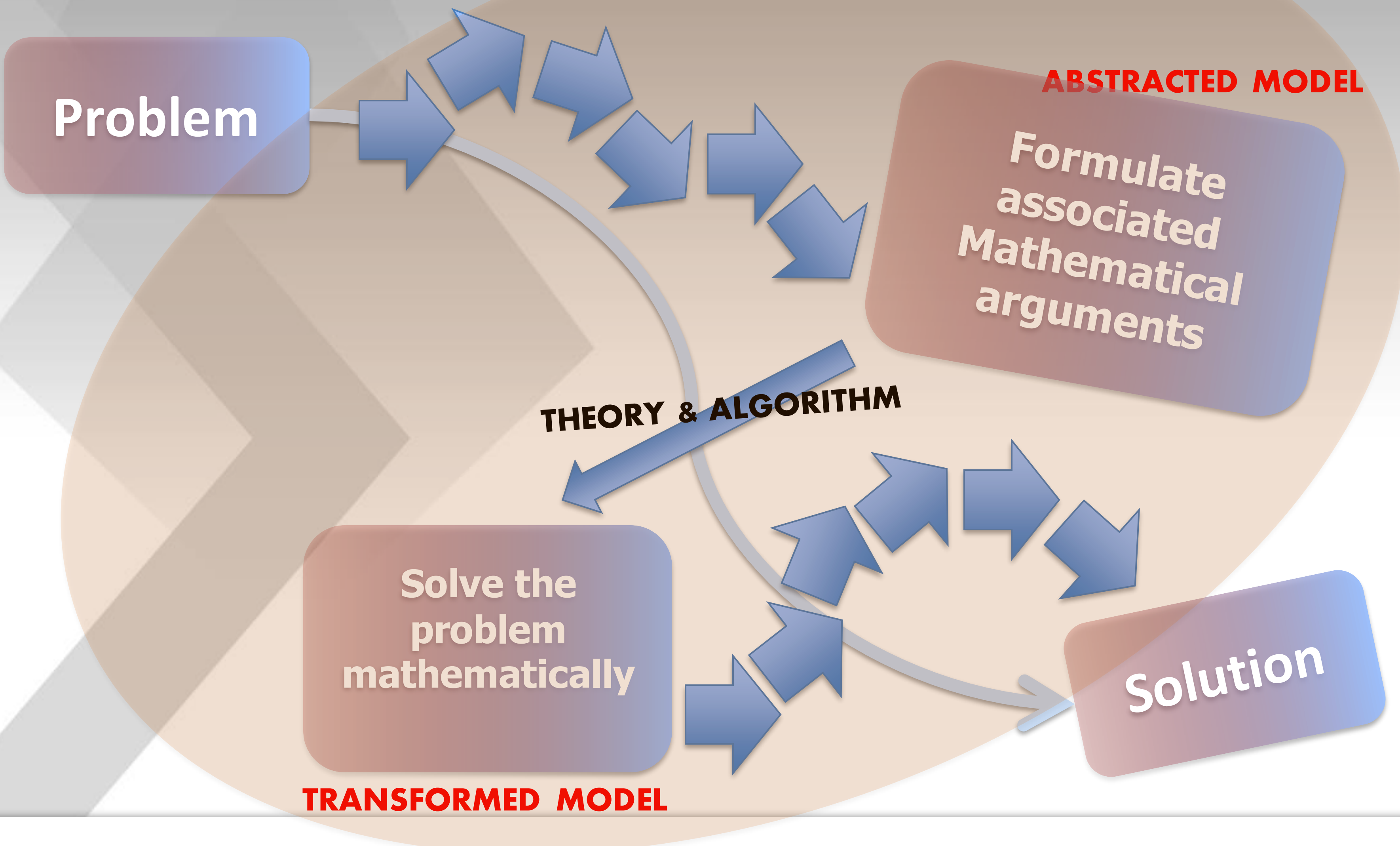
100th Anniversary of Chula Engineering 2013

2110200 Discrete structures

Athasit Surarerks SEC 1

Computer Engineering
19th Floor, Eng.4-1910
Tel: 02-218-6996
Email: athasit.s@chula.ac.th

WHY ?


Problem
ABSTRACTED MODEL
Formulate associated Mathematical arguments
THEORY & ALGORITHM
Solve the problem mathematically
TRANSFORMED MODEL
Solution

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GOALs

Apply the obtained problem-solving skills to model and solve problems in computer engineering & science and other areas.

Applications and Modeling

Specify, verify, and analyze an algorithm.

Algorithmic Thinking

Able to work with discrete structures: sets, graphs, finite-state machines, etc.

Discrete Structure

Combinatorial Analysis

Perform analysis to solve counting problems.

Mathematical Reasoning

Read, comprehend, and construct mathematical arguments.

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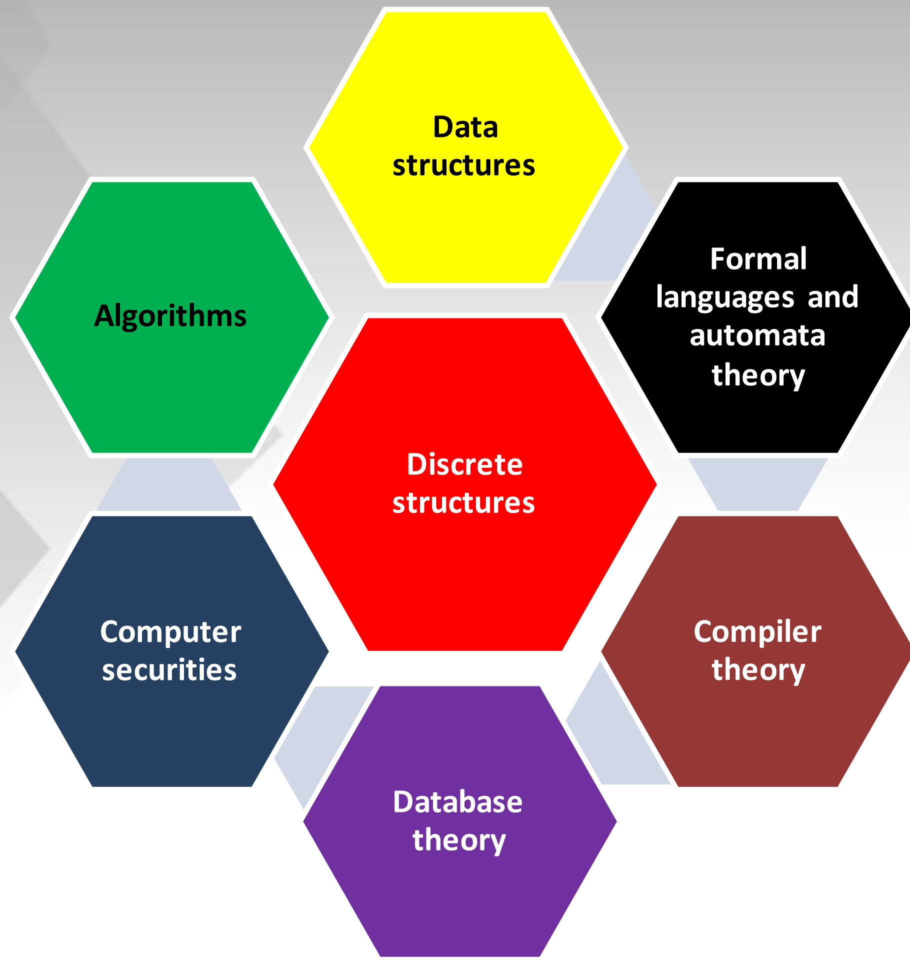
GATEW@Y TO



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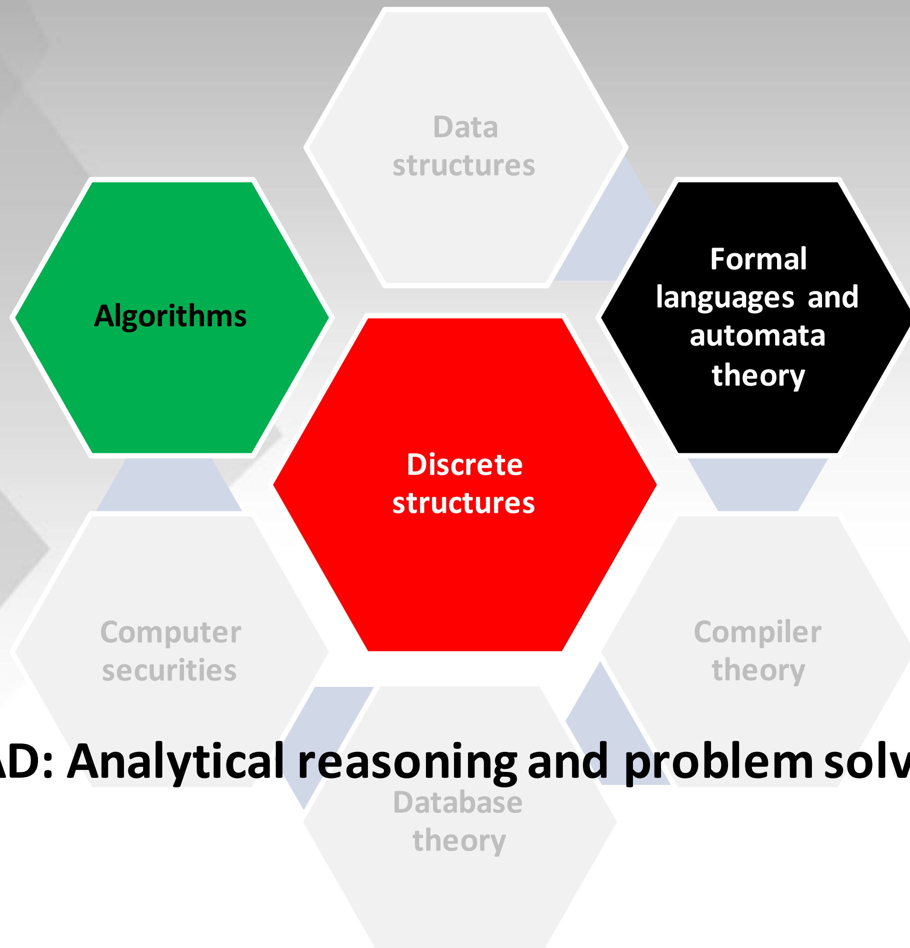

THREAD: Analytical reasoning and problem solving skill

TEXTBOOK

DISCRETE MATHEMATICS
AND ITS APPLICATIONS
SEVENTH EDITION
GLOBAL EDITION
McGrawHill
Kenneth H. Rosen

OUTLINE

1	• The Foundation	15 %
2	• Number Theory	15 %
3	• Counting Technique	15 %
4	• Graphs and Trees	15 %
	FINAL EXAM	**40 %**

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EVALUATION

1	September 16, 2014	15 %
2	October 9, 2014	15 %
3	November 4, 2014	15 %
4	November 25, 2014	15 %
	FINAL EXAM	40 %

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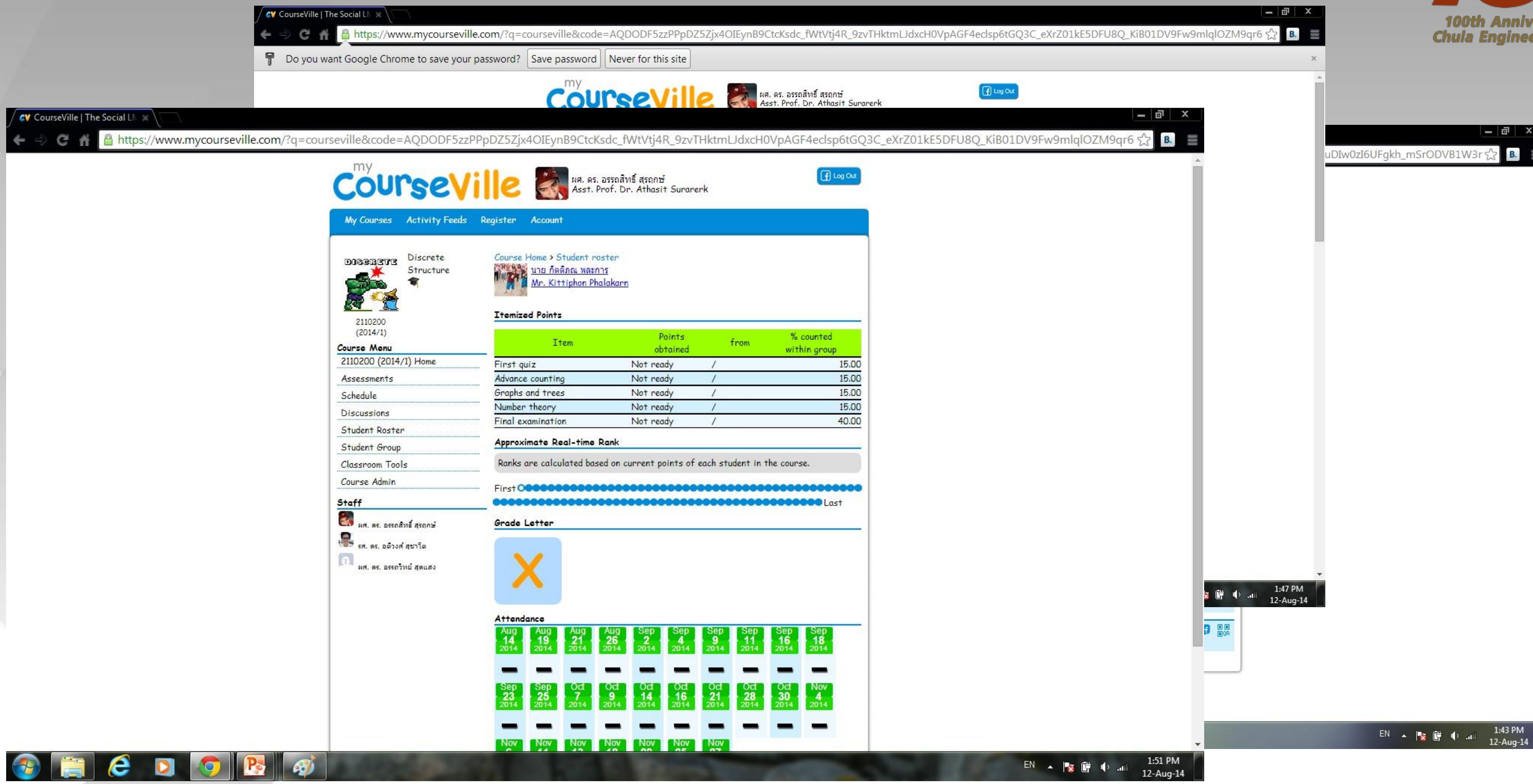

my CourseVille

LOGIC

Rules of logic gives precise meaning to mathematical statements

LOGIC

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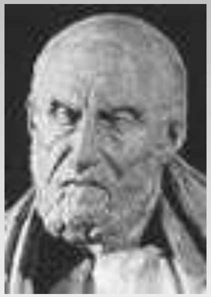
STATE OF THE ARTs





Aristotle (384-322 B.C.)	SYLLOGISTIC REASONING ~ก.ถ่ายทอด
Euclid of Alexandria (325-265 B.C.)	DEDUCTIVE REASONING
Chrysippus of Soli (279-206 B.C.)	MODAL LOGIC
George Boole (1815-1864 A.D.)	PROPOSITIONAL LOGIC
Augustus De Morgan (1806-1871 A.D.)	DE MORGAN's LAWs

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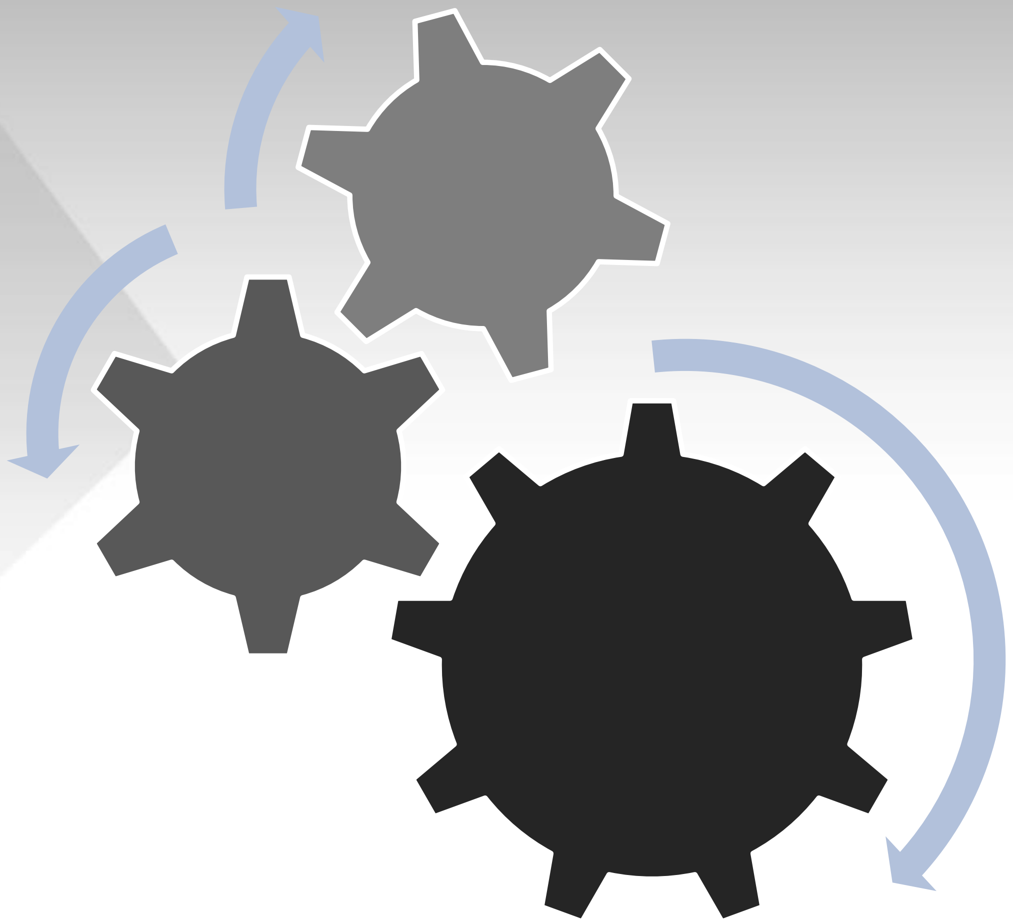
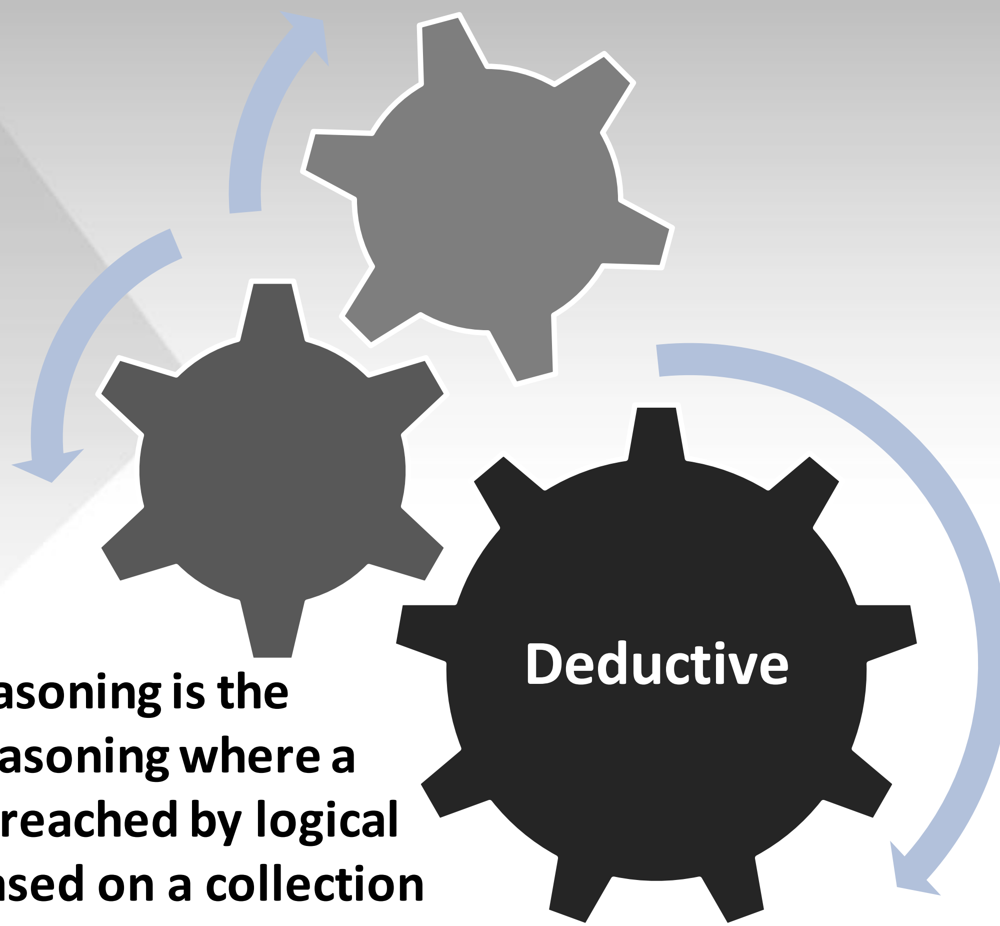
REASONING MODELS



REASONING MODELS



Deductive reasoning is the method of reasoning where a conclusion is reached by logical arguments based on a collection of assumptions.

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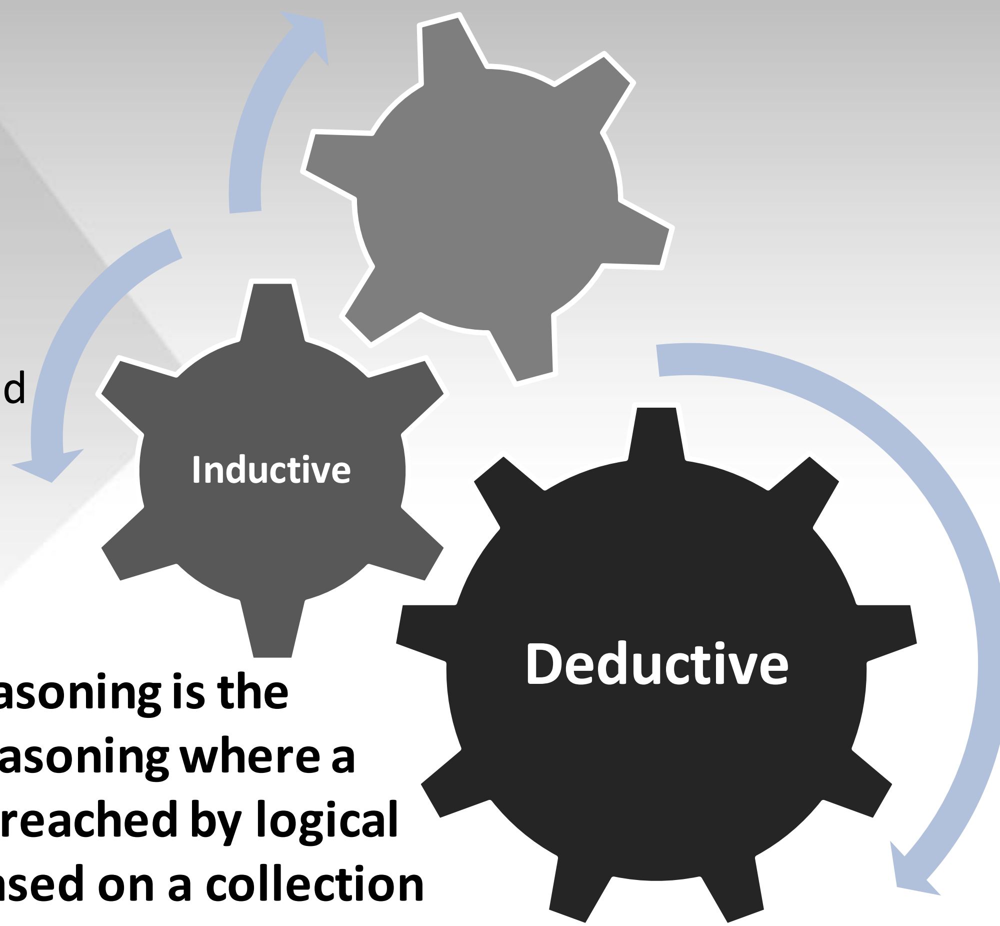
REASONING MODELS



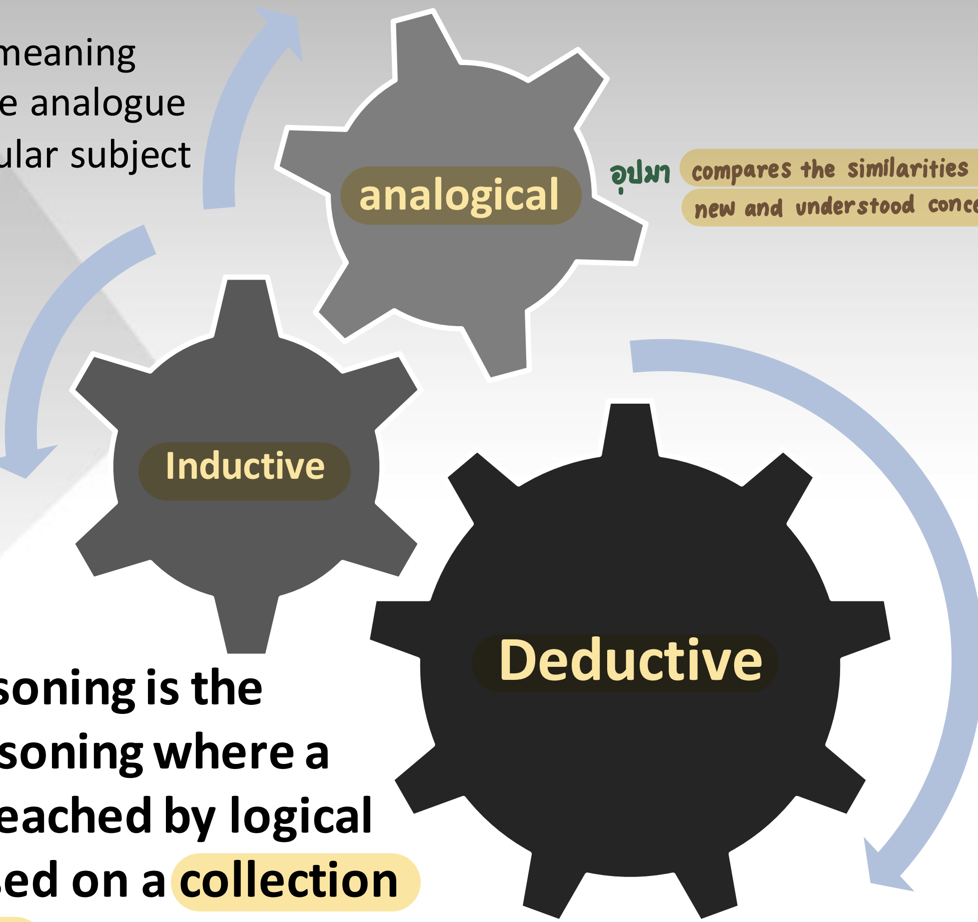
Inductive reasoning is the method of reasoning based on making inferences and conclusions from observations.

Deductive reasoning is the method of reasoning where a conclusion is reached by logical arguments based on a collection of assumptions.

REASONING MODELS

is a cognitive process of transferring information or meaning from a particular subject (the analogue or source) to another particular subject (the target).

อุปมา compares the similarities between new and understood concepts



อุปนัย

Inductive reasoning is the method of reasoning based on making inferences and conclusions from observations.

อนุมาน

Deductive reasoning is the method of reasoning where a conclusion is reached by logical arguments based on a collection of assumptions.

Propositional Logic

บอกเล่า

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

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Negation (not)	$\neg$
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Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	$\neg p$
T	F
F	T

The negation of *p* has opposite truth value to *p*.

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The conjunction of *p* and *q*, is true when, and only when, both *p* and *q* are true.

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The disjunction of *p* and *q*, is true when at least one of *p* or *q* is true.

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Exclusive or = OR but NOT both $p \oplus q = (p \vee q) \wedge \neg(p \wedge q)$

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \leftrightarrow q$ is true when p and q have the same truth value. Intuitively, $p \leftrightarrow q$ is $(p \rightarrow q) \wedge (q \rightarrow p)$

Propositional Logic

Definition

A proposition is a declarative statement that is either true or false but not both.

OPERATORS

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Exclusive or (Xor)	$\oplus$
Implication	$\rightarrow$
Bicondition	$\leftrightarrow$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

It is false when *p* is true and *q* is false, and true otherwise.

General Compound Proposition

$$(p \wedge q) \vee \neg p$$

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \vee \neg p$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$p \wedge \neg q \vee r \rightarrow p \leftrightarrow s$

$\downarrow$

$(((p \wedge (\neg q)) \vee r) \rightarrow p) \leftrightarrow s$

Translating from Natural language

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Example (**Rosen**):

You cannot ride the rollercoaster if you are under 4 feet tall unless you are older than 16 years old.

q: You can ride the rollercoaster
r: You are under 4 feet tall
s: You are older than 16 years old

$$(r \wedge \neg s) \rightarrow \neg q$$

q: You can ride the rollercoaster
$\neg r$: You are at least 4 feet tall
s: You are older than 16 years old

$$\neg(\neg r \vee s) \rightarrow \neg q$$

Translating from Natural language

Example :

A statement S is a proposition if the truth value of S is either true or false but not both.

q: A statement S is a proposition
r: The truth value of S is true
s: The truth value of S is false

$$((r \vee s) \wedge (\neg(r \wedge s)) \rightarrow q$$

$$((r \wedge \neg s) \vee (\neg r \wedge s)) \rightarrow q$$

Conditional Statements

Contrapositive

The ***contrapositive*** of an implication $p \rightarrow q$ is:

$$\neg q \rightarrow \neg p$$

which has the same truth values as $p \rightarrow q$.

Conditional Statements

Only if

The only-if statement "*q only if p*" means

$$\neg p \rightarrow \neg q$$

which has the same truth values as $q \rightarrow p$.

Conditional Statements

Only if

Only if I studied more would have a chance of passing.

MEANING: Studying more would be the only way for me to pass.

p I studied more.

q I would have a chance of passing.

$$q \text{ only if } p \equiv \neg p \to \neg q$$
$$\equiv q \to p.$$

NECESSARY CONDITION

Conditional Statements

NECESSARY & SUFFICIENT CONDITION

Definition: A necessary condition for some state of affairs q is a condition that must be satisfied in order for q to obtain.

If p is the **necessary condition** for q, this means that $\neg p \rightarrow \neg q$.

Definition: A sufficient condition for some state of affairs q is a condition that, if satisfied, guarantees that q obtains.

If p is the **sufficient condition** for q, this means that $p \rightarrow q$.

Conditional Statements

Converse and Inverse

The *converse* of an implication $p \rightarrow q$ is:

$$q \rightarrow p$$

The *inverse* of an implication $p \rightarrow q$ is:

$$\neg p \rightarrow \neg q$$

DO NOT have the same truth values as $p \rightarrow q$

VALID ARGUMENT

Definition

An argument is a sequence of statements. All statements excluded the final one are called "hypotheses", the final statement is called "conclusion". A argument is the form:

$$p\ ;\ q\ ;\ r\ ;\quad \ldots \quad \therefore f$$ **(read therefore)**

An argument is valid means that if all hypotheses are true, the conclusion is also true.

VALID ARGUMENT

Example

Given an argument $p \vee (q \vee r)$; $\neg r$; $\therefore p \vee q$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$\neg r$	$p \vee q$
T	T	T	T	T	F	T
T	T	F	T	**T**	**T**	T
T	F	T	T	T	F	T
T	F	F	F	**T**	**T**	T
F	T	T	T	T	F	T
F	T	F	T	**T**	**T**	T
F	F	T	T	T	F	F
F	F	F	F	F	T	F

VALID ARGUMENT

Example

Given an argument $p \vee (q \vee r)\ ;\ \neg r\ ;\quad \therefore p \vee q$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$\neg r$	$p \vee q$
T	T	T	T	T	F	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	T	T	T
F	T	T	T	T	F	T
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	F	F	T	F

VALID ARGUMENT

VALID

Example

Given an argument $p \vee (q \vee r)$; $\neg r$; $\therefore p \vee q$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$\neg r$	$p \vee q$
T	T	T	T	T	F	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	T	T	T
F	T	T	T	T	F	T
F	T	F	T	T	T	T
F	F					
F	F					

TRUE

$$(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$$

Consistency

- Translating natural language to logical expressions is essential to specifying system spec.
- Specifications are ***"consistent"*** when they do not conflict with one another. i.e.:

There must be an assignment of truth values to every expression that make all the expression true.

Consistency

EXAMPLE

- Whenever the system is being upgraded, users cannot access the file system.
- If users can access the file system, they can save new files.
- If users cannot save new files, the system is not being upgraded.

Consistency

p	q	r	$p \rightarrow \neg q$	$q \rightarrow r$	$\neg r \rightarrow \neg p$
T	F	T	T	T	T

- Whenever the system is being upgraded, users cannot access the file system.

$p \rightarrow \neg q$

- If users can access the file system, they can save new files.

$q \rightarrow r$

- If users cannot save new files, the system is not being upgraded.

$\neg r \rightarrow \neg p$

Consistency

EXAMPLE

p	q	r	$p \to \neg q$	$q \to r$	$\neg r \to \neg p$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

TAUTOLOGY
CONTRADICTION
CONTINGENCY

- A compound proposition that is always *true* is called a "***tautology***".
- A compound proposition that is always *false* is called a "***contradiction***".
- If neither a tautology nor a contradiction, it is called a "***contingency***".

Logical Equivalences

The propositions p and q are called "**logical equivalent**" ($p \equiv q$) if $p \leftrightarrow q$ is a tautology

Logical Equivalences

Showing Logically Equivalent propositions

1. Show that the truth values of these propositions are always the same.

Construct truth tables.

Logical Equivalences

Showing Logically Equivalent propositions

- Example (Rosen):

  Show that $p \rightarrow q \equiv \neg p \vee q$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Logically Equivalent

Logical Equivalences

Showing Logically Equivalent propositions

1. Show that the truth values of these propositions are always the same.

   Construct truth tables.

2. Use series of established equivalences.

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Commutative laws สลับที่

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Associative laws จัดหมู่

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Distributive laws กระจาย

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r$$

THEOREM

Logical Equivalences, given any propositions p,q and r, a tautology T and a contradiction F, the following logical equivalences hold:

Identity laws เอกลักษณ์

$$p \wedge \mathrm{T} \equiv p$$
$$p \vee \mathrm{F} \equiv p$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Domination laws (Universal bound laws)

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Idempotent laws

$$p \wedge p \equiv p$$
$$p \vee p \equiv p$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Negation laws

$$p \lor \neg p \equiv \mathrm{T}$$
$$p \land \neg p \equiv \mathrm{F}$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Double negation law

$$\neg(\neg p) \equiv p$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

De Morgan's laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

THEOREM

Logical Equivalences, given any propositions *p*,*q* and *r*, a tautology T and a contradiction F, the following logical equivalences hold:

Absorption laws

$(p\wedge T)\vee(p\wedge q) \equiv p\wedge(\overset{T}{\cancel{T\vee q}})$

$$p\vee(p\wedge q) \equiv p$$

$$p\wedge(p\vee q) \equiv p$$

PROPOSITIONAL LOGIC

EXAMPLE

Show that $\neg(p \lor (\neg p \land q))$ and $(\neg p \land \neg q)$ are logically equivalent.

Proof:

PROPOSITIONAL LOGIC

EXAMPLE

Show that $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Proof: $\neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$	By the De Morgan law
$\equiv \neg p \wedge (\neg\neg p \vee \neg q)$	By the De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	By the Double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	By the distributive law
$\equiv F \vee (\neg p \wedge \neg q)$	By the negation law
$\equiv (\neg p \wedge \neg q) \vee F$	By the commutative law
$\equiv (\neg p \wedge \neg q)$	By the identity law

Q.E.D.

PROPOSITIONAL LOGIC

EXAMPLE

Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Proof:

PROPOSITIONAL LOGIC

EXAMPLE

Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Proof: $(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg (p \wedge q) \vee (p \vee q)$	Since $\boxed{(x \rightarrow y) \equiv (\neg x \vee y)}$
$\equiv \neg p \vee \neg q \vee (p \vee q)$	By the De Morgan law
$\equiv (\neg p \vee p) \vee (\neg q \vee q)$	By the associative law By the commutative law
$\equiv T \vee T$	By the negative law
$\equiv T$	By the domination law

Q.E.D.

EXERCISE 1

Let p, q, and r be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p, q, r, and logical connectives,

a) You get an A in this class, but you do not do every exercise in this book.

$$r \wedge \neg q$$

EXERCISE 1

Let *p*, *q*, and *r* be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using *p*, *q*, *r*, and logical connectives,

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

$$p \wedge q \wedge r$$

EXERCISE 1

Let *p*, *q*, and *r* be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using *p*, *q*, *r*, and logical connectives,

c) To get an A in this class, it is necessary for you to get an A on the final.

$$\neg p \rightarrow \neg r$$

EXERCISE 1

Let p, q, and r be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p, q, r, and logical connectives,

d) You get an A on the final, but you don't do every exercise in this book, nevertheless, you get an A in this class.

$$p \wedge \neg q \wedge r$$

EXERCISE 1

Let p, q, and r be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p, q, r, and logical connectives,

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$$(p \land q) \rightarrow r$$

EXERCISE 1

Let *p*, *q*, and *r* be the propositions:

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using *p*, *q*, *r*, and logical connectives,

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

$$r \leftrightarrow (p \vee q)$$

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EXERCISE 2

Construct a truth table for each of these compound propositions.

a) $(p \to q) \land (\neg p \to r)$

P	q	r	$P \to q$	$\neg p$	$\neg p \to r$	$(p \to q) \land (\neg p \to r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

EXERCISE 2

Construct a truth table for each of these compound propositions.

b) $(\neg p \vee q) \wedge (p \to q)$

p	q	$\neg p$	$\neg p \vee q$	$p \to q$	$(\neg p \vee q) \wedge (p \to q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

EXERCISE 3

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- **If I play hockey, then I am sore the next day.**
- **I use the whirlpool if I am sore.**
- **I did not use the whirlpool.**

I am not sore.

I did not play hockey the day before.

EXERCISE 3

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- If I work, it is sunny.
- I worked last Monday or I worked last Friday.
- It was not sunny on Tuesday.
- It was not sunny on Friday.

I did not work on Tuesday.
I did not work on Friday.
I worked last Monday.
It was sunny on Monday.

EXERCISE 3

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- **All insects have six legs.**
- **Dragonflies are insects.**
- **Spiders do not have six legs.**
- **Spiders eat dragonflies.**

Dragonflies have six legs.

Spiders are not insects.

EXERCISE

Is the following assertion a proposition?

This statement is fal

No, since this statement is neither "true" nor "false".

Proof

Suppose that the statement is true, this contradicts with its assertion.

It is then the statement is false. This also contradicts with its assertion. **Q.E.D.**

EXERCISE

The n^{th} statement in a list of 100 statements is

Exactly *n* statements are false.

What conclusion can you draw from these statements?

At most one statement can be true, then 99 statements are false.
That is only the 99th statement is true.

EXERCISE

The n^{th} statement in a list of 100 statements is

At least *n* statements are false.

What conclusion can you draw from these statements?

50 first statements are true.
The others are false.

EXERCISE

The n^{th} statement in a list of 99 statements is

At least *n* statements are false.

Can we conclude anything from these statements?

CONTRADICTION

EXERCISE

Given two logical operators,

$p \mid q$ means $\neg(p \wedge q)$

$p \downarrow q$ means $\neg(p \vee q)$

Find a simple proposition for $(p \downarrow q) \downarrow (p \downarrow q)$.

$(p \vee q)$

EXERCISE 5

Given two logical operators,

$p \mid q$ means $\neg(p \wedge q)$

$p \downarrow q$ means $\neg(p \vee q)$

Find a ... tion for $(p \downarrow q) \downarrow (p \downarrow q)$.

Find a proposition equivalent to $p \rightarrow q$ using only $\downarrow$.

$((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$

Consider the following statements:

All students go to school.

John is a student.

Diana is a student.

……

Of course we can **conclude** that

John goes to school.

Diana goes to school.

……

PREDICATE LOGIC

The statement "All students go to school" has two parts:

students (denoted by variable x)

go to school (the predicate)

This statement can be denoted by P(x), where P denotes the predicate "go to school".

P(x) is said to be the value of the propositional function P at x.

Once a value has been assigned to the variable x, the statement P(x) becomes a proposition and has a truth value.

PREDICATE LOGIC

UNIVERSAL QUANTIFIER

A statement $\forall x\, P(x)$

means P(x) for all values of x in the universal of discourse.

i.e., $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$
when all elements in the universe of discourse can be listed as $(x_1, x_2, \ldots, x_n)$.

Note that if the universal of discourse (domain) is empty, then this statement is true for any propositional function P(x) because there are no elements x in the domain for which P(x) is false.

PREDICATE LOGIC

EXISTENTIAL QUANTIFIER

A statement $\exists x\ P(x)$

means

There exists an element x in the universal of discourse such that P(x).

$P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$
when all elements in the universal of discourse can be listed as $(x_1, x_2, \ldots, x_n)$.

PREDICATE LOGIC

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Example (Rosen):

What is the truth value of $\forall x P(x^2 \geq x)$, when the universe of discourse consists of:

1) all real numbers?
2) all integers?

Since $x^2 \geq x$ only when $x \leq 0$ or $x \geq 1$, $\forall x P(x^2 \geq x)$ is false if the universe consists of all real numbers. However, it is true when the universe consists of only the integers.

PREDICATE LOGIC

Example (Rosen):

What is the truth value of $\exists x P(x)$ where $P(x)$ is the statement $x^2 > 10$, and the universe of discourse consists of the positive integers not exceeding 4?

Since the elements in the universe can be listed as $\{1,2,3,4\}$, $\exists x P(x)$ is the same as $P(1) \vee P(2) \vee P(3) \vee P(4)$. There for $\exists x P(x)$ is true since $P(4)$ is true.

PREDICATE LOGIC

PRECEDENCE OF QUANTIFIERS

The quantifiers have higher precedence than all logical operators from propositional calculus.

For example, $\forall x P(x) \vee Q(x)$ means

$$(\forall x P(x)) \vee Q(x).$$

PREDICATE LOGIC

NEGATING QUANTIFIED EXPRESSIONS

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

Negation of
"Every 2nd year students loves Discrete math." is
"There is a 2nd year student who does not love Discrete math."

Negation of
"Some student in this class get 'A'." is
"None of the students in this class get 'A'."

PREDICATE LOGIC

NEGATING QUANTIFIED EXPRESSIONS

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

What is the negation of the statement
"There is an honest politician" ?

Solution:
From $\neg\exists x P(x) \equiv \forall x \neg P(x)$, then the negation is
"All politicians are not honest."

PREDICATE LOGIC

EXAMPLES FROM LEWIS CARROLL

All lions are fierce.
Some lions do not drink coffee.

$\therefore$ Some fierce creatures do not drink coffee.

P(x) x is lion.
Q(x) x is fierce.
R(x) x drinks coffee.

$$\forall x\,(P(x) \rightarrow Q(x))$$
$$\exists x\,(P(x) \wedge \neg R(x))$$
$$\therefore\ \exists x\,(Q(x) \wedge \neg R(x))$$

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PREDICATE LOGIC

QUANTIFIERS

- Universal quantification $\forall$
- Existential quantification $\exists$
- Unique existential quantification $\exists!$ มีจริงได้ตัวเดียว

Consider a statement $\forall x\ (P(x) \to Q(x))$.

CONTRAPOSITION

Its contrapositive is $\forall x\ (\neg Q(x) \to \neg P(x))$.

INVERSE

Its inverse is $\forall x(\ \neg P(x) \to \neg Q(x))$.

CONVERSE

Its converse is $\forall x\ (Q(x) \to P(x))$.

PREDICATE LOGIC

THE ORDER OF QUANTIFIERS EXAMPLE

$$\forall x\, \exists y\ \ P(x,y)$$

$$\exists y\, \forall x\ \ P(x,y)$$

Given a predicate $P(x,y)$: $x + y = 0$

PREDICATE LOGIC

THE ORDER OF QUANTIFIERS
NESTED QUANTIFIERS

$\forall x\, P(x) \wedge \exists y\, Q(y)$

$\forall x\, \exists y\, (\, P(x) \wedge Q(y)\,)$

$\exists y\, \forall x\, (\, P(x) \wedge Q(y)\,)$

$\forall x\, P(x) \wedge \forall x\, Q(x)$

$\forall x\, \forall\, y\, (\, P(x) \wedge Q(y)\,)$

Prenex normal form (PNF)

PREDICATE LOGIC

THE ORDER OF QUANTIFIERS

PRENEX NORMAL FORM

Find a prenex normal form for

$\forall x(\exists y R(x, y) \land \forall y \neg S(x, y) \rightarrow \neg(\exists y R(x, y) \land P)).$

$\forall x(\neg(\exists y R(x, y) \land \forall y \neg S(x, y)) \lor \neg(\exists y R(x, y) \land P))$

$\forall x(\forall y \neg R(x, y) \lor \exists y S(x, y) \lor \forall y \neg R(x, y) \lor \neg P).$

$\forall x(\forall y_1 \neg R(x, y_1) \lor \exists y_2 S(x, y_2) \lor \forall y_3 \neg R(x, y_3) \lor \neg P)$

$\forall x \forall y_1 \exists y_2 \forall y_3 (\neg R(x, y_1) \lor S(x, y_2) \lor \neg R(x, y_3) \lor \neg P).$

PREDICATE LOGIC
EXERCISE

Express the following theorem using the first order predicate logic.

Mathematical induction

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\frac{P}{\therefore\ P \lor Q}$$

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\frac{P \wedge Q}{\therefore\ P}$$

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\begin{array}{l} P \\ Q \\ \hline \therefore P \land Q \end{array}$$

PREDICATE LOGIC
RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- **Modus ponens**
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\begin{array}{l} P \rightarrow Q \\ P \\ \hline \therefore Q \end{array}$$

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- **Modus tollens**
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\begin{array}{l} P \rightarrow Q \\ \neg Q \\ \hline \therefore \neg P \end{array}$$

PREDICATE LOGIC
RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- **Hypothetical syllogism**
- Disjunctive syllogism
- Resolution
- Dilemma

$$P \rightarrow Q$$
$$Q \rightarrow R$$
$$\overline{\therefore\ P \rightarrow R}$$

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$\begin{array}{l} P \lor Q \\ \neg P \\ \hline \therefore Q \end{array}$$

PREDICATE LOGIC
RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- **Resolution**
- Dilemma

$$\begin{array}{l} P \lor Q \\ \neg P \lor R \\ \hline \therefore\ Q \lor R \end{array}$$

PREDICATE LOGIC

RULES OF INFERENCE

- Disjunctive addition
- Conjunctive simplification
- Conjunction addition
- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Resolution
- Dilemma

$$P \lor Q$$
$$P \to R$$
$$Q \to R$$

$$\therefore\ R$$

PREDICATE LOGIC
RULES OF INFERENCE
UNIVERSAL MODUS PONENS

Consider a statement $\forall x\,(P(x) \rightarrow Q(x))$.

For a particular e,

P(e) is true,

therefore Q(e) is true.

Modus ponens

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad} \\ \therefore q \end{array}$$

PREDICATE LOGIC

RULES OF INFERENCE

UNIVERSAL MODUS TOLLENS

Consider a statement $\forall x\,(P(x) \rightarrow Q(x))$.

For a particular e,

$\neg Q(e)$ is true,

therefore $\neg P(e)$ is true.

Modus tollens

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

PREDICATE LOGIC

RULES OF INFERENCE

Universal instantiation

$\forall x P(x) \quad \therefore P(c)$ if $c \in U$.

Universal generalization

$P(c)$ for an arbitrary $c \in U \;\therefore\; \forall x P(x)$

Existential instantiation

$\exists x P(x) \;\therefore\; P(c)$ for some element $c \in U$

Existential generalization

$P(c)$ for some element $c \in U \;\therefore\; \exists x P(x)$

IMPORTANT

PROPOSITION is a declarative statement that is either true or false but not both.

OPERATORS

- ✓ Negation (not)
- ✓ Conjunction (and)
- ✓ Disjunction (or)
- ✓ Exclusive or (Xor)
- ✓ Implication (if…then)
- ✓ Bicondition
- ✓ logically equivalence

THEOREMS

- The identity laws
- The domination laws
- The Idempotent laws
- The double negation laws
- The commutative laws
- The associative laws
- The De Morgan's laws
- The negation laws

RULES OF INFERENCE

- Modus ponens
- Modus tollens
- Hypothetical syllogism
- Disjunctive syllogism
- Addition
- Simplification
- Conjunction
- Resolution

KEYWORDS

Valid arguments, Contrapositive, Only if-statement, Necessary condition, Sufficient condition, Converse, Inverse, Consistency, Tautollogy, Contradiction, Contigency, Logically equivalence, Tautology, Contradiction,

You can find many more detail and examples on many websites.

Example (Rosen Ex. 6, P.67)

Propositional Logic

จงแสดงว่าสมมติฐานต่อไปนี้

- บ่ายวันนี้อากาศไม่แจ่มใสและหนาวกว่าเมื่อวาน (p, q) $\neg p \wedge q$
- เราจะไปว่ายน้ำเมื่ออากาศแจ่มใสเท่านั้น (r, only if) $\neg p \rightarrow \neg r$
- ถ้าเราไม่ไปว่ายน้ำเราจะไปพายเรือ (s) $\neg r \rightarrow s$
- ถ้าเราไปพายเรือแล้วเราจะกลับถึงบ้านก่อนพระอาทิตย์ตกดิน (t) $s \rightarrow t$

สรุปได้ว่า

- เราจะถึงบ้านก่อนพระอาทิตย์ตกดิน $\therefore t$

1 $\neg p \wedge q$
2 $\neg p \rightarrow \neg r$
3 $\neg r \rightarrow s$
4 $s \rightarrow t$

$\therefore t$

	ขั้นตอน	เหตุผล
1)	$\neg p \wedge q$	สิ่งที่กำหนดให้ 1
2)	$\neg p$	Conjuctive simplification (1)
3)	$\neg p \rightarrow \neg r$	สิ่งที่กำหนดให้ 2
4)	$\neg r$	Modus ponens (2) และ (3)
5)	$\neg r \rightarrow s$	สิ่งที่กำหนดให้ 3
6)	s	Modus ponens (4) และ (5)
7)	$s \rightarrow t$	สิ่งที่กำหนดให้ 4
8)	t	Modus ponens (6) และ (7) Q.E.D.

THE END