Politecnico di Milano

Prova finale: Introduzione all’analisi di missioni spaziali
AA 2023-2024

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Data di consegna: 11/12/23

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1 Introduction

The assignment requires to develop an orbital transfer between two points on different orbits. There are several strategies available, ranging from standard impulsive maneuvers to more complex ones involving continuous thrust. Different maneuvers will be compared focusing on two main aspects:

- ΔV, the total cost of maneuver.
- Δt, the total time spent maneuvering from the starting point to the final point.

2 Orbits Characterization

2.1 Initial Orbit

The starting point is defined by the following position and velocity vectors:

- $\mathbf{r_i} = (-7894.6436, -854.6173, 2641.2167)$Km
- $\mathbf{v_i} = (-0.3252, -6.7530, -1.1450)$Km/s

Which is on an orbit with the following Keplerian elements:

- Semi-major axis: $a_i = 8262$Km
- Eccentricity: $e_i = 0.0935$
- Inclination: $i_i = 21.8439°$
- RAAN: $\Omega_i = 62.2514°$
- Argument of periapsis: $\omega_i = 18.8093°$
- True anomaly: $\theta_i = 103.1696°$

2.2 Final Orbit

The final point can be described using the following Keplerian elements:

- Semi-major axis: $a_f = 13490$Km
- Eccentricity: $e_f = 0.3593$
- Inclination: $i_f = 44.2151°$
- RAAN: $\Omega_f = 50.6667°$
- Argument of periapsis: $\omega_f = 98.7206°$
- True anomaly: $\theta_f = 117.1698°$

Which defines the final point position and velocity:

- $\mathbf{r_f} = (-2649.2446, 12549.6521, 5745.6387)$Km
- $\mathbf{v_f} = (3.6450, -1.2467, -3.5119)$Km/s

2.3 Orbits visualization

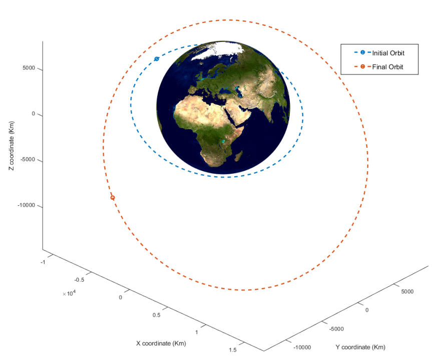


Figure 1: Orbits with initial and last point highlighted.

3 Orbital transfers

3.1 Preliminary observations

Looking at the α between initial and final orbit we can conclude that a bi-elliptic plane change might be an inefficient strategy ($\|\alpha\| = 23.1539° < 38.94°$) thus we will stick to standard maneuvers and variations of it.

3.2 Standard maneuver

3.2.1 Transfer's aim

The standard transfer strategy consists of a four-impulse maneuvering sequence, where (1) is the orbital plane change, (2) is the change of argument of periapsis and (3) is the bi-tangent transfer. Several cases are possible, as maneuvers (1), (2), and (3) allow each two distinct suitable points where the maneuvers can occur. About the possible choices, we follow a reasonable time-over-cost optimization strategy, which will be discussed more specifically in the following paragraph.

3.2.2 Transfer description

The transfer starts with a plane change maneuver: the time optimal point in our case is also ΔV optimal being it the furthest away, at $\theta = 140.3288°$ on the initial orbit. We are now on orbit 1. The next maneuver consists in the change of argument of periapsis, we chose the closest available point, which is located at $\theta = 215.0000°$ on orbit 1, moving on orbit 2 at $\theta = 144.9991°$. The last maneuver is a bitangent transfer, as required we perform a PA (perigee - apogee) transfer (although AP would greatly improve Δt), reaching the final orbit. The maneuver requires $\Delta V = 4.5924\text{Km/s}$ and $\Delta t = 20442s$.

3.2.3 Transfer plot

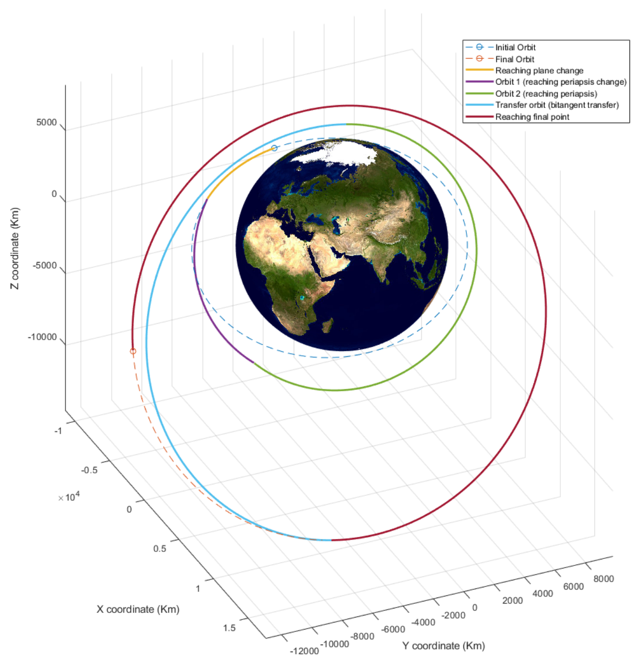


Figure 2: Standard transfer.

3.3 Inverse transfer

3.3.1 Transfer's aim

The focus of this transfer is reducing the plane change cost, performing it further from the planet. This is accomplished by a bi-elliptic transfer (1) (which can be either AP or PA), followed by the plane change (2) and a change of periapsis (3).

3.3.2 Transfer description

The first maneuver required is the bi-elliptic transfer, as already stated there are two available choices: we consider both a AP and PA, the first one puts us on a true anomaly ($\theta = 0°$) where the cost optimal plane change point ($\theta = 140.3264°$) is also the closest one, while the second one will guarantee better overall positioning ($\theta = 180°$) after the plane change at the closest available point ($\theta = 320.3264°$). Once the plane change is completed in the selected points the last required maneuver is the change of periapsis which will be executed at the first available points to minimize the time taken. There's a considerable difference in both ΔV and Δt between the two proposed solutions: AP maneuver requires $\Delta V = 5.388 Km/s$ and $\Delta t = 29942s$ while PA maneuver requires $\Delta V = 6.631 km/s$ and $\Delta t = 26258s$.

3.3.3 Transfer plot

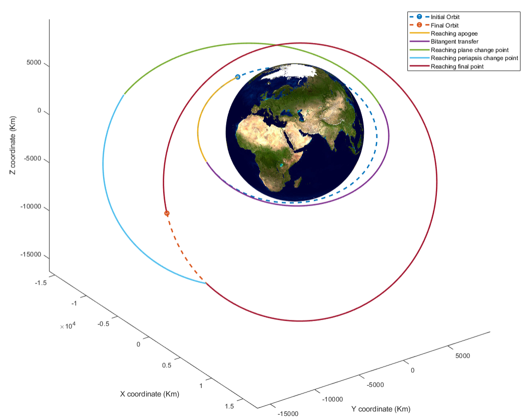


Figure 3: AP transfer.

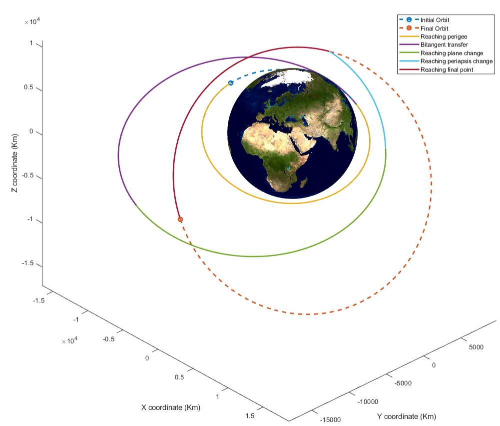


Figure 4: PA transfer.

3.4 Direct transfer

3.4.1 Transfer's aim

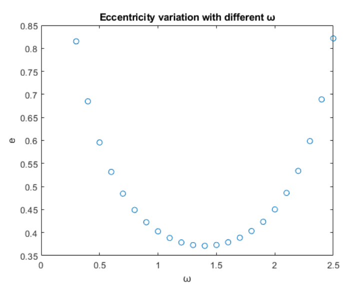
The purpose of this transfer is to minimize the Δt required, by finding a cofocal orbit that connects the initial and final point.

3.4.2 Transfer description

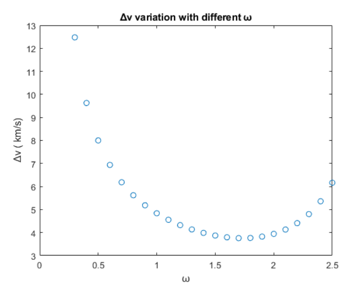
We first need to find the plane that contains our orbit: the vector normal to the plane can be found by vector product: $\mathbf{n} = \mathbf{r_i} \times \mathbf{r_f}$. The ascending node is then calculated as $\mathbf{NA} = (0, 0, 1) \times \mathbf{n}$. Using their definition it is easy to obtain the orbital elements $i = 33.7663°$ and $\Omega = 36.0122°$. Since there isn't a unique orbit we can span ω values between 0 and 2π to explore different possibilities. The other orbital elements can then be calculated solving the system:

$$\begin{cases} p - e\|\mathbf{r_i}\|cos(th_i) = r_i \\ p - e\|\mathbf{r_f}\|cos(th_f) = r_f \\ p = a(1 - e^2) \\ 0 < e < 1 \end{cases} \tag{1}$$

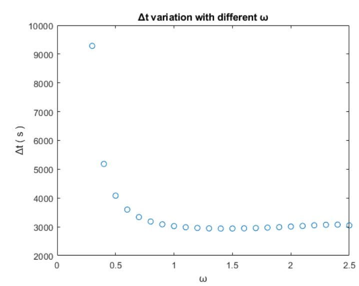
Once we know the various available transfer orbits, we can evaluate the ΔV and Δt, finding the optimal ones.



(a) Eccentricity variation with ω



(b) ΔV variation with ω



(c) Δt variation with ω

As we can see the minimum in ΔV required also provides an almost optimal Δt, this lead us to choose an ω of 1.6. The selected strategy is better than previous ones both in terms of $\Delta t = 2947s$ and $\Delta V = 3.7962$Km/s.

3.4.3 Transfer plot

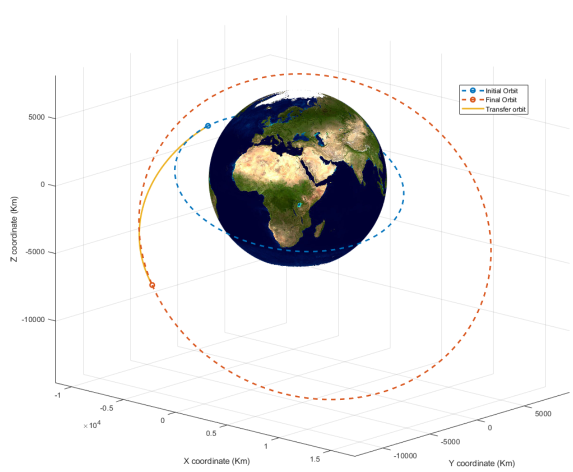


Figure 6: Direct transfer.

3.5 Optimal control transfer

3.5.1 Transfer's aim

Since we have been analyzing impulsive transfers only, we will now try a different approach, where the problem will be solved using continuous low thrust propulsion and an optimal control approach.

3.5.2 Optimal control theory

The purpose of optimal control theory is, given a certain dynamical system $\mathbf{F}(\mathbf{x}(\mathrm{t}), \mathbf{u}(\mathrm{t}), t)$, to find the input $\mathbf{u}$ which minimizes a certain objective function $J(\mathbf{x}, \mathbf{u}, t)$ while subject to some general either linear or nonlinear, equality or inequality constraints on $\mathbf{x}$ and $\mathbf{u}$. The most general formulation of an optimal control problem is:

$$\min_{\mathbf{x}(t)\in\mathbb{R}^n, \mathbf{u}(t)\in\mathbb{R}^m} J(\mathbf{x}(t), \mathbf{u}(t), t) \tag{2}$$

subject to

$$\dot{\boldsymbol{x}} = \mathbf{F}(\mathbf{x}(\mathrm{t}), \mathbf{u}(\mathrm{t}), \mathrm{t}) \tag{3}$$

$$\mathbf{c}(\mathbf{x}(\mathrm{t}), \mathbf{u}(\mathrm{t}), \mathrm{t}) < \mathbf{0} \tag{4}$$

$$\mathbf{g}(\mathbf{x}(\mathrm{t}), \mathbf{u}(\mathrm{t}), \mathrm{t}) = \mathbf{0} \tag{5}$$

There are different approaches to solving this kind of problem: we chose to use a direct method, which consists in using a discrete approximation of the optimal control problem and solving a NLP problem. This will be directly explained while applying it to our problem.

3.5.3 Optimal control problem

We will first start by defining our system dynamics, using the classic two body problem equations and $\mathbf{x} = (\mathbf{r}, \mathbf{v})$ as our state variable and $\mathbf{u} = \mathbf{a_c}$ as our control variable:

$$\dot{\mathbf{x}} = \begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} + \mathbf{a_c} \end{cases} \tag{6}$$

Where our state variables will be $\mathbf{x} = (\mathbf{r}, \mathbf{v})$ and our control variable will be the acceleration provided by thrust $\mathbf{u} = \mathbf{a_c}$. The next step is choosing our objective function that will need to be minimized, the obvious choice is the maneuver's ΔV, which will be calculated as:

$$J = \int_{t_0}^{t_f} \|\mathbf{a_c}\| dt \tag{7}$$

It is now required to set the required constraints, such as fixing the starting $\mathbf{x_i} = (\mathbf{r_i}, \mathbf{v_i})$ and final state variables $\mathbf{x_f} = (\mathbf{r_f}, \mathbf{v_f})$, ensuring that we don't get too close to earth with $\|\mathbf{r}\| > 7000$Km and limiting the control variable by providing an upper and lower bound for $\mathbf{a_c}$ components, to improve optimization stability. Discretizing the problem as below:

$$\begin{aligned} N &= \text{number of grid points} \\ t &\rightarrow [t_0, t_1 ... t_N] \\ \mathbf{x}(t) &\rightarrow [\mathbf{x}_0, \mathbf{x}_1 ... \mathbf{x}_N] \\ \mathbf{u}(t) &\rightarrow [\mathbf{u}_0, \mathbf{u}_1 ... \mathbf{u}_N] \\ &\text{where:} \\ \mathbf{x_k} &= \mathbf{x}(t_k) \\ \mathbf{u_k} &= \mathbf{u}(t_k) \end{aligned}$$

We end up with the final formulation of our NLP problem:

$$\min_{\mathbf{u}} \sum_{k=0}^{N-1} \frac{\|\mathbf{u_{k+1}}\| + \|\mathbf{u_k}\|}{2}(t_{k+1} - t_k) \tag{8}$$

$$\mathbf{x_{k+1}} = \mathbf{x_k} + \frac{t_{k+1} - t_k}{2}(\mathbf{F}(\mathbf{x_{k+1}}, \mathbf{u_{k+1}}) + \mathbf{F}(\mathbf{x_k}, \mathbf{u_k})) \tag{9}$$

$$\begin{aligned} \mathbf{x_0} &= (\mathbf{r_i}, \mathbf{v_i}) \\ \mathbf{x_N} &= (\mathbf{r_f}, \mathbf{v_f}) \\ \mathbf{u_0} &= \mathbf{0} \\ \sqrt{\mathbf{x_k}(1)^2 + \mathbf{x_k}(2)^2 + \mathbf{x_k}(3)^2} &> 7000 \qquad \text{for } k = 0, 1, \ldots, N \\ \|\mathbf{u_k}\|_\infty &< 5 \qquad \text{for } k = 0, 1, \ldots, N \end{aligned} \tag{10}$$

Using ICLOCS2, a MATLAB toolbox for optimal control problems, we obtained the following solution :

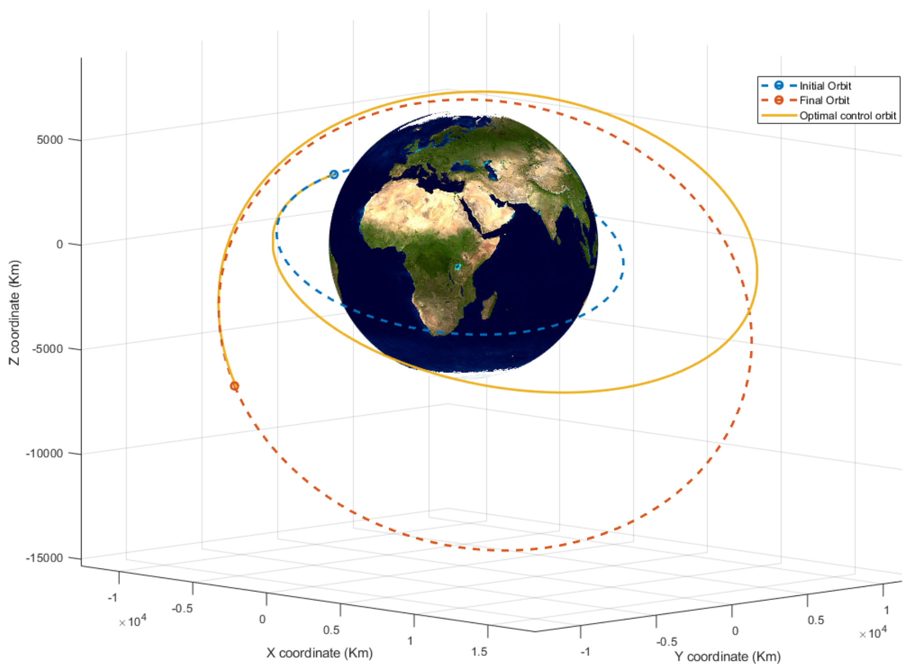
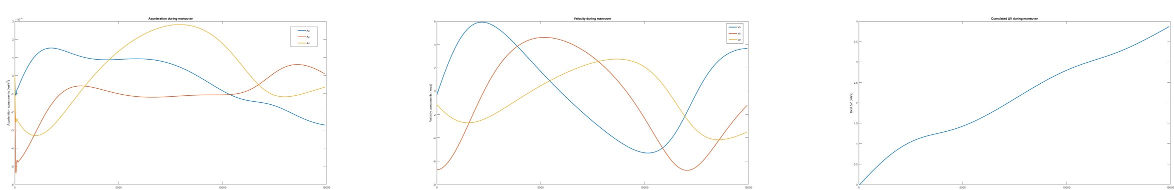


Figure 7: OCP trajectory



(a) Acceleration profile

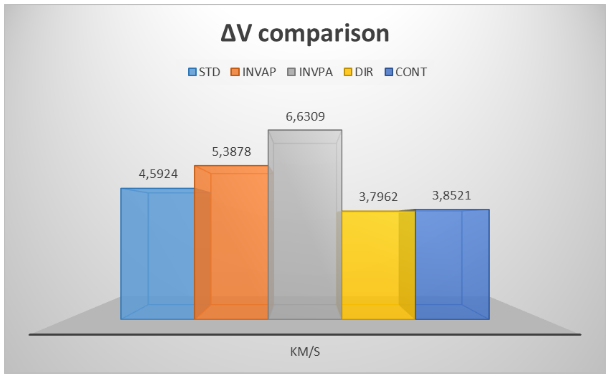
(b) Velocity profile

(c) Cumulated ΔV

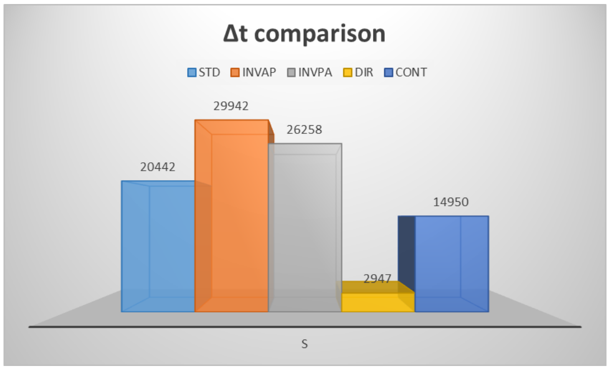
This kind of maneuver would be optimal for low thrust propulsion system such as an electromagnetic propulsor. With a time of 14950 seconds, we obtain a ΔV of 3.85 km/s, about 1.6% higher than the direct maneuver, which requires the assumption of impulsive maneuvers.

4 Final considerations

Following our initially stated objectives, in conclusion we choose the direct transfer as it is the fastest. It is also the cheapest transfer, as a consequence of the low Δi and $\Delta\Omega$ between the two orbital planes and the distance between initial and final point.



(a) ΔV comparison



(b) Δt comparison

5 Appendix

Table 1: Transfer 1

t(s)	a(km)	e(-)	i(deg)	Ω(deg)	ω(deg)	θ(deg)	Δv(m/s)
0	8262,4662	0,0935	21,8412	62,2519	18,8102	103,1696	0
811	8262,4662	0,0935	21,8412	62,2519	18,8102	140,3288	2,5985
	8262,4662	0,0935	44,2151	50,6666	28,7166	140,3288	
2510	8262,4662	0,0935	44,2151	50,6666	28,7166	215	0,7481
	8262,4662	0,0935	44,2151	50,6666	98,7206	144,9991	
7044	8262,4662	0,0935	44,2151	50,6666	98,7206	0	1,0645
	12913,4878	0,4199	44,2151	50,6666	98,7206	0	
14346	12913,4878	0,4199	44,2151	50,6666	98,7206	180	0,1811
	13490	0,3593	44,2151	50,6666	98,7206	180	
20442	13490	0,3593	44,2151	50,6666	98,7206	117,1698	0

Table 2: Transfer 2(p-a)

t(s)	a(km)	e(-)	i(deg)	Ω(deg)	ω(deg)	θ(deg)	Δv(m/s)
0	8262,4661	0,0935	21,8412	62,2519	18,8102	103,1696	0
5450	8262,4662	0,0935	21,8412	62,2519	18,8102	0	1,0645
	12913,4878	0,4199	21,8412	62,2519	18,8102	0	
12753	12913,4878	0,4199	21,8412	62,2519	18,8102	180	1,0517
	13490	0,3593	21,8412	62,2519	18,8102	180	
19540	13490	0,3592	21,8412	62,2519	18,8102	320,3264	2,9844
	13490	0,3592	44,2151	50,6666	28,719	320,3264	
21431	13490	0,3592	44,2151	50,6666	28,719	35,0008	2,4008
	13490	0,3593	44,2151	50,6666	98,7206	324,9991	
26258	13490	0,3593	44,2151	50,6666	98,7206	117,1698	0

Table 2: Transfer 2(a-p)

t(s)	a(km)	e(-)	i(deg)	Ω(deg)	ω(deg)	θ(deg)	Δv(m/s)
0	8262,4661	0,0935	21,8412	62,2519	18,8102	103,1696	0
1713	8262,4662	0,0935	21,8412	62,2519	18,8102	180	0,2440
	8838,9783	0,0221	21,8412	62,2519	18,8102	180	
5848	8838,9783	0,0221	21,8412	62,2519	18,8102	0	1,0517
	13490	0,3593	21,8412	62,2519	18,8102	0	
11084	13490	0,3592	21,8412	62,2519	18,8102	140,3264	1,6913
	13490	0,3592	44,2151	50,6666	28,719	140,3264	
15923	13490	0,3592	44,2151	50,6666	28,719	215	2,4008
	13490	0,3593	44,2151	50,6666	98,7206	144,9991	
29942	13490	0,3593	44,2151	50,6666	98,7206	117,1698	0

Table 3: Transfer 3

t(s)	a(km)	e(-)	i(deg)	Ω(deg)	ω(deg)	θ(deg)	Δv(m/s)
0	8262,4662	0,0935	21,8412	62,2519	18,8102	103,1696	2,5959
	11962	0,3789	33,7644	36,0104	91,6732	53,7263	
2947	11962	0,3789	33,7644	36,0104	91,6732	135,6764	1,2003
	13490	0,3593	44,2151	50,6666	98,7206	117,1698	