UniTs - University of Trieste

Faculty of Scientific and Data Intensive Computing

Department of mathematics informatics and geosciences

Introduction to Galaxies and Astrophysics

Lecturers:
Prof. Alexandro Saro // Prof. Cebrolini Matteo

Author:
Andrea Spinelli

April 1, 2025

github.com/Spina02 andreaspinelli2002@gmail.com

Preface

As a student of Scientific and Data Intensive Computing, I've created these notes while attending the **Introduction to Galaxies and Astrophysics** course.
This is an optional course for the second semester of the first year my master's degree, but this course is also available for students of the Physics department, anyway, next year it will probably be removed from both curricula.
The course is held by Prof. Alexandro Saro, a researcher at the INAF Astronomical Observatory of Trieste; the lecture of the course are taken in Italian, but the notes are written in English conformly to the language of the master's degree; however, some images or text could be in Italian due to the original language of the slides.

The first part of the course will be focused on gravity and Einstein's theory of general relativity:
- Non euclidean geometry
- Tensors
- Principles and equations of Einstein
- Gravitational waves

The second part of the course will be focused on cosmology:
- Robertson-Walker metric
- Hubble law
- Friedmann equations
- Cosmological models
- Precision cosmology (hints)

Contents

1
Introduction

The theory we are going to study is a *geometric theory*:

Euclid's 5 Postulates:

1. For any two points A and B, there is exactly one line that passes through them.
2. Every line can be extended indefinitely in both directions.
3. Given a point O and a radius R, there exists exactly one circle centered at O with radius R.
4. All right angles are congruent.
5. Given a line R and a point P not on R, there exists exactly one line through P that is parallel to R.

The fifth postulate is more complex than the others; over time, mathematicians tried several times to prove the fifth postulate based on the other four. It was later discovered that the fifth is independent of the others.

Consequences:

1. There is no line parallel to R through P
   $\to$ Planar elliptic geometries $[S^2]$
2. There exist two or more lines parallel to R through P
   $\to$ Planar hyperbolic geometries $[H^2]$

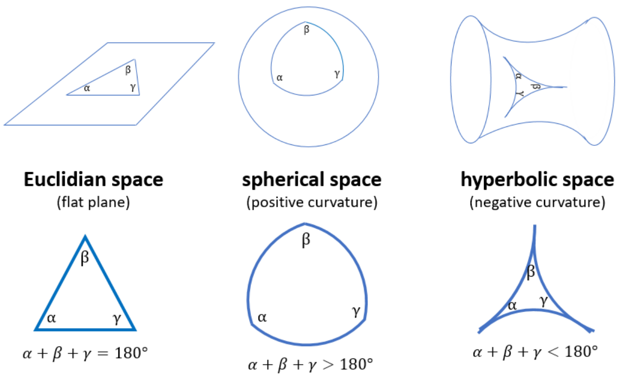

Figure 1.1: Elliptic and hyperbolic geometries

Elliptic Geometry

Planar elliptic geometry is a form of non-Euclidean geometry that rejects Euclid's fifth postulate, which in Euclidean geometry guarantees the existence of exactly one parallel line through a given point. In elliptic geometry, no parallel lines exist; instead, every pair of lines eventually intersects. A classic example of this is spherical geometry, where the "lines" are represented by great circles on a sphere.

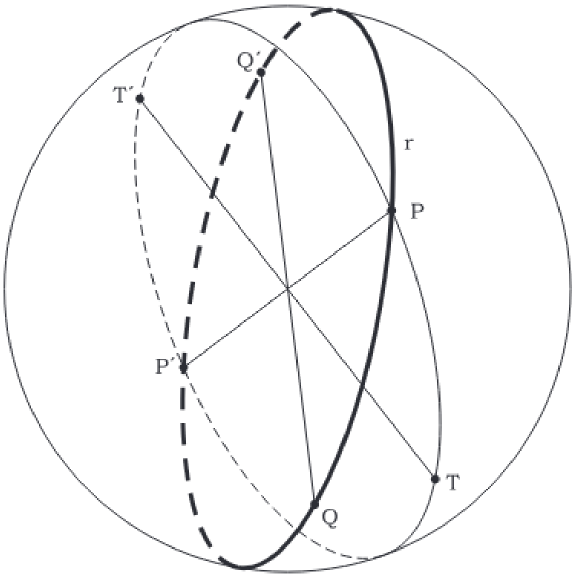

Figure 1.2: Spherical geometry

In this context, triangles, known as *spherical triangles*, display an intriguing property: the sum of their interior angles exceeds 180° (a phenomenon known as spherical excess), with the excess being proportional to the area of the triangle.

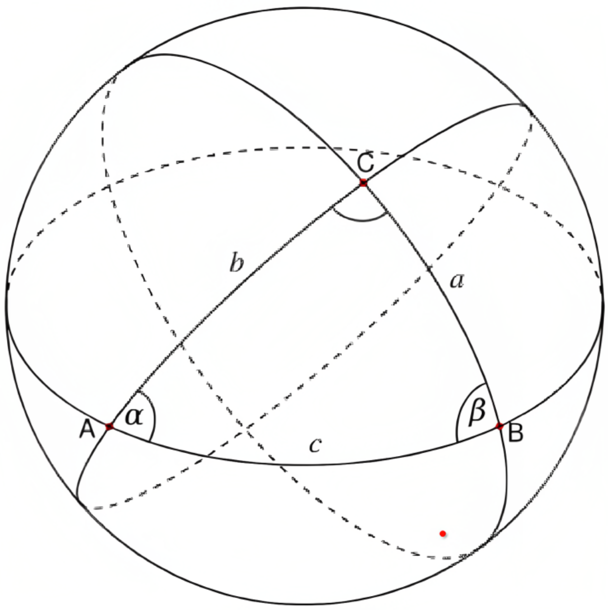

Figure 1.3: Spherical triangle

Hyperbolic Geometry

Planar hyperbolic geometry is a form of non-Euclidean geometry that rejects Euclid's fifth postulate, which in Euclidean geometry guarantees the existence of exactly one parallel line through a given point. In hyperbolic geometry, multiple parallel lines exist, and the sum of the angles in a triangle is always less than 180°.

In this context, triangles—known as hyperbolic triangles—display an intriguing property: the sum of their interior angles is less than 180°, with the deficit being proportional to the area of the triangle.

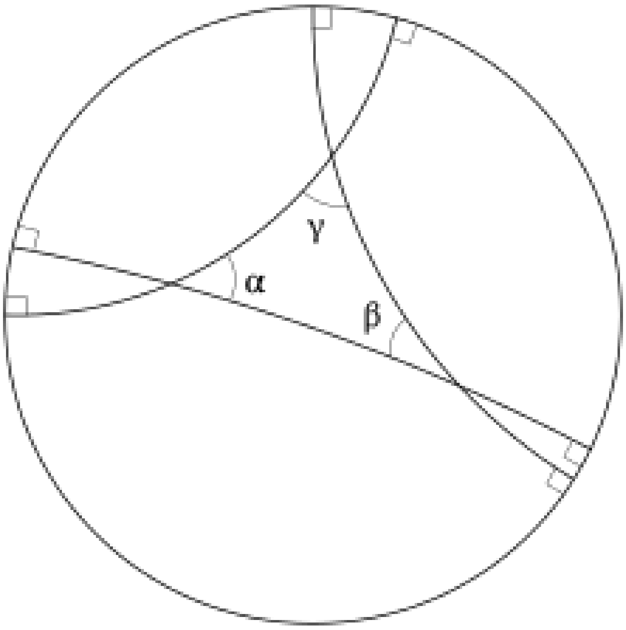

Figure 1.4: Hyperbolic triangle

Curva piana