1. Sol: 1 Hz processor means that this computer have 1 clock cycle per second, 2.5 GHz means have 2.5×10^9 clock cycle per second.

so: all clock cycle: $6\times2.5\times10^9 = 1.5\times10^{10}$, total operation: $1.5\times10^{10}\times4 = 6\times10^{10}$

solution is $6n^5$, so: $\underbrace{6n^5 = 6\times10^{10}}$, thus, $n=10^2$

INT202 Complexity of Algorithms

Tutorial 1

1. What is the theoretical maximum input size *n* of a problem to be solved in 6 seconds on a 2.5 GHz single core processor that can perform up to 4 operations per clock cycle if the running time of its solution is $6n^5$?

2. Find the asymptotic notation in Big-Oh:

$O(1) \Leftarrow$ (a)
```
int i = 1;        1
    i++;          1
int m = i * i;    1
```

$O(\log n) \Leftarrow$ (b)
```
int i = 1;              1
while (i < n){
    i = i * 2;
}
```
if $i=i+1$, it repeat n times, but $i=i\times2$, so, $\underbrace{1\times2\times\cdots\times2}_{x}=n$, $1\times2^x=n$, $x=\log n$

$O(\log\log n) \Leftarrow$ (c)
```
int i = 2;              1
while (i < n){
    i = i * i;
}
```
$((2^2)^{2\cdots x})^2 = n$

$2^{2^x} = n$ $x=\log\log n$

$O(n) \Leftarrow$ (d)
```
int j = 0;                       1
for (int i = 0; i <= n; i++) {   n+1
    j = i;
    j = j * j;
}
```

$O(n\log n) \Leftarrow$ (e)
```
for (int m = 0; m < n; m++) {   n
    i = 1;
    while (i < n){              log n
        i = i * 2;
    }
}
```

(f)
```
for (int i = 0; i <= n; i++) {     n-1
    for (int j = 0; j <= n; j++) { n-1
        j = i;
        j = j * j;
    }
}
```

$\Downarrow$

$O(n^2)$

3. Multiplying two $n \times n$ square matrices $A = (a_{ij})_{1\le i,j\le n}$ and $B = (b_{ij})_{1\le i,j\le n}$ gives a matrix

$$C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{pmatrix}$$

wherein
$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} \quad \forall 1 \le i, j \le n$$

(a) Write a pseudo-code that performs the operation $C = A \cdot B$

(b) Give its asymptotic notation $T(n) \in O(g(n))$ (i.e $T(n)$ is Big-Oh of $g(n)$)

(a), The multiply of two matrices have such rules: line in first matrix multipy column in second matrix. so, the pseudo-code is:

```
matrix = [n x n]
 sum = 0
for i = 1 -> n :              n
   for j = 1 -> n :           n
      for k = 1 -> n :        n
           sum += A[i,k] · B[k,j]
      matrix[i,j] = sum
      sum = 0
```

(b), according the pseudo-code above, asymptotic notation is $O(n^3)$