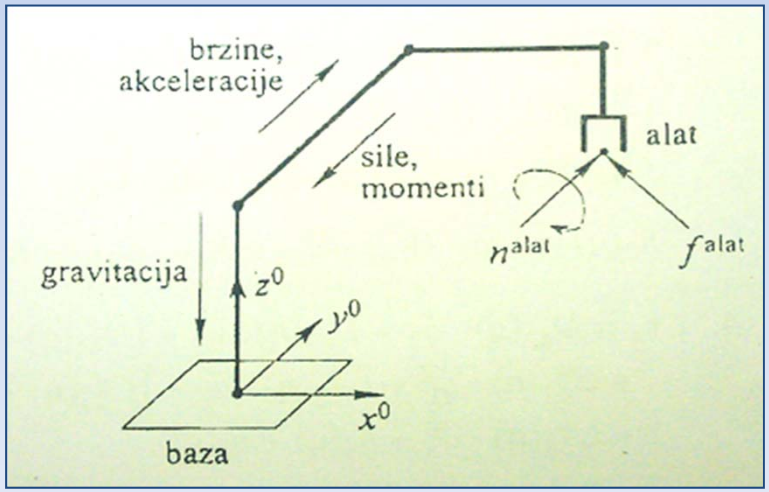
Sveučilište u Zagrebu
Fakultet elektrotehnike i računarstva
Zavod za automatiku i računalno inženjerstvo

OSNOVE ROBOTIKE

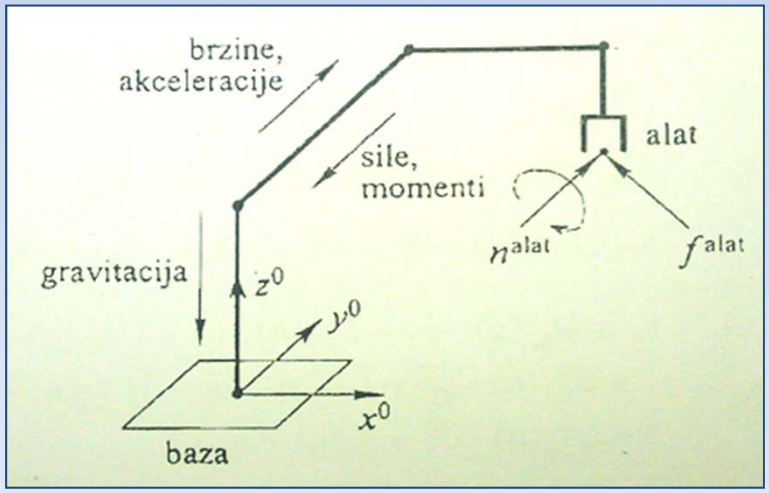
3. domaća zadaća, grupa D

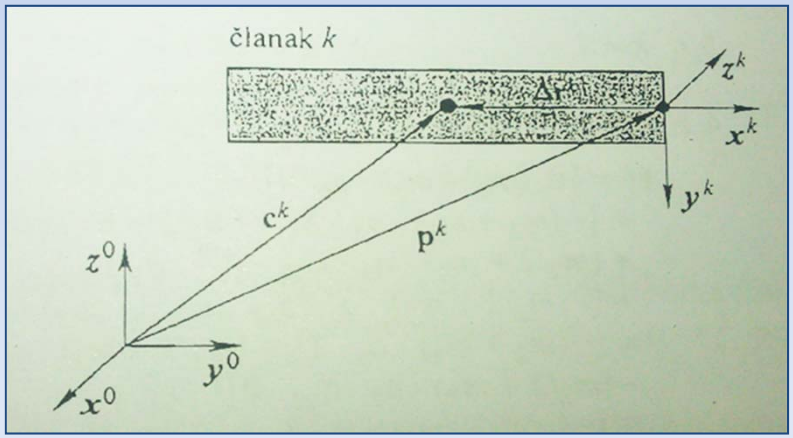
Newton-Eulerov dinamički model: direktna dinamika



- $\omega_k = \omega_{k-1} + \xi_k \cdot \dot{q}_k \cdot z_{k-1}$
- $\dot{\omega}_k = \dot{\omega}_{k-1} + \xi_k \cdot \{\ddot{q}_k \cdot z_{k-1} + \omega_{k-1} \times (\dot{q}_k \cdot z_{k-1})\}$
- $\Delta s_k = p_k - p_{k-1}$
- $v_k = v_{k-1} + \omega_k \times \Delta s_k + (1 - \xi_k) \cdot \dot{q}_k \cdot z_{k-1}$
- $\dot{v}_k = \dot{v}_{k-1} + \dot{\omega}_k \times \Delta s_k + \omega_k \times (\omega_k \times \Delta s_k) + (1 - \xi_k)\{\ddot{q}_k \cdot z_{k-1} + 2 \cdot \omega_k \times [\dot{q}_k \cdot z_{k-1}]\}$
- $\xi_k = 1$ => rotacijski članak, $\xi_k = 0$ => translacijski članak.

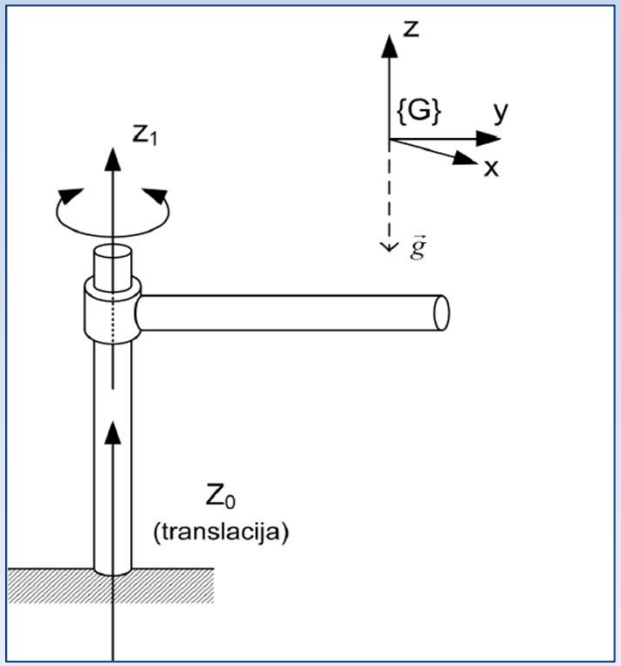
Newton-Eulerov dinamički model: inverzna dinamika

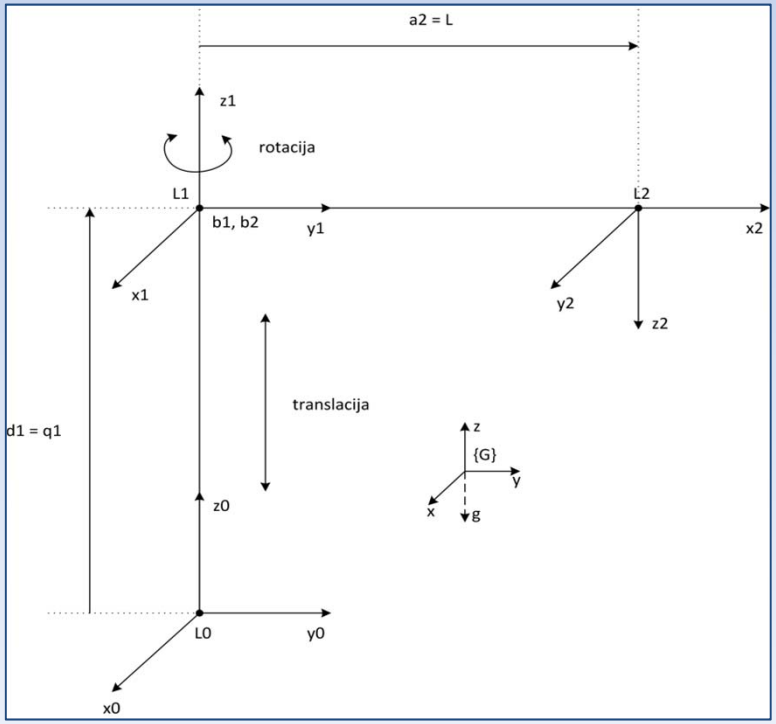




- $\Delta r_k = c_k - p_k$
- $f_k = f_{k+1} + m_k \cdot [\dot{v}_k + \dot{\omega}_k \times \Delta r_k + \omega_k \times (\omega_k \times \Delta r_k)]$
- $n_k = n_{k+1} + (\Delta s_k + \Delta r_k) \times f_k - \Delta r_k \times f_{k+1} + D_k \cdot \dot{\omega}_k + \omega_k \times (D_k \cdot \omega_k)$
- $\tau_k = \xi_k \cdot (n_k)^T \cdot z_{k-1} + (1 - \xi_k) \cdot (f_k)^T \cdot z_{k-1} + b_k(\dot{q}_k)$

Zadatak: Konfiguracija TR





D-H parametri

θ	d	a	α
0	q_1	0	0
q_2	0	L	$-\pi$

Zadatak: Konfiguracija TR

Matrice transformacija

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} \cos(q_2) & \sin(q_2) & 0 & L \cdot cos(q_2) \\ \sin(q_2) & -\cos(q_2) & 0 & L \cdot \sin(q_2) \\ 0 & 0 & -1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Tenzori inercije

$$D_{c1} = m_1 \cdot \frac{q_1^2}{12} \cdot diag(1,1,0)$$

$$D_{c2} = m_2 \cdot \frac{L^2}{12} \cdot diag(0,1,1)$$

Položaji centara masa članaka u odnosu na L_k

$$\Delta c_1 = \begin{bmatrix} 0 & 0 & -\frac{q_1}{2} & 1 \end{bmatrix}^T$$

$$\Delta c_2 = \begin{bmatrix} -\frac{L}{2} & 0 & 0 & 1 \end{bmatrix}^T$$

Zadatak: Konfiguracija TR

1. Početni uvjeti:

$\boldsymbol{T_0^0 = I}$, $\boldsymbol{v_0 = 0}$, $\boldsymbol{\dot{v}_0} = [0 \quad 0 \quad g]^T$, $\boldsymbol{\omega_0 = 0}$, $\boldsymbol{\dot{\omega}_0 = 0}$, k=1

2. Brzine i akceleracije članaka

a) k=1, prvi članak, $\xi_1 = 0$

$z_0 = R_0^0 \cdot i_3 = [0 \quad 0 \quad 1]^T$
$\omega_1 = [0 \quad 0 \quad 0]^T$
$\dot{\omega}_1 = [0 \quad 0 \quad 0]^T$
$\Delta s_1 = H_1 \cdot (T_0^1 - T_0^0) \cdot i_4 = [0 \quad 0 \quad q_1]^T$
$\dot{v}_1 = [0 \quad 0 \quad \ddot{q}_1 + g]^T$

Zadatak: Konfiguracija TR

2. Brzine i akceleracije članaka

b) k=2, drugi članak, $\xi_2 = 1$

$$z_1 = R_0^1 \cdot i_3 = [0 \quad 0 \quad 1]^T$$

$$\omega_2 = [0 \quad 0 \quad \dot{q}_2]^T$$

$$\dot{\omega}_1 = [0 \quad 0 \quad \ddot{q}_2]^T$$

$$\Delta s_2 = H_1 \cdot (T_0^2 - T_0^1) \cdot i_4 = [L \cdot \cos(q_2) \quad L \cdot \sin(q_2) \quad 0]^T$$

$$\dot{v}_2 = \begin{bmatrix} -L \cdot cos(q_2) \cdot \dot{q}_2^2 - L \cdot \ddot{q}_2 \cdot sin(q_2) \\ -L \cdot sin(q_2) \cdot \dot{q}_2^2 + L \cdot \ddot{q}_2 \cdot cos(q_2) \\ \ddot{q}_1 + g \end{bmatrix}$$

Zadatak: Konfiguracija TR

3. Rubni uvjeti u vrhu alata

$f_3 = 0, n_3 = 0, k = 2$

4. Sile i momenti članaka

a) k=2, drugi članak, $\xi_2 = 1$

$$\Delta r_2 = H_1 \cdot T_0^2 \cdot (\Delta c_2 - i_4) = \left[-\frac{L}{2}\cos(q_2) \quad -\frac{L}{2}\sin(q_2) \quad 0\right]^T$$

$$f_2 = \begin{bmatrix} -\frac{L \cdot m_2}{2} \cdot (\cos(q_2) \cdot \dot{q}_2^2 + \sin(q_2) \cdot \ddot{q}_2) \\ -\frac{L \cdot m_2}{2} \cdot (\sin(q_2) \cdot \dot{q}_2^2 - \cos(q_2) \cdot \ddot{q}_2) \\ m_2 \cdot (\ddot{q}_1 + g) \end{bmatrix}$$

$$D_2 = R_0^2 \cdot D_{c2} \cdot (R_0^2)^T = \frac{m_2 \cdot L^2}{12} \cdot \begin{bmatrix} \sin(q_2)^2 & -\sin(q2) \cdot \cos(q_2) & 0 \\ -\sin(q2) \cdot \cos(q_2) & \cos(q_2)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Zadatak: Konfiguracija TR

4. Sile i momenti članaka

a) k=2, drugi članak, $\xi_2 = 1$

$$n_2 = \begin{bmatrix} \frac{m_2 L}{2} \cdot \sin(q_2) \cdot (\ddot{q}_1 + g) \\ -\frac{m_2 L}{2} \cdot \cos(q_2) \cdot (\ddot{q}_1 + g) \\ \frac{m_2 L^2}{3} \cdot \ddot{q}_2 \end{bmatrix}$$

$$\tau_2 = \frac{m_2 L^2}{3} \cdot \ddot{q}_2$$

Zadatak: Konfiguracija TR

4. Sile i momenti članaka

b) k=1, prvi članak, $\xi_1 = 0$

$$\Delta r_1 = H_1 \cdot T_0^1 \cdot (\Delta c_1 - i_4) = \left[0 \quad 0 \quad -\frac{q_1}{2}\right]^T$$

$$f_1 = \begin{bmatrix} -\frac{L \cdot m_2}{2} \cdot (\cos(q_2) \cdot \dot{q}_2^2 + \sin(q_2) \cdot \ddot{q}_2) \\ -\frac{L \cdot m_2}{2} \cdot (\sin(q_2) \cdot \dot{q}_2^2 - \cos(q_2) \cdot \ddot{q}_2) \\ (m_1 + m_2) \cdot (\ddot{q}_1 + g) \end{bmatrix}$$

$$D_1 = R_0^1 \cdot D_{c1} \cdot (R_0^1)^T = \frac{m_1 q_1^2}{12} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Zadatak: Konfiguracija TR

4. Sile i momenti članaka

b) k=1, prvi članak, $\xi_1 = 0$

$$n_1 = \begin{bmatrix} \frac{Lm_2q_1}{2} \cdot (\sin(q_2) \cdot \dot{q}_2^2 - \ddot{q}_2 \cdot \cos(q_2)) + \frac{Lm_2}{2} \cdot \sin(q_2) \cdot (\ddot{q}_1 + g) \\ -\frac{Lm_2q_1}{2} \cdot (\cos(q_2) \cdot \dot{q}_2^2 + \ddot{q}_2 \cdot \sin(q_2)) - \frac{Lm_2}{2} \cdot \cos(q_2) \cdot (\ddot{q}_1 + g) \\ \frac{m_2L^2}{3} \cdot \ddot{q}_2 \end{bmatrix}$$

$$\tau_1 = (m_1 + m_2) \cdot (\ddot{q}_1 + g)$$

DODATAK: Matlab skripte i funkcije

dh.m – Skripta za određivanje matrica transformacija

```
syms theta_k alfa_k a_k d_k

q1 = sym('q1','real');
q2 = sym('q2','real');
L  = sym('L','real');

theta = [ 0,        q2];
alfa  = [ 0,  sym(-pi)];
d     = [q1,         0];
a     = [ 0,         L];

T_k = [ ...
        cos(theta_k), -cos(alfa_k)*sin(theta_k),  sin(alfa_k)*sin(theta_k), a_k*cos(theta_k) ;

        sin(theta_k),  cos(alfa_k)*cos(theta_k), -sin(alfa_k)*cos(theta_k), a_k*sin(theta_k) ;

                   0,               sin(alfa_k),               cos(alfa_k),              d_k ;

                   0,                         0,                         0,                1 ;
      ];

T_00 = sym(eye(4));
T_01 = subs(T_k, [theta_k, d_k, a_k, alfa_k], [theta(1), d(1), a(1), alfa(1)]);
T_12 = subs(T_k, [theta_k, d_k, a_k, alfa_k], [theta(2), d(2), a(2), alfa(2)]);
T_02 = T_01 * T_12;
```

DODATAK: Matlab skripte i funkcije

NewtonEulerForward.m – Funkcija za računanje direktne dinamike

```
function [zk_1 wk wkd dsk vkd] = NewtonEulerForward(T0k, T0k_1, wk_1, wk_1d, ksik, vk_1d, k)

H1 = [eye(3) zeros(3,1)];

zk_1 = T0k_1(1:3,1:3)*[0 0 1]';

wk = wk_1 + ksik*eval(['sym(''q' mat2str(k) 'd'',''real'')'])*zk_1;

wkd = wk_1d + ksik*(eval(['sym(''q' mat2str(k) 'dd'',''real'')'])*zk_1+...
      cross(wk_1,eval(['sym(''q' mat2str(k) 'd'',''real'')'])*zk_1));

dsk = H1*(T0k-T0k_1)*[0 0 0 1]';

vkd = vk_1d + cross(wkd,dsk) + cross(wk, cross(wk,dsk)) + ...
      (1-ksik)*(eval(['sym(''q' mat2str(k) 'dd'',''real'')'])*zk_1 + ...
      2*cross(wk, eval(['sym(''q' mat2str(k) 'd'',''real'')'])*zk_1));

end
```

DODATAK: Matlab skripte i funkcije

NewtonEulerInverse.m – Funkcija za računanje inverzne dinamike

```
function [fk, nk, tauk] = NewtonEulerInverse(T0k, wk, wkd, ksik, vkd, fkp1, nkp1, dck, dsk, Dk_, zk_1, k)

H1 = [eye(3) zeros(3,1)];

deltaRk = H1 * T0k*(dck-[0 0 0 1]');

fk = fkp1 + eval(['sym(''m' mat2str(k) ''',''real'')'])*(vkd + cross(wkd,deltaRk) + cross(wk,cross(wk,deltaRk)));
fk = simple(fk);

Dk = T0k(1:3,1:3) * Dk_ * T0k(1:3,1:3)';

nk = nkp1 + cross(dsk+deltaRk, fk) - cross(deltaRk,fkp1) + Dk*wkd + cross(wk,Dk*wk);
nk = simple(nk);

tauk = ksik*nk'*zk_1 + (1-ksik)*fk'*zk_1;
tauk = simple(tauk);


end
```

DODATAK: Matlab skripte i funkcije

dz3.m – Skripta koja se koristi prethodnim funkcijama za određivanje din. modela robota

```
% Prvi clanak translacijski, drugi rotacijski
ksi = [0 1];
% Racunamo potrebne matrice transformacija
dh
% Simbolicke definicije pojedinih parametara
g  = sym( 'g','real'); % gravitacijsko ubrzanje, g0 = 9.80665 m/s^2
m1 = sym('m1','real'); % masa prvog clanka
m2 = sym('m2','real'); % masa drugog clanka
% Pocetni uvjeti za direktnu dinamiku
v0  = [0 0  0]';
v0d = [0 0  g]';
w0  = [0 0  0]';
w0d = [0 0  0]';
% Pocetni uvjeti za inverznu dinamiku
f3  = [0 0  0]';
n3  = [0 0  0]';
% Tenzori inercije clanaka u odnosu na koordinatne sustave L1, tj. L2
D1_ = m1 * q1^2/12 * diag([1 1 0]);
D2_ = m2 * L^2/12 * diag([0 1 1]);
% Položaji centara masa clanaka u odnosu na koordinatne sustave L1, tj. L2
dc1 = [0 0 -q1/2 1]';
dc2 = [-L/2 0 0 1]';

[z0 w1 w1d ds1 v1d] = NewtonEulerForward(T_01, T_00, w0, w0d, ksi(1), v0d, 1);
[z1 w2 w2d ds2 v2d] = NewtonEulerForward(T_02, T_01, w1, w1d, ksi(2), v1d, 2);

[f2, n2, tau2] = NewtonEulerInverse(T_02, w2, w2d, ksi(2), v2d, f3, n3, dc2, ds2, D2_, z1, 2);
[f1, n1, tau1] = NewtonEulerInverse(T_01, w1, w1d, ksi(1), v1d, f2, n2, dc1, ds1, D1_, z0, 1);
```