Qualitative Reasoning Model for Simulating a Container System

Janne Spijkervet
Vrije Universiteit Amsterdam
265525
janne.spijkervet@student.uva.nl

Tim Smit
Vrije Universiteit Amsterdam
2564567
tim.smit2@student.uva.nl

April 12, 2019

1 Introduction

Change is a characteristic found in every aspect of the physical world. Qualitative Reasoning defines a framework for describing changes, employing physical aspects, such as time and space, for solving problems using qualitative information over quantitative information. Concepts of a physical model are usually qualitative, but lay embedded within a vastly more complex framework established by continuous, real-valued variables. Humans are inclined to use qualitative, causal calculus when reasoning about changes and behavior of the physical world [1]. Consequently, the goal of this field of research is to derive these complex concepts from a simpler, yet still formal qualitative foundation. A qualitative model replaces continuous, real-valued variables with qualitative values to describe the physical world. These values only retain essential information to describe the states of a system, i.e. $[+, -, 0]$. To model change and behavior, confluence is used, i.e. a qualitative differential equation. Consequently, there are two main concepts in qualitative physics: state and confluence.

In this research, we analyze a container system using these concepts. The model of a bathtub is used as the contextual artifact for the container system.

Both the states and confluence between the states are modeled using our algorithm designed to qualitatively reason about a system in the physical world. With a given set of Quantities, Quantity Spaces and Dependencies, an exhaustive search algorithm is used to find all valid states and confluences such a container system.

2 Method

2.1 System Description

The causal model for a container system, visualized in Figure 1, has the following Quantities: Inflow (of water into the container), Outflow (of water out of the container) and Volume (of the water in the container). We employ the following Quantity Spaces for Inflow: $[0, +]$, and Outflow and Volume: $[0, +, \max]$. There are five dependencies between the three Quantities: the Inflow has a positive Influence on the Volume, the Outflow has a negative Influence on the Volume, the Volume is positively Proportional with respect to Outflow. The Outflow is at its highest value when the Volume is at its highest value and there is no Outflow when there is no Volume. This translates to:

I^+(Inflow, Volume)
I^-(Outflow, Volume)
P^+(Volume, Outflow)
VC(Volume(max), Outflow(max))
VC(Volume(0), Outflow(0))

Table 1: System Description

2.2 Assumptions

Our solution to the container system problem is postulated using several assumptions about the plausible states and confluences in the physical world. We distinguish two types of assumptions: (1) intra-state assumptions, i.e. assumptions about the plausible states within a state, and (2) inter-state, i.e. assumptions about the plausible transitions between two states. We make the following intra-state assumptions and illustrate its impact on the solution of the problem:

1. We allow, at any time, for an exogenous force to open or close the Inflow and Outflow of the model. This results in a vastly greater amount of transitions.
2. We do not model an overflow of Inflow, Volume and Outflow, i.e. when the Magnitude reaches a maximum value, it cannot have a positive derivative.

3. We do not model an underflow of Inflow, Volume and Outflow, i.e. when the Magnitude reaches zero, it cannot have a negative derivative.

4. The Inflow magnitude cannot be zero while the Volume magnitude has a maximum value and the Volume derivative is zero.

5. The Inflow cannot be zero while the Volume magnitude is maximum.

6. The Inflow magnitude cannot be positive while the Volume magnitude is zero and the Volume derivative value is not positive.

We make the following inter-state assumptions and illustrate its impact on the solution of the problem:

1. The magnitude value of state 1 is not equal to state 2 if the derivative of state 1 is zero, i.e. we assume a derivative has a direct impact on the magnitude.

2. The magnitude of state 2 is smaller or equal than the magnitude of state 1 if the derivative of state 1 is not positive.

3. The derivative of state 1 cannot be maximum if the magnitude value of state 1 is zero and the magnitude value of state 2 is not positive.

4. The derivative of state 2 cannot be negative if the derivative of state 1 is maximum.

5. The magnitude of state 2 cannot be greater or equal to state 1 if the derivative of state 1 is negative.

6. The magnitude value of state 2 cannot be positive if the derivative of state 1 is negative and the magnitude value is positive.

7. The Volume derivative of state 2 cannot be maximum if the Inflow derivative of state 1 is negative.

8. The Volume derivative of state 2 cannot be maximum if the Inflow magnitude of state 1 is maximum.

9. The Volume derivative of state 1 cannot be zero and Volume derivative of state 2 cannot be maximum if the Inflow magnitude of state 1 is maximum and the Inflow derivative of state 1 is negative.

2.3 Representation

The system's model is constructed using a class containing two connected state Quantities and their Dependency. This model is then used in the Pruning algorithm to check whether a state is valid and if a transition between the two states are valid. Each state is constructed using a class containing the state's Quantities, i.e. Inflow, Volume and Outflow, and a list containing the classes of states it can transition to.

2.4 Generation

All states are generated using the Cartesian Product of the three possible Quantities, Inflow (I), Volume (V) and Outflow (O), generating all possible values each Quantity can hold. After this process, the invalid states are pruned using the Pruning algorithm. The resulting set is then used to generate all possible transitions.

$$I \times V \times O = \{(i, v, o) \mid i \in I \text{ and } v \in V \text{ and } o \in O\}$$

The same process is used to generate all possible transitions between two states, with set A and B both containing all states. After this process, the invalid transitions are pruned from the set using the Pruning algorithm.

2.5 Pruning

All invalid states and transitions are pruned from the total set of generated states and transitions using our assumptions. This is done by both a calculus derived from the assumptions in Qualitative Reasoning and manually encoding the rules of the Quantities and Dependencies the intra- and inter-states hold.

2.6 Trace

An insightful trace is printed with every iteration in the Pruning algorithm. A trace is provided for both the pruning of invalid intra-states as well as invalid transitions. It provides information of all valid states in the model, as well as all valid transitions between these states.

2.7 Graph

All valid states and transitions are visualized in a graph as shown in Figure 2. Besides an image, a browser-based visualization is also provided in an HTML file to allow for the user to explore the graph in more detail.

3 Experiment Results

Our state generation algorithm generates 486 possible states. Of these states, only 24 are valid when the pruning algorithm is run using the intra-state assumptions. With the 24 valid states, there are 576 possible transitions ($24 * 23$). Of these transitions, only 102 are valid when the pruning algorithm is run using the inter-state assumptions. The resulting, valid container system is plotted in a state-graph shown in Figure 2.

4 Conclusion

In this paper, we modeled a container system using concepts derived from the field of Qualitative Reasoning. The concepts of a physical model are often qualitative, but lie in a more complex framework established by continuous, real-valued variables. The goal of this paper was to derive these more complex concepts from a simpler, yet still formal qualitative formulation. We replaced quantitative information with qualitative values, solely retaining the essential information needed to describe the states of the system.

References

[1] Daniel G. Bobrow. Qualitative reasoning about physical systems: An introduction. In Daniel G. Bobrow, editor, *Qualitative Reasoning About Physical Systems*, pages 1 – 5. Elsevier, Amsterdam, 1984.

Appendix

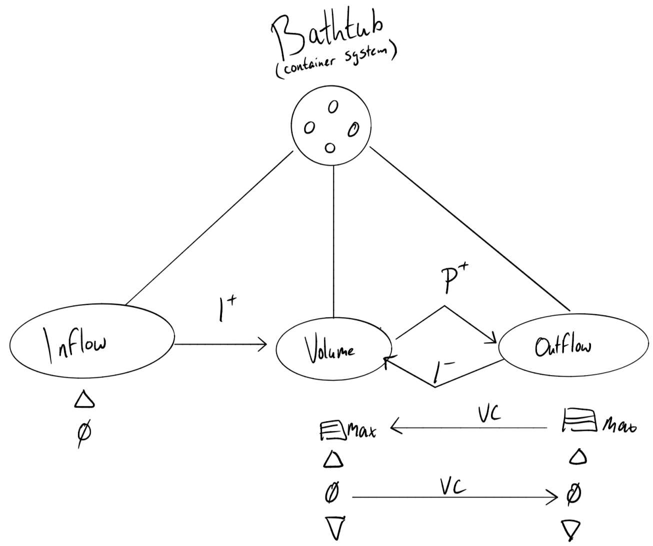

Figure 1: A container system modeled using qualitative reasoning

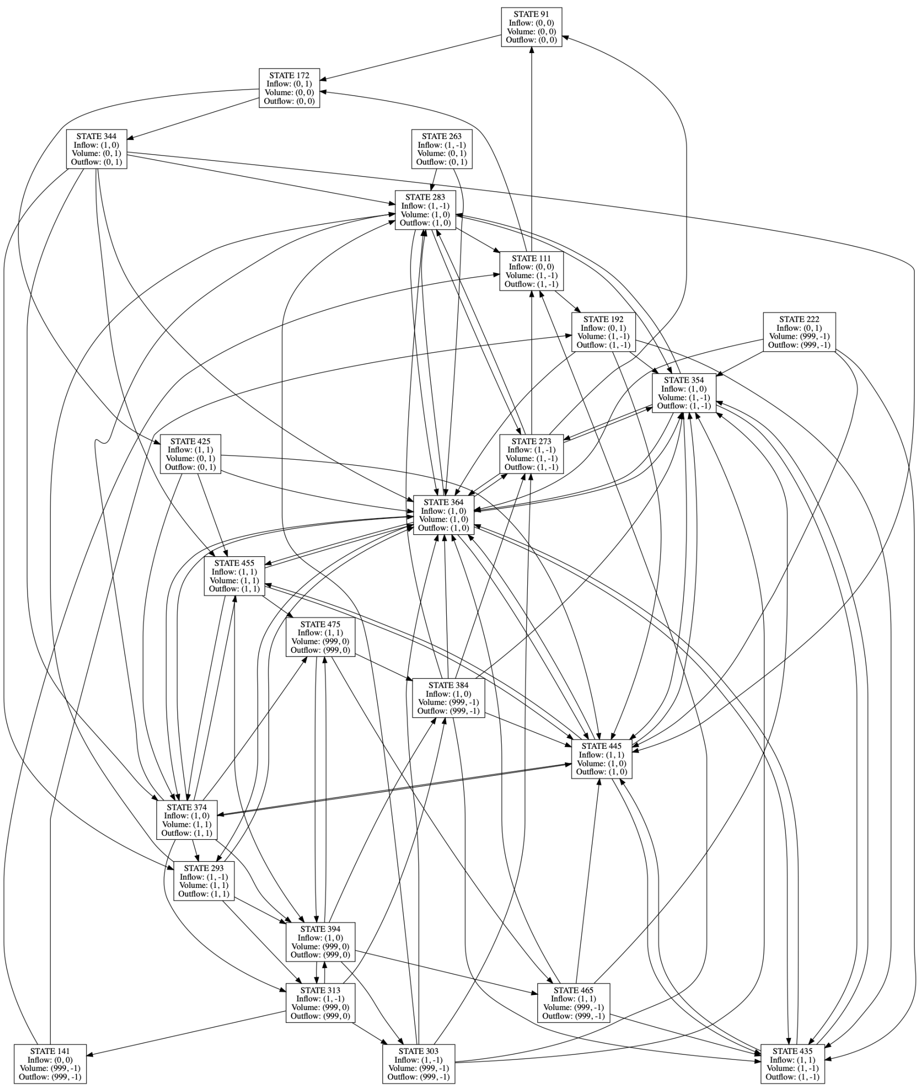

Figure 2: A container system modeled using qualitative reasoning