Xi'an Jiaotong-Liverpool University
西交利物浦大學

PAPER CODE	EXAMINER	DEPARTMENT	TEL
CSE 204		Computer Science and Software Engineering	

2017/18 SEMESTER 2 – RESIT EXAMINATIONS

BACHELOR DEGREE – Year 3

Complexity of Algorithms

TIME ALLOWED: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. **Total marks available are 100. This accounts for 80% of the final mark.**
2. **The number in the column on the right indicates the marks for each question.**
3. **Answer all questions.**
4. **Answers should be written in English in the answer script provided.**
5. **Relevant and clear steps should be included in the answers.**

Notes:

- **To obtain full marks for each question, relevant and clear steps should be included in the answers.**
- **Partial marks may be awarded depending on the degree of completeness and clarity.**

Question 1: Algorithm analysis [20 marks]

1. Consider the following code fragment.

```
function mystery(n)
    r=0
    for i=1 to n-1 do
        for j=i+1 to n do
            for k=1 to j do
                output foobar
    return(r)
```

Let T(n) denote the number of times `foobar' is printed as a function of n.

1) Express T(n) as a summation (actually two nested summations). [5 marks]

2) Simplify the summation, and give the worst-case running time using Big Oh notation. [5 marks]

2. Analyzing Recursion. Note: although the question asks you to give the answers in big-O notation, your answers should be tight up to constant factors. We have the following piece of code, representing a recursive function.

```
function NoIdea (n)
  if n > 1 :
  print `A'
  NoIdea (n/3)
  for i = 1 . . . n
    print `B'
  end for
  NoIdea (n/3)
```

1) What is the runtime of the above function? Express your answer using the big-O notation. [5 marks]

2) Express the number of times that this algorithm prints `A' in terms of n using the big-O notation. [5 marks]

Question 2: AVL Tree and Heap [15 marks]

1. Answer the following questions:

1) Insert the following sequence of elements into an AVL tree, starting with an empty tree: 10, 20, 15, 25, 30, 16, 18, 19. Draw the AVL tree that results when the above elements are added (in the given order) to an empty AVL tree. [4 marks]

2) Delete 30 in the AVL tree that you got. [3 marks]

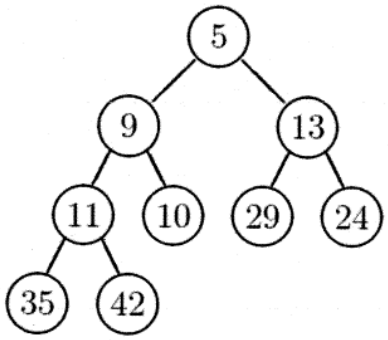
3) What is the running time of a deletion operation in an AVL tree? [2 marks]

2. Given the following pre-order and in-order tree traversals, draw the tree. [2 marks]

Pre-order A, B, C, D, E, F

In-order B, A, D, E, C, F

3. Given the following heap, draw the heap that would result after deleting the minimum element. [4 marks]

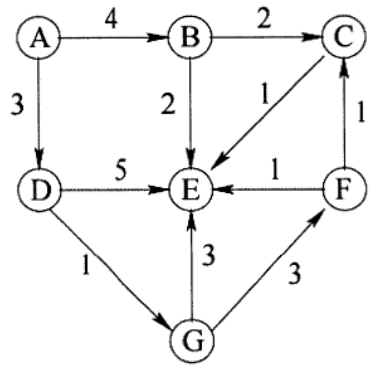


Question 3: Graph [15 marks]

1. Execute Dijkstra's algorithm on the following graph starting at vertex A. If there are any ties, the vertex with the lower-alphabetic-order letter comes first.

1) List the vertices in the order in which they are deleted from the priority queue and for each the shortest distance from A to the vertex. [4 marks]

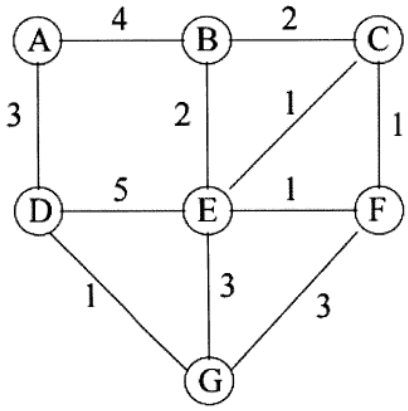
2) Draw the shortest paths tree that results. [3 marks]



2. Prim and Kruskal Executions

1) Execute Prim's algorithm on the following graph starting at vertex A. If there are any ties, the vertex with the lower letter comes first. List the edges in the order in which they are added to the tree. [4 marks]

2) Execute Kruskal's algorithm on the following graph starting at vertex A. Assume that equal weight edges are ordered lexicographically by the labels of their vertices assuming that the lower labeled vertex always comes first when specifying an edge, e.g. (C, E) is before (C, F) which in turn is before (D, G). List the edges in the order in which they are added to the developing forest. [4 marks]



Question 4: Number Theory and Cryptography [20 marks]

1. Given two integers a=134 and b=52,

1) compute gcd(a,b); [4 marks]

2) find another two integers s and t so that s*a+t*b=gcd(a,b). [4 marks]

2. My toy RSA key is N = 187; e = 107. You observe a ciphertext c = 2. What is the plaintext? [6 marks]

3. The Ceasar army was composed of more than 1000 and less than 3000 soldiers. When counted in groups of 11, the remainder was 0; in goups of 9, the remainder was 5, and in groups of 13, the remainder was 8. How many soldiers were there in that army? Explain clearly your reasoning leading to your answer. [6 marks]

Question 5: NP-Hardness [30 marks]

1. State the definition of the Vertex Cover (VC) problem for undirected graphs (Let G be an undirected graph. Let S be a set of vertices. Let E be an edge set). [10 marks]
2. Approx-Vertex-Cover is a polynomial time 2-approximation algorithm for Vertex Cover. Give the pseudocode of the Approx-Vertex-Cover function. [10 marks]
3. Consider the following problem: Let G be a directed graph. Let S be a subset of vertices of G such that deletion of S (vertices) and the edges incident to vertices in S results in a graph G' with no directed cycles. Such an S is a feedback node set. The size of S is the number of vertices in S. The Feedback Node Set (FNS) decision problem is "to determine for a given integer k if G has a feedback node set of size at most k". Show how VC can be reduced to FNS in polynomial time. [10 marks]

END OF EXAM PAPER