9.5.2024.

8. tjedan - auditorne

LEVEL CROSSING RATE (LCR)

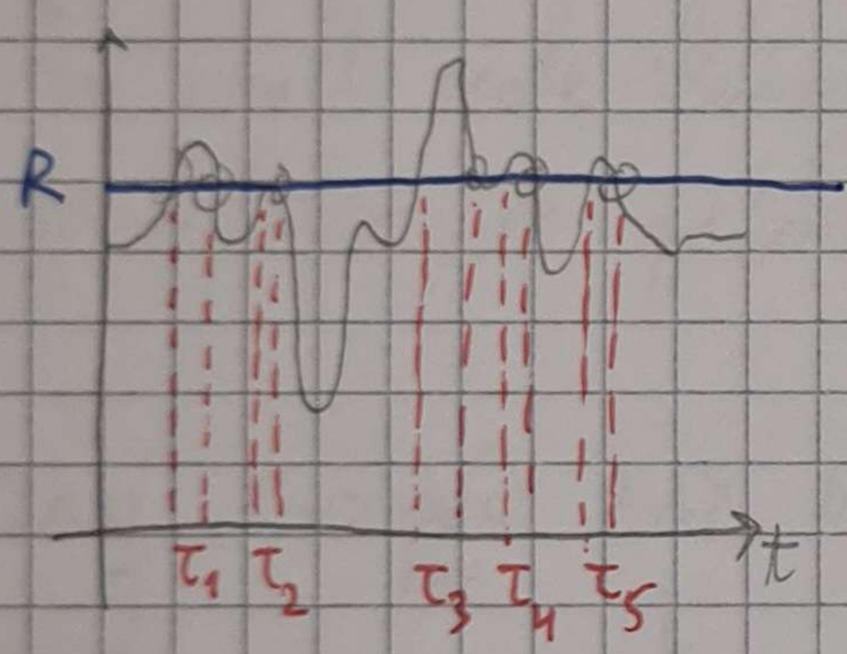
AVERAGE FADE DURATION (AFD)

STATISTIKE 2. reda

↳ broj prolazaka nekim pragom
↕
SIGNALA

↳ KOLIKO DUGO SE SIGNAL ZADRŽAVA ISPOD NEKE RAZINE

o = kad padne ispod R



krivo dan ili označeno

↑
ZAPRAVO JE OBRNUTO

→ τ_i = od 0 do sljedećeg dizanja R

AFD

$$\bar{\tau} = \frac{1}{N_R} \sum \tau_i$$

$$N_R = \sqrt{2\pi}\, f_{0,max}\, \rho\, e^{-\rho^2} \rightarrow LCR$$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho\, f_{0,max} \sqrt{2\pi}}$$

$$\rho = \frac{R}{R_{rms}}$$

→ rms

↳ OMJER PRAGOVA

① $N_R = ?$

$\xi = 1$

$f_{D,max} = 20\,Hz$

$f_c = 900\,MHz \rightarrow \lambda = \frac{1}{3}\,m$

$$\boxed{f_{D,max} = \frac{v}{\lambda}\cos\alpha}$$

max za je $\alpha = 0 \rightarrow \cos = 1$

$$N_R = \sqrt{2\pi}\cdot f_{D,max}\,\xi\,e^{-\xi^2} = \sqrt{2\pi}\cdot 20\,Hz\cdot 1\,e^{-1} = 18.44\ \text{prolaz}/s$$

$$f_{D,max} = \frac{v}{\lambda}\cdot\underbrace{\cos\alpha}_{=1} \Rightarrow \underline{v = 6.67\,m/s}$$

②

$\xi = 0.01,\ \xi = 0.1,\ \xi = 1$

$f_{D,max} = 200\,Hz$

$$\bar{\tau} = \frac{e^{\xi^2} - 1}{\xi\cdot f_{D,max}\cdot\sqrt{2\pi}}$$

$\xi = 0.01 \rightarrow \bar{\tau} = 19.95$

$\xi = 0.1 \rightarrow \bar{\tau} = 200.47\,\mu s$

$\xi = 1 \rightarrow \bar{\tau} = 3.43\,ms$

- Znaj interpretirati graf !!
- Znaj usporediti graf s drugim grafovima

(3)

a) $\rho = 0.707$

$f_{D,max} = 20\,Hz$

$\bar{\tau} = 18.3\,ms$

b) $R_B = 50\,bit/s$, $\rho = 0.1$

$N_R = \sqrt{2\pi} \cdot 20 \cdot 0.1 \cdot e^{-0.1^2}$

$= 4.96$ prolaza/s

$\approx 5 \rightarrow$ 5 puta u 1 sekundi prođemo ispod prihvatljive granice

$BER = \frac{5}{50} = \frac{1}{10} = 0.1$

→ BIT ERROR RATE

(4) $v = 80\,km/h = 22.22\,m/s$

$f = 6\,GHz$

$t = 5\,s$

$\rho = \frac{R}{R_{rms}} = \frac{R_{rms}}{R_{rms}} = 1$

a) $N_R = \sqrt{2\pi} \cdot f_{D,max} \cdot \rho \cdot e^{-\rho^2} = 409.7$ prolaza/s

$f_{D,max} = \frac{v}{\lambda} \underbrace{\cos\alpha}_{1} = 444.4\,Hz$

$N_R \cdot t = 409.7 \cdot 5 = 2048.5$ prolaza/5 sekundi

b) PROSJEČNO TRAJANJE FEDINGA

$\bar{\tau} = 1.54\ ms$

a) -||- na 20dB ISPOD AMPLITUDE OVOJNICE

RMS VRIJ. =

$\rho = \frac{R}{R_{RMS}} \Rightarrow \rho_{dB} = R_{dB} - R_{RMS,dB} = -20dB = 0.01$ → ABS IZNOS IDE U FORMULE

$\underline{\bar{\tau} = 8.98\ \mu s}$

(5)

U KANALU S GAUSSOVIM ŠUMOM

$P_{pogreške} = \int_0^\infty P_{AWGN}(x) \cdot p(x)\,dx$

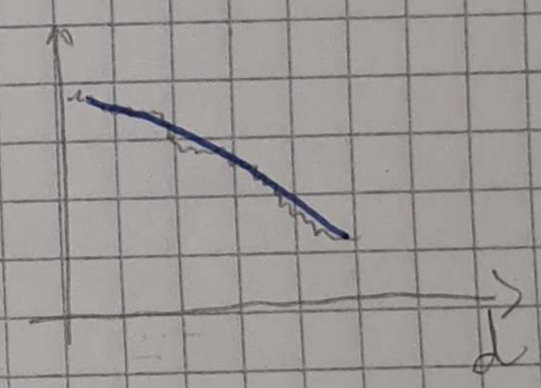
u kanalu s RAY. FEDINGOM

$x = \frac{E_b}{N_0} \cdot \alpha^2$

E_b → ENERGIJA BITA

$\alpha^2 = 1$

N_0 → NEŠ SPEKTRALNO



(6) DPSK MODULACIJA, AWGN

$P_{AWGN} = \frac{1}{2} e^{-\frac{E_b}{N_0}}$

$= \frac{1}{2} e^{-x}$

; a) $P_{RAY} = ?$

RAYLEIGOV razdioba

$p(x) = \frac{1}{\Gamma} \cdot e^{-\frac{x}{\Gamma}}$; $x \geq 0$

↳ srednji SNF

$P_{RAY} = \int_0^\infty P_{AWGN}(x) \cdot p(x)\,dx$

$= \int_0^\infty \left[\frac{1}{2} e^{-x}\right] \cdot \left[\frac{1}{\Gamma} \cdot e^{-\frac{x}{\Gamma}}\right] dx$

$$= \frac{1}{2\Gamma} \cdot \int_0^\infty e^{-\frac{\Gamma \cdot x + x}{\Gamma}} \, dx = \frac{1}{2\Gamma} \cdot \int_0^\infty e^{-x\left[\frac{\Gamma+1}{\Gamma}\right]} \, dx$$

$$= \frac{1}{2\cancel{\Gamma}} \cdot \left(-\frac{\cancel{\Gamma}}{\Gamma+1}\right) \cdot \underbrace{e^{-x\left[\frac{\Gamma+1}{\Gamma}\right]}}_{e^{-\infty}=0} \Bigg|_0^\infty = \underline{\frac{1}{2(\Gamma+1)}}$$

$$e^0 = 1$$

b) $\Gamma = 30\,dB$

$$BER = ? = P_{RAY} = \frac{1}{2(\Gamma+1)} = \frac{1}{2(1000+1)} \approx \underline{\underline{5 \cdot 10^{-4}}}$$

↳ <u>alto realmente!</u>

$\Gamma = 30\,dB$
$= 1000$

9. Tjedan

16.5.2024.

$$P_{e,ray} = \int_0^\infty P_{e,AWGN}(x) \cdot p(x)\,dx \quad ; \quad p(x) = \frac{1}{\Gamma} \cdot \exp\left(-\frac{x}{\Gamma}\right)$$

↓ gibka ↓ ray rasipanja

↳ PROSJEČNI OMJER $\frac{S}{N} = \Gamma$

NA ISPITU ĆE BITI INTEGRALI KAO SA PROŠLIH AUDITORNIH!

① $p(x) = e^{-x}, \; x > 0$

RAYLEIGHOV FEDING, DPSK MODULACIJA

$$p(x) = e^{-x} = \frac{1}{\Gamma} e^{-\frac{x}{\Gamma}}$$

$\Gamma = 1$ (jer $1 = \frac{1}{\Gamma}$) ; $\boxed{P_{e,RAY} = \frac{1}{2(\Gamma+1)} = \frac{1}{4}}$

② DPSK SIGNAL

$\boxed{P_{e,ray} = 10^{-3}}$ $\quad \Gamma = \frac{\overline{E_b}}{N_0} = ?$; AKO NIJE ZADANA FORMULA, MOŽEŠ BILO KOJU OD 2 UZETI (iz tablice)

$$P_{e,ray} = \frac{1}{2(\Gamma+1)} \rightarrow 2 \cdot P_e + 2 P_e \Gamma = 1$$

$$\rightarrow \Gamma = \frac{1 - 2P_e}{P_e} = 499$$

$L_F = \left| 20 \log\left(\frac{\lambda}{4\pi d}\right) \right| = \underline{118.89\,dB}$

$= 20 \log\left(\frac{4\pi d}{\lambda}\right)$

$\lambda = \frac{c}{f}$

$A(f,d) =$ <u>odredjuje se sa crvenog grafa</u>

$= 34\,dB$

OR $G(h_{TX}) = 20 \log\left(\frac{h_{TX}}{200}\right) = -13.98\,dB$

↳ 200m po FORMULI

$G(h_{RX}) = 10 \log\left(\frac{h_{RX}}{3}\right) = -1.76\,dB$

$G_{AREA} =$ odredjuje se s plavog grafa

$= \underline{0}$ → za veliki grad!

(NIJE NA PAPIRU!)

PRIGRADSKO ! = GRADSKO PODRUČJE

$L_{50}(dB) = 168.63\,dB$

$1kW = 1000W \Rightarrow 60\,dBm = 30\,dB$

$P_{TX} = 60\,dBm \rightarrow P_{RX} = P_{TX} - L_{50}(dB) = \underline{-108.63\,dBm}$

HATA

(4) ISTI PARAM KAO U (3.) zad

$$L(dB) = A + B \cdot \log(d(km)) - E$$

$$A = 69.55 + 26.16 \cdot \log(f(MHz)) - 13.82 \log(h_{Tx}(m))$$

$$= 134.32\ dB$$

$$B = 44.9 - 6.55 \cdot \log(h_{Tx}(m)) = 34.4\ dB$$

$$E = 3.2\left[\log(11.75 \cdot h_{Rx})\right]^2 - 4.97 = 1.05\ dB$$

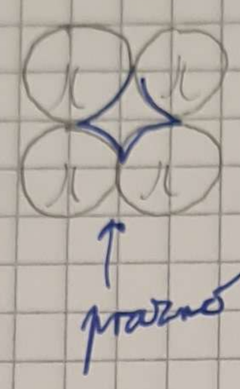
pazi koju formulu uzimaš!

$f = 2100\ MHz \rightarrow$ VELIKI GRAD

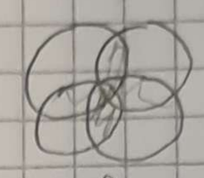
$$L(dB) = 167.67\ dB$$

NAJČEŠĆI OBLIK ĆELIJE ?

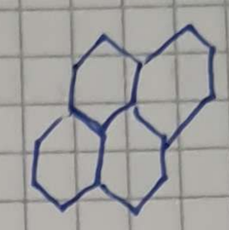
na početku

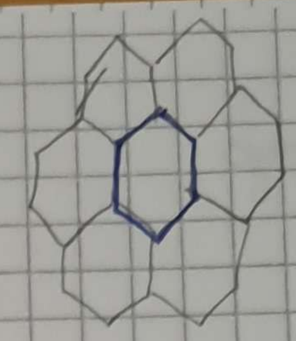


moguć



nah...





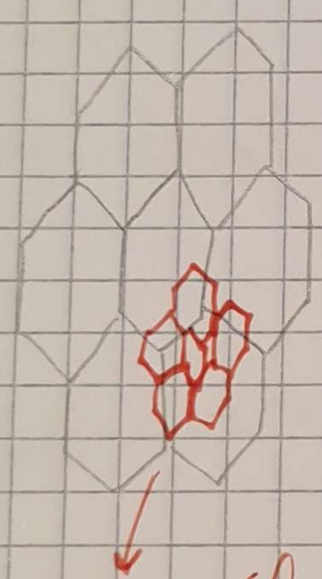
=> 7 ĆELIJA

6-KUTI

4 ĆELIJE = 1 GROZD

ili 3 ć = 1 GROZD

ili 7 ĆELIJA



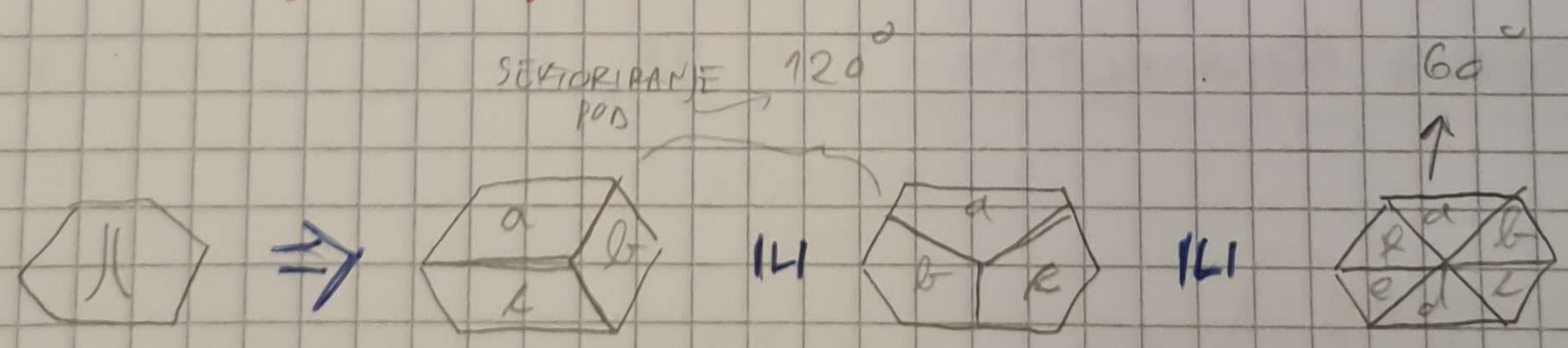
„manje ćelije u većim ćelijama"

- NA SELIMA DI JE MANJA GUSTOĆA STVARI/BILJAKA

VRVEĆA → veće i jače ćelije i BS

→ na GUŠĆIM PODRUČJIMA manje ĆELIJE i slabije ĆELIJE

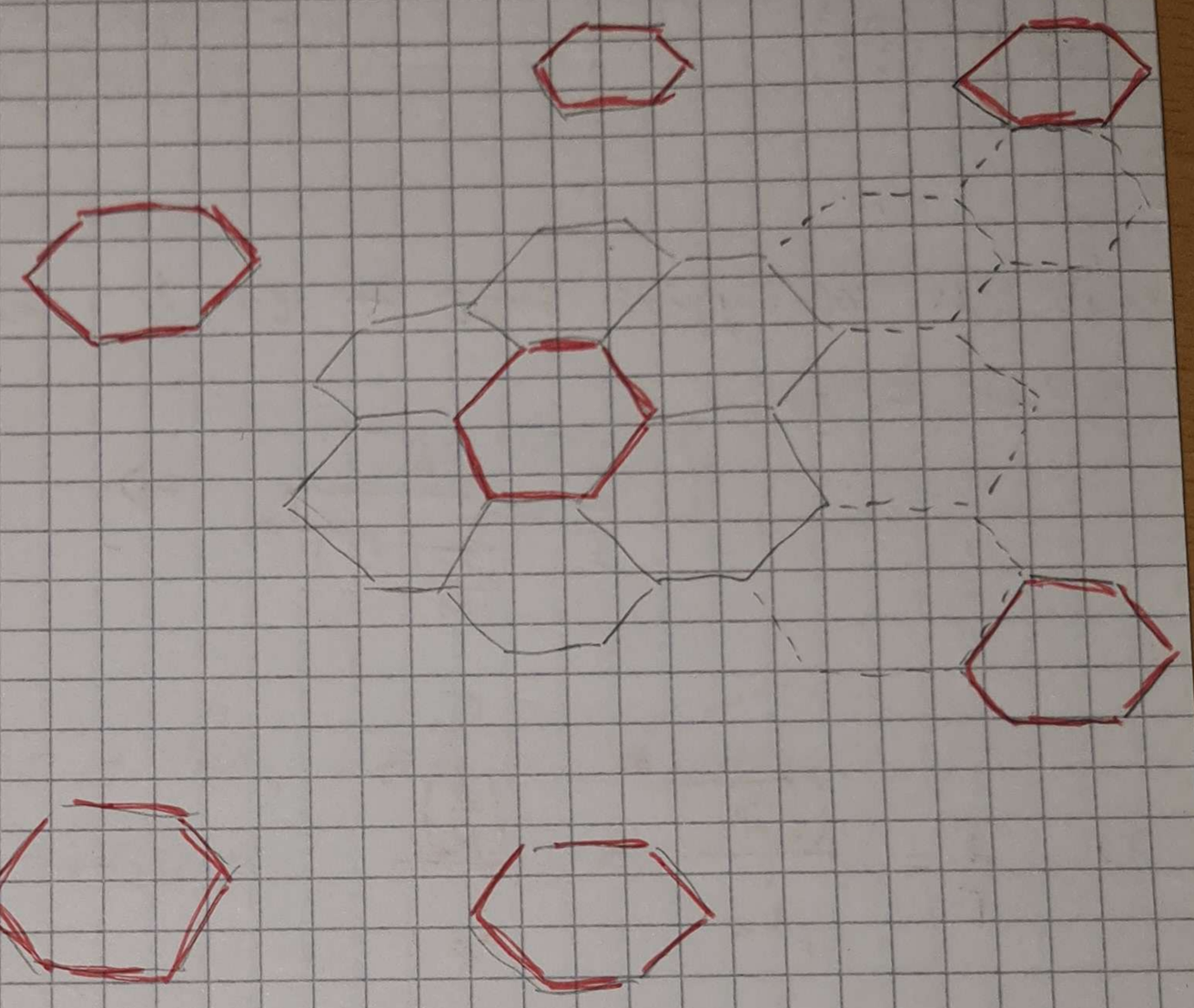
SEKTORIRANJE ĆELIJA



→ SVAKI SEKTOR IMA SVOJU f

23.5.2024.

10. tjedan - auditorne



ISTOKANALNA INTERFERENCIJA

■ → ista f

→ želimo odvojenost

→ ako je veličina svake ćelije otp. ista i ako BS odašilju istu snagu
↳ ISTOKANALNA INTERFER.

JE neovisna o broju odašiljanja

7. predavanje

D je udalj. od centra do centra

→ R , D
↳ RADIUS OPISANE KRUŽNICE

$Q = \frac{D}{R} = \sqrt{3N}$
↳ N = BROJ ĆELIJA U GROZDU

UDALJENOST → FAKTOR POTISKIVANJA

→ CARRIER

→ SIGNAL

$\frac{S}{I} = SIR = \frac{C}{I} = \frac{1}{6}\left(\frac{D}{R}\right)^n$

↳ INTERFER

→ SIGNAL TO INTERFER. RATIO

$I = \sum_{i=1}^{i_0} I_i$

i_0 = BROJ ISTOKANALNIH INTERFERIRAJUĆIH ĆELIJA

$P_{RX} \sim (D_i)^{-m} \left(\frac{1}{D_i^m}\right)$; m = EKSPONENT PRIGUŠENJA UDALJENE SNAGE

- ako su ODAŠILJAČKE SNAGE ∀ BS iste i ako je m ISTI U SVAKOM CIJELOM PROSTORU

$\hookrightarrow \frac{S}{I} = \frac{R^{-m}}{\sum_{i=1}^{i_0} (D)^{-m}} \Rightarrow ; \frac{S}{I} = \frac{\left(\frac{D}{R}\right)^m}{i_0}$!

T). $\frac{S}{I} = \frac{(\sqrt{3N})^m}{i_0} = \frac{\left(\frac{D}{R}\right)^m}{i_0}$

1

$\frac{S}{I} = 15\,dB$; $m = 4$; OPTIMALNI N = ? ; b) 120° ; c) 60°

SEKTORIRANJE

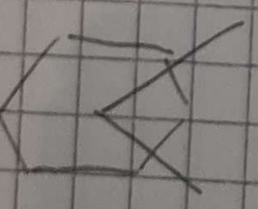
a) OMNI DIREKCIJSKA ANTENA

a) $i_0 = 6$ (SVI SU SMETAJU)

$\hookrightarrow \frac{S}{I} = \frac{(\sqrt{3N})^m}{i_0} > 15\,dB = 31.623$

$N > 4.59 \rightarrow N = 7$! jer su 6 KUTI ! $\left(\frac{NE}{5}\right)$

b) SEKTORIRANJE 120°



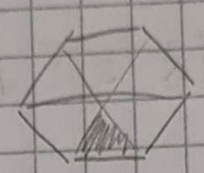
$i_0 = 2$ (samo 2 smetaju)

3, 4, 7, 12 SU POTENCIJALNI ODGOVORI

$\frac{S}{I} = \frac{(\sqrt{3N})^m}{i_0} > 15\,dB = 31.623$

$m > 2.65 \rightarrow N = 3$ E grozdovi od 3 i 4 ćelije

4) ramo i smeta

$i_0 = 1$ $\quad 60^\circ \rightarrow$ 

$N > 1{,}87 \rightarrow \underline{N = 3}$

② OMNIDIREKCIJSKA BS

$\frac{S}{I} = 18\,dB = 63{,}096$, $\quad n = 3{,}8$, $\quad D = 4.8\,km = 4800\,m$, $\quad R = ?$ $\quad i_0 = 6$

$$\frac{S}{I} = \frac{\left(\frac{D}{R}\right)^n}{i_0} \rightarrow \sqrt[n]{\frac{S}{I} \cdot i_0} = \frac{D}{R} \rightarrow R = \frac{D}{\sqrt[n]{\frac{S}{I} \cdot i_0}} \approx 1\,km$$

$$= \frac{1}{6}\left(\frac{D}{R}\right)^n$$

LAKŠE ↑

$$10 \log \left(\frac{D}{R}\right)^n = 10 \log\left(6 \cdot \frac{S}{I}\right) \Rightarrow n \cdot 10 \log\left(\frac{D}{R}\right) = 10 \log 6 + \underbrace{10 \log \frac{S}{I}}_{18}$$

$\frac{D}{R} \approx 4{,}769$

$R = 1\,km$

③ OMNIDIREKCIJSKA BS, nedostaje 3dB ZA POUZDANI prijam (μ $\frac{S}{I}$)

prelazi 120°, JEL TO DOBRO ILI NE?
$i_0 = 2$

$$\left(\frac{S}{I}\right)_6 = \frac{1}{6}\left(\frac{D}{R}\right)^m \quad \text{i} \quad \left(\frac{S}{I}\right)_2 = \frac{1}{2}\left(\frac{D}{R}\right)^m$$

$6 = i_0$

$i_0 = 2$
120°

(a bio je 3dB -)

$$\frac{\left(\frac{S}{I}\right)_2}{\left(\frac{S}{I}\right)_6} \approx 3 = \underline{4.8dB}$$ dobro je jer je tako 4.8dB više, pa je to ✓

ERLANZI

ERLANG B / ERLANG C SUSTAV ; * trunking

→ STAVAJA IH U REP ČEKANJA

odbacuje KORISNIKE

→ mjera prometa: ERLANG → 1 ERL = zauzeće 1 radio kanala
1 punu sat
→ najprometniji

$T_H = T_{HOLD}$ = prosječno trajanje poziva → u min → pretvori u SATE

λ = BROJ POZIVA U GLAVNOM PROMETNOM SATU

A_p = PONUDENI PROMET $= \lambda \cdot T_H$

% blokiranja

A = STVARNI PROMET $= A_p \left[1 - p_N(B)\right]$

PROPAGACIJSKI UDJELI

OKOMURA-HATA

- NEDOPUNA S ANALITIČKIM IZRAZIMA
- BOLJI ZA RAČUNALNE PRORAČUNE

REZ BI TREBALI BITI JAKO SLIČNI

L_{50}
↳ jer je 50 kao medijan

OKOMURA MODEL

- ČISTO EMPIRIJSKI

$$\rightarrow L_{50}(dB) = \underbrace{L_F}_{\text{medijan PRIGUŠENJA}} + \overbrace{A(f,d)}^{*} - \underbrace{G(h_{TX})}_{**} \underbrace{- G(h_{RX})}_{\text{PRIGUŠENJE SLOB. PROST.}} - G_{AREA}$$

↳ KOREK. FAKTOR ZBOG OKOLIŠA

$*$ = MEDIJAN PRIGUŠENJA relativno prema slobodnom prostoru

$**$ = KOREKCIJSKI FAKTORI ZBOG VISINA ANTENA

(3) VELIKI GRAD, OKOMURA MODEL

$d = 10\,km$, $h_{TX} = 40\,m$, $h_{RX} = 2\,m$, $P_{TX} = 1000\,W$, $f = 2{,}1\,GHz$

$P_{RX} = ?$, $L_{50}(dB) = ?$

$$L_{50}(dB) = L_F + A(f,d) - G(h_{TX}) - G(RX) - G_{AREA}$$

6.6.2024.

12. tjedan - auditorne

(1) UKUPNO $2 \cdot 10^6$ STANOVNIKA

A, B, C → DAVATELJI USLUGA

A → 394 ĆELIJE, SVAKA 19 KANALA

B → 98 ĆELIJA, SVAKA 57 KANALA

C → 49 ĆELIJA, SVAKA 100 KANALA

$N_{A,UK}$

$N_{B,UK}$; $N_{UK} = ?$

$N_{C,UK}$

↳ BROJ KANALA (UKUPNI), NA - BROJ KANALA PO ĆELIJI

$p(B) = 2\%$

$\lambda = 2$ POZIVA PO SATU

$T_H = 3$ min prosječ. trajanje poziva

(A) → $A_p = \lambda \cdot T_H = 2 \cdot \frac{3}{60} = 0.1$ ERL PO KORISNIKU

$A_{p,UK} = 12.333$ ERL → promet ERL

↳ UKUPNI PODJAELNI PROMET PO ĆELIJI (tablica)

$N_A = \frac{A_{p,UK}}{A_p} = 123.33$ TJ. 123 KORISNIKA PO ĆELIJI

$N_{A,UK} = N_A \cdot 394 = 123 \cdot 394 = 48462$ (394 → ĆELIJE)

(B) $A_p = \lambda \cdot T_H = 0.1$ ERL

$A_{p,UK} = 46.816$ ERL (TABLICA)

$N_B = \frac{A_{p,UK}}{A_p} = 468.16$ KORISNIKA PO ĆELIJI $= 468$

$N_{B,UK} = 468 \cdot 98$ (98 → ĆELIJE)

$= 45864$ KORISNIKA UK.

(c) $A_N = 0.1$ ERL , $A_{N,UK} = 87.972$ ERL , $N_c = 879$ KORISNIKA PO ĆELIJI

(za 2% i 100)

$N_{c,UK} = 43071$ KORISNIKA

$N_{UK} = N_{A,UK} + N_{B,UK} + N_{C,UK} = 137\,397$ KORISNIKA

„ZASTUPLJENOST NA TRŽIŠTU" →

$A \rightarrow \frac{48462}{10^6 \cdot 2} = 2.42\%$

$B = \frac{45864}{2 \cdot 10^6} = 2.29\%$

$C = \frac{43071}{2 \cdot 10^6} = 2.15\%$

$\frac{137\,397}{10^6} = 6.87\%$

---

(2) $\lambda = 3$

$T_H = \frac{4}{60}$

1 ĆELIJA

a) $A_N = ?$

b) $N = ?$ $\pi(B) = 1\%$; 5 KANALA

↳ BROJ KORISNIKA (PO ĆELIJI)

c) $N+2$

$\pi(B) = ?$

NOVI

a) $A_N = \lambda \cdot T_H = 3 \cdot \frac{4}{60} = 0.2$ ERL

b) $A_{N,UK} = 1.361$ (TABLICA) , $N = \frac{A_{N,UK}}{A_N} = 6 \leftarrow (6.805)$

c) $N+2 = 8$

$A_{N,UK} = 8 \cdot A_N = 1.6$ ERL → $\pi(B) = 2\%$

TABLICA

$A_{N,UK} = N \cdot A_N$

PRIHVATLJIVO (ispod 5%)

5 KANALA, $\pi(B) = 2\%$ = 1.657 ERL

③ 57 KANALA

EIRP = 32 W

R = 10 km (KRUŽNICA)

$\pi(B) = 5\%$

$T_H = \frac{2}{60}\,h$, $\lambda = 2/h$

→ RASPODJELITI ĆELIJU U 4 NOVE ZA POVEĆAN KAPACITET

$P_{RX} = P_{TX} \cdot R^{-m}$ ($m = 4$, m = EKSPONENT PRIGUŠENJA)

---

a) $N = ?$, $A_N = \lambda \cdot T_H = 2 \cdot \frac{2}{60} = 0.067\ Erl$

$A_{N,UK} = 51.548\ Erl$ (TABLICA, 57, 5%)

$$N = \frac{A_{N,UK}}{A_N} = \frac{51.548\ \cancel{Erl}}{0.067\ \cancel{Erl}} = \underline{\underline{769}}$$

b) R_{max}, $P_{TX,max} = ?$

POČ. POVRŠINA $r^2\pi$

NOVA POVRŠINA $\left(\frac{r}{2}\right)^2 \pi = \frac{r^2}{4}\pi$

→ ZA ISTI SNR (dmaxa) na rubu ćelije mora biti jednaka (dmax) na rubu (ORIGINALNE ĆELIJE)

$$P_{TX1} \cdot R^{-4} = P_{TX2} \cdot \left(\frac{R}{2}\right)^{-4}$$

$$\hookrightarrow P_{TX2} = \frac{P_{TX1}}{16} \rightarrow \boxed{P_{TX2} = 2\,W}$$

c) KOLIKO KANALA ZA fleksib. STABILNOST?

(dijelenje) → ne mijenjamo broj kanala na ćelji

(4) 300 KANALA U SUSTAVU

7 ĆELIJA U GROZDU = N

$p(B) = 2\%$

a) $A_{p,uk} = ?$ BROJ KANALA PO ĆELIJI $= \frac{300}{7} = 42.85 = 42 =$

$A_{p,uk} = 32.836$ ERL (TABLICA, 42, 2%)

b) $\frac{S}{I} = 10 \log \left[\frac{\sqrt{3N}}{i_0}^{m} \right] dB = 18.66 dB$; $i_0 = 6$, $m = 4$, $N = 7$

$m = 4$

$i_0 = 6$

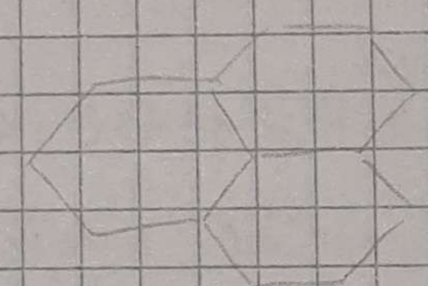
→ BROJ SMETAJUĆIH ODAŠILJAČA

NE KOD NJIH 7 TO JE $i_0 \gg 6$

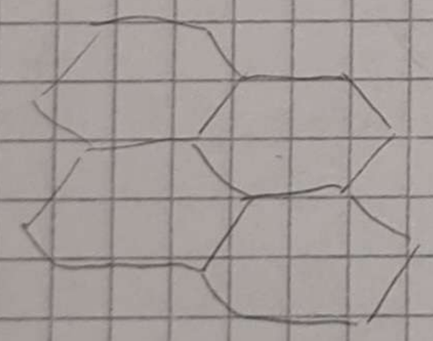
c) 3 ĆELIJE U GROZDU

4 ĆELIJE U GROZDU

→ SEKTORI 120° i 60°



BROJ KANALA PO ĆELIJI → $\frac{300}{3} = 100$



120° → 3 SEKTORA PO ĆELIJI

60° → 6 SEKTORA

$\frac{300}{4} = 75$

BROJ KANALA PO SEKTORU

120° → 3 S PO Ć	60° → 6 S PO Ć
$\left\lfloor \frac{75}{3} \right\rfloor = 25$	$\left\lfloor \frac{75}{6} \right\rfloor = 12$
$A_p = 17.505$ ERL	$A_p = 6.615$ ERL
$A_{p,uk} = 17.505 \cdot 3$	$A_{p,uk} = 6.615 \cdot 6$
$A_{p,uk} = 52.515$ ERL	$A_{p,uk} = 39.69$ ERL

$\frac{100}{3} = 33.33 \Rightarrow 33$	$\frac{100}{6} = 16$
$A_p = 24.626$ ERL	$A_p = 9.828$ ERL
$A_{p,uk} = 24.626 \cdot 3$	$A_{p,uk} = 9.828 \cdot 6$
$= 73.38$ ERL	$= 58.968$ ERL

13. 6. 2024.

13. tjedan - auditorne

$\boxed{\text{MIMO}}$

↳ povećanje pouzdanosti, brzine, područja pokrivanja

SISO → $y = h \cdot x + n$

y ↑ IZLAZ; x ↳ ULAZNI SIGNAL; n ↳ ŠUM

korak. karala
→ ŽELIMO DA nisu KORELIRANI
↓
ortogonalni

MIMO → 2×2

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

↓

→ problem korelacije vrijednosti u matrici H

→ želimo da su vrij. samo na dijagonali (DIJAGONALIZACIJA)

mat H
↳ prijenosna matr.

↳ rastav na singularne vrij.

PRISJETI SE POSTUPKA ZA PITANJA NI

$$A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$$

$$A = U \Sigma V^T$$

1) $$A^T \cdot A = \begin{bmatrix} 4 & 3 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 25 & -15 \\ -15 & 25 \end{bmatrix}$$

2) SVOJSTVENE VRIJ. i VEKTORI

$$A\vec{v} = \lambda \vec{v}$$

matrica · vektor = skalar · vektor

JEDINIČNA MAT

$$A\vec{v} = (\lambda \cdot I)\cdot \vec{v}$$

$$(A - \lambda I)\cdot \vec{v} = 0$$

$$\det(A^T \cdot A - \lambda I) = 0 \rightarrow \det\left(\begin{bmatrix} 25-\lambda & -15 \\ -15 & 25-\lambda \end{bmatrix}\right) = 0$$

$$(25-\lambda)^2 - 15^2 = 0$$

$$= \lambda^2 - 50\lambda + 400 = 0$$

SVOJSTVENE VRIJEDNOSTI $\rightarrow$ $\lambda_1 = 10$ $\quad$ $\lambda_2 = 40$

$\lambda_1 = 10$

$$A^T A - 10 \cdot I = \begin{bmatrix} 25 & -15 \\ -15 & 25 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 15 & -15 \\ -15 & 15 \end{bmatrix}$$

$\rightarrow$ SVESTI NA "row-echelon" formu $\rightarrow$ $\begin{bmatrix} 15 & -15 \\ -15 & 15 \end{bmatrix} + \rightarrow \begin{bmatrix} 15 & -15 \\ 0 & 0 \end{bmatrix} /:15$

$$\rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 - v_2 = 0 \rightarrow \boxed{v_1 = v_2}$

$0v_1 + 0v_2 = 0$

UOBIČAJNA KONVENCIJA
$\hookrightarrow$ UZMI 1

$1 = 1 = v_1 = v_2$

SVOJSTVENE VEKTORE TU TRAŽIMO

→ JEDINIČNA FORMA

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

PODIJELIO S NORMOM

$$\sqrt{1^2+1^2} = \sqrt{2}$$

$$\lambda_2 = 40$$

$$A^T A - 40 \cdot I = \begin{bmatrix} -15 & -15 \\ -15 & -15 \end{bmatrix} -$$

$$\begin{bmatrix} -15 & -15 \\ 0 & 0 \end{bmatrix} /:-15 \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow v_1 + v_2 = 0 \rightarrow v_1 = -v_2$$

SVOJSTVENI VEKTOR $\rightarrow$ $-1 = v_1$, $+1 = v_2$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} \end{bmatrix}$$

TU UPIŠEŠ VRIJ. ZA VEĆI λ

$$V = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \Leftarrow \text{SVOJSTVENA VRIJ.}$$

ZA $\lambda = 40$ ZA $\lambda = 10$

3)

Σ = matrica singularnih vrij.,

KORIJENI IZ SVOJSTVENIH VRIJ.

$$\Sigma = \begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{40} \end{bmatrix}$$

velika vrij

(RA $\lambda = 40$)

4) $U = ?$ $\quad / \cdot V, \cdot \Sigma^{-1}$

$A = U \Sigma V^T$; $V = V^T$ vrijedi

$A V \Sigma^{-1} = U$

X^{-1} = INVERZ MATRICE

$$X = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow X^{-1} = \frac{1}{\det X} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

$$U = \begin{bmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$$

$$\Sigma^{-1} = \frac{1}{\sqrt{40} \cdot \sqrt{40} - 0} \cdot \begin{bmatrix} \sqrt{40} & -0 \\ -0 & \sqrt{40} \end{bmatrix}$$

$$\Sigma^{-1} = \frac{1}{20} \begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{40} \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} \frac{\sqrt{10}}{20} & 0 \\ 0 & \frac{\sqrt{10}}{10} \end{bmatrix}$$

$A = U \Sigma V^T$

$$A = \underbrace{\begin{bmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{V^T}$$

→ TAK SE RADI TA DIJAGONALIZACIJA

→ TAK SU ONDA NEOVISNI

→ smanjuje rasipanje energije

NI,

5G TEHNOLOGIJE

Što je novo, koji je napredak, nacrtaj

→ male ćelije → mala valna duljina, visoka f

→ masivni MIMO → biti će stotinjak nezavisnih kanala

→ beamforming → usmjerenje prema korisniku

→ OBLIKOVANJE DIJAGRAMA ZRAČENJA

→ mm valovi su bitni

→ virtualizacija mrežnih