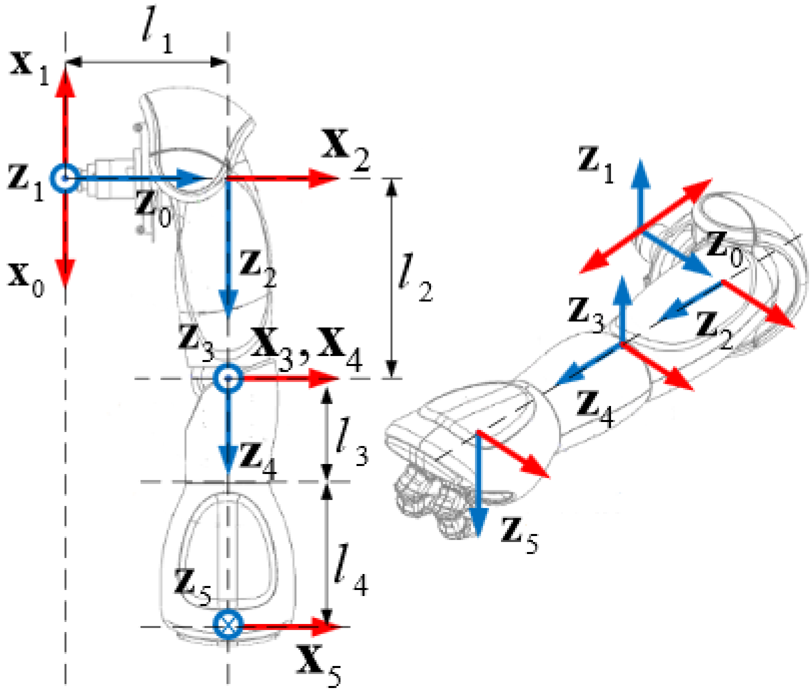
Ivan Rezo 0036466940 Grupa P01	FAKULTET ELEKTROTEHNIKE I RAČUNARSTVA SVEUČILIŠTA U ZAGREBU Zavod za automatiku i računalno inženjerstvo **Osnove robotike** Dinamički model manipulatora Zadaća broj 4 i 5	15.01.2016.

Zadatak 1.

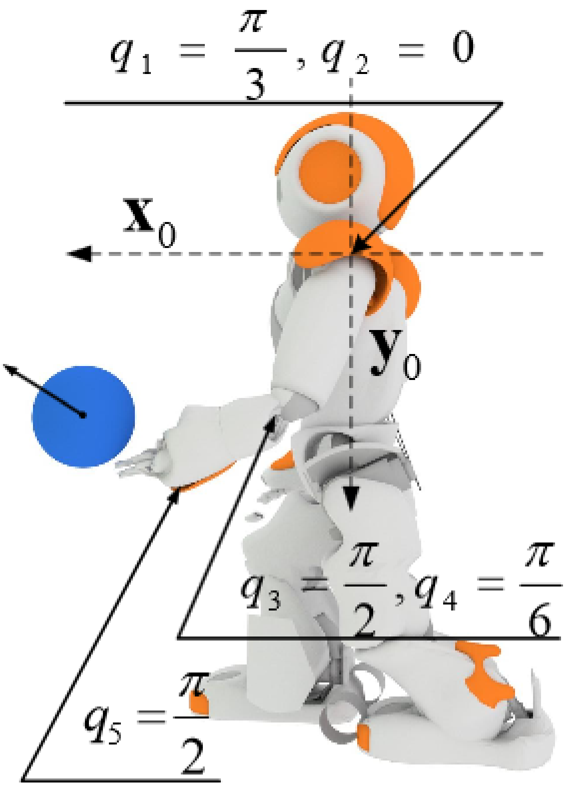


Slika 1. Zglobovi lijeve ruke *Nao* robota u početnom položaju

Parametri *Denavit-Hartenbergove* tablice su:

Tablica 1. Parametri *DH* tablice robota

	θ	d	α	a
1	$q_1(180°)$	0	$-90°$	0
2	$q_2(-90°)$	0	$90°$	l_1
3	$q_3(0°)$	l_2	$-90°$	0
4	$q_4(0°)$	0	$90°$	0
5	$q_5(0°)$	$l_3 + l_4$	$90°$	0



Slika 2. Položaj zglobova lijeve ruke *Nao* robota u trenutku izbačaja kuglice

Kako je zadan položaj zglobova lijeve ruke *Nao* robota u trenutku izbačaja kuglice, potrebno je uračunati i te vrijednosti pa se dobije:

$$q = \begin{bmatrix} \pi + \frac{\pi}{3} & -\frac{\pi}{2} & \frac{\pi}{2} & \frac{\pi}{6} & \frac{\pi}{2} \end{bmatrix}$$

Odnosno,

$$q = \begin{bmatrix} \frac{4\pi}{3} & -\frac{\pi}{2} & \frac{\pi}{2} & \frac{\pi}{6} & \frac{\pi}{2} \end{bmatrix}$$

Sljedeći korak je odrediti vrijednosti matrica $T_0^1, T_0^2, T_0^3, T_0^4$ i T_0^5. Taj dio je izveden pomoću skripte *matriceT.m*. Rezultati su sljedeći:

$$T_0^1 = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & \frac{l_2}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} l_2 \\ 0 & 0 & -1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & \frac{l_2}{2} \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} l_2 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{l_2}{2} + \frac{\sqrt{3}}{2}(l_3 + l_4) \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{l_3 + l_4}{2} + \frac{\sqrt{3}}{2} l_2 \\ -1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Iz prethodnih matrica može se iščitati vektor **p** (prva tri retka četvrtog stupca).

$$p_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$p_2 = \begin{bmatrix} 0 & 0 & l_1 \end{bmatrix}^T$$

$$p_3 = \begin{bmatrix} \frac{l_2}{2} & \frac{\sqrt{3}}{2} l_2 & l_1 \end{bmatrix}^T$$

$$p_4 = \begin{bmatrix} \frac{l_2}{2} & \frac{\sqrt{3}}{2} l_2 & l_1 \end{bmatrix}^T$$

$$p_5 = \begin{bmatrix} \frac{l_2}{2} + \frac{\sqrt{3}}{2}(l_3 + l_4) & \frac{l_3 + l_4}{2} + \frac{\sqrt{3}}{2} l_2 & l_1 \end{bmatrix}^T$$

Također je potrebno isčitati i vektor **z** (prva tri retka trećeg stupca):

$$z_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{bmatrix}^T$$

$$z_2 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{bmatrix}^T$$

$$z_3 = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$$

$$z_4 = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{bmatrix}^T$$

$$z_5 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}^T$$

Dodatno, za izračun brzina će biti potrebna i matrica T_0^0 koja glasi:

$$T_0^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Iz matrice T_0^0 se može isčitati vektor **p** i vektor **z** koji glase:

$$p_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$z_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

Brzina kuglice jednaka je brzini vrha ruke *Nao* robota. Kako bi se odredila brzina vrha ruke robota, potrebno je odrediti brzine svakog prethodnog članka. Prema tome:

Članak I

$$\vec{\omega}_0 = 0$$
$$\vec{v}_0 = 0$$
$$\vec{\omega}_1 = \vec{\omega}_0 + \dot{q}_1 \cdot \vec{z}_0 = \begin{bmatrix} 0 & 0 & \dot{q}_1 \end{bmatrix}^T$$
$$\vec{v}_1 = \vec{v}_0 + \frac{\partial}{\partial t}\left(\Delta\vec{s}_1\right) = \vec{v}_0 + \vec{\omega}_1 \times \Delta\vec{s}_1 + \frac{\partial}{\partial t}\left\|\Delta\vec{s}_1\right\| \cdot \Delta\hat{s}_1 = 0$$

*Vrijedi:

$$\Delta\vec{s}_1 = \vec{p}_1 - \vec{p}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$
$$\vec{\omega}_1 \times \Delta\vec{s}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \dot{q}_1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$
$$\left\|\Delta\vec{s}_1\right\| = 0$$

Članak II

$$\vec{\omega}_2 = \vec{\omega}_1 + \dot{q}_2 \cdot \vec{z}_1 = \vec{\omega}_1 + \dot{q}_2 \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} \frac{\sqrt{3}}{2}\dot{q}_2 & -\frac{1}{2}\dot{q}_2 & \dot{q}_1 \end{bmatrix}^T$$
$$\vec{v}_2 = \vec{v}_1 + \frac{\partial}{\partial t}\left(\Delta\vec{s}_2\right) = \vec{v}_1 + \vec{\omega}_2 \times \Delta\vec{s}_2 + \frac{\partial}{\partial t}\left\|\Delta\vec{s}_2\right\| \cdot \Delta\hat{s}_2 = \begin{bmatrix} -\frac{1}{2}\dot{q}_2 \cdot l_1 & -\frac{\sqrt{3}}{2}\dot{q}_2 \cdot l_1 & 0 \end{bmatrix}^T$$

*Vrijedi:

$$\Delta\vec{s}_2 = \vec{p}_2 - \vec{p}_1 = \begin{bmatrix} 0 & 0 & l_1 \end{bmatrix}^T$$
$$\vec{\omega}_2 \times \Delta\vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\sqrt{3}}{2}\dot{q}_2 & -\frac{1}{2}\dot{q}_2 & \dot{q}_1 \\ 0 & 0 & l_1 \end{vmatrix} = -\frac{1}{2}\dot{q}_2 \cdot l_1 \cdot \vec{i} - \frac{\sqrt{3}}{2}\dot{q}_2 \cdot l_1 \cdot \vec{j}$$
$$\left\|\Delta\vec{s}_2\right\| = l_1$$

Članak III

$$\vec{\omega}_3 = \vec{\omega}_2 + \dot{q}_3 \cdot \vec{z}_2 = \vec{\omega}_2 + \dot{q}_3 \cdot \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3) & \dot{q}_1 \end{bmatrix}^T$$

$$\vec{v}_3 = \vec{v}_2 + \frac{\partial}{\partial t}(\Delta\vec{s}_3) = \vec{v}_2 + \vec{\omega}_3 \times \Delta\vec{s}_3 + \frac{\partial}{\partial t}\|\Delta\vec{s}_3\| \cdot \Delta\hat{s}_3 = \begin{bmatrix} -\frac{\sqrt{3}}{2}\dot{q}_1 \cdot l_2 - \frac{1}{2}\dot{q}_2 \cdot l_1 & \frac{1}{2}\dot{q}_1 \cdot l_2 - \frac{\sqrt{3}}{2}\dot{q}_2 \cdot l_1 & l_2 \cdot \dot{q}_2 \end{bmatrix}^T$$

*Vrijedi:

$$\Delta\vec{s}_3 = \vec{p}_3 - \vec{p}_2 = \begin{bmatrix} \frac{l_2}{2} & \frac{\sqrt{3}}{2}l_2 & 0 \end{bmatrix}^T$$

$$\vec{\omega}_3 \times \Delta\vec{s}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3) & \dot{q}_1 \\ \frac{l_2}{2} & \frac{\sqrt{3}}{2}l_2 & 0 \end{vmatrix} = -\frac{\sqrt{3}}{2}\dot{q}_1 \cdot l_2 \cdot \vec{i} + \frac{1}{2}\dot{q}_1 \cdot l_2 \cdot \vec{j} + l_2 \cdot \dot{q}_2 \cdot \vec{k}$$

$$\|\Delta\vec{s}_3\| = l_2$$

Članak IV

$$\vec{\omega}_4 = \vec{\omega}_3 + \dot{q}_4 \cdot \vec{z}_3 = \vec{\omega}_3 + \dot{q}_4 \cdot \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T = \begin{bmatrix} \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3) & \dot{q}_1 - \dot{q}_4 \end{bmatrix}^T$$

$$\vec{v}_4 = \vec{v}_3 + \frac{\partial}{\partial t}(\Delta\vec{s}_4) = \vec{v}_3 + \vec{\omega}_4 \times \Delta\vec{s}_4 + \frac{\partial}{\partial t}\|\Delta\vec{s}_4\| \cdot \Delta\hat{s}_4 = \begin{bmatrix} -\frac{\sqrt{3}}{2}\dot{q}_1 \cdot l_2 - \frac{1}{2}\dot{q}_2 \cdot l_1 & \frac{1}{2}\dot{q}_1 \cdot l_2 - \frac{\sqrt{3}}{2}\dot{q}_2 \cdot l_1 & l_2 \cdot \dot{q}_2 \end{bmatrix}^T$$

*Vrijedi:

$$\Delta\vec{s}_4 = \vec{p}_4 - \vec{p}_3 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$\vec{\omega}_4 \times \Delta\vec{s}_4 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3) & \dot{q}_1 - \dot{q}_4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\|\Delta\vec{s}_4\| = 0$$

Članak V

$$\vec{\omega}_5 = \vec{\omega}_4 + \dot{q}_5 \cdot \vec{z}_4 = \vec{\omega}_4 + \dot{q}_5 \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3 + \sqrt{3}\dot{q}_5) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3 + \dot{q}_5) & \dot{q}_1 - \dot{q}_4 \end{bmatrix}^T$$

$$\vec{v}_5 = \vec{v}_4 + \frac{\partial}{\partial t}(\Delta\vec{s}_5) = \vec{v}_4 + \vec{\omega}_5 \times \Delta\vec{s}_5 + \frac{\partial}{\partial t}\|\Delta\vec{s}_5\| \cdot \Delta\hat{s}_5 = \begin{bmatrix} -\frac{\sqrt{3}}{2}\dot{q}_1 \cdot l_2 - \frac{1}{2}\dot{q}_2 \cdot l_1 - \frac{l_3 + l_4}{2} \cdot (\dot{q}_1 - \dot{q}_4) \\ \frac{1}{2}\dot{q}_1 \cdot l_2 - \frac{\sqrt{3}}{2}\dot{q}_2 \cdot l_1 + \frac{\sqrt{3}}{2}(l_3 + l_4) \cdot (\dot{q}_1 - \dot{q}_4) \\ l_2 \cdot \dot{q}_2 + \frac{1}{2} \cdot (l_3 + l_4) \cdot (\sqrt{3}\dot{q}_2 - \dot{q}_3) \end{bmatrix}$$

*Vrijedi:

$$\Delta\vec{s}_5 = \vec{p}_5 - \vec{p}_4 = \begin{bmatrix} \frac{\sqrt{3}}{2}(l_3 + l_4) & \frac{l_3 + l_4}{2} & 0 \end{bmatrix}^T$$

$$\vec{\omega}_5 \times \Delta\vec{s}_5 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2}(\sqrt{3}\dot{q}_2 + \dot{q}_3 + \sqrt{3}\dot{q}_5) & \frac{1}{2}(-\dot{q}_2 + \sqrt{3}\dot{q}_3 + \dot{q}_5) & \dot{q}_1 - \dot{q}_4 \\ \frac{\sqrt{3}}{2}(l_3 + l_4) & \frac{l_3 + l_4}{2} & 0 \end{vmatrix} = \begin{bmatrix} -\frac{l_3 + l_4}{2} \cdot (\dot{q}_3 - \dot{q}_4) \\ \frac{\sqrt{3}}{2}(l_3 + l_4) \cdot (\dot{q}_3 - \dot{q}_4) \\ \frac{1}{2} \cdot (l_3 + l_4) \cdot (\sqrt{3}\dot{q}_2 - \dot{q}_3) \end{bmatrix}^T \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix}$$

$$\|\Delta\vec{s}_5\| = l_3 + l_4$$

Nakon što se uvrste zadane vrijednosti brzina zglobova i duljina članaka, za brzinu kuglice pri izbačaju se dobije:

$$\vec{\omega}_5 = -173.206\vec{i} - 100\vec{j}$$
$$\vec{v}_5 = 18.187\vec{i} - 10.5\vec{j}$$

Zadatak 2.

Opći oblik *Jacobijan* matrice *n*-osnog manipulatora glasi:

$$J(q) = \begin{bmatrix} A(q) \\ B(q) \end{bmatrix} = \begin{bmatrix} \frac{\partial p(q)}{\partial q_1} & \frac{\partial p(q)}{\partial q_2} & \dots & \frac{\partial p(q)}{\partial q_n} \\ z_0(q) & z_1(q) & \dots & z_{n-1}(q) \end{bmatrix}$$

Potrebno je iščitati vektor **p** iz matrice T_0^5 koji glasi:

$$p_5 = \begin{bmatrix} \frac{l_2}{2} + \frac{\sqrt{3}}{2}(l_3 + l_4) & \frac{l_3 + l_4}{2} + \frac{\sqrt{3}}{2}l_2 & l_1 \end{bmatrix}^T$$

Nakon toga, odredi se se *Jacobijan* matrica za naš slučaj prema principu:

$$J(q)=\begin{bmatrix} A(q) \\ B(q) \end{bmatrix}=\begin{bmatrix} \frac{\partial p_5(q)}{\partial q_1} & \frac{\partial p_5(q)}{\partial q_2} & \frac{\partial p_5(q)}{\partial q_3} & \frac{\partial p_5(q)}{\partial q_4} & \frac{\partial p_5(q)}{\partial q_5} \\ z_0(q) & z_1(q) & z_2(q) & z_3(q) & z_4(q) \end{bmatrix}$$

Potom se linijske brzine izračunaju kao umnožak *A(q)* dijela *Jacobijan* matrice i zadanih brzina zglobova, a kutne brzine kao umnožak *B(q)* dijela *Jacobijan* matrice i zadanih brzina zglobova. Kompletan postupak se obavlja pokretanjem skripte *drugi.m.*

Zadatak 3.

Brzina kuglice je izračunata kao:

$$\vec{v}=18.187\vec{i}-10.5\vec{j}$$

Kut pod kojim je kuglica bačena iznosi:

$$\alpha = arctg\left(\frac{10.5}{18.187}\right)=30°$$

Početna brzina kuglice iznosi:

$$v_0=\sqrt{18.187^2+10.5^2}=21$$

Kod kosog hica, za prijeđeni put po *x* i *y* osi vrijedi:

$$x=v_0 t\cdot\cos\alpha$$
$$y=v_0 t\cdot\sin\alpha-\frac{gt^2}{2}$$

Izražavanjem *t* iz prve jednadžbe te uvrštavanjem u drugu jednadžbu, dobije se:

$$y=x\cdot tg\alpha-\frac{gx^2}{2v_0^2\cdot\cos^2\alpha}$$

Nakon uvrštavanja svih poznatih vrijednosti (*y*=0.05 [m]), kao rješenje se dobije da je Edith od Renea na udaljenosti:

$$x_1=0.087\,[\mathrm{m}]$$
$$x_2=38.845\,[\mathrm{m}]$$

Pogledom na animaciju (kuglica je u silaznoj putanji), kao rješenje se uzima $x_2=38.845\,[\mathrm{m}]$.

Zadatak 4.

Tenzori inercije robota glase:

$$D_1^{'} = m_1 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D_2^{'} = \frac{m_2 l_1^2}{12} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_3^{'} = \frac{m_3 l_2^2}{12} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_4^{'} = m_4 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D_5^{'} = \frac{m_5 (l_3 + l_4)^2}{12} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ukupna kinetička energija Reneove lijeve ruke može se izračunati kao:

$$T(q, \dot{q}) = \frac{\dot{q}^T \cdot \sum_{k=1}^{n} \left[\left(A^k\right)^T \cdot m_k \cdot A^k + \left(B^k\right)^T \cdot D_k \cdot B^k \right] \cdot \dot{q}}{2}$$

No prije toga je potrebno za svaki članak odrediti koordinate njegovog centra mase u odnosu na koordinatni sustav baze prema formuli:

$$c^i(q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot T_0^i(q) \cdot \Delta c^i$$

Gdje Δc^i predstavlja koordinate centra mase *i*-tog članka u odnosu na koordinatni sustav L_i. Nakon toga, preostaje odrediti tenzor inercije svakog članka u odnosu na koordinatni sustav baze prema formuli:

$$D_i(q) = R_0^i(q) \cdot D_i^{'} \cdot \left(R_0^i(q)\right)^T$$

Uz to je potrebno i odrediti *Jacobijan* matricu prema izrazu:

$$J^k(q) = \begin{bmatrix} A^k(q) \\ B^k(q) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial c^k(q)}{\partial q_1} & \dots & \dfrac{\partial c^k(q)}{\partial q_k} & 0 \\ z_0(q) & \dots & z_{k-1}(q) & 0 \end{bmatrix}$$

Iz *Jacobijan* matrice se izraze $A^k(q)$ i $B^k(q)$ dio te se uvrštavaju u izraz za ukupnu kinetičku energiju ruke robota. Kompletan postupak se provodi pokretanjem skripte *cetvrti.m*. Konačno rješenje glasi:

$$T(q,\dot{q}) = 73.5m_3 + 220.5m_4 + 220.5m_5$$

Kako je četvrti članak beskonačno malen, masa m_4 se zanemaruje pa konačna energija glasi:

$$T(q,\dot{q}) = 73.5m_3 + 220.5m_5$$