

① Qubit se nalazi u stanju

$$|\Phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

2020.

Želimo li taj qubit dovesti u stanje

$$|\Phi'\rangle = \beta|0\rangle + \alpha|1\rangle,$$

u prikazu stanja na Blochovoj sferi moramo provesti...

- a) rotaciju za π oko x-osi,
- b) rotaciju za π oko y-osi,
- c) rotaciju za π oko z-osi,
- d) rotaciju za $\pi/2$ oko z-osi,
- e) nešto od navedenog.

1. način

traženje generatora ("matricno")

$$|\Phi'\rangle = M|\Phi\rangle$$

$$\left(\begin{array}{l} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X \text{ } \pi \text{ oko x-osi} \\ \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y \text{ } \pi \text{ oko y-osi} \\ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z \text{ } \pi \text{ oko z-osi} \end{array} \right)$$

$$\begin{bmatrix} \beta \\ \alpha \end{bmatrix} = M \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \rightarrow M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \left(* \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \cdot \alpha + 1 \cdot \beta \\ 1 \cdot \alpha + 0 \cdot \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \right)$$

↳ generator NOT(X) → odgovara rotaciju za π oko x-osi

2. način

Blochova sfera

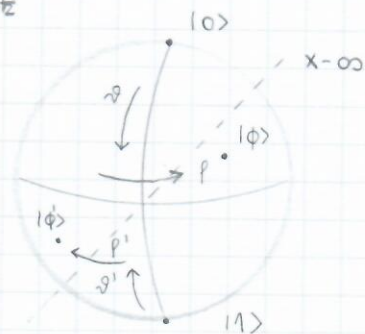
$$|\Phi\rangle = \cos \frac{\theta}{2} e^{i\phi} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha = \cos \frac{\theta}{2} e^{i\phi}, \beta = \sin \frac{\theta}{2} e^{i\phi}$$

$$|\Phi'\rangle = \cos \frac{\theta'}{2} e^{i\phi'} |0\rangle + \sin \frac{\theta'}{2} e^{i\phi'} |1\rangle = \beta|0\rangle + \alpha|1\rangle \rightarrow \alpha = \sin \frac{\theta'}{2} e^{i\phi'}, \beta = \cos \frac{\theta'}{2} e^{i\phi'}$$

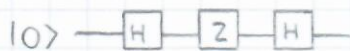
$$\alpha \rightarrow \cos \frac{\theta}{2} = \sin \frac{\theta'}{2} \\ \cos \frac{\theta}{2} = \cos(\frac{\theta'}{2} - \frac{\theta'}{2}) / \arccos 1/2 \\ \theta = \pi - \theta'$$

$$\alpha \rightarrow e^{i\phi} = e^{-i\phi'} \\ \phi = -\phi'$$

što imadi za β !



② Na osnovu iz logičkog kruga



stane qubita istovremeno je stanje ...

2020.

- a) $|0\rangle$,
 b) $|1\rangle$,
 c) $|+\rangle$,
 d) $|-\rangle$,
 e) miša od navedenog.

$H \rightarrow$ Hadamardov generator $\left(* H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right)$
 $Z \rightarrow$ Z generator

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + (-1) \cdot 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

$$ZH|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \cdot \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 \\ 0 \cdot 1 + (-1) \cdot 0 \end{bmatrix} + \begin{bmatrix} 1 \cdot 0 + 0 \cdot 1 \\ 0 \cdot 0 + (-1) \cdot 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

$$HZH|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \frac{1}{2} \cdot \left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + (-1) \cdot 0 \end{bmatrix} - \begin{bmatrix} 1 \cdot 0 + 1 \cdot 1 \\ 1 \cdot 0 + (-1) \cdot 1 \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \right) = \frac{1}{2} (|0\rangle + |1\rangle - (|0\rangle - |1\rangle))$$

$$= \frac{1}{2} \cdot 2 \cdot |1\rangle = |1\rangle$$

$$(* |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle))$$

$$(* H|0\rangle = |+\rangle, H|1\rangle = |-\rangle, H|+\rangle = |0\rangle, H|-\rangle = |1\rangle, Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle)$$

② Razmatramo kvantni logički krug

$$|0\rangle \rightarrow \boxed{H} - \boxed{R(\phi)} - \boxed{H} - \boxed{\text{X}}$$

Kolika je vjerojatnost da u mjerenju dobijemo vrijednost $|1\rangle$?

- a) 0
 b) $\frac{1}{2}(1 + \cos\phi)$
 c) $\frac{1}{2}(1 - \cos\phi)$
 d) $\cos\phi$
 e) $\cos\phi^2$

2019.

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + (-1) \cdot 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |\psi\rangle$$

$$R(\phi)H|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 \\ 0 \cdot 1 + e^{i\phi} \cdot 0 \end{bmatrix} + \begin{bmatrix} 1 \cdot 0 + 0 \cdot 0 \\ 0 \cdot 0 + e^{i\phi} \cdot 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{i\phi} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + e^{i\phi} |1\rangle)$$

$$HR(\phi)H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ e^{i\phi} \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ e^{i\phi} \end{bmatrix} \right)$$

$$\checkmark = \frac{1}{2} \left(\begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + (-1) \cdot 0 \end{bmatrix} + \begin{bmatrix} 1 \cdot 0 + 1 \cdot e^{i\phi} \\ 1 \cdot 0 + (-1) \cdot e^{i\phi} \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{i\phi} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$|\psi\rangle = \frac{1}{2} \left(\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) + e^{i\phi} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \right) = \frac{1}{2} ((|0\rangle + |1\rangle) + e^{i\phi} (|0\rangle - |1\rangle))$$

$$= \frac{1}{2} ((1 + e^{i\phi})|0\rangle + (1 - e^{i\phi})|1\rangle) = \frac{1}{2} ((1 + \cos\phi + i\sin\phi)|0\rangle + (1 - \cos\phi - i\sin\phi)|1\rangle)$$

$$P_{\psi \rightarrow 1} = |\langle \psi | 1 \rangle|^2$$

$$\langle \psi | = \text{konj. kompleksni } \psi = \frac{1}{2} ((1 + \cos\phi - i\sin\phi)|0\rangle + (1 - \cos\phi + i\sin\phi)|1\rangle)$$

$$P_{\psi \rightarrow 1} = \left| \left(\frac{1}{2} ((1 + \cos\phi - i\sin\phi)\langle 0| + (1 - \cos\phi + i\sin\phi)\langle 1|) \right) \cdot (0 \cdot |0\rangle + 1 \cdot |1\rangle) \right|^2$$

= staje umnožak vrijednosti uz $\langle 0| \cdot |0\rangle$ te uz $\langle 1| \cdot |1\rangle$

$$= \left| \frac{1}{2} (1 - \cos\phi + i\sin\phi) \right|^2 = \frac{1}{4} (1 - 2\cos\phi + \cos^2\phi + \sin^2\phi) = \frac{1}{4} (2 - 2\cos\phi)$$

$$= \frac{1}{2} (1 - \cos\phi)$$

② Razmatramo kvantni logički krug



gdje je generator Φ definiran s $|0\rangle \rightarrow |0\rangle$ i $|1\rangle \rightarrow e^{i\Phi}|1\rangle$ pri čemu je faza Φ realan broj. Kolika je vjerojatnost da u mjerenju dobijemo vrijednost 0, tj. da je qubit u stanju $|0\rangle$?

- a) 0
- b) $\frac{1}{2}(1 - \cos \Phi)$
- c) $\frac{1}{2}(1 + \cos \Phi)$
- d) $\cos \Phi$
- e) $\cos^2 \Phi$

2018.

$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$(*H|0\rangle = |+\rangle, H|1\rangle = |-\rangle, H|+\rangle = |0\rangle, H|-\rangle = |1\rangle)$$

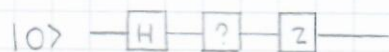
$$\Phi H|0\rangle = \Phi \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \right) = \frac{1}{\sqrt{2}}(\Phi|0\rangle + \Phi|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\Phi}|1\rangle)$$

$$\begin{aligned} H\Phi H|0\rangle &= H \left(\frac{1}{\sqrt{2}}(|0\rangle + e^{i\Phi}|1\rangle) \right) = \frac{1}{\sqrt{2}}(H|0\rangle + e^{i\Phi}H|1\rangle) = \frac{1}{\sqrt{2}}(|+\rangle + e^{i\Phi}|-\rangle) \\ &= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + e^{i\Phi} \cdot \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right) = \frac{1}{2}((1 + e^{i\Phi})|0\rangle + (1 - e^{i\Phi})|1\rangle) \\ &= \frac{1}{2}((1 + \cos \Phi + i\sin \Phi)|0\rangle + (1 - \cos \Phi - i\sin \Phi)|1\rangle) \end{aligned}$$

$$= |\psi\rangle \rightarrow \langle \psi| = \frac{1}{2}((1 + \cos \Phi - i\sin \Phi)\langle 0| + (1 - \cos \Phi + i\sin \Phi)\langle 1|)$$

$$\begin{aligned} P_{\psi \rightarrow 0} &= |\langle \psi|0\rangle|^2 = \left| \frac{1}{2}((1 + \cos \Phi - i\sin \Phi)\langle 0| + (1 - \cos \Phi + i\sin \Phi)\langle 1|) \cdot (|0\rangle) \right|^2 \\ &= \left| \frac{1}{2}(1 + \cos \Phi - i\sin \Phi) \right|^2 = \frac{1}{4}(1 + 2\cos \Phi + \cos^2 \Phi + \sin^2 \Phi) = \frac{1}{4}(2 + 2\cos \Phi) \\ &= \frac{1}{2}(1 + \cos \Phi) \end{aligned}$$

③ Ako na ulazu kvantnog logičkog kruga



dobuamo stanje

$$\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle),$$

generator označen upitnikom je?

a) X

b) Y

c) Z

d) S

e) T

$$(* X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix})$$

2019.

Označimo s $|\psi\rangle$ stanje prije ulaska u neoznati generator. Stanje $|\psi\rangle$ dobivamo djelovanjem generatora H na početno stanje $|0\rangle$.

$$\begin{aligned} |\psi\rangle &= H|0\rangle & (* H|0\rangle &= |+\rangle) \\ &= |+\rangle \\ &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \end{aligned}$$

Označimo s $|\psi'\rangle$ stanje prije ulaska u generator Z. Stanje nakon djelovanja generatora Z jednako je rezultatnom stanju.

$$\begin{aligned} Z|\psi'\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \\ Z(\alpha\psi'|0\rangle + \beta\psi'|1\rangle) &= \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \end{aligned}$$

Za generator Z vrijedi $Z|0\rangle = |0\rangle$ i $Z|1\rangle = -|1\rangle$ pomoću čega dobivamo...

$$\begin{aligned} \alpha\psi' &= \frac{1}{\sqrt{2}} \rightarrow |\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \\ \beta\psi' &= i\frac{1}{\sqrt{2}} \end{aligned}$$

Neoznati generator možemo dobiti pomoću stanja $|\psi\rangle$ i $|\psi'\rangle$.

$$\begin{aligned} ?|\psi\rangle &= |\psi'\rangle \\ ?(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)) &= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \\ \frac{1}{\sqrt{2}}(?|0\rangle + ?|1\rangle) &= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \end{aligned}$$

Neoznati generator trebalo bi stanje $|0\rangle$ ostaviti nepromijenjeno, a stanje $|1\rangle$ pretvoriti u $i|1\rangle$. To radi generator S.

$$(* |L\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle), |R\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle))$$

③ Měření stavu qubitu má vstupu z logického kruhu



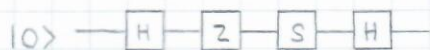
výhodnost 0 dostaneme u 50% případů. Z toho můžeme zkusit
da qubit označen vnitřním křídlem je qubit...

- a) X,
- b) Y,
- c) Z,
- ☒ d) H,
- e) S.

2018.

Qubit H za vstupu stavu $10 \rangle$ vrací výstupní stav $1+ \rangle$, což je
výhodnost u vstupu stavu měřícího qubitu. Za qubit H
také platí $H|1+ \rangle = |10 \rangle$ což značí da li měření stavu qubitu
vždy dostane výhodnost 0. Z toho, qubit H je
měřící qubit.

(4) Na ulazu iz logičkog kruga



stanje qubitа istojetno je stanju...

a) 10 ,

b) 11 ,

c) $1+$,

d) $1-$,

e) miša od navedenog.

2018.

$$H10 = 1+ = \frac{1}{\sqrt{2}}(10 + 11)$$

$$ZH10 = Z\left(\frac{1}{\sqrt{2}}(10 + 11)\right) = \frac{1}{\sqrt{2}}(Z10 + Z11) = \frac{1}{\sqrt{2}}(10 - 11) = 1-$$

$$SZH10 = S\left(\frac{1}{\sqrt{2}}(10 - 11)\right) = \frac{1}{\sqrt{2}}(S10 - S11) = \frac{1}{\sqrt{2}}(10 - i11)$$

$$HSZH10 = H\left(\frac{1}{\sqrt{2}}(10 - i11)\right) = \frac{1}{\sqrt{2}}(H10 - iH11)$$

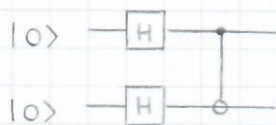
$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(10 + 11) - i\frac{1}{\sqrt{2}}(10 - 11)\right)$$

$$= \frac{1}{2}((1-i)10 + (1+i)11)$$

$$(* H10 = 1+, H11 = 1-, Z10 = 10, Z11 = -11, S10 = 10, S11 = i11)$$

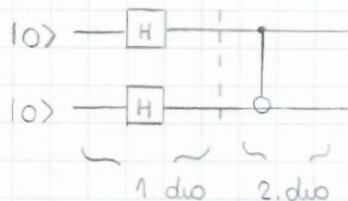
$$(* 1+ = \frac{1}{\sqrt{2}}(10 + 11), 1- = \frac{1}{\sqrt{2}}(10 - 11))$$

③ Na desnoj (izlasknoj) strani kvantnog logičkog kruga.



dobivamo stanje...

- a) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$,
 b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,
 c) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$,
 d) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,
 e) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.



$$\begin{aligned} 1. \text{ dio} &\rightarrow H|0\rangle \otimes H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ &= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\ &= |\psi_1\rangle \end{aligned}$$

$$\begin{aligned} 2. \text{ dio} &\rightarrow \text{CNOT}|\psi_1\rangle = \text{CNOT}\left(\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)\right) \\ &= \frac{1}{2}(\text{CNOT}|00\rangle + \text{CNOT}|01\rangle + \text{CNOT}|10\rangle + \text{CNOT}|11\rangle) \\ &= \frac{1}{2}(|00\rangle + |01\rangle + |11\rangle + |10\rangle) \\ &= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \end{aligned}$$

$$(* (\lambda_1|0\rangle + \mu_1|1\rangle) \otimes (\lambda_2|0\rangle + \mu_2|1\rangle) = \lambda_1\lambda_2|00\rangle + \lambda_1\mu_2|01\rangle + \mu_1\lambda_2|10\rangle + \mu_1\mu_2|11\rangle)$$

$$\left(\begin{aligned} * |00\rangle &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned} \right)$$

$$(* \text{CNOT}|00\rangle = |00\rangle, \text{CNOT}|01\rangle = |01\rangle, \text{CNOT}|10\rangle = |11\rangle, \text{CNOT}|11\rangle = |10\rangle)$$

④ State sustava na ulaznoj strani kvantnog logičkog kruga



2...

2019.

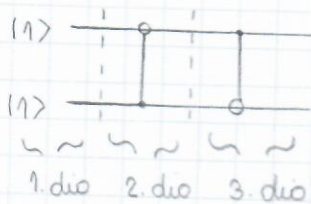
a) $|01\rangle$,

b) $|10\rangle$,

c) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,

d) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$,

e) $|11\rangle$.

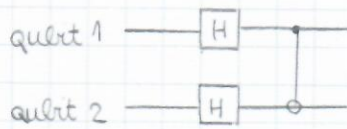


$$1. \text{ dio} \rightarrow |1\rangle \otimes |1\rangle = |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$2. \text{ dio} \rightarrow \text{preokrenuti CNOT} |11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$$

$$3. \text{ dio} \rightarrow \text{CNOT} |01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$$

(5) Shatimo li logički krug



2019.

kao jedan generator, njegov matricni prikaz je...

a) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$,

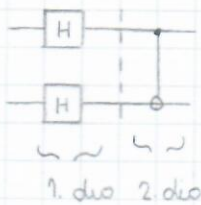
b) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$,

c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$,

d) $\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$,

e) $\frac{1}{2} \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$

Promatramo sledeći dio kruga kao jedan generator...

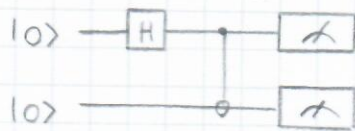


1. dio $\rightarrow H \otimes H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

2. dio $\rightarrow \text{CNOT}(H \otimes H) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$

$\rightarrow \downarrow$

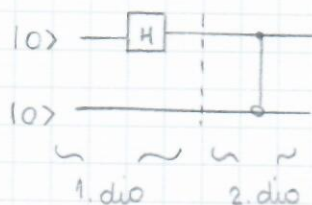
(6) Kolika je vjerojatnost da ma ulazu iz kvantnog logičkog kruga



sustav isprimo u stanje $|01\rangle$?

- a) 0
- b) $\frac{1}{4}$
- c) $\frac{1}{2}$
- d) $\frac{1}{\sqrt{2}}$
- e) 1

2019.



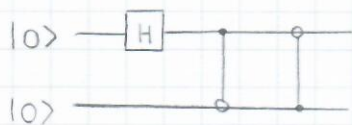
$$1. \text{ dio} \rightarrow H|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\begin{aligned} 2. \text{ dio} \rightarrow \text{CNOT}(H|0\rangle \otimes |0\rangle) &= \text{CNOT}\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right) \\ &= \frac{1}{\sqrt{2}}(\text{CNOT}|00\rangle + \text{CNOT}|10\rangle) \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{aligned}$$

↳ vjerojatnost za $|01\rangle$ je 0!

$$(* \text{ CNOT}|00\rangle = |00\rangle, \text{CNOT}|01\rangle = |01\rangle, \text{CNOT}|10\rangle = |11\rangle, \text{CNOT}|11\rangle = |10\rangle)$$

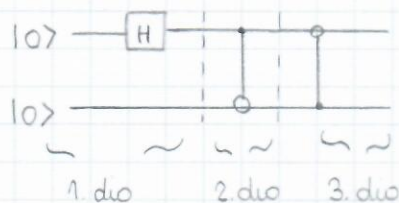
⑤ State systava na vložnoj (desnoj) strani kvantnog logičkog kruga



2018.

&...

- a) $\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$,
- b) $\frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$,
- c) $\frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$,
- d) $\frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$,
- e) $\frac{1}{\sqrt{2}} (|00\rangle + |01\rangle)$.



$$1. \text{ dio} \rightarrow H|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)$$

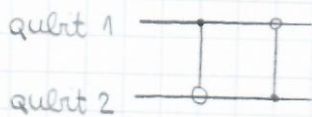
$$2. \text{ dio} \rightarrow \text{CNOT}(H|0\rangle \otimes |0\rangle) = \text{CNOT}\left(\frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)\right) = \frac{1}{\sqrt{2}} (\text{CNOT}|00\rangle + \text{CNOT}|10\rangle) = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$3. \text{ dio} \rightarrow \text{preokrenuti CNOT}(\text{CNOT}(H|0\rangle \otimes |0\rangle)) = p.\text{CNOT}\left(\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)\right) = \frac{1}{\sqrt{2}} (p.\text{CNOT}|00\rangle + p.\text{CNOT}|11\rangle) = \frac{1}{\sqrt{2}} (|00\rangle + |01\rangle)$$

$$(* \text{ CNOT}|00\rangle = |00\rangle, \text{CNOT}|01\rangle = |01\rangle, \text{CNOT}|10\rangle = |11\rangle, \text{CNOT}|11\rangle = |10\rangle)$$

$$(* p.\text{CNOT}|00\rangle = |00\rangle, p.\text{CNOT}|01\rangle = |11\rangle, p.\text{CNOT}|10\rangle = |10\rangle, p.\text{CNOT}|11\rangle = |01\rangle)$$

6) Skratimo li kvantni logički krug



2018.

Kao jedan generator, njege matricni prikaz je ...

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

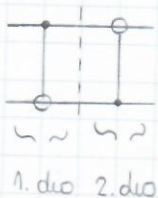
b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$,

d) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

e) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

Promatramo navedeni dio kruga kao jedan generator...

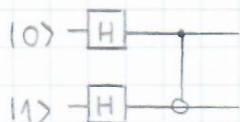


1. dio \rightarrow CNOT = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

2. dio \rightarrow preokrenuti CNOT(CNOT) = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

\rightarrow \downarrow

7) Na demoj (izlaskoj) strani kvantnog logičkog kruga



dobuamo stanje...

2018

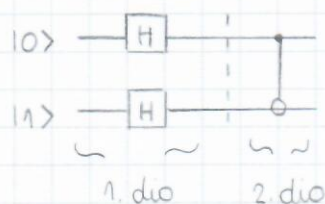
a) $|01\rangle$,

b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,

c) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$,

d) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,

e) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.



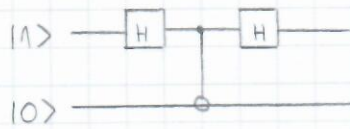
$$\begin{aligned} 1. \text{ dio} \rightarrow H|0\rangle \otimes H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\ &= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \end{aligned}$$

$$\begin{aligned} 2. \text{ dio} \rightarrow \text{CNOT}(H|0\rangle \otimes H|1\rangle) &= \text{CNOT}\left(\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)\right) \\ &= \frac{1}{2}(\text{CNOT}|00\rangle - \text{CNOT}|01\rangle + \text{CNOT}|10\rangle - \text{CNOT}|11\rangle) \\ &= \frac{1}{2}(|00\rangle - |01\rangle + |11\rangle - |10\rangle) \\ &= \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) \end{aligned}$$

$$(* H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$$

$$(* \text{CNOT}|00\rangle = |00\rangle, \text{CNOT}|01\rangle = |01\rangle, \text{CNOT}|10\rangle = |11\rangle, \text{CNOT}|11\rangle = |10\rangle)$$

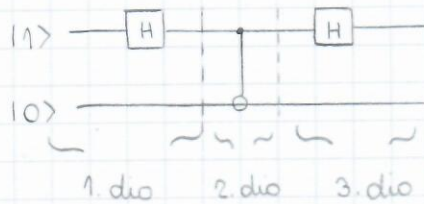
(11) Na desnoj (izlaznoj) strani kvantnog logičkog kruga



2017.

dobivamo stanje...

- a) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,
- b) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$,
- c) $\frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle)$,
- (d) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$,
- e) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle - |11\rangle)$.



$$1. \text{ dio} \rightarrow H|1\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$$

$$\begin{aligned} 2. \text{ dio} \rightarrow \text{CNOT}(H|1\rangle \otimes |0\rangle) &= \text{CNOT}\left(\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)\right) \\ &= \frac{1}{\sqrt{2}}(\text{CNOT}|00\rangle - \text{CNOT}|10\rangle) \\ &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \end{aligned}$$

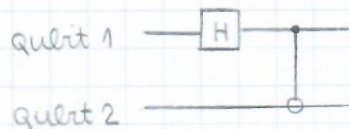
$$3. \text{ dio} \rightarrow (H \otimes I)(\text{CNOT}(H|1\rangle \otimes |0\rangle)) = \left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \cdot \left(\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \right)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$

10) Skratimo li kvantni logički krug



2017.

kao jedan generator, njegov matricni prikaz je...

a) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$,

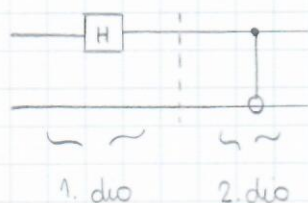
b) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$,

c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$,

d) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$,

e) $\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$.

Promatramo sledeći dio kruga kao jedan generator...

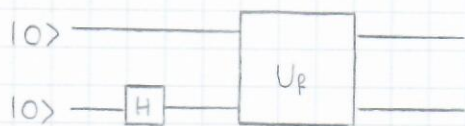


1. dio $\rightarrow H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

2. dio $\rightarrow CNOT(H \otimes I) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$

→ ↓

- 4) U kvantnom logičkom krugu na dvi ulaza U_f predstavljaju implementaciju usmotešene funkcije $f: \{0,1\} \rightarrow \{0,1\}$.



Stanje drugog (donjeg) bita na ulaznoj (desnoj) strani je...

a) $|0\rangle$,

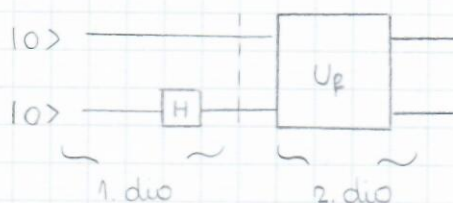
b) $|1\rangle$,

c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$,

d) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$,

e) nije moguće prikazati vektorom stanja.

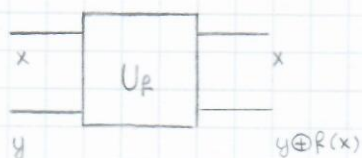
2020



$$1. \text{ dio} \rightarrow |0\rangle \otimes H|0\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

$$2. \text{ dio} \rightarrow U_f(|0\rangle \otimes H|0\rangle) = U_f\left(\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)\right) \\ = \frac{1}{\sqrt{2}}(U_f|00\rangle + U_f|01\rangle)$$

Za U_f vrijedi...



\rightarrow prvi element se ne mijenja.

\rightarrow drugi element se obrađuje $\circ f(x)$ po modulu 2

$$U_f(|0\rangle \otimes H|0\rangle) = \frac{1}{\sqrt{2}}(U_f|00\rangle + U_f|01\rangle) \\ = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0 \oplus f(0)\rangle + |0\rangle \otimes |1 \oplus f(0)\rangle) \\ = |0\rangle \otimes \frac{1}{\sqrt{2}}(|0 \oplus f(0)\rangle + |1 \oplus f(0)\rangle) \\ = |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

\searrow ukupne vrijednosti, jedna je sigurno $|0\rangle$, a druga $|1\rangle$

- ⑤ Ako nota U_f predstavlja implementaciju funkcije f sa svojstva $f(0)=1$ i $f(1)=1$ te ako na ulaznoj (desnoj) strani kvantnog logičkog kruga.



imamo stanje $|00\rangle$, možemo zaključiti da na ulazu u krug imamo stanje...

a) $|00\rangle$,

b) $|10\rangle$,

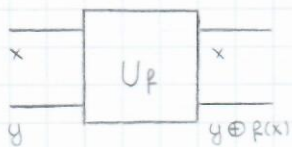
c) $|10\rangle$,

d) $|11\rangle$,

e) takva situacija nije moguća.

2020.

Za implementaciju funkcije f vrijedi...



Na temelju zadatog kruga zaključujemo...

$$\hookrightarrow |x\rangle = |0\rangle$$

$$\hookrightarrow |y \oplus f(x)\rangle = |0\rangle$$

$$|y \oplus 1\rangle = |0\rangle \rightarrow |y\rangle = |1\rangle$$

$$\text{ulazno stanje} \rightarrow |x\rangle \otimes |y\rangle = |0\rangle \otimes |1\rangle = |01\rangle$$

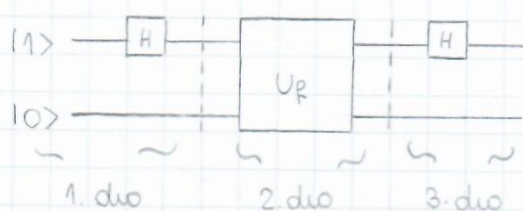
- ⑥ U kvantnom logičkom krugu na slici isata U_f su implementacija funkcije f za koju vrijedi $f(0) = f(1) = 1$.



2020

Stanje sustava na izlaznoj (desnoj) strani kruga je...

- a) $|00\rangle$,
- b) $|01\rangle$,
- c) $|10\rangle$,
- ④ d) $|11\rangle$,
- e) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.



$$1. \text{ dio} \rightarrow H|1\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$$

$$2. \text{ dio} \rightarrow U_f(H|1\rangle \otimes |0\rangle) = U_f\left(\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)\right) = \frac{1}{\sqrt{2}}(U_f|00\rangle - U_f|10\rangle)$$

$$= \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$$

$$\begin{aligned} U_f|00\rangle &= |0\rangle \otimes |0\rangle + f(0)|1\rangle \\ &= |0\rangle \otimes |1\rangle \\ &= |01\rangle \end{aligned}$$

$$3. \text{ dio} \rightarrow (H \otimes I)(U_f(H|1\rangle \otimes |0\rangle)) = (H \otimes I) \cdot \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$$

$$= (H \otimes I) \cdot \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |1\rangle)$$

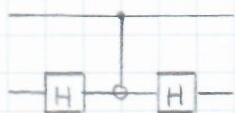
$$= \frac{1}{\sqrt{2}}(H|0\rangle \otimes |1\rangle - H|1\rangle \otimes |1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle - \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle\right)$$

$$= \frac{1}{2}(|01\rangle + |11\rangle - |01\rangle - (-|11\rangle))$$

$$= |11\rangle$$

⑦ Kvantni logički krug prikazan slikom



2020

je implementacija generatora

$$U_{\Phi}|x\rangle = e^{i\Phi}(-1)^{f(x)}|x\rangle, \quad \Phi \in \mathbb{R}, \quad x = 00, 01, 10, 11$$

gdje je $f(x) = 0$ za svaki x osim za $x = w$ za koj vrijedi $f(w) = 1$.
Odredi w .

a) $w = 00$

b) $w = 01$

c) $w = 10$

d) $w = 11$

e) ništa od navedenog

$$U_{\Phi}|x\rangle = \underbrace{e^{i\Phi}}_{\text{mektno (fazi faktor)}} \underbrace{(-1)^{f(x)}}_{\text{Graevov algoritam, tražimo winner-a}}|x\rangle$$

mektno (fazi faktor)

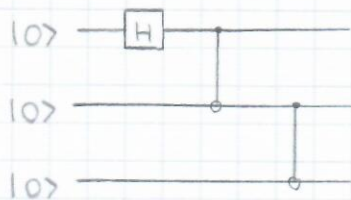
Graevov algoritam, tražimo winner-a

Izračun matricnog prikaza...

$$\begin{aligned} (I \otimes H) \text{CNOT} (I \otimes H) &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \rightarrow w = 11 \end{aligned}$$

Ako rezultatna matrica ima samo jedinice u dijagonali, winner se čita uz pomoć mreke, u kojem je (jedina) negativna vrijednost.

8) Na ulazu iz kvantnog logičkog kruga



2020

stanje sustava je...

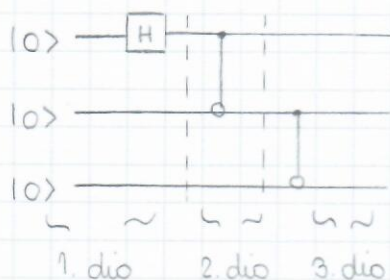
a) $|000\rangle$,

b) $|111\rangle$,

c) $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$,

d) $\frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$,

e) $\frac{1}{\sqrt{2}}(|000\rangle + |110\rangle)$.



$$1. \text{ dio} \rightarrow H|0\rangle \otimes |0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \otimes |0\rangle \\ = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$$

$$2. \text{ dio} \rightarrow \text{CNOT}(1. \text{ na } 2. \text{ bit}) (H|0\rangle \otimes |0\rangle \otimes |0\rangle) = \frac{1}{\sqrt{2}}(|000\rangle + |110\rangle)$$

$$3. \text{ dio} \rightarrow \text{CNOT}(2. \text{ na } 3. \text{ bit}) (\text{CNOT}(H|0\rangle \otimes |0\rangle \otimes |0\rangle)) = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

- 10) Za pohranu matricnog prikaza qubitnog geratora 16-qubitnog kvantnog računala u memoriji (mr. pri simulaciji izvođenja kvantnog algoritma) potrebno je prelišno (uzmite da je svaki skalar matrice kompleksan broj te da za pohranu jednog realnog broja koristimo 8 bajta)...

- a) 0,5 MB,
- b) 1 MB,
- c) 35 GB,
- d) 70 GB,
- e) još (znatno) više.

2020.

$$m = 16$$

$$N = 2^m = 2^{16}$$

matricni prikaz geratora \rightarrow matrica $N \times N \rightarrow$ mat. $2^{16} \times 2^{16}$

\hookrightarrow uzimajući u obzir da je za pohranu jednog kompleksnog broja potrebno 16 bajta, onda je ukupno potrebno...

$$2^{16} \cdot 2^{16} \cdot 16 = 68,72 \cdot 10^9 \text{ B} \approx 70 \text{ GB}$$

- 11) Pretražujemo li bazu veličine 10^{10} Groverom, potrebno je računalo s prelišno koliko qubita?

$$\log_2(10^{10}) = 33,22 \rightarrow \text{uzima se 33 ili 34}$$

- 12) Pretražujemo li bazu veličine 10^6 , Groverov gerator mora delovati koliko puta?

$$m = \log_2(10^6) = 19,93 \approx 20$$

$$R = \sqrt{2^m} = 1024 \approx 1000$$