

INT202 Complexity of Algorithms

Tutorial 2

Q1. Compute the following expressions that are given in postfix notation.

(a) $1\ 2\ +\ 4\ *\ 3\ +$

(b) $5\ 7\ *\ 3\ 4\ 1\ +\ *\ -$

(c) $3\ 4\ 2\ *\ 1\ 5\ -\ 2\ ^\wedge\ /\ +$

Note: The $^$ symbol means exponentiation, i.e. $x\ y\ ^$ means x^y in the usual infix notation.

Q2. Compute Big-Oh notation.

a) `void func(int n){`

`int i = 0;`

`while(i * i * i <= n)`

`i++;`

`}`

b) `void recursive(int n, int m, int o) {`

`if (n <= 0) {`

`printf("%d, %d\n", m, o);`

`} else {`

`recursive(n - 1, m + 1, o);`

`recursive(n - 1, m, o + 1);`

`}`

`}`

c)

`void func(int n) {`

`for (int i = 1, s = 0; i <= n; ++i) {`

`int t = 1;`

`for (int j = 1; j <= i; ++j)`

`t = t * j;`

`}`

`}`

Q3. State whether the following statement is true or false, and give a brief justification for your answer.

a) If $f(n)$ and $g(n)$ are both $O(h(n))$, then $f(n)+g(n)$ is $O(h(n))$.

b) The asymptotic complexity of $\sum_{i=1}^n \log i$ is $O(\log n)$

Q4. Consider the following code fragment.

```
for i=1 to n do
    for j=i to 2*i do
        output hello
```

Let $T(n)$ denote the number of times 'hello' is printed as a function of n .

1) Express $T(n)$ as a summation;

2) Simplify the summation and give the worst-case running time using Big-Oh notation.

Q5. Consider the following code fragment.

```
for i=1 to n-1 do
    for j=i+1 to n do
        for k=1 to j do
            output hello
```

Let $T(n)$ denote the number of times 'hello' is printed as a function of n .

1) Express $T(n)$ as a summation;

2) Simplify the summation, and give the worst-case running time using Big-Oh notation.

Q6. Give a tight bound of the runtime complexity class for the following code fragment in Big-Oh notation, in terms of the variable N .

```
int sum = N;
for (int i = 0; i < 1000; i++) {
    for (int j = 1; j <= i; j++) {
        sum += N;}
    for (int j = 1; j <= i; j++) {
        sum += N;}
    for (int j = 1; j <= i; j++) {
        sum += N;}
}
```

Q7. Prove that $3\log n + \log \log n$ is $\Theta(\log n)$