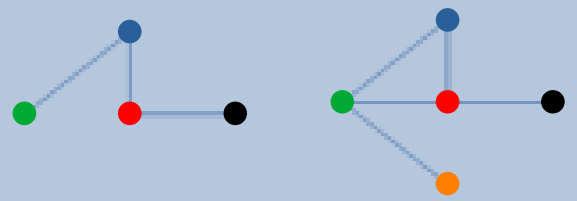
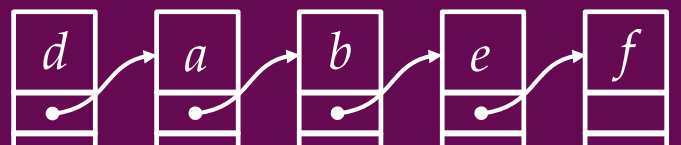
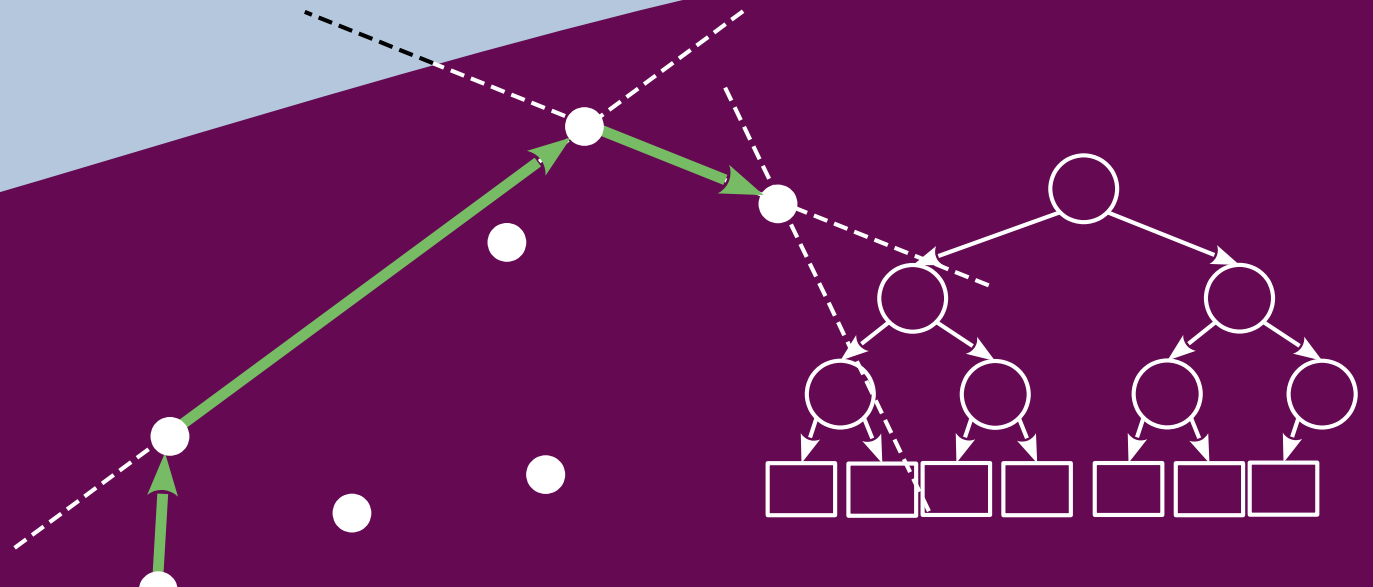
Napredni algoritmi i strukture podataka

Dodatak: Simplex – grafička analogija







Problem

Riješite:

$$\max 3x_1 + 5x_2$$

$$\begin{aligned} \text{uz} \quad & x_1 + 5x_2 \leq 40 \\ & 2x_1 + x_2 \leq 20 \\ & x_1 + x_2 \leq 12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

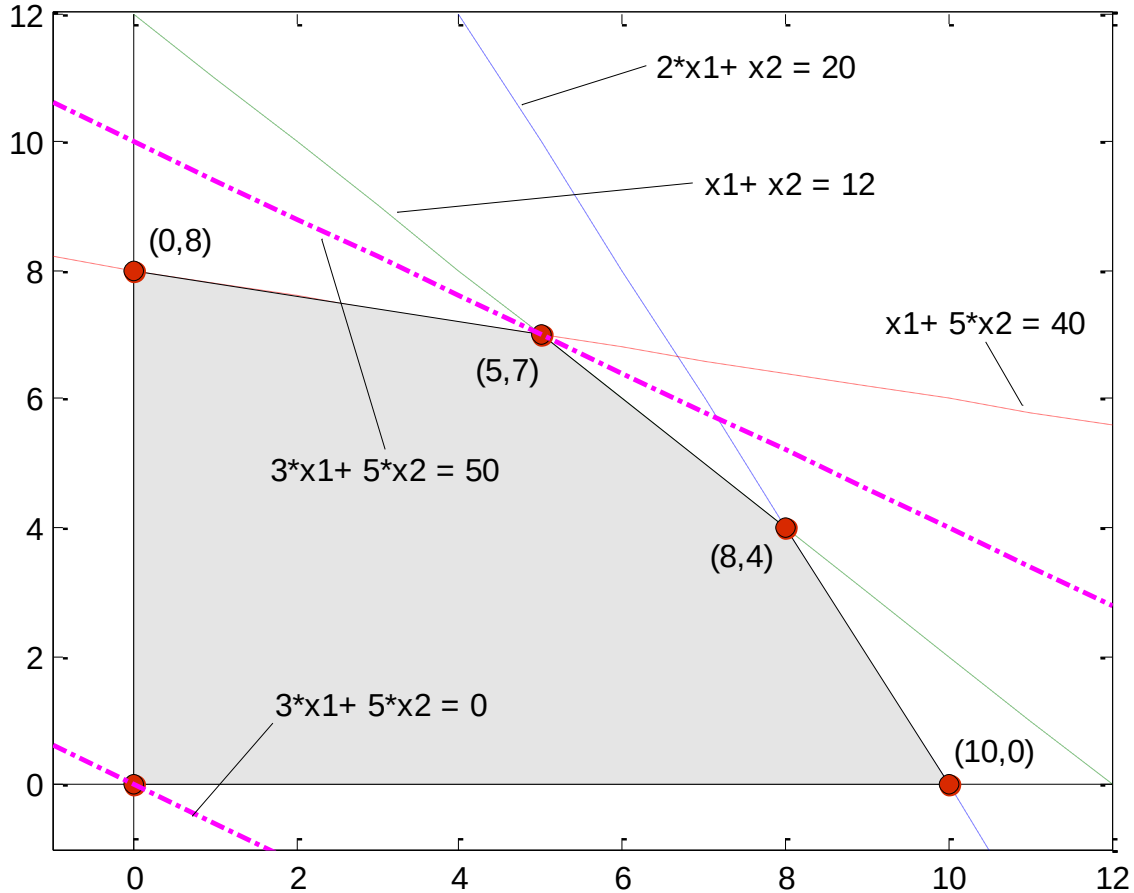
Rješenje grafičkom metodom

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{uz} \quad & x_1 + 5x_2 \le 40 \\ & 2x_1 + x_2 \le 20 \\ & x_1 + x_2 \le 12 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Grafičko rješenje

$f_{min} = 0; \quad x_1 = 0, x_2 = 0$

$f_{max} = 50; x_1 = 5, x_2 = 7$



Simplex – init (iteracija 0)

Pretvorba u standardnu formu:

c^T

$$\begin{aligned} &\min && -3x_1 - 5x_2 \\ &\text{uz} && x_1 + 5x_2 + x_3 = 40 \\ & && 2x_1 + x_2 + x_4 = 20 \\ & && x_1 + x_2 + x_5 = 12 \\ & && x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

b

- Izražavanje rješenja u bazi stupčanih vektora matrice A

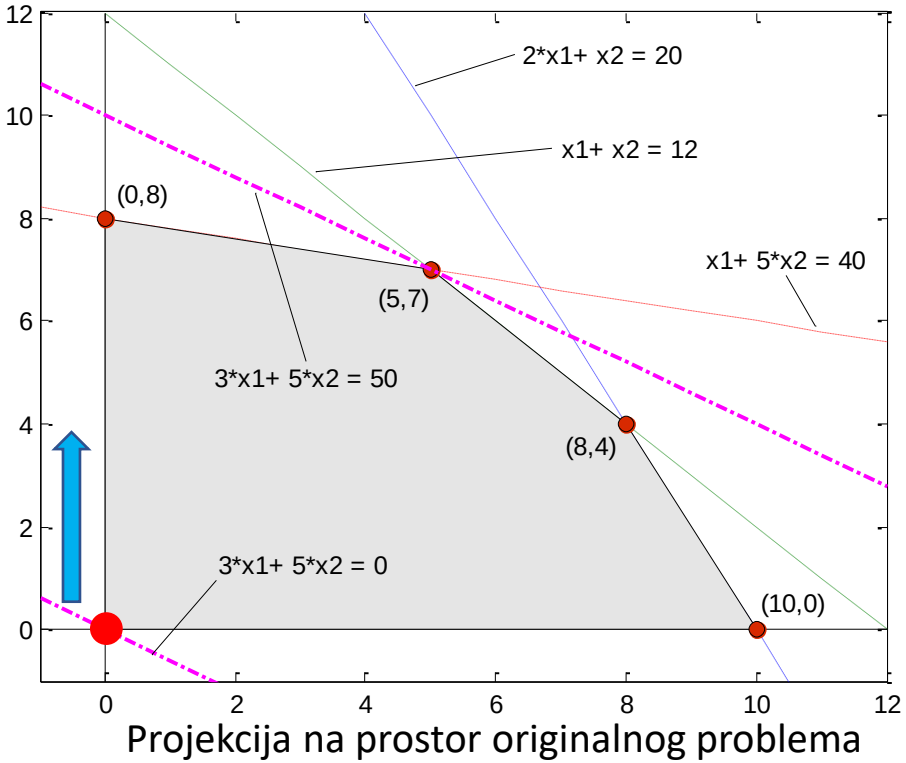
$$\overset{a_1}{x_1}\begin{bmatrix}1\\2\\1\end{bmatrix} + \overset{a_2}{x_2}\begin{bmatrix}5\\1\\1\end{bmatrix} + \overset{a_3}{x_3}\begin{bmatrix}1\\0\\0\end{bmatrix} + \overset{a_4}{x_4}\begin{bmatrix}0\\1\\0\end{bmatrix} + \overset{a_5}{x_5}\begin{bmatrix}0\\0\\1\end{bmatrix} = \overset{b}{\begin{bmatrix}40\\20\\12\end{bmatrix}}$$

- Kao polazno rješenje uzimamo ekstrem $\mathbf{x}_0 = [0, 0, 40, 20, 12]^T$. $f(x_0)=0$
- Bazične varijable (x_3,x_4,x_5)

Simplex – iteracija 1

a_1	a_2	a_3	a_4	a_5	RHS
–3	–5	0	0	0	0
1	5	1	0	0	40
2	1	0	1	0	20
1	1	0	0	1	12

$x_{(0)}=[0,0,40,20,12]^T$, $f(x_{(0)})=0$



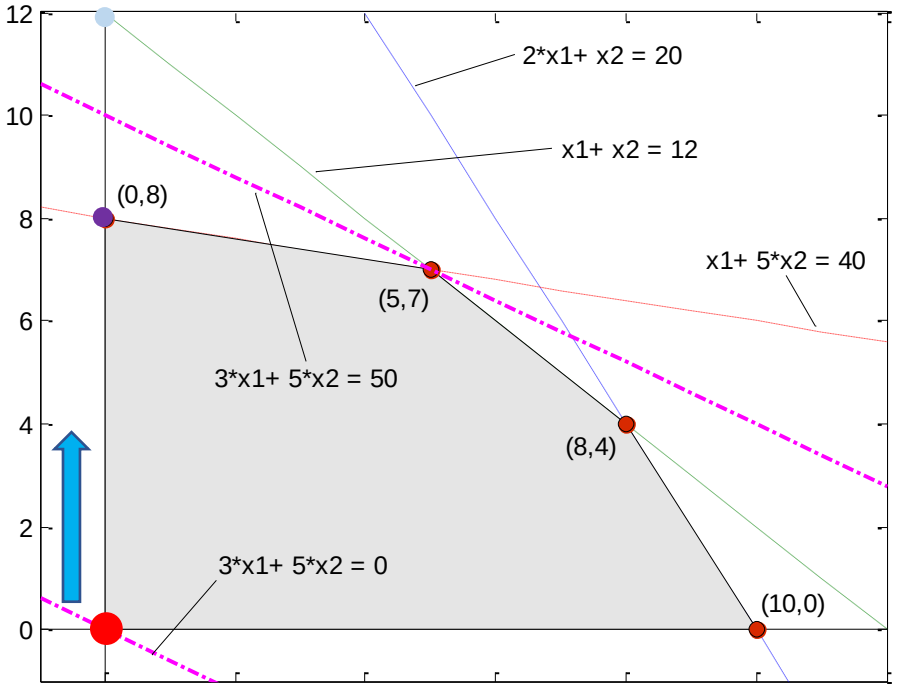
Projekcija na prostor originalnog problema

Simplex – iteracija 1

a_1	a_2	a_3	a_4	a_5	RHS	Q
–3	–5	0	0	0	0	
1	5	1	0	0	40	8
2	1	0	1	0	20	20
1	1	0	0	1	12	12

Pivot: (1,2)

*Iteracija se završava pivotiranjem



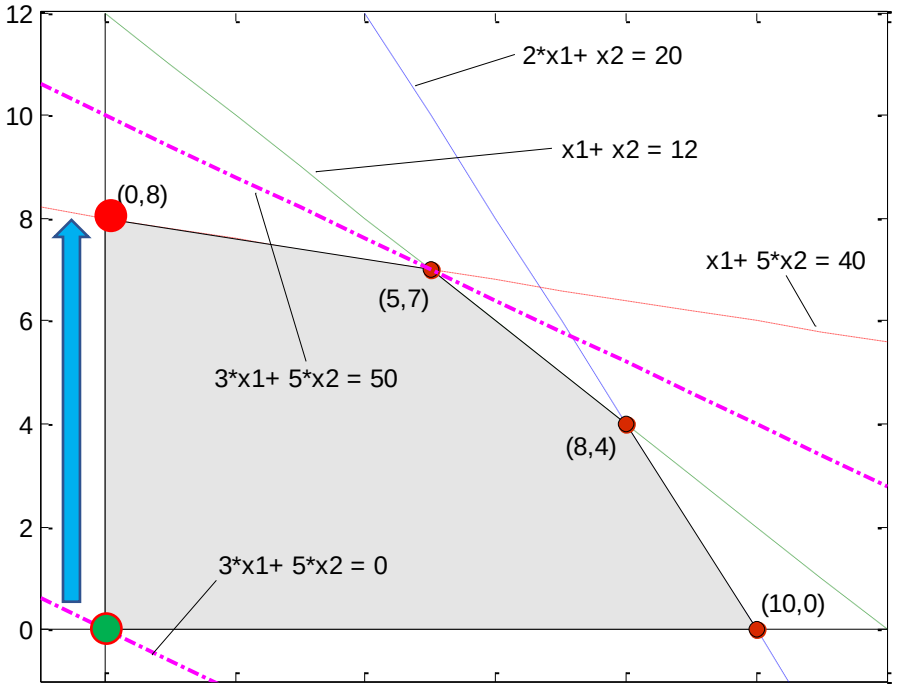
Projekcija na prostor originalnog problema

Simplex – iteracija 1

a_1	a_2	a_3	a_4	a_5	RHS
–2	0	1	0	0	40
1/5	1	1/5	0	0	8
9/5	0	-1/5	1	0	12
4/5	0	-1/5	0	1	4

Skok iz $x_{(0)}$ u:

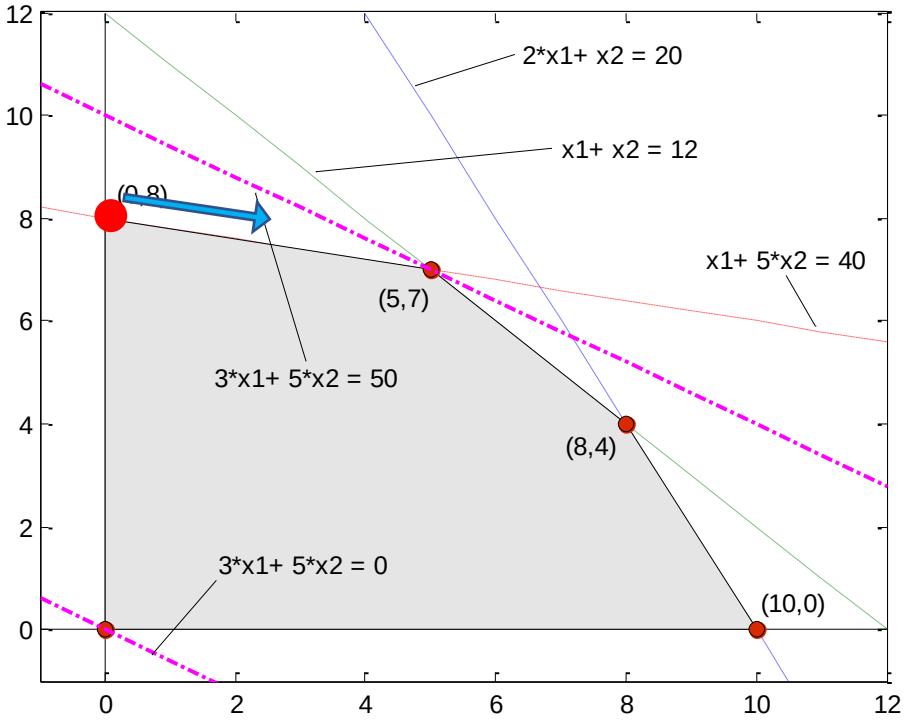
$x_{(1)}=[0,8,0,12,4]^T$, $f(x_{(1)})=-40$



Projekcija na prostor originalnog problema

Simplex – iteracija 2

a_1	a_2	a_3	a_4	a_5	RHS
–2	0	1	0	0	40
1/5	1	1/5	0	0	8
9/5	0	-1/5	1	0	12
4/5	0	-1/5	0	1	4

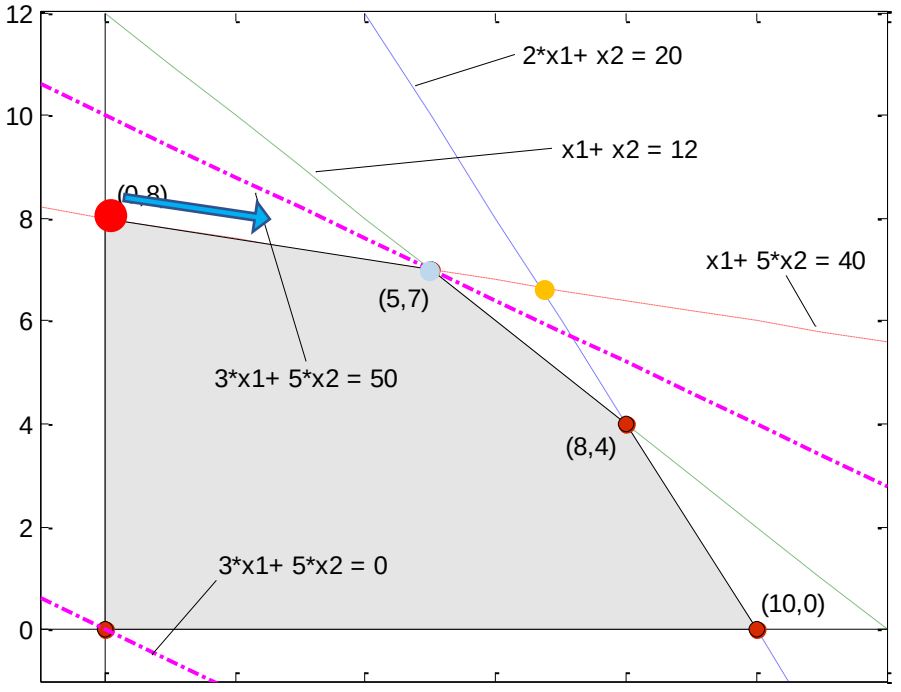


Projekcija na prostor originalnog problema

Simplex – iteracija 2

a_1	a_2	a_3	a_4	a_5	RHS	Q
–2	0	1	0	0	40	
1/5	1	1/5	0	0	8	40
9/5	0	-1/5	1	0	12	60/9
4/5	0	-1/5	0	1	4	5

Pivot: (3,1)



Projekcija na prostor originalnog problema

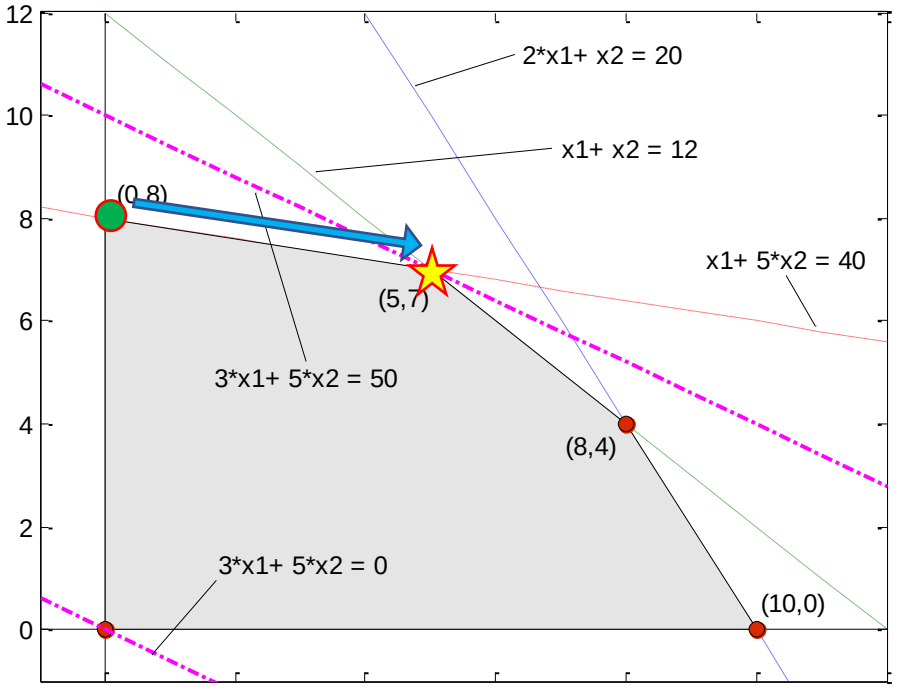
Simplex – iteracija 2

a_1	a_2	a_3	a_4	a_5	RHS
0	0	1/2	0	10/4	50
0	1	1/20	0	-1/4	7
0	0	1/4	1	-9/4	3
1	0	-1/4	0	5/4	5

Skok iz $x_{(1)}$ u:

$x_{(2)}=[5,7,0,3,0]^T$, $f(x_{(2)})=-50$

OPTIMUM!



Projekcija na prostor originalnog problema