INT202 Complexity of Algorithms

Tutorial 1

1. What is the theoretical maximum input size $n$ of a problem to be solved in 6 seconds on a 2.5 GHz single core processor that can perform up to 4 operations per clock cycle if the running time of its solution is $6n^5$?

2. Find the asymptotic notation in Big-Oh:

(a)
```
int i = 1;
    i++;
int m = i * i;
```

(b)
```
int i = 1;
while (i < n){
    i = i * 2;
}
```

(c)
```
int i = 2;
while (i < n){
    i = i * i;
}
```

(d)
```
int j = 0;
for (int i = 0; i <= n; i++) {
    j = i;
    j = j * j;
}
```

(e)
```
for (int m = 0; m < n; m++) {
    i = 1;
    while (i < n){
        i = i * 2;
    }
}
```

(f)
```
for (int i = 0; i <= n; i++) {
    for (int j = 0; j <= n; j++) {
        j = i;
        j = j * j;
    }
}
```

3. Multiplying two $n \times n$ square matrices $A = (a_{ij})_{1 \le i,j \le n}$ and $B = (b_{ij})_{1 \le i,j \le n}$ gives a matrix

$$C = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{pmatrix}$$

wherein $$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \quad \forall 1 \le i, j \le n$$

(a) Write a pseudo-code that performs the operation C = A · B

(b) Give its asymptotic notation T(n) $\in$ O(g(n)) (i.e T(n) is Big-Oh of g(n))