Nasp - MI (Bože pomozi)

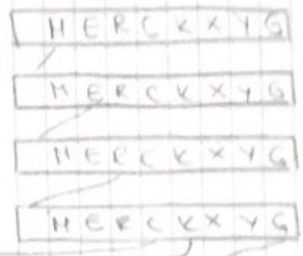
MI 2014

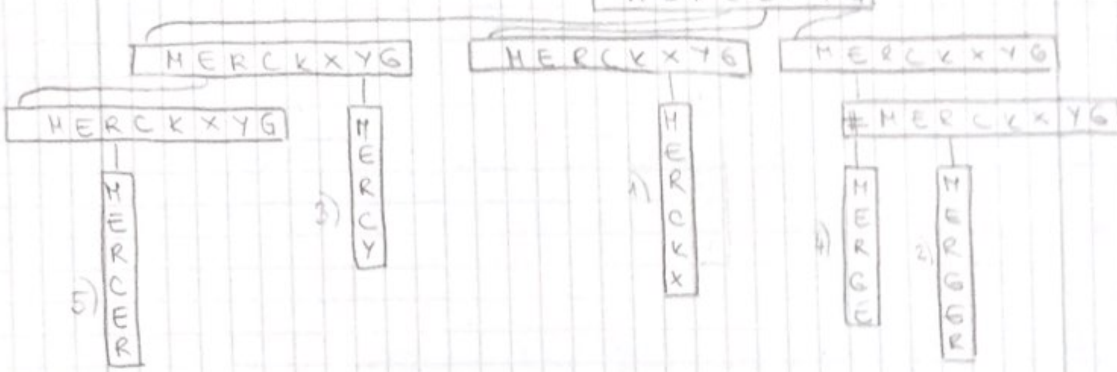
STRUKTURA TRIE

- za pretraživanje se koriste samo dijelovi ključa pohranjenih podataka
- 2 vrste čvorova:
  - unutarnji = samo kodovi (svi djelovi svih ključeva) i pokazivači
  - listovi = podaci
- visina = najduža riječi
- ne ovisi o redoslijedu upisa podataka

(1)

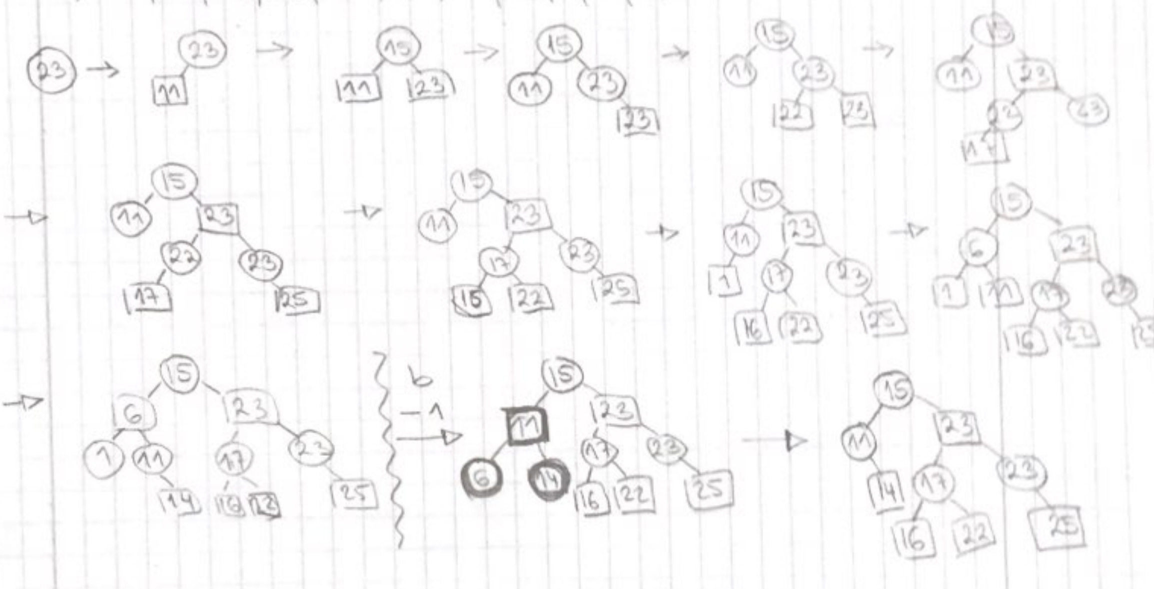
Razine:	1	2	3	4	5	6
1)	M	E	R	C	K	X
2)	M	E	R	C	E	R
3)	M	E	R	C	Y	
4)	M	E	R	G	E	
5)	M	E	R	G	E	R





(2) C-B

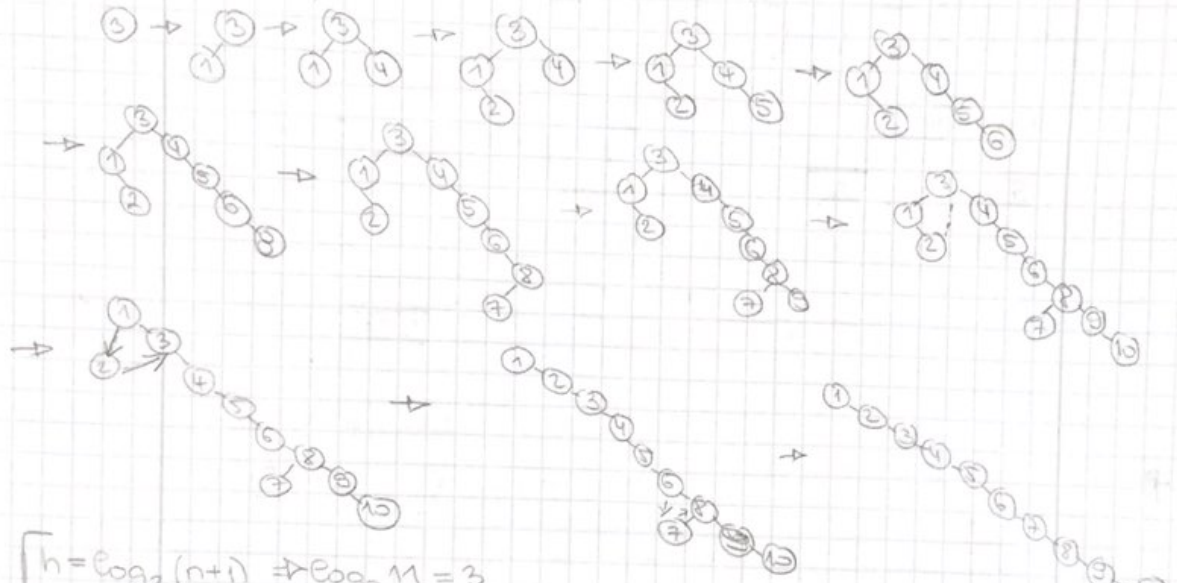
a) 23, 11, 15, 23, 22, 17, 25, 16, 1, 6, 14



③ DSW - daje savršeno uravnoteženo stablo

MI 2015

Ⓝ 3, 1, 4, 2, 5, 6, 8, 7, 9, 10



$h = \lceil \log_2(n+1) \rceil \Rightarrow \lceil \log_2 11 \rceil = 3$

$k = 2^3 - 1 = 7$ → broj čvorova u punim razinama

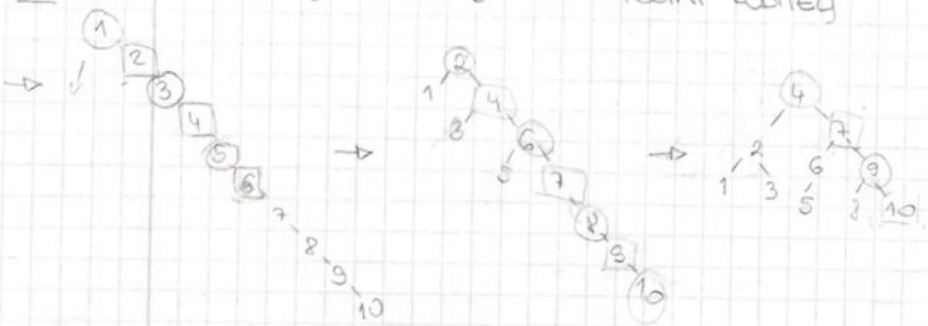
$n - k$ rotacija ulijevo za svaki dr. čvor od vrha.

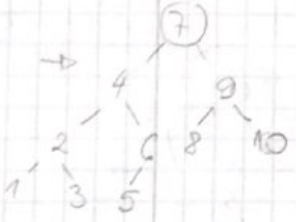
dok ($k > 1$)

$k = k/2$

k rot ulijevo za svaki drugi čvor od vrha

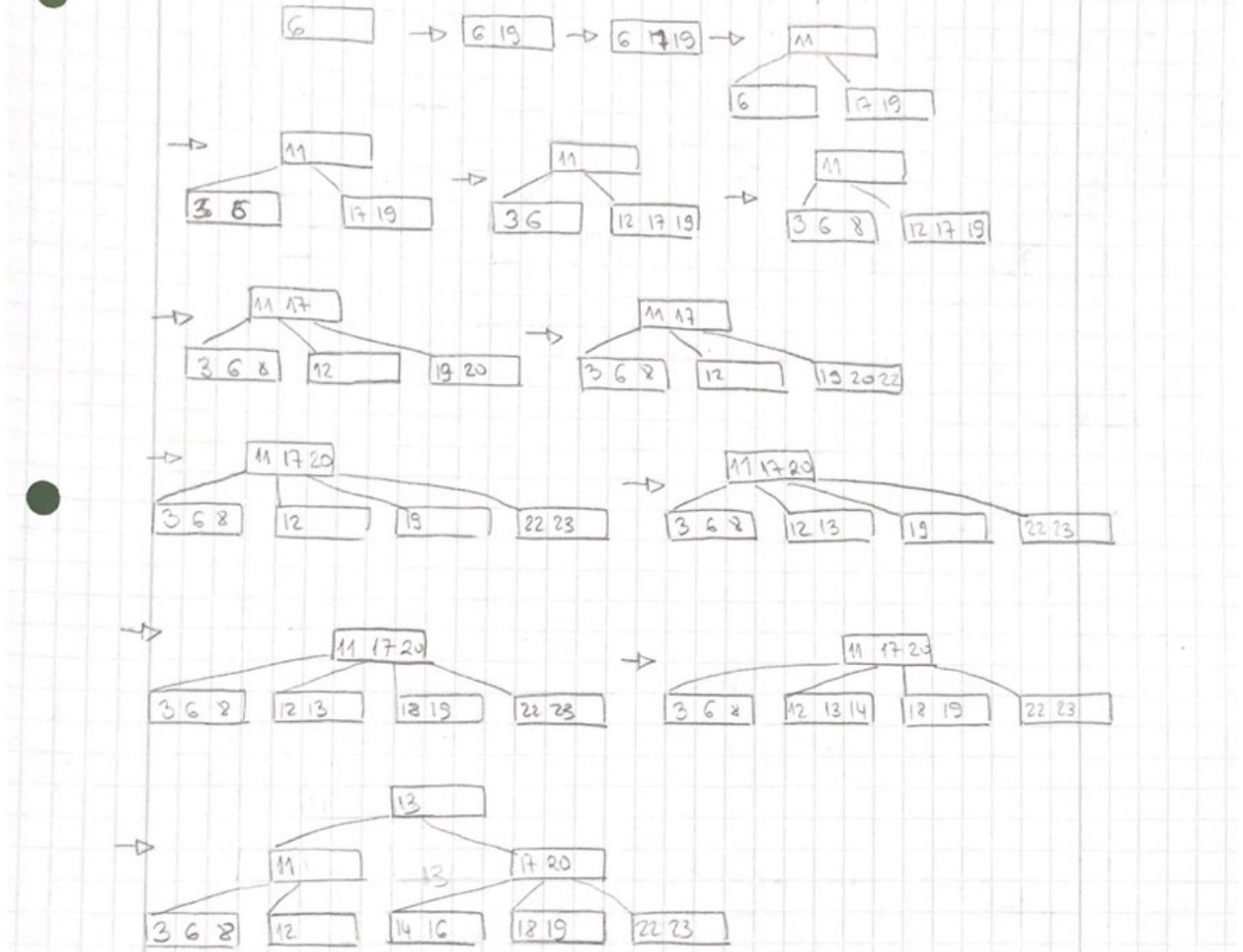
○ → RODITELJ □ → DITE KOJE MORA POSTAT RODITELJ





② B-stablo 4. reda

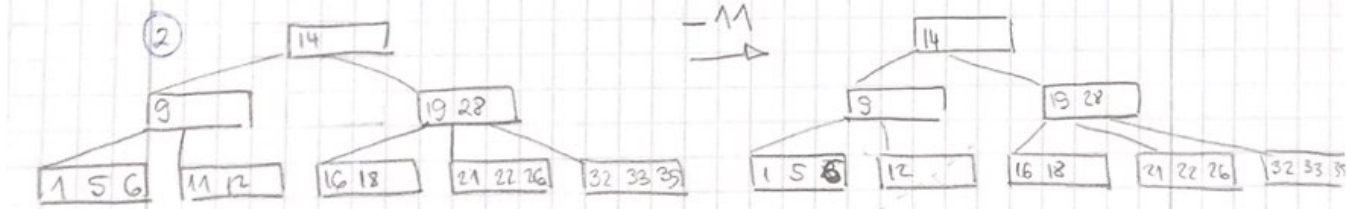
6, 19, 17, 11, 3, 12, 8, 20, 22, 23, 13, 18, 14, 16

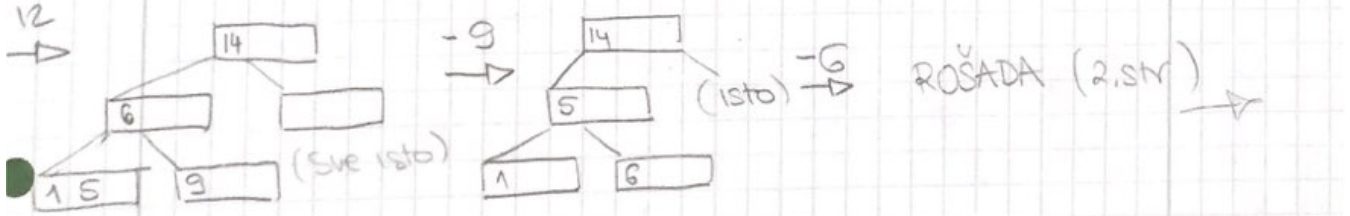


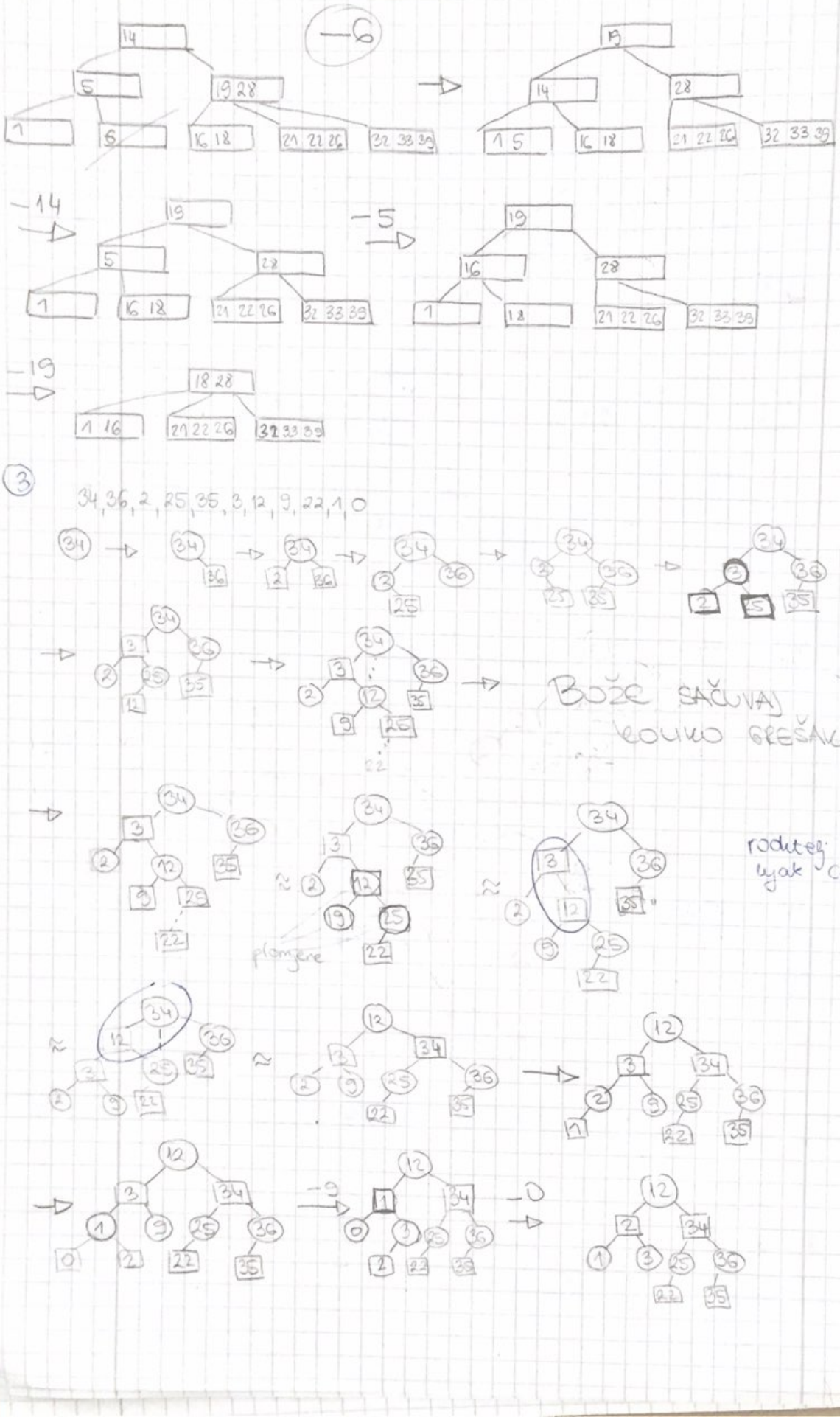
HI 2016

① DA

②

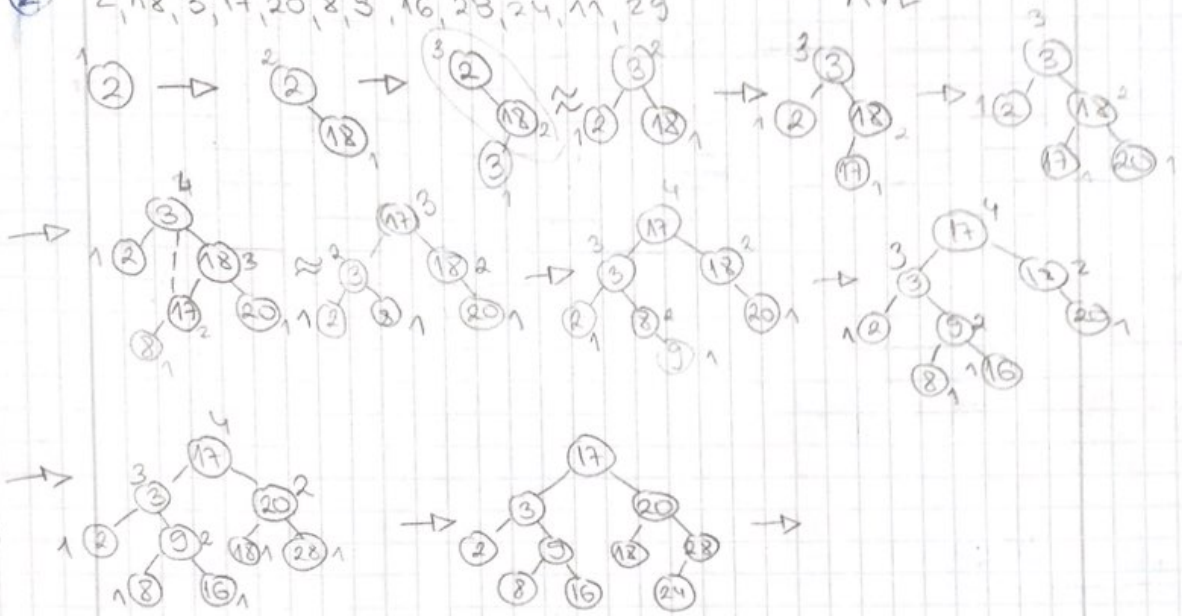




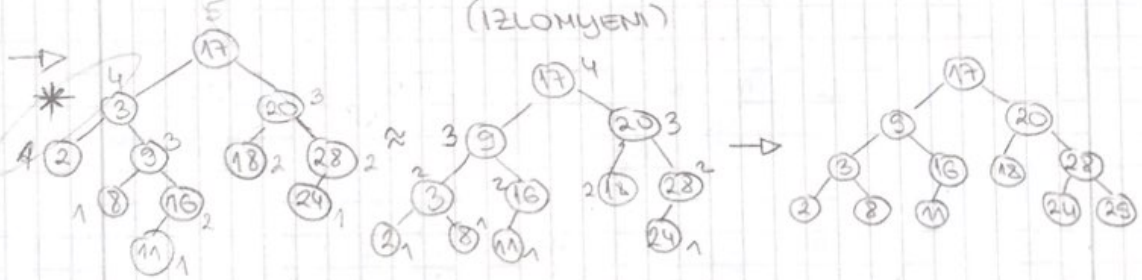

-6
-14
-5
-19
3
34,36,2,25,35,3,12,9,22,1,0
BOŽE SAČUVAJ KOLIKO GREŠAKA
roditelj crven
uyak crn
plamjere

MI 2017

② 2, 18, 3, 17, 20, 8, 9, 16, 28, 24, 11, 29 AVL

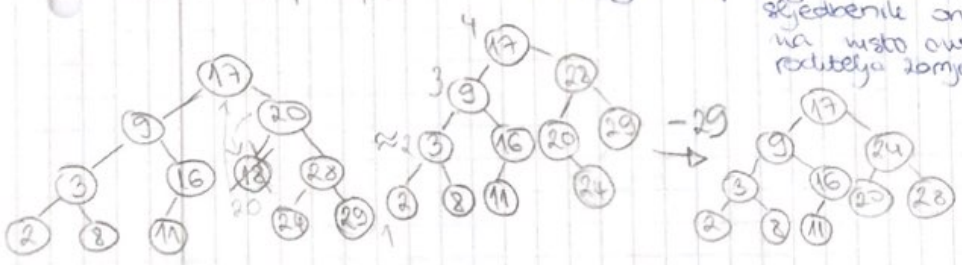


(IZLOMJENI)

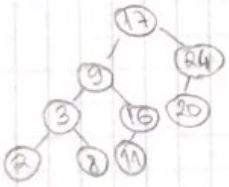


BRIŠI 18, 29, 28

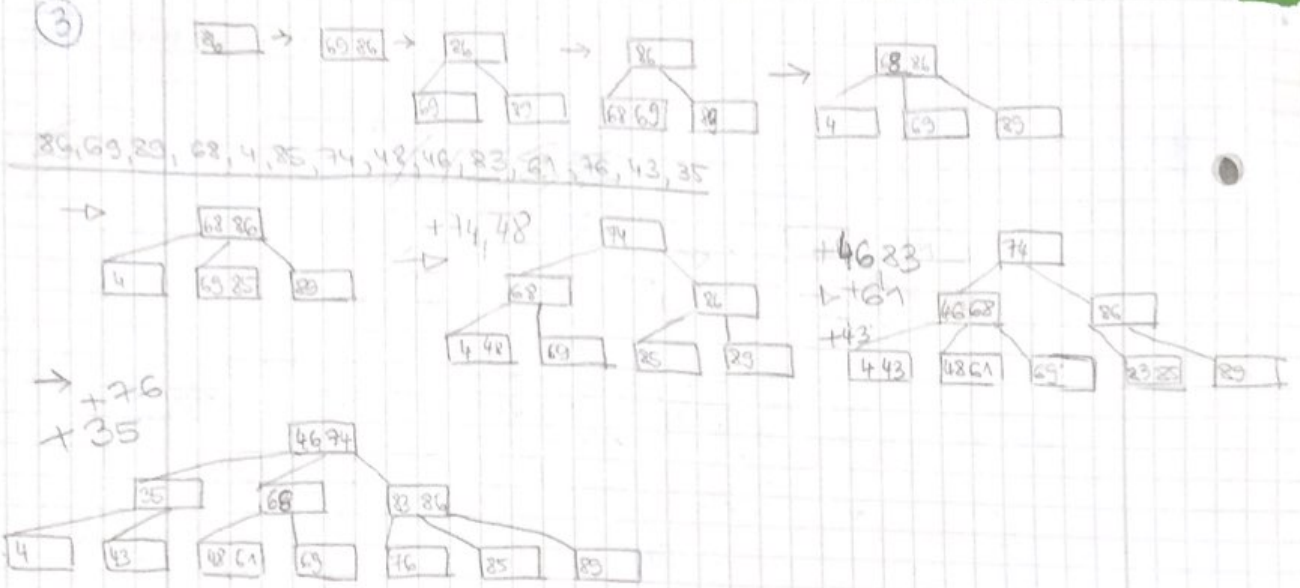
•Brisanje kopiranjem ⇒ zamjenski (prethodnik ili sljedbenik onog koji se briše, stavit ga na mjesto onog koji se briše, pokazivač roditelja zamjenskog na djeda mu



-28



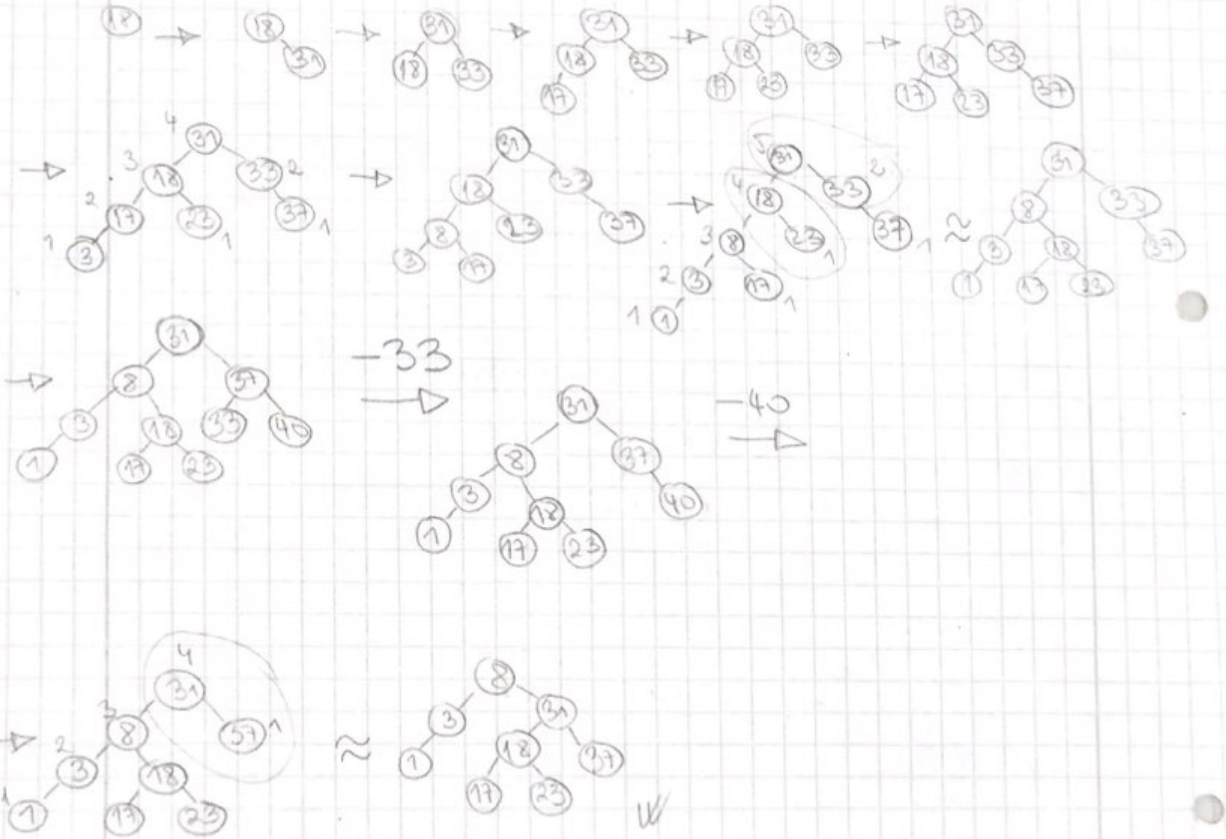
③



MI 2018

① DA (ali isto pitanje ko 2016?)

② AVL : 18, 31, 33, 17, 23, 37, 3, 2, 1, 40



Neuronske mreže:

→ FEEDFORWARD ARH
↳ podaci putuju od ulaza prema

→ 3 osnovna slaja u mreži: ulazni, skriveni i izlazni
① ② ③

① broj neurona odgovara broju ulazni var.
② Proizvolni broj podsloyeva (prema izlazu sve manje neurona)
③ broj neurona odgovara broju izlaznih var.

NEURON → mapa jednostavnog preslikavanja
→ više ulaza, 1 izlaz
→ akt. fja - izlazna fja:
- Adaline $f(z)=z$
- sigmoid $\sigma(z)=1/(1+e^{-z})$
- tanh $f(z)=2\sigma(2z)-1$
- ReLU $f(z)=\max(z,0)$

Adaline: $y=f\left(\sum w_i x_i\right)$

$y=w^T x = x^T w$

→ uyežbavanje: $\min \frac{1}{2}\sum\left(y_i - y_{d_{ji}}\right)^2 = \frac{1}{2}\sum_{i=1}^{p} e_i^2$

$= \frac{1}{2}\|e\|^2$

* želimo savršeno uyežbavanje t.d $y=y_d$

$Ax=b$

→ 3 slučaja: $p=n$ → jedinstveno rješenje
$p<n$ → beskonačno rješenja
$p>n$ → postoj max 1 rješenje (točno)

↳ $\|b-Ax\|$

↳ bira se $\|x\|$ (tj. najmanje norme)

→ $x=A^+b$

Gradijentna metoda optimizacije:

$\nabla E(\vec{w}) = \sum e_i \nabla e_i$

↳ skupno uyežbavanje: EPOHA - jedno izračunavanje gradijenta ciljne fje

$\nabla E(\vec{w}) \approx p \cdot e_i \cdot \nabla e_i$

↳ iterativno uyežbavanje:
→ približno računanje gradijenta na temelju samo jedne točke skupa

$W_{k+1} = W_k - \alpha_k e_i \nabla e_i$

ADALINE KORAČNO

↳ $e_i = X_{d,i}^T \cdot W - y_{d,i}$

$\nabla e_i = x_{d,i}$

$W_{k+1} = W_k + \alpha_k e_i^{(k)} x_{d,i}$

pr. (slajd 77)

$X_d^T \cdot W = y_d$

→ zatvorena forma $(X_d^T)^{-1} \cdot y_d$

$p > n$:

$W = (X_d X_d^T)^{-1} X_d y_d = \begin{bmatrix} 0.5833 \\ 0 \end{bmatrix}$

Gradijentna metoda

$X_d (X_d^T \cdot \vec{w} - y_d)$

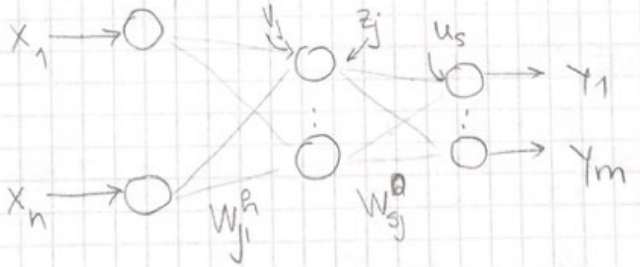
Vježbavanje cijele mreže:

2 faze: - učenje

- obrada podataka

Podaci: - trening skup
- validacijski skup
- testni skup

- n – ulazi $i = 1, \dots n$
- m – izlazi $s = 1, \dots m$
- ℓ – neurona skrivenog sloja $j = 1, \dots \ell$
- f_n – akt. fja skrivenog sloja
- f_b – " izlaznog "
- w_{ji}^h – težine ulazni u skriveni
- w_{sj}^o – težine skrivenog u izlazni sloj



$$v_j = \sum_{i=1}^{n} w_{ji}^h x_i \qquad z_j = f_j^h\left(\sum_{i=1}^{n} w_{ji}^h x_i\right) = f_j^h(v_j)$$

MI 2018.

⑤

	A	B	C	D	E	F
c	6	5	4	3	1	2
v	1200	1100	900	1000	400	800

:100

	A	B	C	D	E	F
1	0	0	0	0	4	4
2	0	0	0	0	4	4
3	0	0	0	10	10	12
4	0	0	9	10	14	14
5	0	11	11	11	14	18
6	12	12	12	12	15	22
7	12	12	12	19	19	22
8	12	12	12	21	23	23
9	12	12	20	23	25	27
10	12	12	21	22	26	31
11	12	23	23	23	26	33
12	12	23	23	30	30	34

C=12

A,D,E,F 34

MI 2017

①

	A	B	C	D	E	F
c	4	3	1	9	12	5
v	11	6	2	18	30	8

	E	D	F	A	B	C
1	0	0	0	0	0	2
2	0	0	0	0	0	2
3	0	0	0	0	6	6
4	0	0	0	11	11	11
5	0	0	8	11	11	13
6	0	0	8	11	11	13
7	0	0	8	11	17	17
8	0	0	8	11	17	19
9	0	18	18	19	19	19
10	0	18	18	19	19	21
11	0	18	18	19	19	21
12	30	30	30	30	30	30
13	30	30	30	30	30	32
14	30	30	30	30	30	32
15	30	30	30	30	36	36
16	30	30	30	41	41	38
17	30	30	38	41	41	43
18	30	30	38	41	41	43
19	30	30	38	41	47	47

47 → B A E

MI 2016

⑥

	A	B	C	D	E	F
c	8	5	5	6	9	2
v	10	4	5	3	13	3

→ 15 L

	E	A	D	C	B	F
1	0	0	0	0	0	0
2	0	0	0	0	0	3
3	0	0	0	0	0	3
4	0	0	0	0	0	3
5	0	0	0	5	5	5
6	0	0	3	5	5	8
7	0	0	3	5	5	8
8	0	10	10	10	10	10
9	13	13	13	13	13	13
10	13	13	13	13	13	13
11	13	13	13	13	13	16
12	13	13	13	13	13	16
13	13	13	13	15	15	16
14	13	13	13	18	18	18
15	13	13	16	18	18	18

18 → C, E

MI 2015

⑤

	A	B	C	D	E	F	G	H
C	1	9	1	3	1	1	4	2
V	8	45	6	19	3	2	10	8

a) $\Sigma = 10$

b) $\Sigma_2 = 8$

	B	G	D	H	A	C	E	F
1	0	0	0	0	8	8	8	8
2	0	0	0	8	8	14	14	14
3	0	0	19	19	19	19	19	19
4	0	10	19	19	27	27	27	27
5	0	10	19	27	27	33	33	33
6	0	10	19	27	35	35	36	36
7	0	10	29	29	35	41	41	41
8	0	10	29	29	37	41	44	44
9	45	45	45	45	45	43	44	46
10	45	45	45	45	53	53	53	53

a) 53 - B, A

b) 44 - E, C, A, H, D

MI 2014

⑥

	A	B	C	D	E	F
C	2	1	3	2	4	2
V	10	6	19	7	10	8

$\Sigma = 6$

	E	C	A	F	D	B
1	0	0	0	0	0	6
2	0	0	10	10	10	10
3	0	19	19	19	19	19
4	10	19	19	19	19	25
5	10	19	29	29	29	29
6	10	19	29	29	29	35

$35 \rightarrow B, A, C$

MI 2013

⑦

	A	3	C	D	E	F
C	7	3	1	3	6	6
V	16	8	6	7	13	10

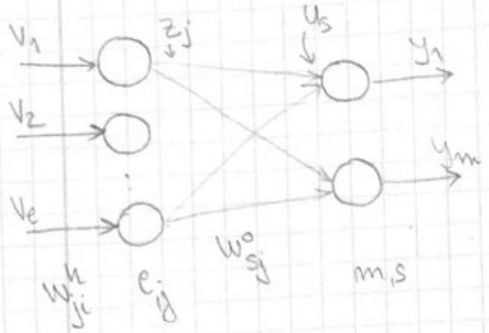
$\Sigma = 10$

	A	E	F	B	D	C
1	0	0	0	0	0	6
2	0	0	0	0	0	6
3	0	0	0	8	8	8
4	0	0	0	8	8	14
5	0	0	0	8	8	14
6	0	13	13	13	15	15
7	16	16	16	16	16	21
8	16	16	16	16	16	22
9	16	16	16	21	21	22
10	16	16	16	24	24	27

$27 \rightarrow C, B, E$

- Algoritam rasprostiranja unatrag

→ prirodno „rekurzivna struktura
→ izračun parcijalne derivacije po težinama između 2 susjedna sloja



v_j → ulazni ulaz u j-ti neuron prethodnog sloja

z_j → izlaz j-tog neurona prethodnog sloja

u_s → ulazni ulaz u s-ti neuron promatranog sloja

y_s → izlaz s-tog neurona promatranog sloja

Nelinearna aktivacijska fja
↳ poželjno derivabilna, sigmoid ⇒ $f(x) = \frac{1}{1+e^{-x}}$

I. Aktivnost sloja
→ derivacija s obzirom na izlaz izlaznog sloja y

$$\boxed{EA^o = y_i - y_{di}} \quad (m \times 1)$$

II. Ulaz u sloj
→ derivacija s obzirom na ulaz u izlazni sloj u

$$\boxed{E_1 = EA^o .* y .* (1-y)}$$
$(m\times1)\ (m\times1)\ (m\times1)\ (m\times1)$

III Težine poveznica

$$\boxed{EW^o = E_1 \cdot z^T} \quad (m \times e)$$

→ Komponenta gradijenta pogreške, koristi se u osvježavanju parametara (težina poveznica)

IV. Izlaz prethodnog sloja

$$\boxed{EA^h = (W_o)^T E_1}$$
$e\times e \quad e\times m \quad m\times e$

→ ponavljaju se koraci II. – IV. dok se ne dođe do ulaza

+ Uvođenje bias ulaza u mrežu (za svaki neuron)

→ ulaz u svaki neuron ne-ulaznog koja se mijenja sa $W^T x$ na $W^T x - \Theta$

→ translatira sve ulaze u neuron za neki iznos

*** Osvježavanje parametara sa biasima

$$W^o_{k+1} = W^o_k - \alpha_k E W^o$$

$$W^h_{k+1} = W^h_k - \alpha_k E W^h$$

$$\Theta^o_{k+1} = \Theta^o_k - \alpha_k E \Theta^o$$

$$\Theta^h_{k+1} = \Theta^h_k - \alpha_k E \Theta^h$$

$2 \times 2 \times 1$ → 2 ulaza × 2 skrivena × 1 izlaz

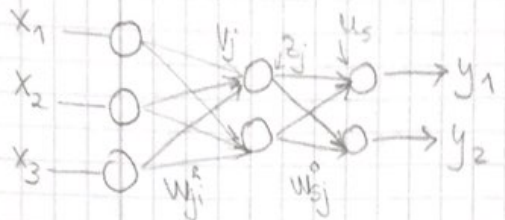
f5.

2013 JR

(6)

ul1	ul2	ul3	rez1	rez2
1	0	1	0	1
1	1	0	0	1
0	0	1	1	0
1	1	1	1	1

a) 3 x 2 x 2



+ SKRIVENI SLOJ - sigmoid

b) $\alpha = 1$ (jer nije zadan)

$$w^h = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad w^o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

I) $x_{d_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad y_{d_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$v = w^h \cdot x - \theta^h = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad z = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$u = w^o \cdot z - \theta^o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

θ^o ne postoji

$$EA^o = y - y_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$EI^o = EA^o .* y .* (1-y)$ → nema ovog jer je riječ o izlaznom sloju

$$EI^o = EA^o = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$EW^o = EI^o \cdot z^T = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1/2 & -1/2 \end{bmatrix}$$

$$w^{o(1)} = w^{o(0)} - \alpha EW^o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -1/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$$

$$EA^h = (w^o)^T \cdot EI^o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$EI^h = EA^h .* z .* (1-z) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$EW^h = EI^h \cdot x^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

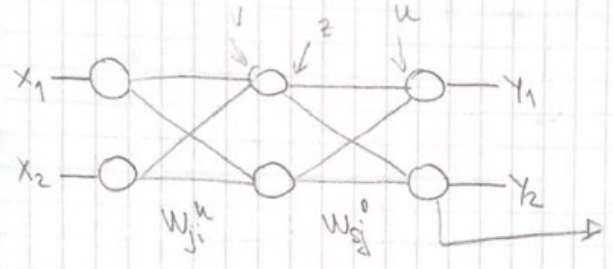
$$E\theta^h = -EI^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \theta^{(1)} = \theta^o$$

MI 2017

④

2×2×2

a)



u_1	u_2	l_1	l_2
2	3	0.5	0.25
4	1	0	0
-1	4	0.25	0
3	0	0	0.25

b) $W^o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $W^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Theta^o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_d = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $y_d = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$

$v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $z = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$EA^o = y - y_d = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$

$E1^o = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} . * \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0625 \end{bmatrix}$

$EW^o = \begin{bmatrix} 0 \\ 0.0625 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1/32 & 1/32 \end{bmatrix}$

$W^{o(1)} = W^{o(0)} - \alpha EW^o$

$W^{o(1)} = \begin{bmatrix} 0 & 0 \\ -1/32 & -1/32 \end{bmatrix}$

$EA^h = (W^o)^T E1^o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$E1^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$EW^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$W^{h(1)} = W^{h(0)} - \alpha EW^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

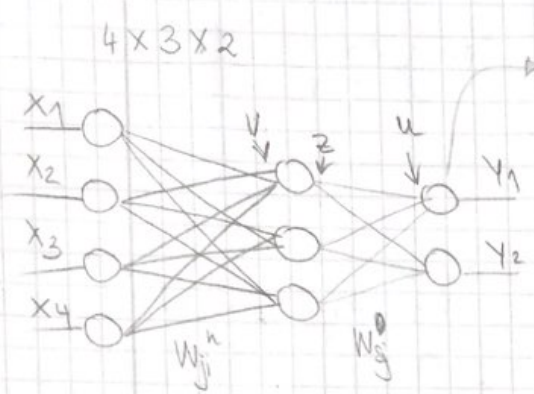
$E\Theta^o = \begin{bmatrix} 0 \\ -0.0625 \end{bmatrix}$ $E\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Theta^{o(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \alpha E\Theta^o = \begin{bmatrix} 0 \\ 1/16 \end{bmatrix}$ $\Theta^{h(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c) Postupak nastavljamo dalje sa novim Θ^h, Θ^o i W^h, W^o, pri čemu je $x_d = [4\ 1]$ i $y_d = [0\ 0]$ te kada ga sve izračunamo onda za ovde isto do kraja

MI 2018

④ 4x3x2



↦ 1 uvećan neuron
S-sigmoida

U_1	U_2	U_3	U_4	l_{z1}	l_{z2}
2	1	-4	3	0.3	0.75
4	2	3	1	0	0
-1	0	1	4	0.25	0
3	-1	-1	0	0	0.25

b) $x_d = \begin{bmatrix} 2 \\ 1 \\ -4 \\ 3 \end{bmatrix}$ $y_d = \begin{bmatrix} 0.3 \\ 0.75 \end{bmatrix}$ $\Theta^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Theta^h = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $W^h = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$W^0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$v = W^h \cdot x - \Theta^h = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $z = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$

$u = W^0 \cdot z - \Theta^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$EA^0 = y - y_d = \begin{bmatrix} 0.2 \\ -0.25 \end{bmatrix}$

$El^0 = \begin{bmatrix} 0.2 \\ -0.25 \end{bmatrix} .* \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} .* \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1/20 \\ -1/16 \end{bmatrix}$

$EW^0 = \begin{bmatrix} 1/20 \\ -1/16 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/40 & 1/40 & 1/40 \\ -1/32 & -1/32 & -1/32 \end{bmatrix}$

$W^{0(1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1/40 & 1/40 & 1/40 \\ -1/32 & -1/32 & -1/32 \end{bmatrix} = \begin{bmatrix} -1/40 & -1/40 & -1/40 \\ 1/32 & 1/32 & 1/32 \end{bmatrix}$

$EA^h = (W^0)^T \cdot El^0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/20 \\ -1/16 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$El^h = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$EW^h = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $W^{h(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

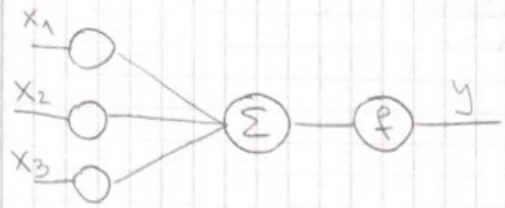
$E\Theta^0 = -El^0 = \begin{bmatrix} -1/20 \\ 1/16 \end{bmatrix}$ $E\Theta^h = -El^h = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Theta^{0(1)} = \begin{bmatrix} 1/20 \\ -1/16 \end{bmatrix}$ $\Theta^{h(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

HI 2013.

⑤

a)



u_1	u_2	u_3	kl.
1	1	1	3
1	1	0	2

b) $p<n$ (beskonačno rj. odabiremo ono najmanje norme

$$X_d = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \quad y_d = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad w^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$k=0$

$$e^0 = [1\,1\,1] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 3 = -3$$

$\mu = 1$

$$w^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 1 \cdot \frac{-3 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{1^2+1^2+1^2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$k=1$

$$e^1 = [1\,1\,0] \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 2 = 2 - 2 = 0$$

$$W = [1\,1\,1]^T$$

c) Prednost Kacmarzova algoritma je da nema potrebe za invertiranjem matrice, što bi morali da optimalne parametre rješavamo desnim pseudoinverzom

Netočne tvrdnje o GA:

→ a) GA uvijek nalazi globalni optimum

→ b) GA su deterministički algoritmi

c) Dobrota jedinke ne mora biti jednaka cijenoj fji

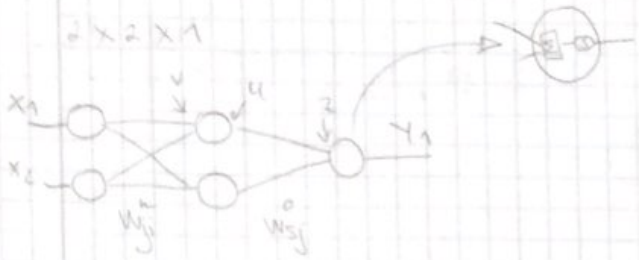
d) Jedno od područja u kojem su GA vrlo uspješni i često bolji od drugih optimizacijskih metoda jesu kombinatorne optimizacije

→ e) Ako tijekom jednog izvođenja (1 usporedbe) GA_1 postigne bolji rez od GA_2 može se zaključiti da je on općenito bolji i očekivati da će uvijek proračiti bolje rješenje od GA_2

MI 2015

④

a) 2 × 2 × 1



b)

ulaz 1	ulaz 2	izlaz
0	0	1
0	1	1
1	0	1
1	1	0

$W^o = \begin{bmatrix} 0 & 0 \end{bmatrix}$ $W^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Theta^o = 0$ $\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y_d = 1$

$v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $z = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$u = \begin{bmatrix} 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = 0$ $y = 1/2$

$E\Theta^o = -1/2$

$E1^o = -1/2 \cdot * 1/2 \cdot 1/2 = -\frac{1}{8}$

$EW^o = -\frac{1}{8} \cdot \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{16} & -\frac{1}{16} \end{bmatrix}$

$W^{o(1)} = \begin{bmatrix} 0 & 0 \end{bmatrix} - \begin{bmatrix} -\frac{1}{16} & -\frac{1}{16} \end{bmatrix} = \begin{bmatrix} \frac{1}{16} & \frac{1}{16} \end{bmatrix}$

$E\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1/8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$E1^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$EW^h = E1^h \cdot x^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$W^{h(1)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$E\Theta^o = \frac{1}{8}$

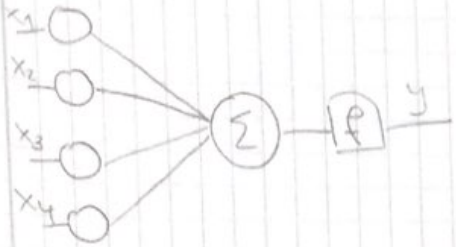
$\Theta^{o(1)} = 0 - \frac{1}{8} = -\frac{1}{8}$

$E\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Theta^{h(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Zad. XY

a)



w_1	w_2	w_3	w_4	izlaz
-1	1	-1	-1	2
-3	-2	0	-1	1

b) Kaczmarzov alg

$$-w_1 + w_2 - w_3 - w_4 = 2$$
$$-3w_1 - 2w_2 - w_4 = 1$$

$x_{d_1} = [-1\ 1\ -1\ -1]^T \qquad x_{d_2} = [-3\ -2\ 0\ -1]^T$

$w^{(0)} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \quad \mu = 1 \qquad e_1^{(0)} = [-1\ 1\ -1\ -1]\cdot\begin{bmatrix}0\\0\\0\\0\end{bmatrix} - 2 = -2$

$$w^{(1)} = w^{(0)} - \frac{-2\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix}}{4} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}-1\\1\\-1\\-1\end{bmatrix} = \begin{bmatrix}-1/2\\1/2\\-1/2\\-1/2\end{bmatrix}$$

$$e_2^{(1)} = [-3\ -2\ 0\ -1]\cdot\begin{bmatrix}-1/2\\1/2\\-1/2\\-1/2\end{bmatrix} - 1 = 1 - 1 = 0$$

$w^{(2)} = w^{(1)}$ //

Rješenje je konačno kada se preklapaju dvije težine rješenja k i $k+1$

c) Osobitost rješenja koje čini Kaczmarzov alg. najpoželjnijim je da se dolazi do istog rješenja kao i računanjem opt. parametara izravno, ali da se izbjegava invertiranje matrice $X_d^T X_d$. Nedostatak je taj da je traženja težina w tek čimes $\to k \Rightarrow \infty$

a) $k = 0$, $w^{(0)} = [0\ 0\ 0\ 0]^T \quad \alpha = 1$

$k = 1 \qquad e_1^{(0)} = -2$

$$w^{(1)} = w^{(0)} - \alpha \cdot x_{d_2} e_1^{(0)} = \begin{bmatrix}-2\\2\\-2\\-2\end{bmatrix}$$

x_1 y_1

$k = 2 \qquad e_2^{(1)} = [-3\ -2\ 0\ -1]\cdot\begin{bmatrix}-2\\2\\-2\\-2\end{bmatrix} - 1 = 4 - 1 = 3$

$$w^{(2)} = \begin{bmatrix}-2\\2\\-2\end{bmatrix} - 3\begin{bmatrix}-3\\-2\\0\\-1\end{bmatrix} = \begin{bmatrix}7\\8\\-2\\1\end{bmatrix}$$

$k = 3 \qquad e_3^{(2)} = [-1\ 1\ -1\ -1]\cdot\begin{bmatrix}7\\8\\-2\\1\end{bmatrix} - 2 = 0$

x_2 y_2

$w^{(3)} = w^{(2)}$ // kraj

III 2015.

③

$f(x) = (x-2)^2 = x^2 - 4x + 4$

$X = 0$

$f'(x) = 2x - 4$

$x_0 = 0$
$i = 1$

$x_1 = x_0 - \alpha \cdot f'(x_0)$

$x_1 = 0 - 0.2\,(0-4) = 0.8$

$x_2 = 0.8 - 0.2\,(1.6-4) = 1.28$

$x_3 = 1.28 - 0.2\,(2.56-4) = 1.568$

$x_4 = 1.568 - 0.2\,(3.136-4) = 1.7408$

$x_5 = 1.7408 - 0.2\,(3.4816-4) = 1.8448$

$x_6 = 1.8448 - 0.2\,(2 \cdot x_5 - 4) = 1.907$

$x_7 = 1.907 - 0.2\,(3.814 - 4) = 1.9442$

Postupak konvergira prema 2.
(jer je to nultočka, min.)

Varijat. strukture TRUE NIJE?

a) Ima dvi vrste čvorova
b) Visina stabla je jednaka dužini najduže grane
→ c) Konačni oblik ovisi o redoslijedu upisa podataka
d) Brisanje je vrlo jednostavno
e) podaci su samo u listovima

Koje bi bile poč. vrijednosti param. mreže kada bismo ih određivali po načelu preporučenom u okviru NASP-a

→ parametri θ svi 0, a težine w nasumični

(?) brojevi u' opsegu u kojem se očekuju njihove konačne vrijednosti

(NEDAMISE)

Zad xy.

$f(x) = (x-3)^2 = x^2 - 6x + 9$

$x^i = 1 \qquad x^{i+1} > 2$

$\alpha = ?$

$f'(x) = 2x - 6$

$x^{i+1} = x^i - \alpha (f'(x^i))$

$2 < 1 - \alpha \cdot (-4)$

$4\alpha > 1$

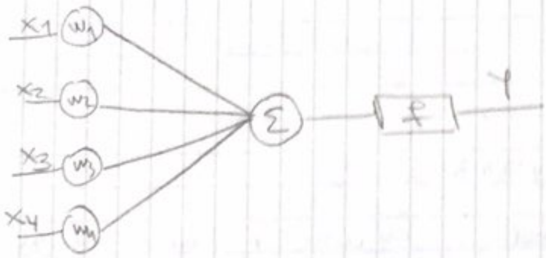
$\alpha > \frac{1}{4}$

Zad xy.

Lin neuron

ul1	ul2	ul3	ul4	izl
1	0	0	0	1
0	1	0	0	1
0	0	1	0	0
0	0	0	1	0

a) skica



P = broj ulaza = 4

n = broj razl. uzoraka = 4

p = n (1 rj.)

c) $X^T \cdot W = Y$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$w_1 = 1$

$w_2 = 1$

$w_3 = w_4 = 0$

d) $\alpha = 1$

$k=0 \qquad e^0 = [1000] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 = -1$

$w^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$k=1 \qquad e^1 = [0100] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 = -1 \qquad w^2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$k=2 \qquad e^2 = [0010] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 0 = 0 \qquad W^3 = W^2$ gotovo

Genetski algoritmi

p.f. $f_6 = 0.5 - \frac{(\sin\sqrt{x^2+y^2})^2 - 0.5}{(1+0.001(x^2+y^2))^2}$

max fje u $[-100, 100] = ?$

$p_i^{(k)} = \frac{g_i^{(k)}}{\sum_i g_i^{(k)}} = \frac{g_i^{(k)}}{G(k)}$ → dobrota ite jedinke u k-toj generaciji

↳ vjerojatnost odabira ite jedinke u ktoj generaciji

Dobrota ⇒ vrijednost fje koju maksimiziramo

Cineoma normalizacija:

1) jedinke se sortiraju po poč. fji dobrote u padajućem redoslijedu
2) najboljoj jedinki pridjeli se dobrota određene vrijednosti
3) svakoj sl. se dobrota umanjuje za neku vrijednost, ali ne ispod min
4) može se krenit i od min pa povećavat

Windowing

1) pronaći min dobrotu u populaciji
2) oduzimamo se dodjeljuje razlika rankova i min. dobrote
3) alternativa: umjesto min dobrote neka druga granica

Elitizam

↳ čuva najbolju jedinku

Uniformno križanje → odabrani bitovi jednog roditelja

zamjenjuju se bitovima drugog roditelja

MI 2018

③ $[-4, 12]$, 3 bitova

a) $2^8 - 1 = 256 - 1 = 255$

udaljenost elemenata

$$d = \frac{12-(-4)}{2^8-1} = \frac{16}{255} = 0.0627$$

b) Četvrti mim prikazivi el. domene?

I. el : -4

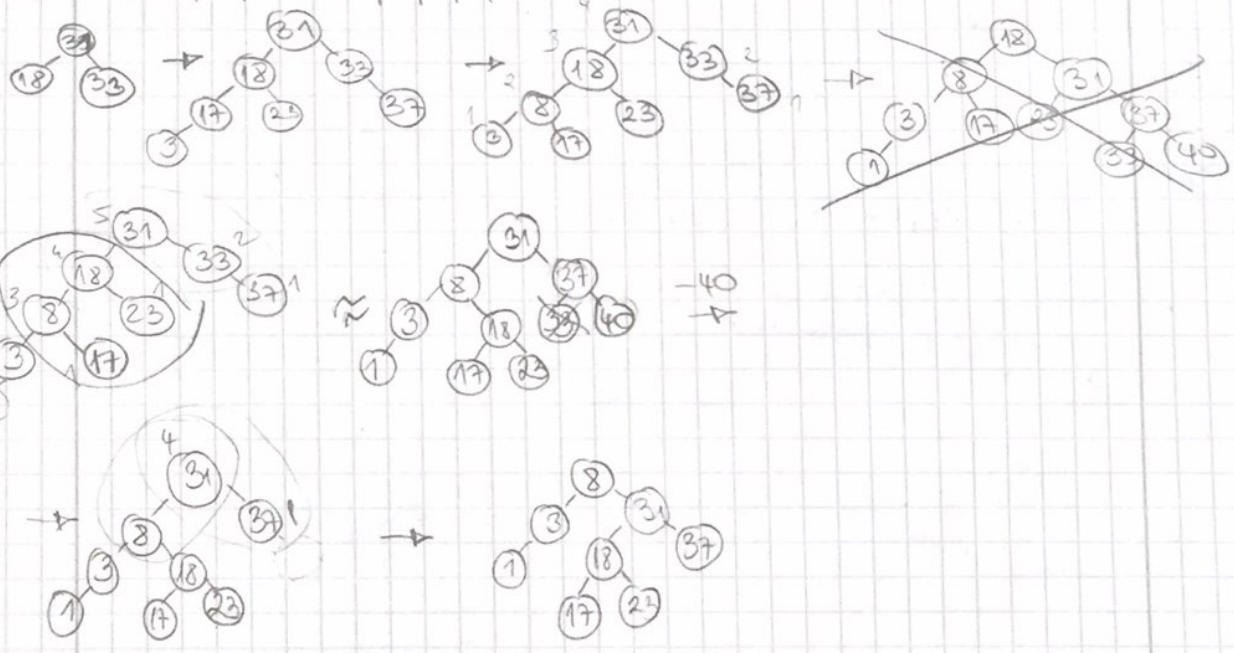
II. el : $-4 + 0.0627 =$

III. el : $-4 + 2 \cdot 0.0627$

IV. el : $-4 + 3 \cdot 0.0627 = -3.8119$

AGAIN

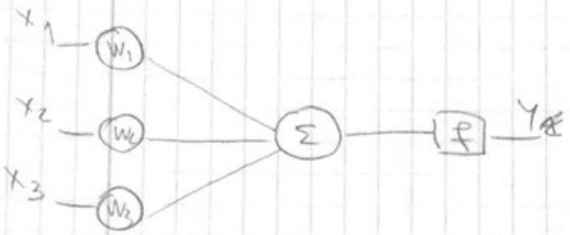
② 18, 31, 33, 17, 23, 37, 3, 8, 1, 40



2016

⑤

ul1	ul2	ul3	izl
1/2	0	0	1
0	-1/4	0	0
-1/2	0	1	-1
0	0	0	0



b) Izravno računanje opt. parametara

~~p~~ - broj ulaza = ~~3~~ 4

~~n~~ - broj uzoraka = ~~3~~

p - broj uzoraka → 4

n - broj ulaza → 3

~~p>n~~ p>n

→ ~~determ~~ pseudoinverz cijeni

$$\left(\begin{bmatrix} 0.5 & 0 & -0.5 & 0 \\ 0 & -0.25 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3\times4} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & -0.25 & 0 \\ -0.5 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{4\times3}\right)^{-1} \cdot A^T$$

$$= \begin{bmatrix} 0.5 & 0 & -0.5 \\ 0 & 0.0625 & 0 \\ -0.5 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 16 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ -0.5 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0.5 & 0 & -1.5 & 0 \\ 0 & -8 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 2 \\ 0 & 16 & 0 \\ 2 & 0 & 2 \end{bmatrix} A^T = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$W \begin{bmatrix} 2 & 0 & 0 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

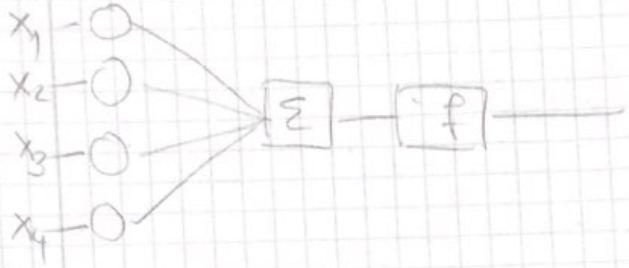
//

HI 2015.

2014.

⑤

w_{i1}	w_{i2}	w_{i3}	w_{i4}	x_i
0.5	0.5	-0.5	-0.5	1
0.5	-0.5	0.5	0.5	-1



~~x~~ $n=4$

$p=2$ $\quad n>p$ desni pseudoinverz

$$A^T(AA^T)^{-1} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \\ -0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix}$$

$4\times 4 \quad 4$

$$w^* = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$k=0 \quad w^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad e^0 = 0 - 1 = -1$

$$w^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 1 \cdot \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$$

$k=1 \qquad e^1 = \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} + 1 = 1$

$$w^2 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$k=2 \qquad e^2 = \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 = 0$

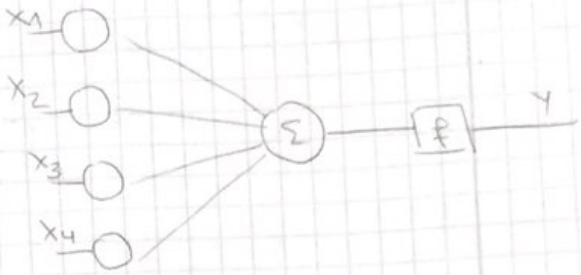
$w^3 = w^2$ // gotovo

MI 2014.

w1	w2	w3	w4	rel
1/2	1/2	-1/2	-1/2	1
1/2	-1/2	1/2	-1/2	-1

p < n

↳ desn: pseudoinversa



b) $A^T(AA^T)^{-1}$

$$W^* = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \\ -0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

c) $W^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \alpha = 1$

$k=0 \qquad e^0 = [0.5 \ 0.5 \ -0.5 \ -0.5] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 = -1$

$$W^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$$

$k=1 \qquad e^1 = [0.5 \ -0.5 \ 0.5 \ -0.5] \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix} + 1 = 1$

$$W^2 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

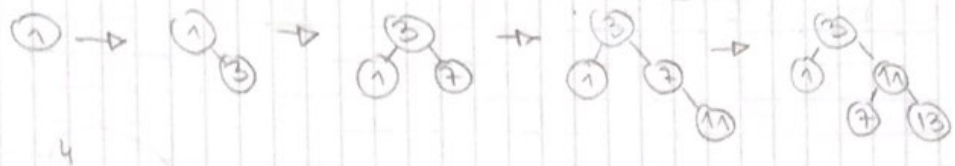
$k=2 \qquad e^2 = [0.5 \ 0.5 \ -0.5 \ -0.5] \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - 1 = 0$

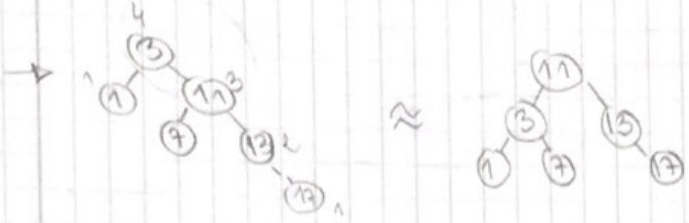
$W^3 = W^2$

//

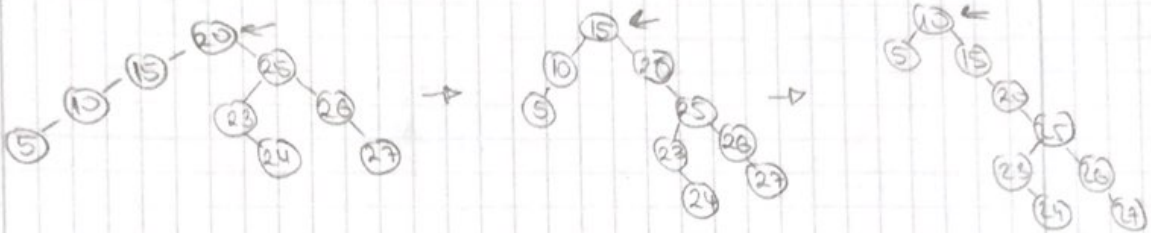
III 2010

③ 1, 3, 7, 11, 13, 17

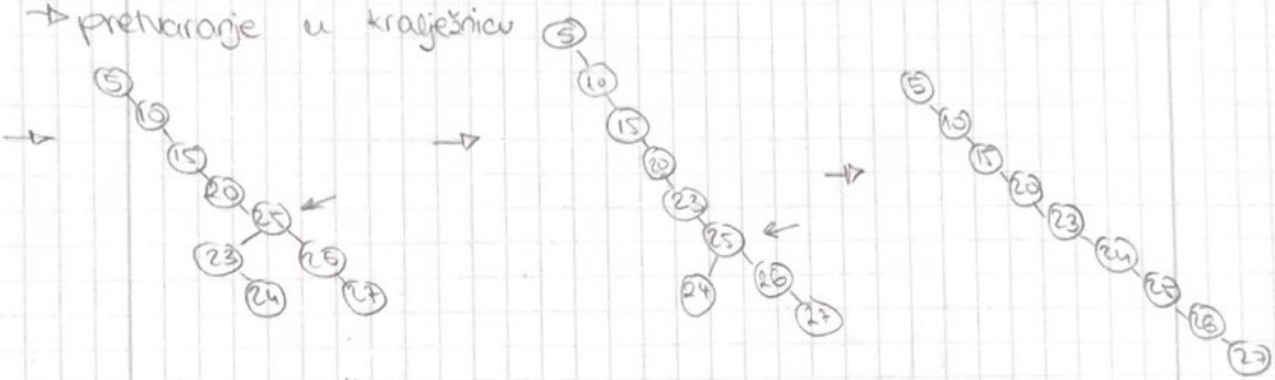




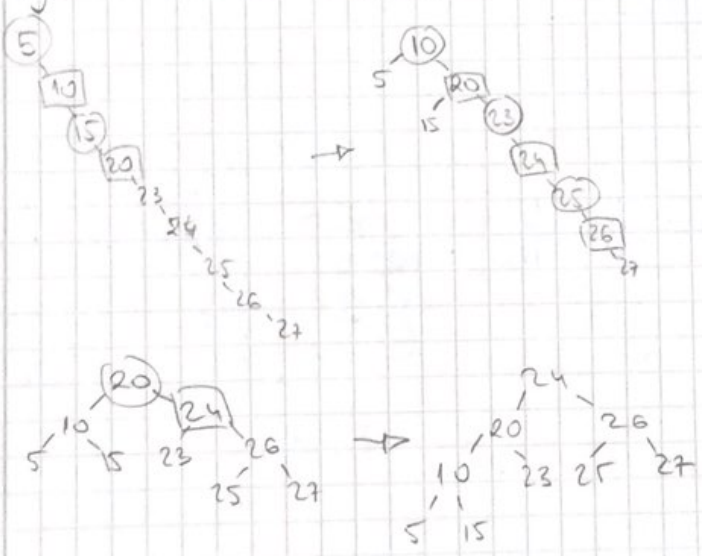
⑤



→ pretvaranje u kralješnicu



→ pretvaranje u zaršeno uravnoteženo

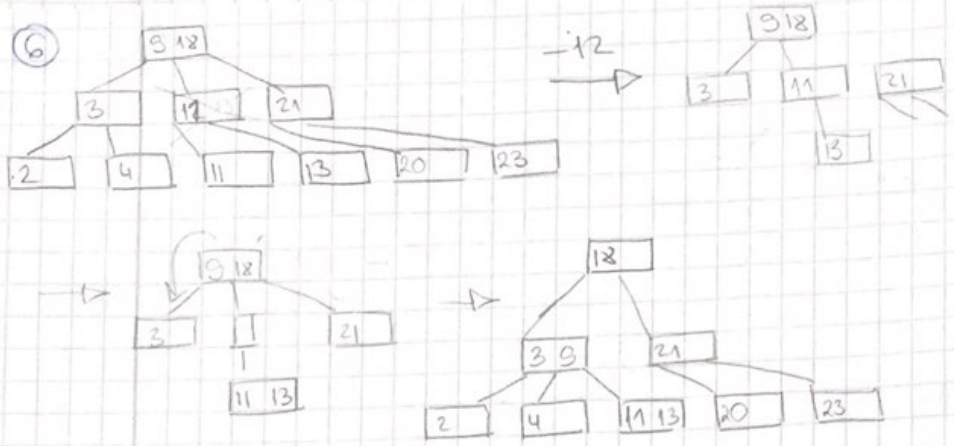


$h = \log_2 (n+1) = \log_2 10 = 3$

$k = 7$

$n-k$ rot (1)

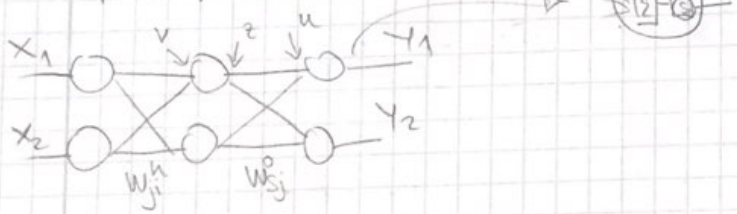
⑥



H12

⑤

w11	w12	v21	v22
0	1	1	0
1	0	0	1



b) $W^{o(0)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $W^{h(0)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Theta^o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\alpha = 1$

$v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $z = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

$EA^o = y - y_0 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$

$EI^o = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -1/8 \\ 1/8 \end{bmatrix}$

$EW^o = \begin{bmatrix} -1/8 \\ 1/8 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} -1/16 & -1/16 \\ 1/16 & 1/16 \end{bmatrix}$ $E\Theta^o = \begin{bmatrix} 1/8 \\ -1/8 \end{bmatrix}$

$W^{o(1)} = \begin{bmatrix} 1/16 & 1/16 \\ -1/16 & -1/16 \end{bmatrix}$ $\Theta^{o(1)} = \begin{bmatrix} -1/8 \\ 1/8 \end{bmatrix}$

$EA^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

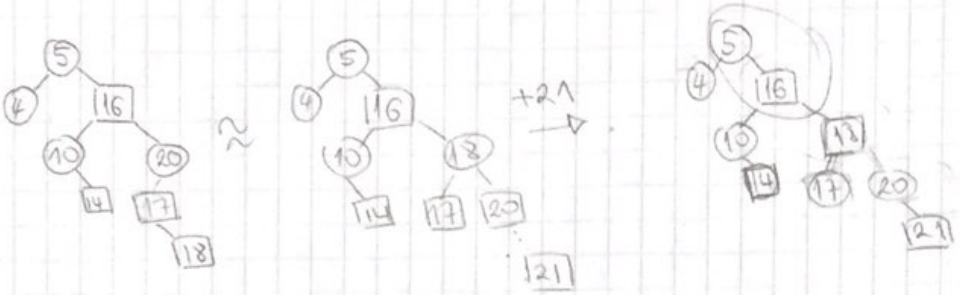
$EI^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

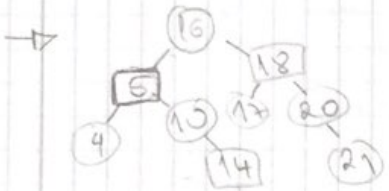
$EW^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$W^{h(1)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $E\Theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Theta^{h(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

MI 2011

②





⑤

jedinka	10110	01111	10101	10111	00001
dobrota	66	39	27	15	3

1) Jednostavni razmjerni odabir
2) križanje s jednom prekidnom točkom
3) mutacija koja mijenja gen na kojoj se odnosi

$p_k = 0.8$ $p_m = 0.1$

0.2, 0.84, 0.05, 0.41, 0.22, 0.62, 0.21, 0.19, 0.44, 0.06, 0.54, 0.76, 0.48, 0.01

1) $p_i^{(k)} = \frac{g_i^{(k)}}{\sum_i g_i^{(k)}}$

	p_1	p_2	p_3	p_4	p_5
	10110	01111	10101	10111	00001
g_i	66	39	27	15	3
p_i =	0.44	0.26	0.18	0.1	0.02

$\sum g_i = 150$

$[0, 0.44\rangle$ $[0.44, 0.70\rangle$ $[0.70, 0.88\rangle$ $[0.88, 0.98\rangle$ $[0.98, 1\rangle$

0.2 → p_1 0.41 → p_1 0.21 → p_1 0.06 → p_1 0.48 → p_2
0.84 → p_3 0.22 → p_1 0.19 → p_1 0.54 → p_2 0.01 → p_1
0.05 → p_1 0.62 → p_2 0.44 → p_2 0.76 → p_3

2) $0.05 < p_k$ → $0.05 < 0.8$ dolazi do križanja | mutacija $w_i < p_m$

gen g_1 g_2 g_3 g_4 g_5

$[0, 0.41\rangle$ $[0.41, 0.67\rangle$

$p = 0.41$ → između g_2 i g_3

10|110 → 10101
10|101 → 10110

① 1 0 1 0 1
0.22 0.62 0.21 0.19 0.44
1 0 1 0 1

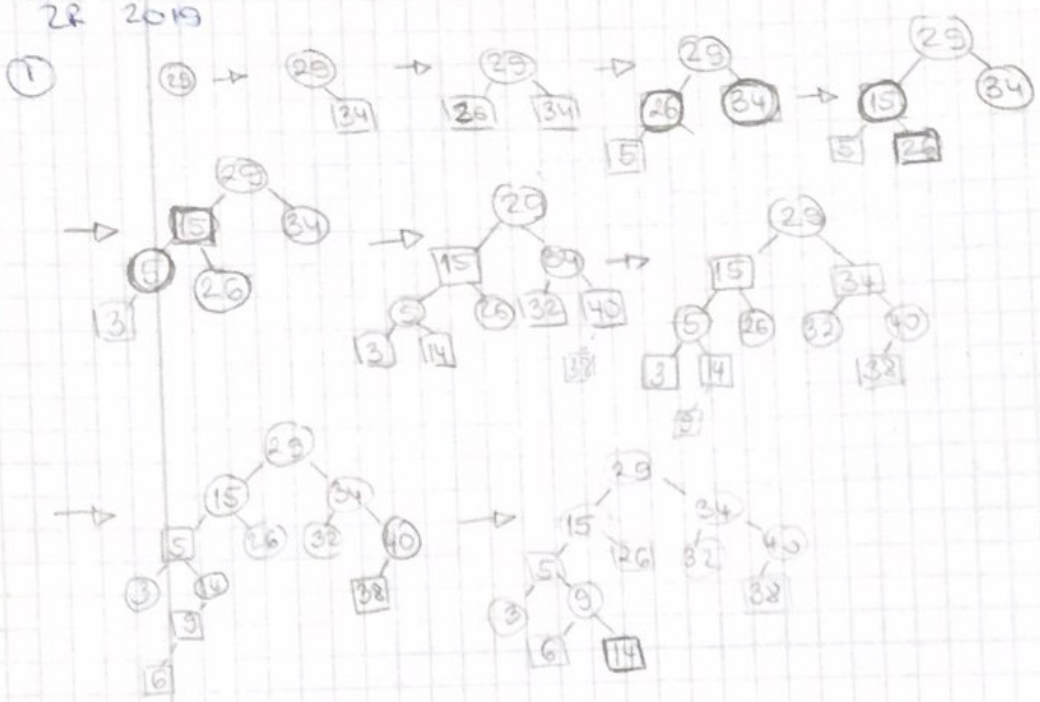
② 1 0 1 1 0
0.06 0.54 0.76 0.48 0.01
0 0 1 1 1

IZLAZ:

1 0 1 0 1
0 0 1 1 1

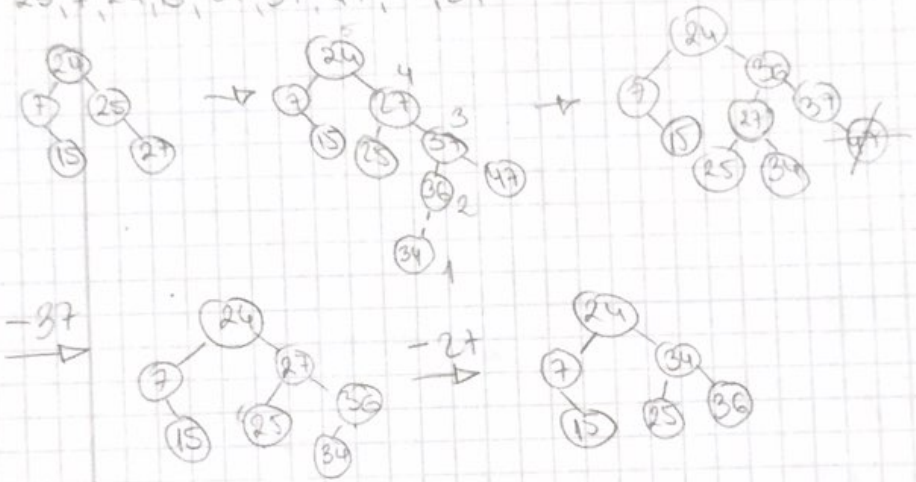
2R 2019

①



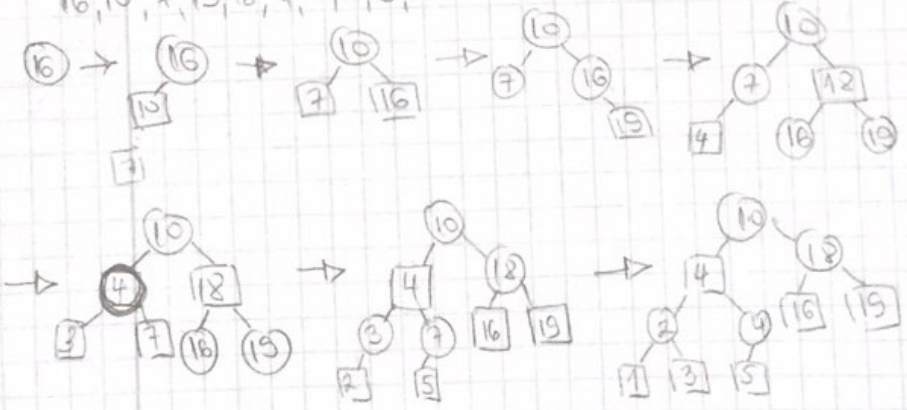
2R 2018

⑭ 25, 7, 24, 15, 27, 37, 47, 36, 34



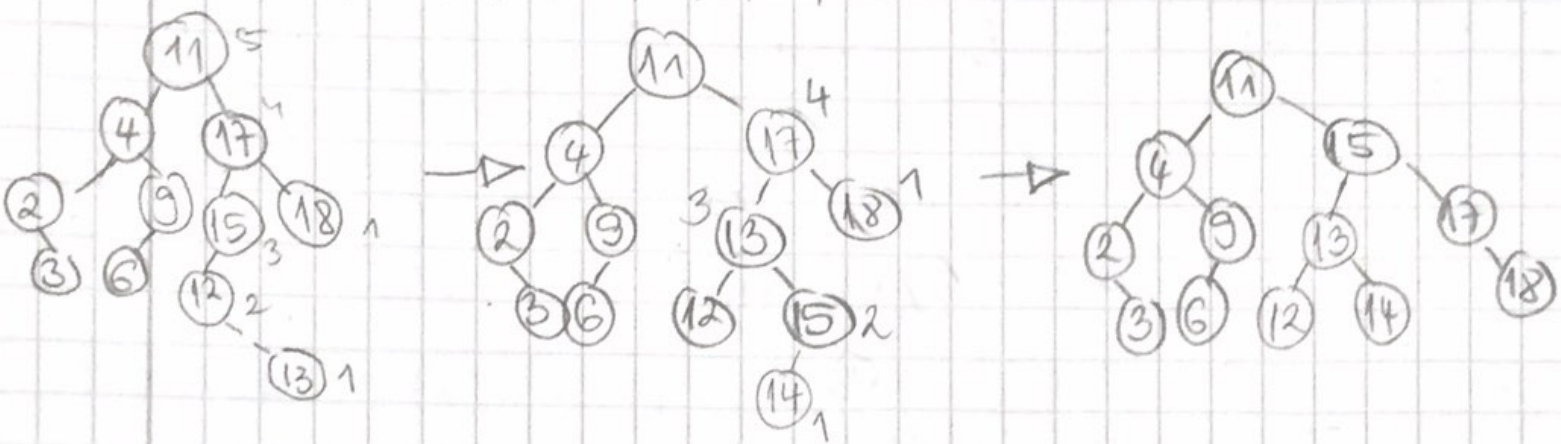
LR 2018

⑤ 16, 10, 7, 19, 18, 4, 3, 2, 5, 1



RR 2018

① 17, 4, 11, 9, 18, 2, 3, 15, 12, 6, 13, 14



NASP

(9) Linearno programiranje i simpleks algoritam

$f(x_1, x_2, \dots)$
$g(x_1, x_2, \dots) \leq 0$
$q(x_1, x_2, \dots) = 0$

f → cijena fja
x_i → varijable odluke
h_i, g_j → ograničenja

LIN. PROG. → sve fje $f(x), g(x), q(x)$ su linearne, $x_i \geq 0$

kanonski lin. problem

$f(x) = c^T x$
$Ax = b$
$x \geq 0$

pr.//

	Transp troškovi				
	O1	O2	O3	O4	Kap
T1	10	9	14	8	32
T2	7	11	9	11	38
T3	8	12	12	9	35
Nar	20	20	25	35	

T1 T2 T3 → pogoni u 3 mjesta
O1 : 4 → naručitelji

x_{ij} - količina iz i-te tvornice j-tom naručitelju
t_{ij} - trošak po i, j

fja: $\min \left(\sum_{ij} x_{ij} t_{ij} \right)$

uvjeti: $\sum_i x_{ij} \leq c(T_i)$

$\sum_j x_{ij} = n_j$

$x_{ij} \geq 0$

pr.// S - strojeva
D - dijelova proizvoda
različiti strojevi, razl. brzina proizvodnje

x_{ij} - broj j-tih dijelova proizvedenih i-tim strojem
t_{ij} - trajanje za i, j

$$\min \left(\sum_{ij} x_{ij} t_{ij} \right)$$

uvjeti: $\sum_i \sum_j x_{ij} = D$

$\sum_i x_{ij} = 1$

$x_{ij} \geq 0$

Pr. N - ruterih postaja

Signal od i-te postaje dobar pri gušen, centralna prima signal snage $\Lambda_i P_i$ $(\Lambda \geq 1)$

signal / šum = ξ

ukupna potrošnja $\propto$ ukupnoj snazi

$$\min\left(\sum_{i=1} P_i\right)$$

$$\frac{\Lambda_i P_i}{\sum_{j \neq i} \Lambda_j P_j} \geq \xi_i$$

$$P_i \geq 0$$

$$\Lambda_i P_i - \xi_i \sum_{j \neq i} \Lambda_j P_j \geq 0$$

METODE RJEŠAVANJA

① Grafička metoda

→ za male probleme s 2-3 dimenzije

→ optimalno rješenje je na rubu, rub se zove LICE

② Algebarska metoda

1. Transf. u kanonski oblik
2. Poveznice kanonske i polazne formulacije
3. Teorijska razmatranja
4. Pretraživanje/kretanje među vršnim/ekstremima
5. Simplex metoda

) negativni b → pomnožit jedn s -1

maximizacija → pretvara u minimizaciju

$\geq$ ograničenja → oduzeti pomoćne dopunske varijable y_i

minimizirati $c^T x$

uvjet $a_{i1}x_1 + \dots - y_i = b_i$

Matrično: $Ax - I_m y = [A \; -I_m][x \; y]^T = b$

$\leq$ ograničenja → dodato pomoćne dop. var. y_i

ograničenja varijabli – transf. sve nenegativne x

- slobodan x – $x \in (-\infty, +\infty) \rightarrow U, V \geq 0$; $x = U - V$
- $x \in (-\infty, g] \rightarrow x' \geq 0$; $x' = -x + g$
- $x \in [\ell, g] \rightarrow x' \geq 0$; $x' = x - \ell$

$x' \leq g - \ell$

23. Rajd.

Pl: max $x_2 - x_1$
uyet: $3x_1 - x_2 = -5$
$|x_2| \leq 2$
$x_1 \leq 0$

1. min $x_1 - x_2$
2. $3x_1 - x_2 = 5$ $\quad / \cdot -1$
$x_2 - 3x_1 = 5$
3. $x_1 \in (-\infty, 0] \Rightarrow x_1' = -x_1$
$x_1' \geq 0$

min. $x_1 - x_2 \Rightarrow -x_2 - x_1'$
uyet $x_2 + 3x_1' = 5$
$x_1' \geq 0$
$|x_2| \leq 2$

4. $-2 \leq x_2 \leq 2 \Rightarrow x_2 \in [-2, 2]$
$x_2' = x_2 + 2$, $x_2' \geq 0$
$x_2' \leq 4$

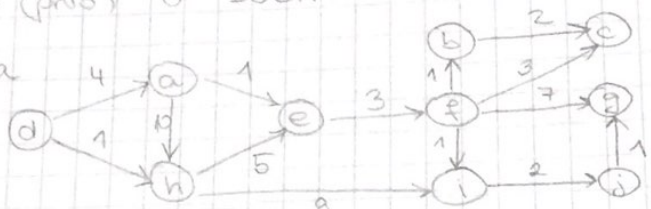
5. $x_2 \leq 4$
$\hookrightarrow x_2' + x_3 = 4$
$x_3 \geq 0$

Rj:
min $-x_2' + 2 - x_1'$
uyet: $x_2' - 2 + 3x_1' = 5 \Rightarrow x_2' + 3x_1' = 7$
$x_1' \geq 0$
$x_2' = x_2 + 2$
$x_2' \geq 0$
$x_2' + x_3 = 4$
$x_3 \geq 0$

→ OBILAZAK GRAFA

alg:
→ (prvo) U ŠIRINU → BFS
→ (prvo) U DUBINU → DFS (nesto sporji jer je rekurzivan)

pr. Dijkstra



akcija	poč	1	2	3	4	5	6	7	8	9	10
		d	h	a	e	f	b	i	c	j	g
a	∞	4	4								
b	∞	∞	∞	∞	∞	9					
c	∞	∞	∞	∞	∞	11	11	11			
d	0										
e	∞	∞	6	5							
f	∞	∞	∞	∞	8						
g	∞	∞	∞	∞	∞	15	15	15	15	12	
h	∞	1									
i	∞	∞	10	10	9	9	9				
j	∞	∞	∞	∞	∞	∞	∞	11	11		

NASP – [redacted] ZI [redacted]

2018

$$\max \quad -5x_1 + 6x_2 + 7x_3$$
$$x_1 + 2x_2 - 2x_3 \leq 5$$
$$-x_2 + 2x_3 \leq 1$$
$$x_2 \leq 6$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

1. Pretvorba u kanonski oblik

$$\min \quad 5x_1 - 6x_2 - 7x_3$$
$$x_1 + 2x_2 - 2x_3 + x_4 = 5$$
$$-x_2 + 2x_3 + x_5 = 1$$
$$x_2 + x_6 = 6$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0$$

2. Prošireni oblik

0. iteracija

baze	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_4	1	2	-2	1	0	0	5
x_5	0	-1	**2**	0	1	0	1
x_6	0	1	0	0	0	1	6
z	5	-6	-7	0	0	0	0

fja cilja →

$$\min_{a>0}\{RHS \mid x_3\} = \min\{-2.5,\ 0.5\} = 0.5$$

1. iteracija

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_4	1	1	0	1	1	0	6
x_3	0	-0.5	1	0	0.5	0	0.5
x_6	0	**1**	0	0	0	1	6
z	5	-9.5	0	0	3.5	0	3.5

2. iteracija

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_4	1	0	0	1	1	-1	0
x_3	0	0	1	0	0.5	0.5	3.5
x_2	0	1	0	0	0	1	6
z	5	0	0	0	3.5	9.5	60.5

x_2, x_3, x_4 čine bazu jer im stupci sadrže jedinične vektore

bazične, ali nebazične, zato što su 0 pa defa

Rj: Vrijednost opt rješenja -60.5, $x_1 = 0$, $x_2 = 6$, $x_3 = 3.5$ ($x_4 = 0$, $x_5 = 0$, $x_6 = 0$) Dajana

pr. prez

$$\max \quad 7x_1 + 6x_2$$
$$2x_1 + x_2 \leq 3$$
$$x_1 + 4x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

$$\min \quad -7x_1 - 6x_2$$
$$2x_1 + x_2 + x_3 = 3$$
$$x_1 + 4x_2 + x_4 = 4$$
$$x_1, x_2, x_3, x_4 \geq 0$$

0. iteracijo

	x_1	x_2	x_3	x_4	RHS
x_3	[2]	1	1	0	3
x_4	1	4	0	1	4
z	-7	-6	0	0	0

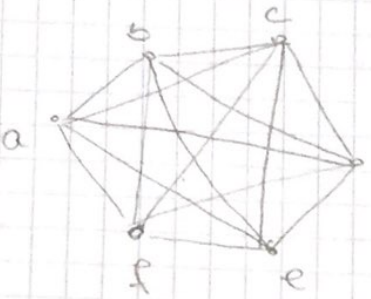
1. iteracija

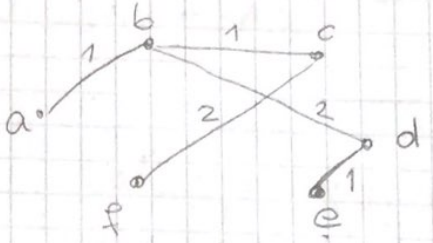
	x_1	x_2	x_3	x_4	RHS
x_1	1	0.5	0.5	0	1.5
x_4	0	[3.5]	-0.5	1	2.5
z	0	-2.5	3.5	0	10.5

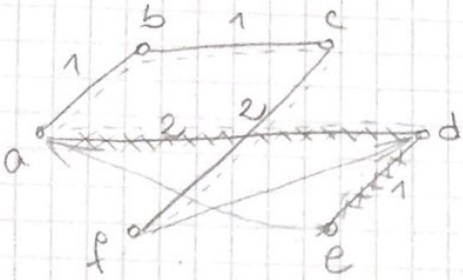
x_1	1	0	4/7	-1/7	8/7
x_2	0	1	-1/7	2/7	5/7
z	0	0	22/7	5/7	26/7

21-2018

5.





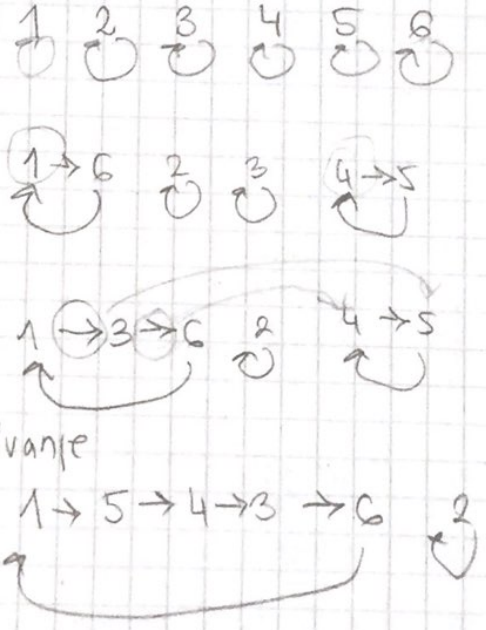


7

abcfde-a

ae 3
ed 1
ad 2

6.



povezivanje u krug

impl						
root	1	2	3	4	5	6
next	1	2	3	4	5	6
lenght	1	1	1	1	1	1
root	1	2	3	4	4	1
next	6	2	3	5	4	1
lenght	2	1	1	2	1	1
	1	2	1	4	4	1
	3	2	6	5	4	1
	3	1	1	2	1	1
	1	2	1	1	1	1
	5	2	6	3	4	1
	5	1	1	2	1	1

21 - 2013.

2. x_1 - meggy (€)
x_2 - keksz (kg)
$x_1 = 7$ kn/€
$x_2 = 50$ kn/kg

$$\max\ 2000x_1 + 350x_2$$
$$-x_1 + 4x_2 \leq 400$$
$$3x_1 + 4x_2 \leq 6800$$
$$x_1, x_2 \geq 0$$

$$\min: -14000x_1 - 17500x_2$$
$$-x_1 + 4x_2 + x_3 = 400$$
$$3x_1 + 4x_2 + x_4 = 6800$$

0. iteráció

0. iteracija

	X_1	X_2	X_3	X_4	RHS
X_3	−1	[4]	1	0	400
X_4	3	4	0	1	6800
z	−14000	−11500	0	0	0

4375

X_2	−1/4	1	1/4	0	100
X_4	[4]	0	−1	1	6400
z	−18375	0	4375	0	1 750 000

459375

X_2	0	1	[3/16]	1/16	500
X_1	1	0	−1/4	1/4	1600
z	0	0	−218.75	4593.75	31 150 000

2 × 4/3 → × 3500/3

X_3	0	16/3	1	1/3	8000/3 = 2666.67
X_1	1	4/3	0	1/3	6800/3 = 2266.67
z	0	1166.67	0	4666.67	31733333.33

$X_1 = 2266,67$

$X_3 = 2666,67$

$X_2 = 0$

$X_4 = 0$

$z = -31733333.33$

b) GRAFIČKI PRIKAZ

$f_1 \Rightarrow 20X_1 + 3.5X_2$

$f_2 \Rightarrow -X_1 + 4X_2 \leq 400$

$f_3 \Rightarrow 3X_1 + 4X_2 \leq 6800$

X_2

800

$f_1 \rightarrow 20X_1 + 3.5X_2 = 0 \rightarrow (0,0)$ (100, 571.43)

$-20X_1 + -3$

$f_2[(0,100),(400,200)]$ $f_3[(1000,950),(1200,$)

$X_1, X_2 \geq 0$ (1. kvadrant)

2. max $7x_1 + 50x_2$

$-x_1 + 4x_2 \leq 400$

$3x_1 + 4x_2 \leq 6800$

$0 \leq x_1 \leq 2000$

$0 \leq x_2 \leq 350$

⇒ (0,0) (−100,14)

⇒ (0,100) (400,200)

⇒ (1900,275) (2000,200)

2a | $x \in (-\infty, +\infty) \rightarrow u, v \geq 0 \quad x = u - v$

$x \in (-\infty, g] \rightarrow x' \geq 0 \quad x' = -x + g$

$x \in [e, g] \rightarrow x' \geq 0 \quad x' = x - e$

$x' \leq g - e$

min $-7x_1 - 50x_2$

$-x_1 + 4x_2 + x_3 = 400$

$3x_1 + 4x_2 + x_4 = 6800$

$x_1' \geq 0 \quad x_1' = -x_1 + 2000 \Rightarrow x_1 = 2000 - x_1'$

$x_2' \geq 0 \quad x_2' = -x_2 + 350 \quad x_2 = 350 - x_2'$

min $7x_1' - 1400 + 50x_2' - 17500$

$7x_1' + 50x_2' - 18900$ ⇒ můžeme ignorovat

$-2000 + x_1' + 1400 - 4x_2' + x_3 = 400$

$6000 - 3x_1' + 1400 - 4x_2' + x_4 = 6800$

min $7x_1' + 50x_2' - 18900$

$x_1' - 4x_2' + x_3 = 1000$

$-3x_1' - 4x_2' + x_4 = -600 \quad / \cdot (-1)$

$3x_1' + 4x_2' - x_4 = 600$

$x_1' \geq 0 \quad x_2' \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0$

x_1'	x_2'	x_3	x_4	RHS
1	−4	1	0	1000
3	4	0	−1	600
7	50	0	0	0

2. max $7x_1 + 50x_2$

$-x_1 + 4x_2 \leq 400$
$3x_1 + 4x_2 \leq 6800$

$0 \leq x_1 \leq 2000$
$0 \leq x_2 \leq 350$

---

min $-7x_1 - 50x_2$

$-x_1 + 4x_2 + x_3 = 400$
$3x_1 + 4x_2 + x_4 = 6800$

	x_1	x_2	x_3	x_4	RHS
x_3	-1	[4]	1	0	400
x_4	3	4	0	1	6800
z	-7	-50	0	0	0

Zeile 1 : 4 ; Zeile 2 − 1 ; Zeile 3 × 12,5

x_2	-1/4	1	1/4	0	100
x_4	[4]	0	-1	1	6400
z	-19,5	0	12,5	0	5000

Zeile 2 : 4 ; Zeile 1 − (−1/4) · Zeile 2 ; Zeile 3 − (−19,5) · Zeile 2

x_2	0	1	3/16	1/16	500
x_1	1	0	-1/4	1/4	1600
z	0	0	61/8	39/8	36200

$x_1 = 1600$

$x_2 = 500$

$z = 36\,200$

$x_3 = x_4 = 0$

Z1 2017

1. max $2x_1 - 3x_2 + 8x_3$

$x_1 \leq 5$

$-x_2 + 2x_3 \leq 3$

$x_1 + x_3 \leq 8$

$x_2 \leq 5$

$x_1, x_2, x_3 \geq 0$

---

min $-2x_1 + 3x_2 - 8x_3$

$x_1 + x_4 = 5$

$-x_2 + 2x_3 + x_5 = 3$

$x_1 + x_3 + x_6 = 8$

$x_2 + x_7 = 5$

$x_1 \dots x_7 \geq 0$

$x_1 + x_2 \geq 3$

$x_1 + x_2 - x_3 = 3$

0 0 +

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
x_4	1	0	0	1	0	0	0	5
x_5	0	-1	[2]	0	1	0	0	3
x_6	1	0	1	0	0	1	0	8
x_7	0	1	0	0	0	0	1	5
z	-2	3	-8	0	0	0	0	0

1:2 ⟩ x -0.5 ⟩ x1

x_4	[1]	0	0	1	0	0	0	5
x_3	0	-1/2	1	0	1/2	0	0	3/2
x_6	1	1/2	0	0	-1/2	1	0	6.5
x_7	0	1	0	0	0	0	1	5
z	-2	-1	0	0	4	0	0	12

⟩ x -1 ⟩ x2

x_1	1	0	0	1	0	0	0	5
x_3	0	-1/2	1	0	1/2	0	0	3/2
x_6	0	[1/2]	0	-1	-1/2	1	0	1.5
x_7	0	1	0	0	0	0	1	5
z	0	-1	0	2	4	0	0	22

1:2 ⟩ x1 ⟩ x -2 ⟩ x2

x_1	1	0	0	1	0	0	0	5
x_3	0	0	1	-1	0	1	0	3
x_2	0	1	0	-2	-1	2	0	3
x_7	0	0	0	+2	+1	-2	+1	2
z	0	0	0	0	3	2	0	25

$x_1 = 5 \quad x_2 = x_3 = 3 \quad x_4 = x_5 = x_6 = x_7 = 0$

$z = -25$

Z1 – 2016

$$\max \quad 5x_1 + 7x_2$$
$$x_1 \leq 2$$
$$x_2 \leq 4$$
$$x_1 + x_2 \leq 7$$
$$x_1, x_2 \geq 0$$

$$\min \quad -5x_1 - 7x_2$$
$$x_1 + x_3 = 2$$
$$x_2 + x_4 = 4$$
$$x_1 + x_2 + x_5 = 7$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

	x_1	x_2	x_3	x_4	x_5	RHS
x_3	1	0	1	0	0	2
x_4	0	**1**	0	1	0	4
x_5	1	1	0	0	1	7
z	−5	−7	0	0	0	0

↓ ×(−1) ↓ ×7

x_3	**1**	0	1	0	0	2
x_2	0	1	0	1	0	4
x_5	1	0	0	−1	1	3
z	−5	0	0	7	0	28

↓ ×(−1) ↓ ×5

x_1	1	0	1	0	0	2
x_2	0	1	0	1	0	4
x_5	0	0	−1	−1	1	1
z	0	0	5	7	0	38

$x_1 = 2$
$x_2 = 4$

$z = -38$

$x_3, x_4, x_5 = 0$

2015.

2)

$$\min\ -7x_1 - 6x_2 - 8x_3$$

$$x_1 \le 12$$

$$3x_2 - x_3 \le 3$$

$$x_1 + x_3 \le 8$$

$$x_3 \le 7$$

$$x_1, x_2, x_3 \ge 0$$

$$x_1 + x_4 = 12$$

$$3x_2 - x_3 + x_5 = 3$$

$$x_1 + x_3 + x_6 = 8$$

$$x_3 + x_7 = 7$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \ge 0$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
x_4	1	0	0	1	0	0	0	12
x_5	0	3	−1	0	1	0	0	3
x_6	1	0	1	0	0	1	0	8
x_7	0	0	**1**	0	0	0	1	7
z	−7	−6	−8	0	0	0	0	0

↑ x−1 ← x_1 ; ↓ x8

x_4	1	0	0	1	0	0	0	12
x_5	0	3	0	0	1	0	1	10
x_6	**1**	0	0	0	0	1	−1	1
x_3	0	0	1	0	0	0	1	7
z	−7	−6	0	0	0	0	8	56

↑ x−1 ; ↓ x7

x_4	0	**3**	0	1	0	−1	1	11
x_5	0	0	0	0	1	0	1	10
x_1	1	0	0	0	0	1	−1	1
x_3	0	0	1	0	0	0	1	7
z	0	−6	0	0	0	7	1	63

1:3 ; ↓ x2

x_4	0	0	0	1	0	−1	1	11
x_2	0	1	0	0	1/3	0	1/3	10/3
x_1	1	0	0	0	0	1	−1	1
x_3	0	0	1	0	0	0	1	7
z	0	0	0	0	2	7	3	83

$x_1 = 1 \quad x_2 = \frac{10}{3} \quad x_3 = 7 \quad x_5 = x_6 = x_7 = 0 \quad z = -83$

$x_4 = 11$

2019 – 2)

geogebra.com

max $7x_1 + 50x_2$

$-x_1 + 4x_2 \leq 400$

$3x_1 + 4x_2 \leq 6800$

$0 \leq x_1 \leq 2000$

$0 \leq x_2 \leq 350$

---

min $-7x_1 - 50x_2$

$-x_1 + 4x_2 + x_3 = 400$

$3x_1 + 4x_2 + x_4 = 6800$

$x_1 + x_5 = 2000$

$x_2 + x_6 = 350$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_3	−1	[4]	1	0	0	0	400
x_4	3	4	0	1	0	0	6800
x_5	1	0	0	0	1	0	2000
x_6	0	1	0	0	0	1	350
z	−7	−50	0	0	0	0	0

: 4 → ×(−1) → ×(−1/4) → x_2

x_2	−1/4	1	1/4	0	0	0	100
x_4	4	0	−1	1	0	0	6400
x_5	1	0	0	0	1	0	2000
x_6	[1/4]	0	−1/4	0	0	1	250
z	−19.5	0	12.5	0	0	0	5000

1.4 → ×(−4) → ×(−16) → x_1 → ×75

x_2	0	1	0	0	0	1	350
x_4	0	0	[3]	1	0	−16	2400
x_5	0	0	1	0	1	−4	1000
x_1	1	0	−1	0	0	4	1000
z	0	0	−7	0	0	78	24500

: 3 → ×(−1/3) → ×1/3 → ×7/3

x_2	0	1	0	0	0	1	350
x_3	0	0	1	1/3	0	−16/3	800
x_5	0	0	0	−1/3	1	28/3	200
x_1	1	0	0	1/3	0	4/3	1200
z	0	0	0	7/3	0	286/3	35100

→ KRAJ

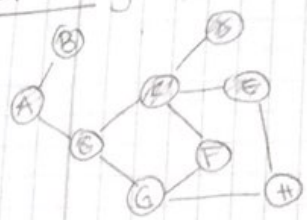
$x_1 = 1000$

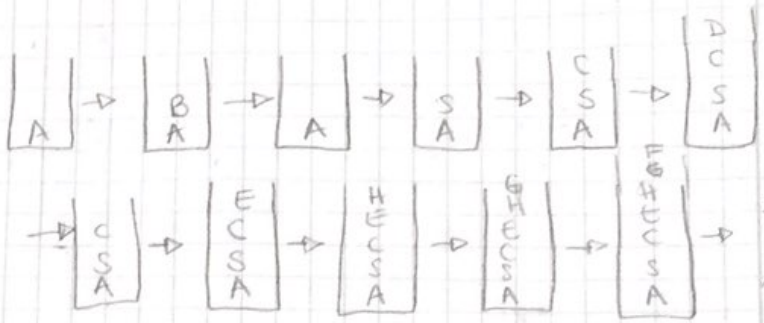
$x_2 = 350$

$z = -24500$

$x_3, x_4, x_5, x_6 \geq 0$

DFS + vježba

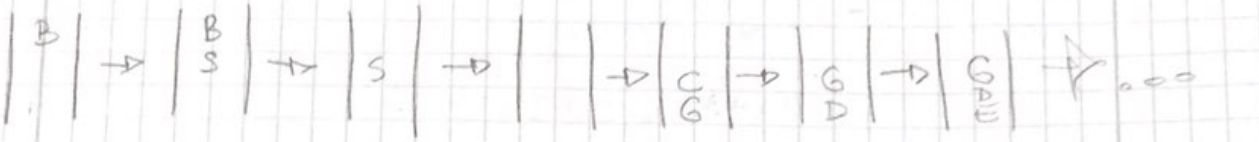




sad ih skida 1 po 1

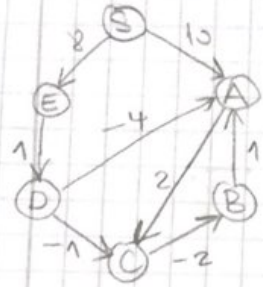
Izlaz: ABSCDEHGF

BFS



Izlaz: ABSCGD

BELLMAN FORD

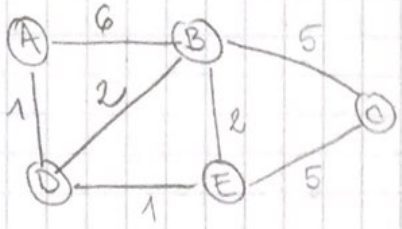


6 čvorova
5 iteracija

	S	A	B	C	D	E
	0	∞	∞	∞	∞	∞
1.iter. S	0	10	∞	∞	∞	8
A	0	10	∞	12	∞	8
C	0	10	10	12	∞	8
E	0	10	10	12	9	8
2.iter	0	~~10~~ 5	10	~~12~~ 8	9	8
	0	5	10	8	9	8
3.iter	0	5	~~10~~ 6	~~8~~ 7	9	8
	0	5	6	7	9	8
4.iter	0	5	~~6~~ 5	7	9	8
	0	5	5	7	9	8

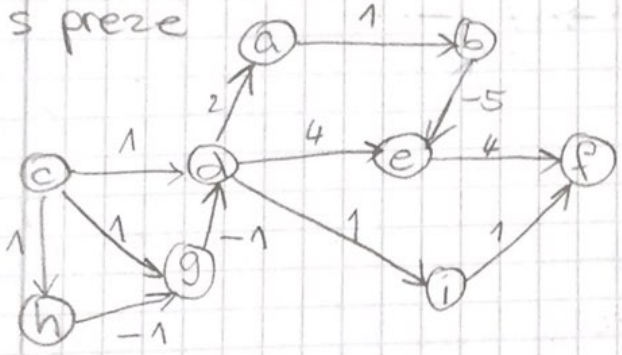
✓

DIJKSTRA



	1.	A	D	E	B	C
A	0					
B	∞	6	3	3		
C	∞	∞	∞	7	7	
D	∞	1				
E	∞	∞	2			

BF s preze



	a	b	c	d	e	f	g	h	i
	∞	∞	0	∞	∞	∞	∞	∞	∞
1 iter.	3	∞	0	~~∞~~	5	~~∞~~	1	1	2
	3	4	0	0	5	3	0	1	2
2. iter	2	4	0	-1	-1	2	0	1	1
3. iter	1	3	0	-1	-2	1	0	1	0
4. iter	1	2	0	-1	-3	1	0	1	0

$c \to h \to g \to d \to a \to b \to e$

$d \to i \to f$

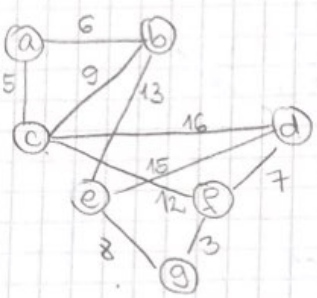
NAJKRAĆA RAZAPINJUĆA STABLA

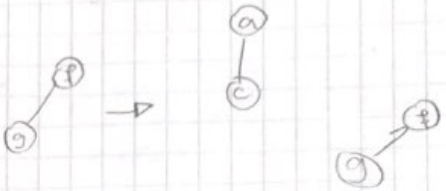
→ povezat sve vrhove s min bridova

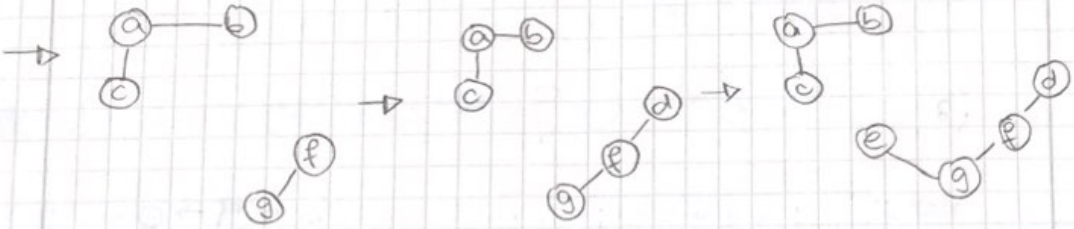
+PRIMOV ALG. fali

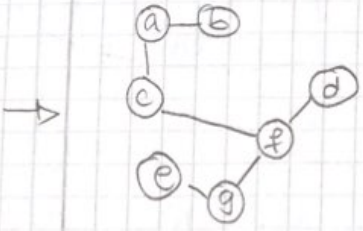
KRUSKALOV ALG

1. Sortirat bridove

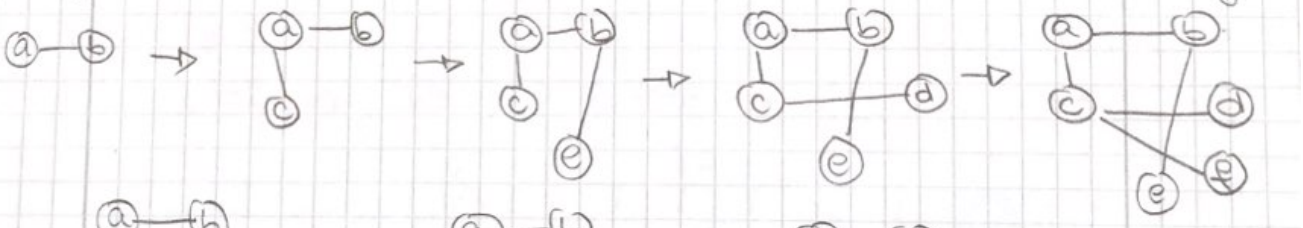


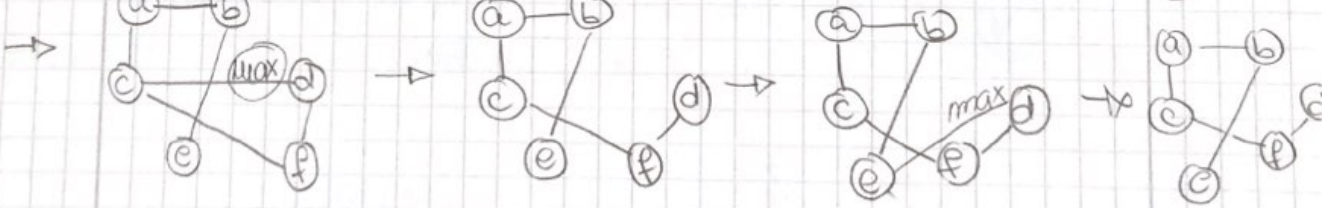






DIJSKTRA → dodavat, ako zaboravimo maknit max u krugu



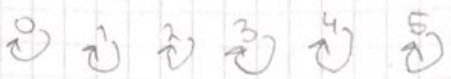


DONE

UNION – FIND

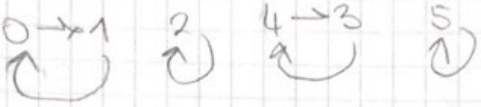
pr.

	0	1	2	3	4	5
ROOT	0	1	2	3	4	5
NEXT	0	1	2	3	4	5
LENGTH	1	1	1	1	1	1



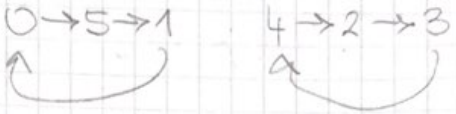
union (0,1) , union (4,3)

	0	1	2	3	4	5
ROOT	0	0	2	4	4	5
NEXT	1	0	2	4	3	5
L.	2	1	1	1	2	1



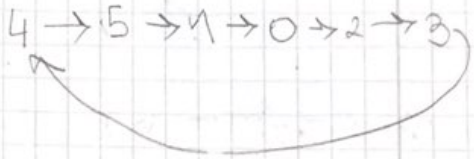
union (2,3) , union (0,5)

	0	1	2	3	4	5
ROOT	0	0	4	4	4	0
NEXT	5	0	3	4	2	1
L	3	1	1	1	3	1



union (2,1) → ISTE, KORIJEN OD DVOJKE OSTAJE

	0	1	2	3	4	5
ROOT	4	4	4	4	4	4
NEXT	2	0	3	4	5	1
L	3	1	1	1	6	1



EULEROVI GRAFOVI

→ povezani graf u kojem postoji putanja kroz svaki brid grafa

Eulerov ciklus → svi vrhovi parnog stupnja

Eulerova putanja → točno 2 vrha neparne

① uzimat brid po brid do zatvaranja ciklusa
② pronać vrh koji još ima bridova i krenuti iz njega
③ ponavljat ① i ② dok ima bridova

FLEURYEV ALGORITAM

→ ① ako se traži Eulerov ciklus krenut iz bilo kojeg vrha, a ako se traži putanja krenut iz jednog od dva neparna

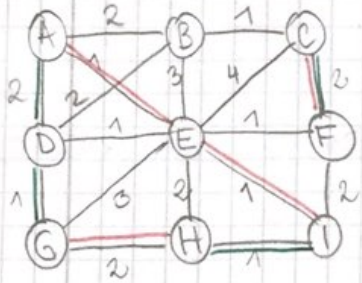
② ako smo prošli V_1, V_k, G_k graf s preostalim bridovima

a) ako je V_k izoliran u G_k, obiđeni čine eulerov ciklus ; KRAJ

b) ako ima još bridova, odabrat neki koji nije MOST

c) ako su svi mostovi, bilo koji

PROBLEM KINESKOG POŠTARA



	A	C	F	G	H	I
A	0	3	2	3	3	2
C		0	2	4	5	4
F			0	3	3	2
G				0	2	3
H					0	1
I						0

ac fg hi → 3 + 3 + 1 = 7
af gh ci → 3 + 3 + 3 = 9 ×
fi gh ac → 3 + 2 + 2 = 7

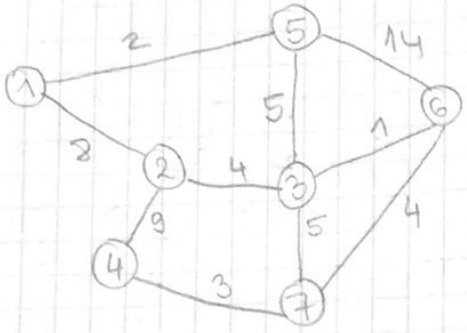
af cg hi → 2 + 4 + 1 = 7
ch gi → 2 + 5 + ×
ci gh → 2 + 4 + 2 ×

ag cf hi → 3 + 2 + 1 = 6 ✓
ch fi 3 + 5 + ×
ci fh 3 + 4 + ×

ah cf gi 3 + 4 + ×
cg fi 3 + 4 + ×
ci fg 3 + 4 + ×

ai cf gh → 2 + 2 + 2 = 6 ✓
cg fh 2 + 4 + ×
ch fg 2 + 5 + ×

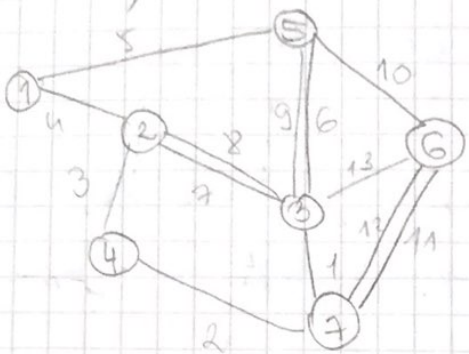
(samo iz tablice kombinacije



	2	5	6	7
2	0	9	5	9
5		0	6	10
6			0	4
7				0

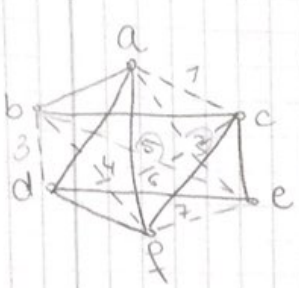
2 - 5 , 6 - 7
2 - 6 , 5 - 7
2 - 7 , 5 - 6

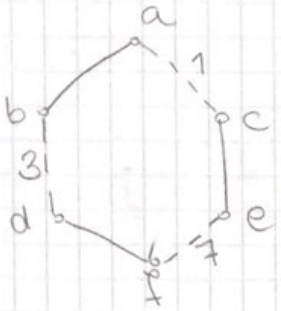
9 + 4 = 13 ←
5 + 10 = 15
9 + 6 = 15

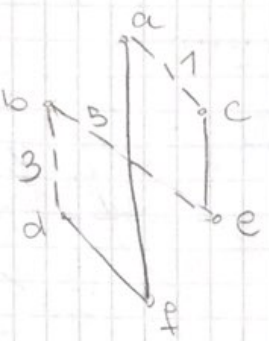


HAMILTONOVI GRAFOVI

→ krećemo iz jednog vrha, obilazimo sve bridove i završimo u istom vrhu



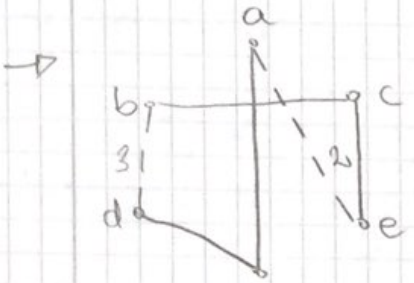


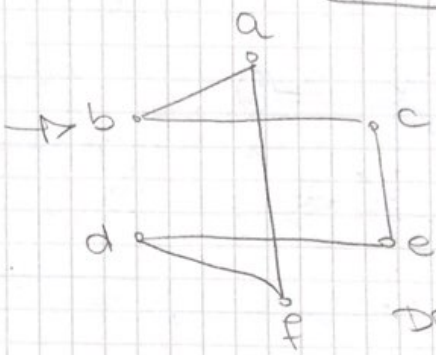


f→d→b→a→c→e

f→a→c→e→b→d

b→d→f→a→c→e



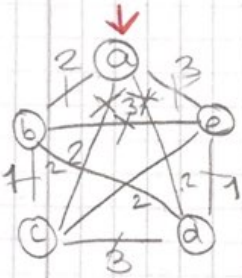


b→c→e→a→f→d)

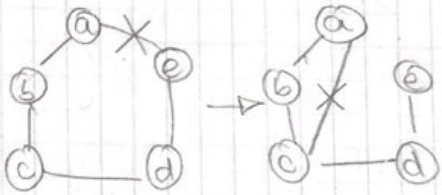
b→a→f→d→e→c)

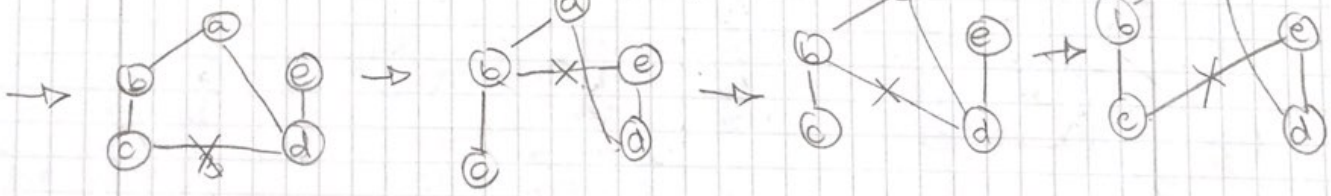
DONE?

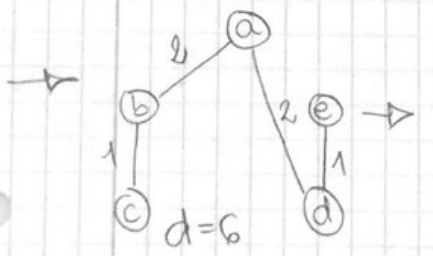
PROBLEM TRGOVAČKOG PUTNIKA



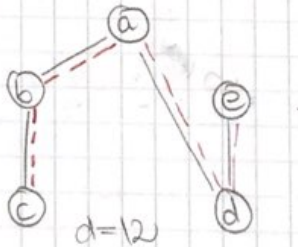
→ DIJKSTRA →



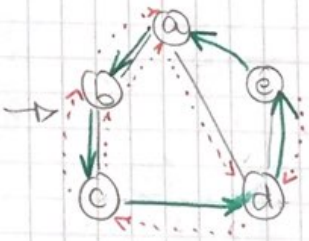




d=6



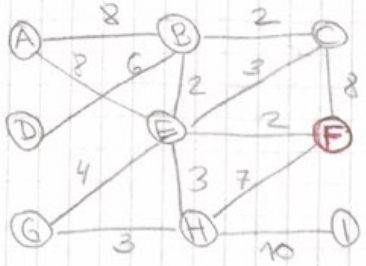
d=12



d=10

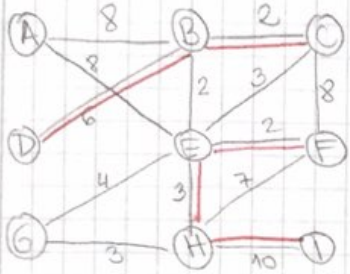
Z1 2019 Problem kin. postara

3.

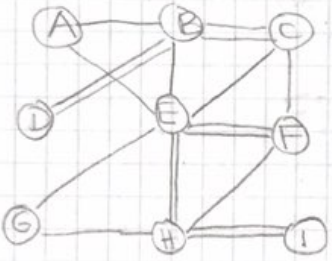


	cd	cf	i
c	0	8	5 16
d		0	10 21
f			0 15
i			0

cd fi → 8 + 15 = 23 ✓
cf di → 5 + 21 = 26
ci df → 16 + 10 = 26

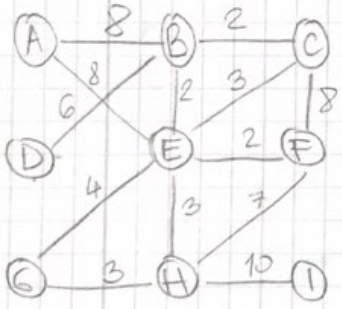


8+2·2+2·6+8+2+3+8+2·2+7
+2·10+2·3+4+3
= 89



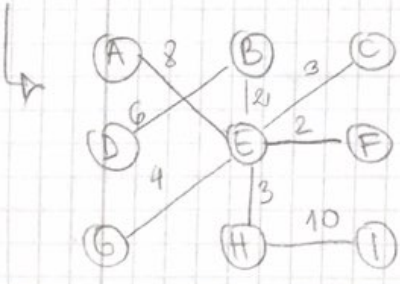
F→C→B→A→E→B→D→B
→C→E→G→H→E→F→H→I
→H→E→F

(4) NAJKRAĆE RAZAPINJUĆE STABLO (IZ F)



	f	e	b	c	h	g	a	d	i
a	∞	∞	10	10	10	10			
b	∞	∞	4						
c	∞	8	5	5					
d	∞	∞	∞	10	10	10	10	10	
e	∞	2							
f	0								
g	∞	∞	6	6	6				
h	∞	7	5	5	5				
i	∞	∞	∞	∞	∞	15	15	15	15

f→e→a = 10
f→e→b = 4
f→e→c = 5
f→e→b→d = 10
f→e = 2
f→e→g = 6
f→e→h = 5
f→e→h→i = 15

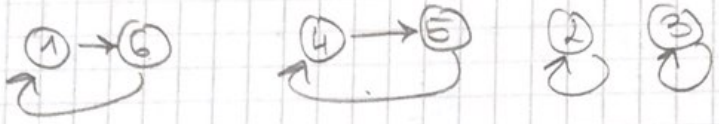


5. MISLIM DA DA

6.

	1	2	3	4	5	6
ROOT	1	2	3	4	5	6
NEXT	1	2	3	4	5	6
L.	1	1	1	1	1	1

1) union (1,6) , union (4,5)

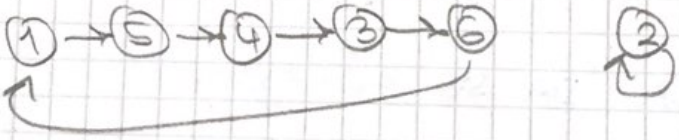


	1	2	3	4	5	6
ROOT	1	2	3	4	4	1
NEXT	6	2	3	5	4	1
L.	2	1	1	2	1	1

2) union (3,6) 3 ima manji L pa njega šestci !

	1	2	3	4	5	6
ROOT	1	2	1	4	4	1
NEXT	3	2	6	5	4	1
L.	3	1	1	2	1	1

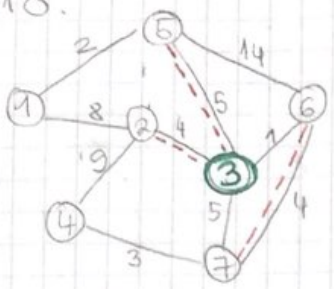
3) union (5,3) 5 trici !



	1	2	3	4	5	6
ROOT	1	2	1	1	1	1
NEXT	5	2	6	3	4	1
L.	5	1	1	2	1	1

21 2018.

3.



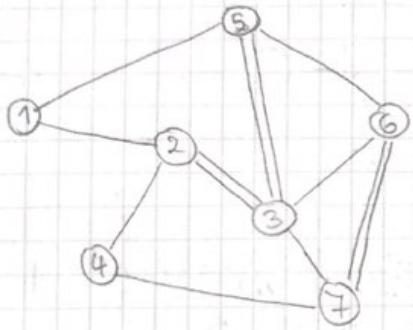
KINESKI POŠTAR, KREĆE IZ ③

	2	5	6	7
2	0	9	5	9
5		0	6	10
6			0	4
7				0

25 67 → 9 + 4 = 13 ✓

26 57 → 5 + 10 = 15

27 56 → 9 + 6 = 15

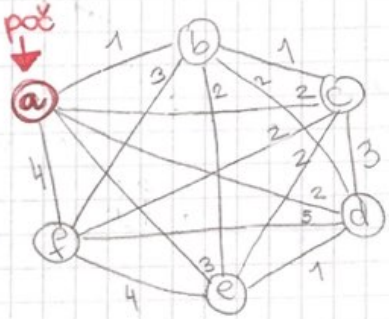


3→2→4→7→3→6→7

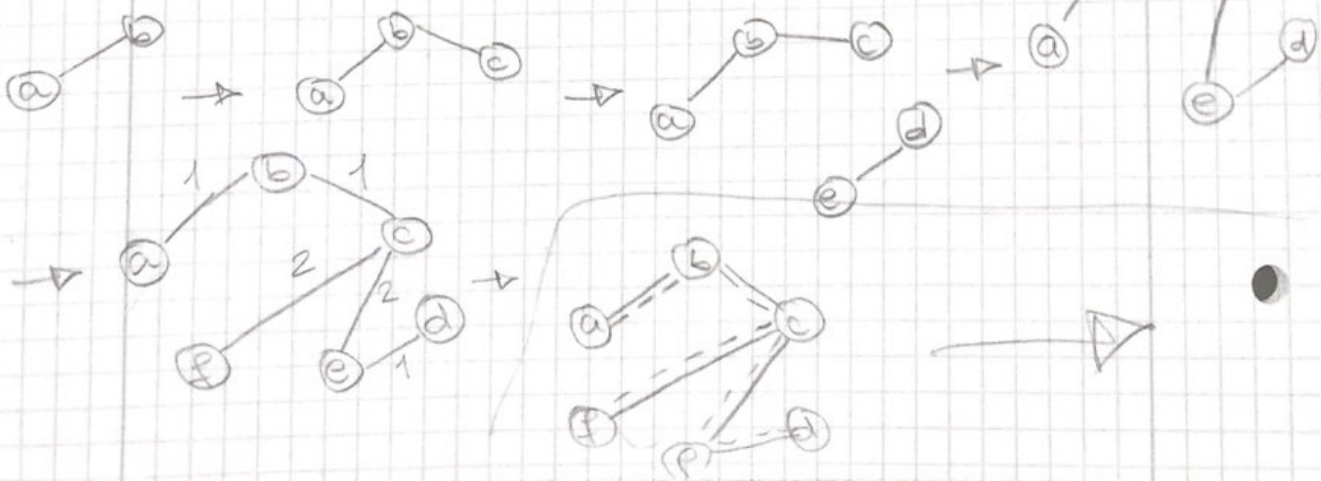
→6→5→1→2→3→5→3

5. TRGOVAČKI PUTNIK → 2 MST HEURISTIKA

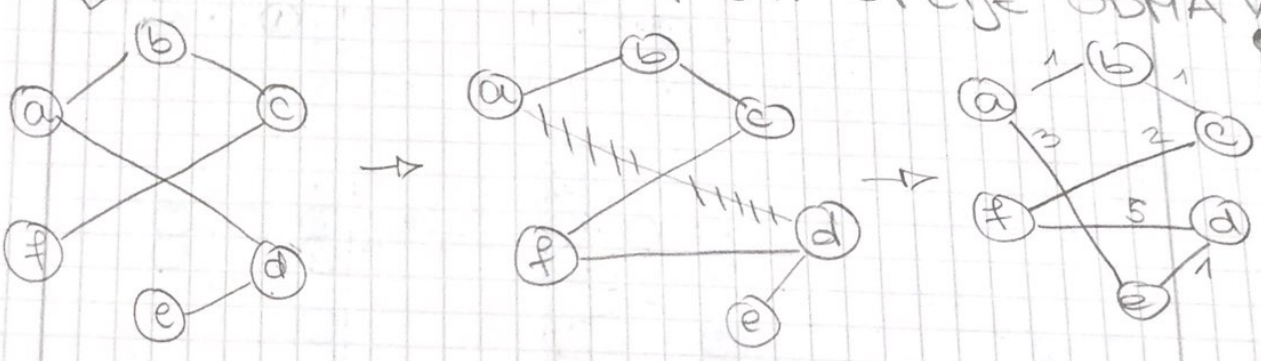
	a	b	c	d	e	f
a	0	1	2	3	3	4
b		0	1	2	2	3
c			0	3	2	2
d				0	1	5
e					0	4
f						0



→ KORISTIT ĆU KRUSKALA (DA SE PODSJETIM)



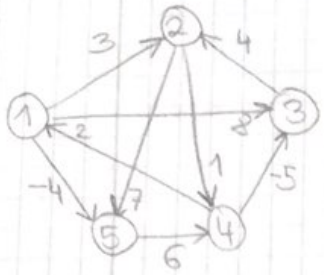
MOŽDA NAJBOLJE POČ. VRHU DAT DVI OPCIJE ODMA ▽



d=13

WFI predavanja primjer

NIL = X



Put između 3 i 5

$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad \Pi^{(0)} = \begin{bmatrix} X & 1 & 1 & X & 1 \\ X & X & X & 2 & 2 \\ X & 3 & X & X & X \\ 4 & X & 4 & X & X \\ X & X & X & 5 & X \end{bmatrix}$$

$$D^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad \Pi^{(1)} = \begin{bmatrix} X & 1 & 1 & X & 1 \\ X & X & X & 2 & 2 \\ X & 3 & X & X & X \\ 4 & 1 & 4 & X & 1 \\ X & X & X & 5 & X \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad \Pi^{(2)} = \begin{bmatrix} X & 1 & 1 & 2 & 1 \\ X & X & X & 2 & 2 \\ X & 3 & X & 2 & 2 \\ 4 & 1 & 4 & X & 1 \\ X & X & X & 5 & X \end{bmatrix}$$

uvijek ostaje — dobro

$$D^{(3)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad \Pi^{(3)} = \begin{bmatrix} X & 1 & 1 & 2 & 1 \\ X & X & X & 2 & 2 \\ X & 3 & X & 2 & 2 \\ 4 & 3 & 4 & X & 1 \\ X & X & X & 5 & X \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix} \qquad \Pi^{(4)} = \begin{bmatrix} X & 1 & 1 & 2 & 1 \\ 4 & X & 4 & 2 & 1 \\ 4 & 3 & X & 2 & 1 \\ 4 & 3 & 4 & X & 1 \\ 4 & 3 & 4 & 5 & X \end{bmatrix}$$

WFI $O(V^3)$

BF $O(EV)$

DJ $O(E + V\log V)\,V$

$(E\log V + V\log V)$

(V^2)

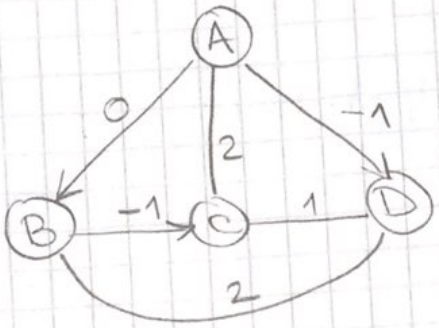
$$D = \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix} \qquad \Pi^{5} = \begin{bmatrix} X & 3 & 4 & 5 & 1 \\ 4 & X & 4 & 2 & 1 \\ 4 & 3 & X & 2 & 1 \\ 4 & 3 & 4 & X & 1 \\ 4 & 3 & 4 & 5 & X \end{bmatrix}$$

BC

FF

LR 2012

WFI ②



D^0 =

	A	B	C	D
A	0	0	2	-1
B	∞	0	-1	2
C	2	∞	0	1
D	∞	2	1	0

Π =

	A	B	C	D
A	X	A	A	A
B	X	X	B	B
C	C	X	X	C
D	X	D	D	X

D^1 =

	A	B	C	D
A	0	0	2	-1
B	∞	0	-1	2
C	2	2	0	1
D	∞	2	1	0

Π^1 =

	A	B	C	D
A	X	A	A	A
B	X	X	B	B
C	C	A	X	C
D	X	D	D	X

D^2 =

	A	B	C	D
A	0	0	-1	-1
B	∞	0	-1	2
C	2	2	0	1
D	∞	2	1	0

Π =

	A	B	C	D
A	X	A	B	A
B	X	X	B	B
C	C	A	X	C
D	X	D	D	X

D^3 =

	A	B	C	D
A	0	0	-1	-1
B	1	0	-1	0
C	2	2	0	1
D	3	2	1	0

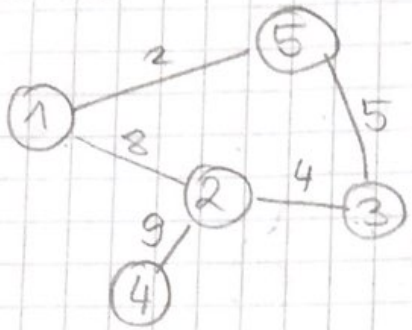
Π^3 =

	A	B	C	D
A	X	A	B	A
B	C	X	B	C
C	C	A	X	C
D	C	D	D	X

D^4 = isto sve ∎

Z-2018

4.



WFI

$$D^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 8 & \infty & \infty & 2 \\ 8 & 0 & 4 & 9 & \infty \\ \infty & 4 & 0 & \infty & 5 \\ \infty & 9 & \infty & 0 & \infty \\ 2 & \infty & 5 & \infty & 0 \end{bmatrix}$$

$$\Pi^0 = \begin{bmatrix} X & 1 & X & X & 1 \\ 2 & X & 2 & 2 & X \\ X & 3 & X & X & 3 \\ X & 4 & X & X & X \\ 5 & X & 5 & X & X \end{bmatrix}$$

$$D^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 8 & 12 & 17 & 2 \\ 8 & 0 & 4 & 9 & \infty \\ \infty & 4 & 0 & 13 & 5 \\ \infty & 9 & 13 & 0 & \infty \\ 2 & 10 & 5 & \infty & 0 \end{bmatrix}$$

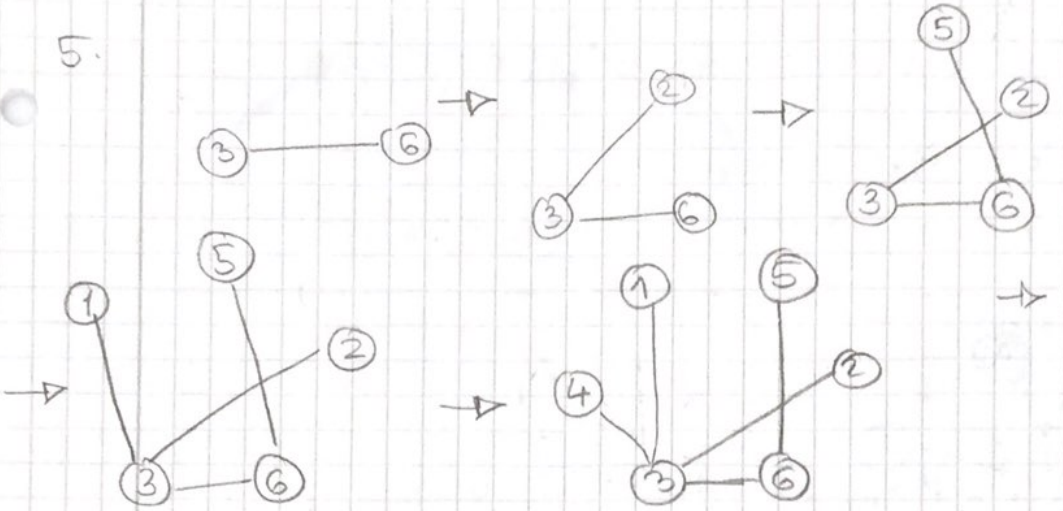
$$\Pi^1 = \begin{bmatrix} X & 1 & 2 & 2 & 1 \\ 2 & X & 2 & 2 & X \\ X & 3 & X & 2 & 3 \\ X & 4 & 2 & X & X \\ 5 & 1 & 5 & X & X \end{bmatrix}$$

$$D^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 8 & 7 & 17 & 2 \\ 8 & 0 & 4 & 9 & 9 \\ 7 & 4 & 0 & 13 & 5 \\ 12 & & & 0 & \\ & & & & 0 \end{bmatrix}$$

$$\Pi^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ X & 1 & 5 & 2 & 1 \\ 2 & X & 2 & 2 & 3 \\ 5 & 3 & X & 2 & 3 \\ & & & X & \\ & & & & X \end{bmatrix}$$

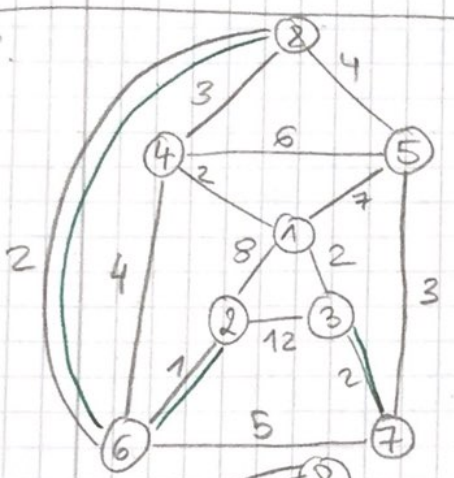
2017

5.



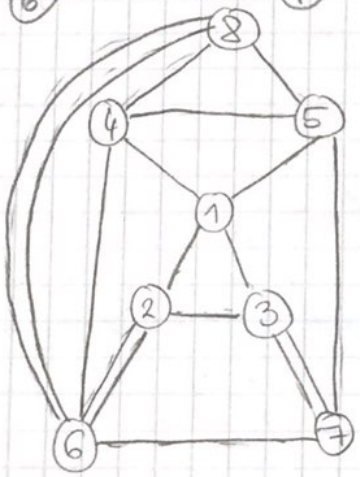
→ sva vrhova spojena, KRAJ!

6.



	2	3	7	8
2	0	8	6	3
3		0	2	9
7			0	7
8				0

28 78 → 8 + 7 = 15
27 38 → 6 + 9 = 15
28 37 → 3 + 2 = 5 ✓



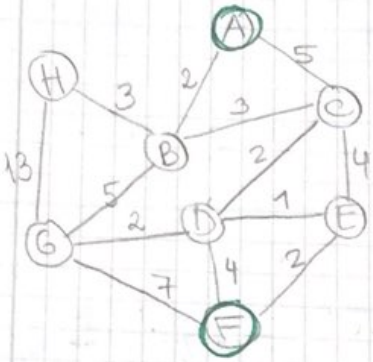
3 → 1 → 5 → 7 → 6 → 8 → 4 → 6
→ 2 → 6 → 8 → 5 → 4 → 1 → 2 → 3 → 7 → 3

2016.

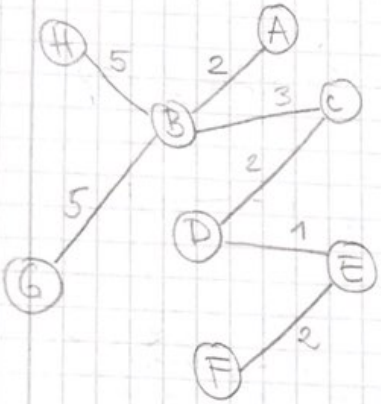
3. Eulerov je, svaki vrh je parnog stupnja

ciklus: $1 \to 2 \to 14 \to 13 \to 2 \to 3 \to 13 \to 12 \to 3 \to 4 \to 12 \to 11$
$\to 4 \to 5 \to 11 \to 10 \to 5 \to 6 \to 10 \to 9 \to 6 \to 7 \to 9 \to 8 \to 7 \to 1$

4.



	A	B	C	H	D	G	E	F
A	0							
B	∞	2						
C	∞	5	5					
D	∞	∞	∞	7	7			
E	∞	∞	∞	9	9	8	8	
F	∞	∞	∞	∞	∞	11	11	10
G	∞	∞	7	∞	7	7		
H	∞	∞	5	5				



$A \to B \to C \to D \to E \to F$

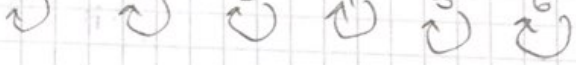
ili

$A \to C \to D \to E \to F$

$d = 10$

21 - 2015

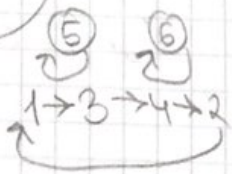
4.



1) (odmah 1,2 i 4,3) $1 \to 2$ $4 \to 3$

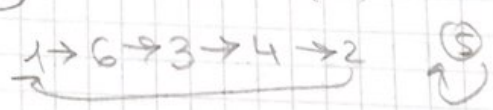
r	1	1	4	4	5	6
n	2	1	4	3	5	6
L	2	1	1	2	1	1

union (2,3)



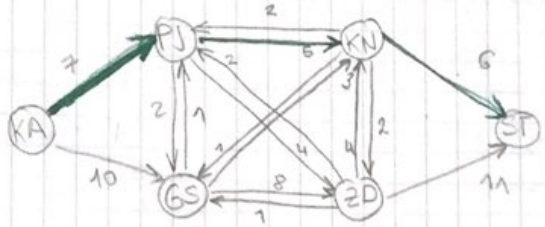
r	1	1	1	1	5	6
n	3	1	4	2	5	6
L	4	1	1	2	1	1

3) union (1,6)



r	1	1	1	1	5	1
n	6	1	4	2	5	3
L	5	1	1	2	1	1

PROTOK U MREŽAMA



nereegativna f_p

$(G=(V,E), c, s \in V, t \in V)$

polazište odredište

FORD FULKERSON

$f(u,v)=0, \forall u,v$

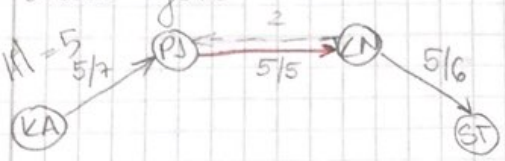
dok u mreži postoji dopunski put p

↳ uvećaj sve tokove na putu p za f_p

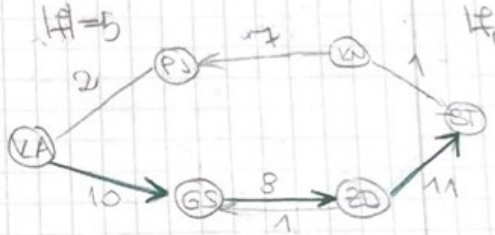
* f_p → preostali kapacitet na putu p

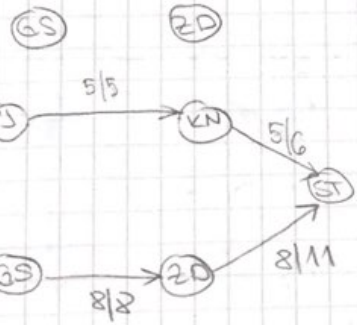
① slika gore

② $|f|=5$



→





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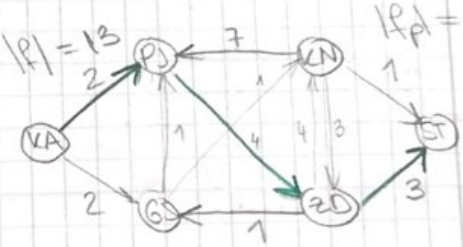
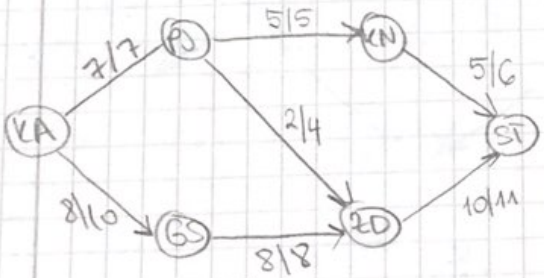
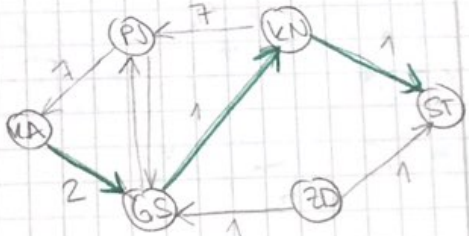
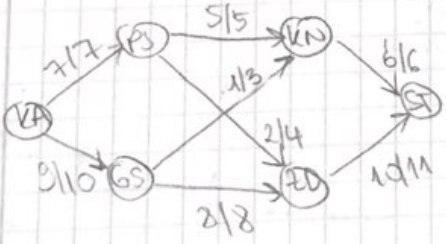
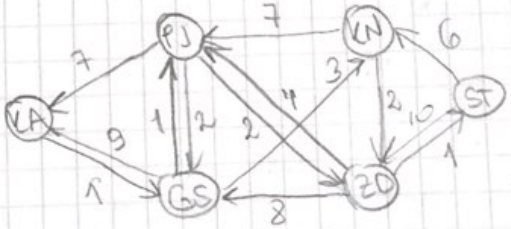








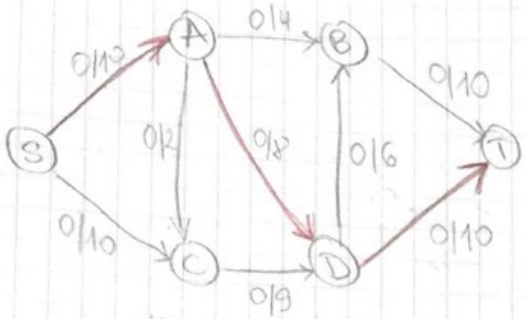


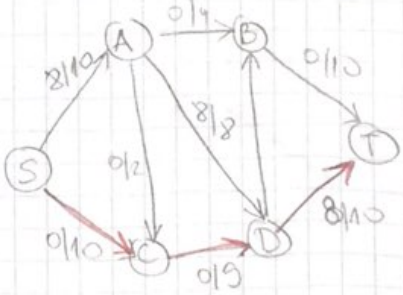
2 opcije ili KA → GS → PJ → ZD → ST

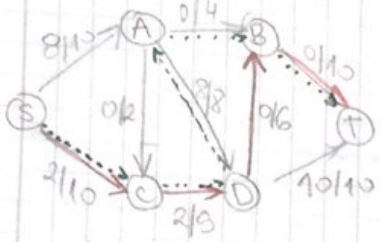
ili KA → GS → KN → ZD → ST

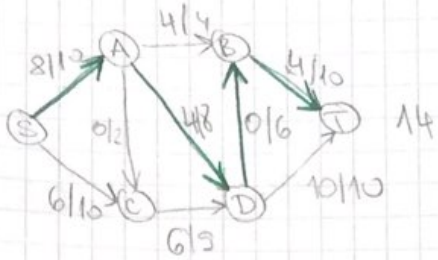
svejedno je

pr

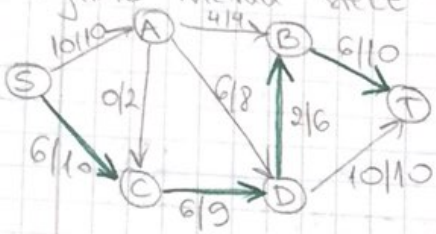


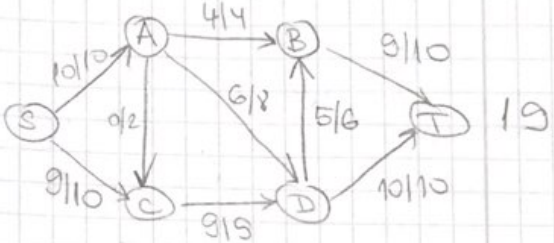






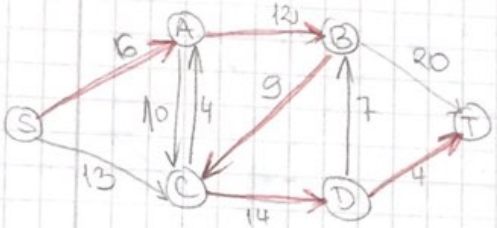
Ne crveni put jer s njime nema sreće

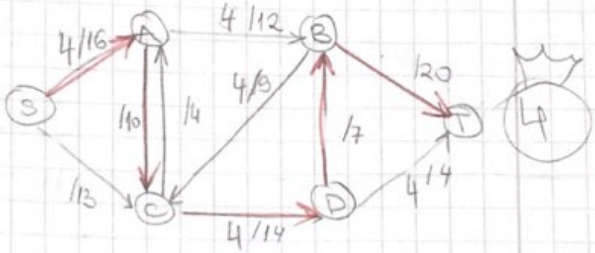


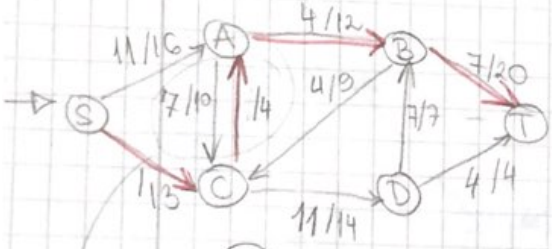


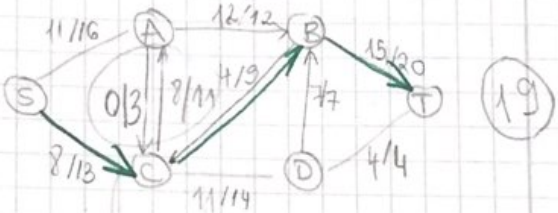
6.3.8

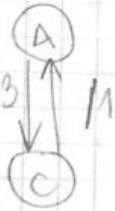
pr

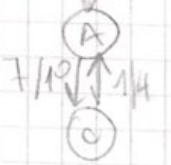


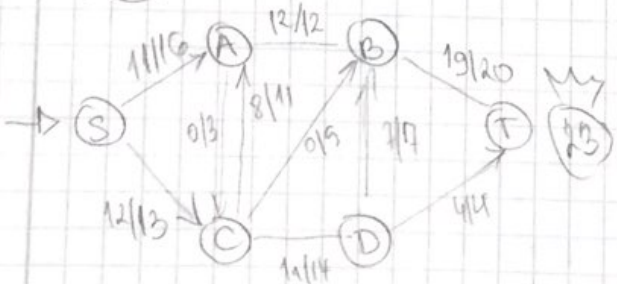












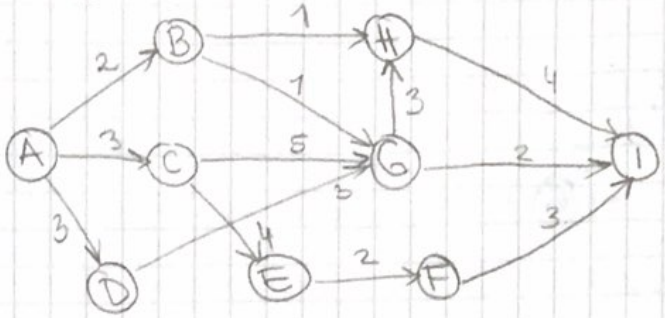
Ista ne ide sad jer

1/ S → A → C → B → T 24

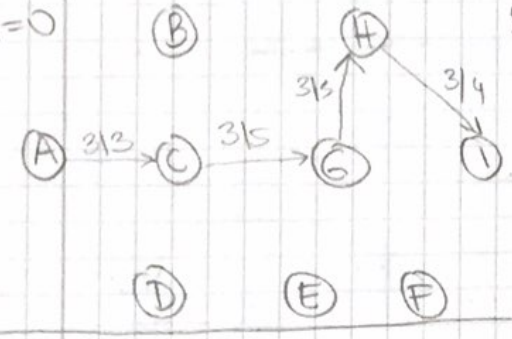
pa bude

2 - 2010

4.

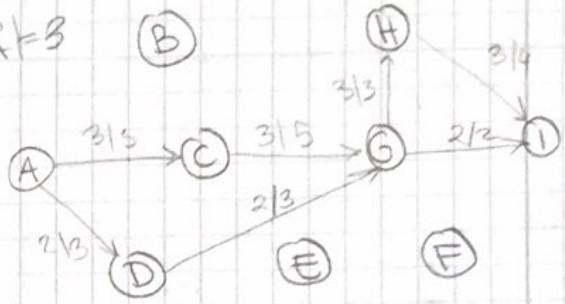


$|f|=0$



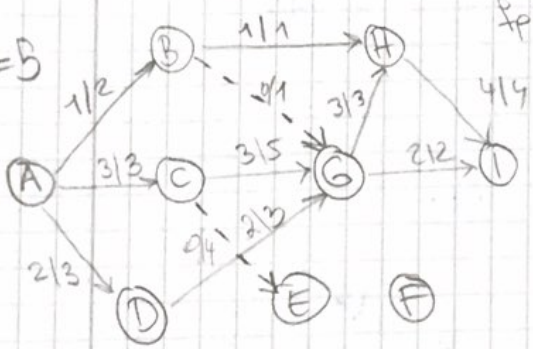
$f_p=3$

$|f|=3$



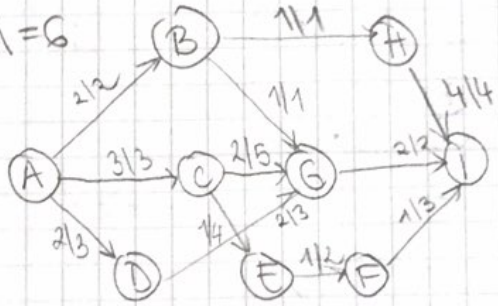
$f_p=2$

$|f|=5$



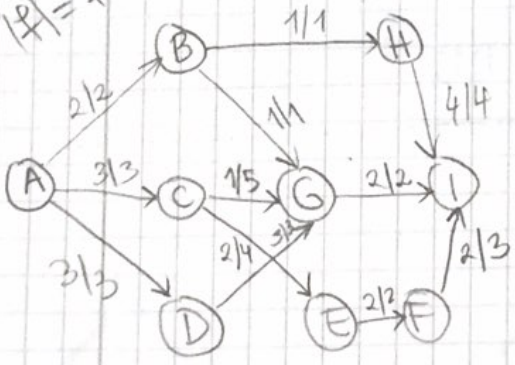
$f_p=1$

$|f|=6$



$f_p=1$

$|f|=7$



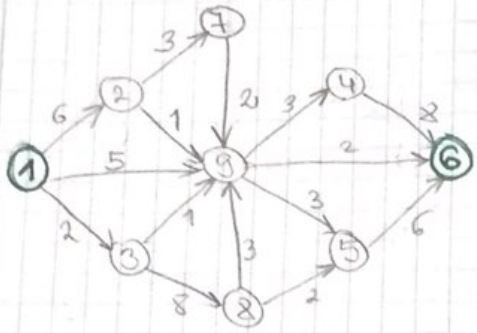
$f_p=1$

$|f|=8$

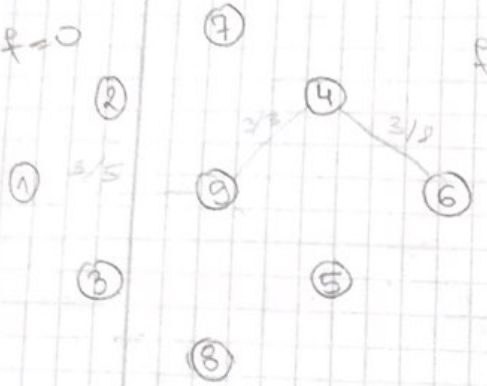
$(4+2+2)$

7 - 2012

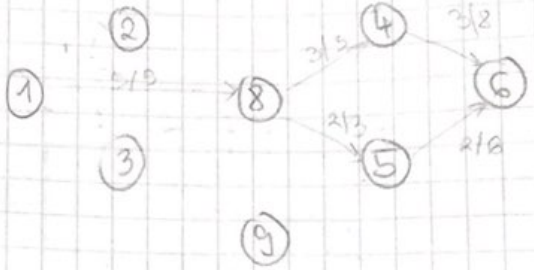
4.



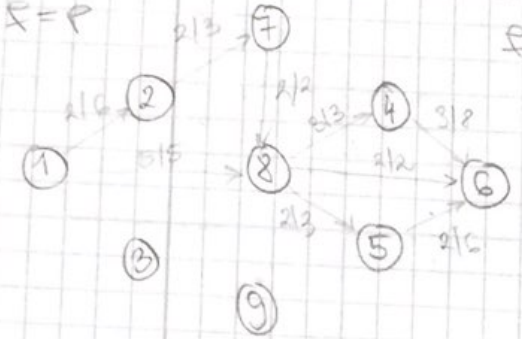
f = 0 $f_p = 3$



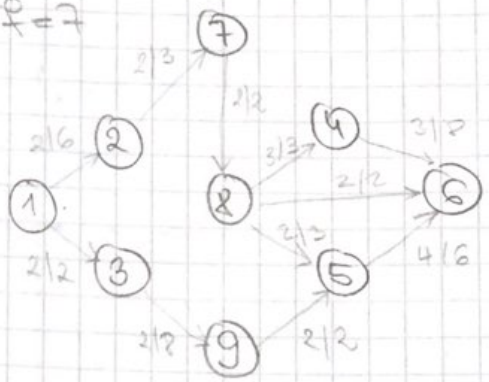
f = 3 $f_p = 2$



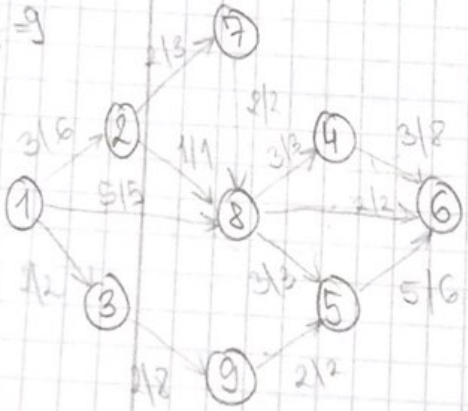
f = 5 $f_p = 2$



f = 7 $f_p = 2$



f = 9 $f_p = 1$

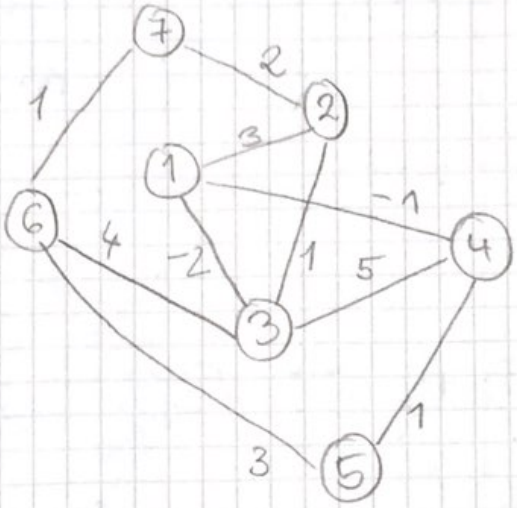


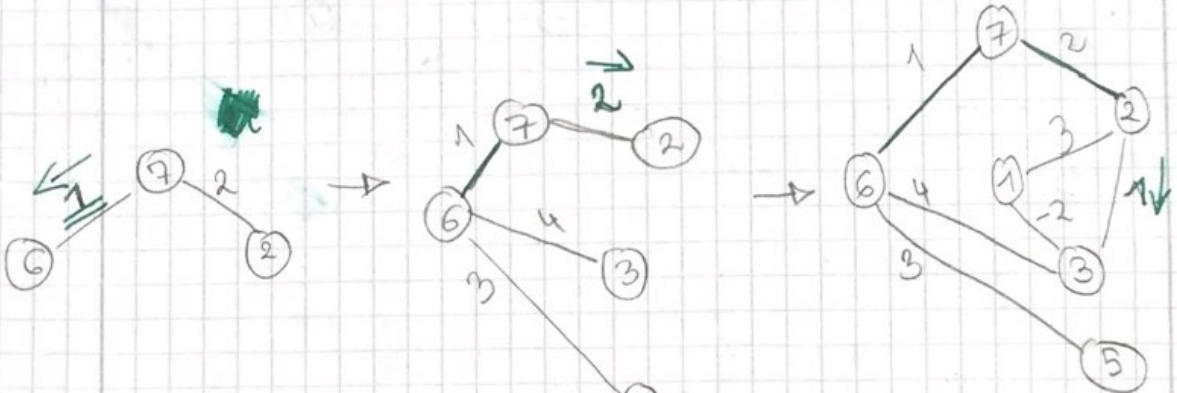
$|S| = 10$

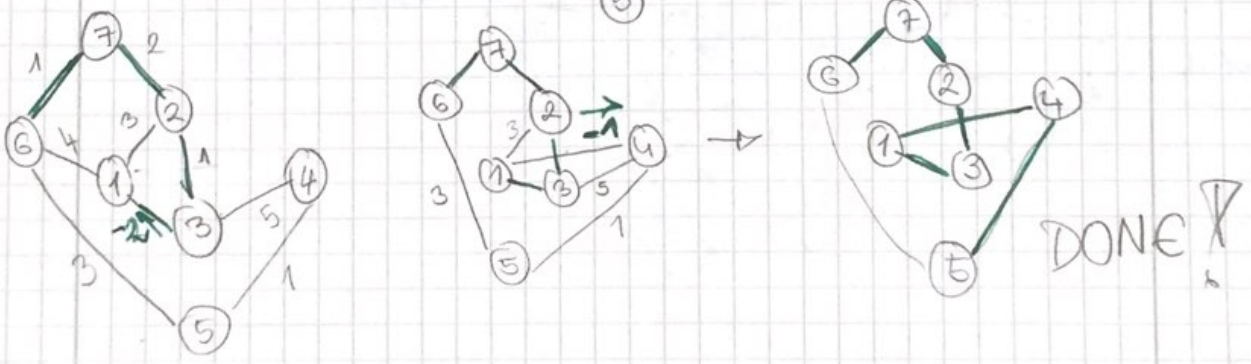
Z - 2013.

3. PRIMOV ALG

→ odaberemo poč. vrh (zadano ode), gledamo njegovu udaljenost od susjednih vrhova i krećemo se u smjeru najmanje udaljenosti cilo vrime, provjeravamo sve udaljenosti koje smo redom označili (ti vrhovi biti u nekom trenutku susjedi

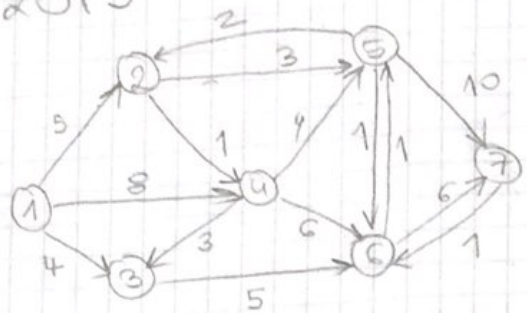






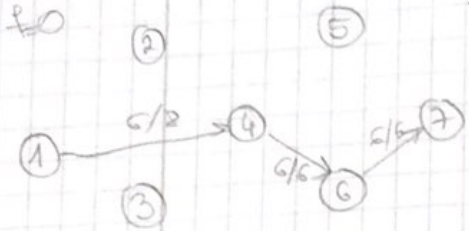
Z1 – 2013

4)



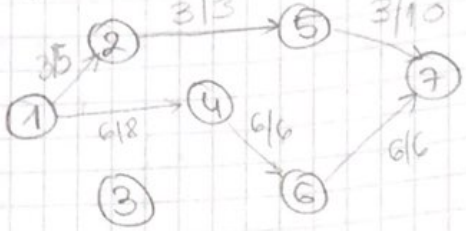
$f_p = 3$

f=0

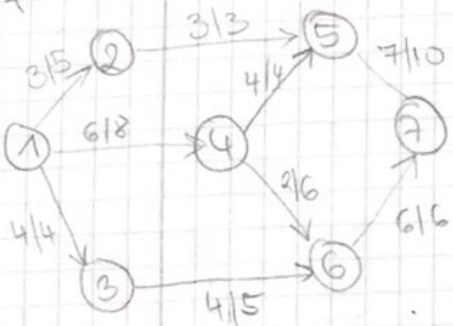


$f_p = 6$

f=6

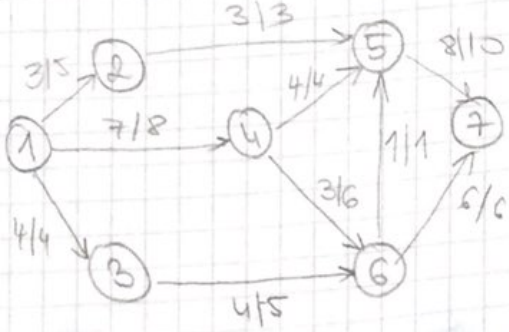


f=9



$f_p = 4$

f=13

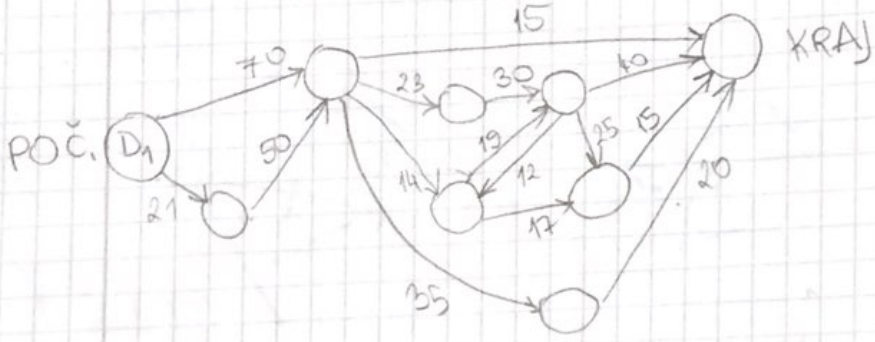


$f_p = 1$

f=14

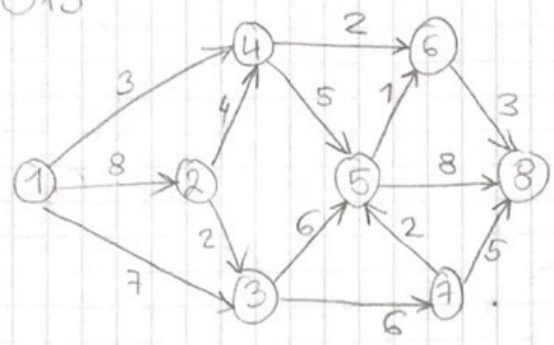
LJR 2013

5. (samo postavit)

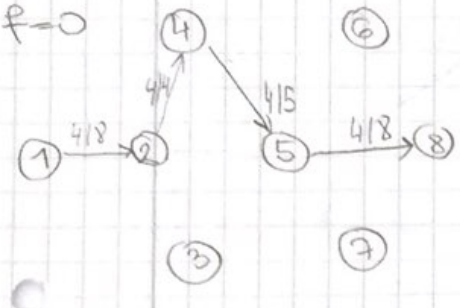


21-2015

5.

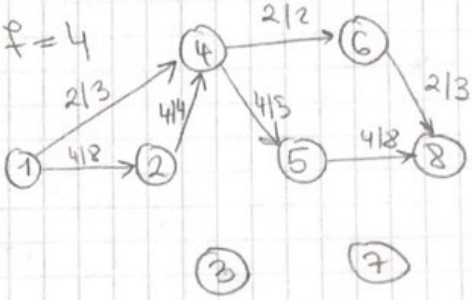


f=0



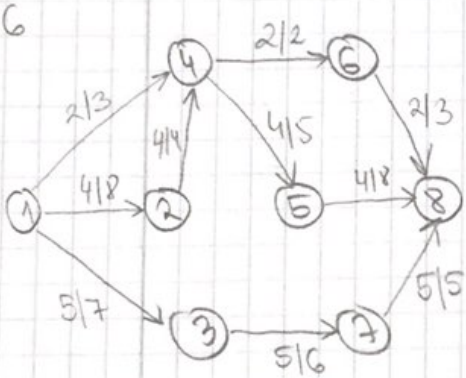
$f_p = 4$

f=4



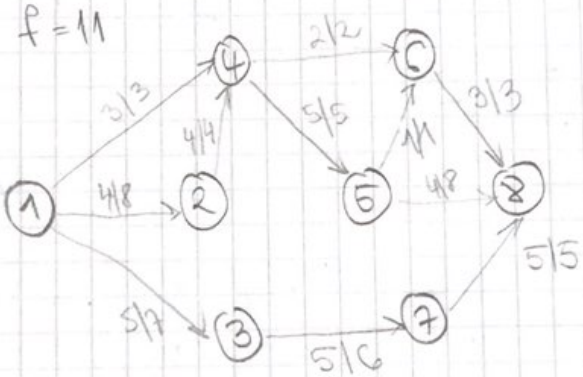
$f_p = 2$

f=6



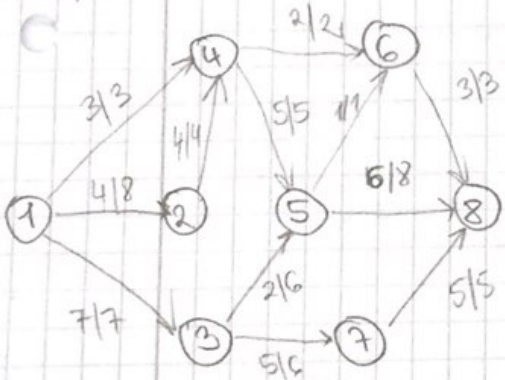
$f_p = 5$

f=11



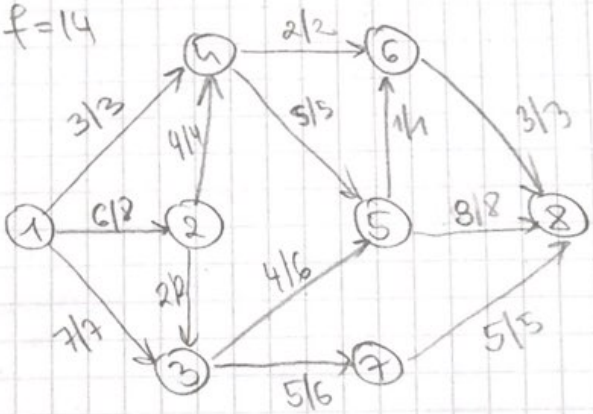
$f_p = 1$

f=12



$f_p = 2$

f=14



$|f| = 16$

2016.

SIMPLEX GRAFIČKI

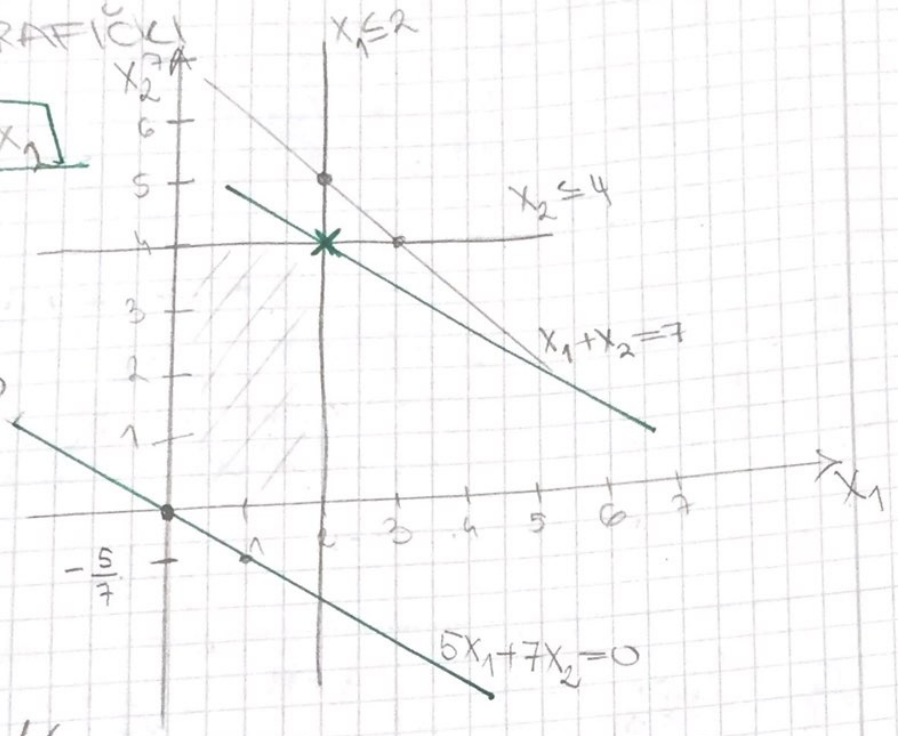
$\max \boxed{5x_1 + 7x_2}$

$x_1 \leq 2$

$x_2 \leq 4$

$x_1 + x_2 \leq 7$

$x_1 \geq 0 \quad x_2 \geq 0$



Točka (2, 4) //