



UniTs - University of Trieste

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Galaxy Astrophysics

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October 28, 2025

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Preface

This course provides a comprehensive overview of galaxy astrophysics, structured as follows:

- **Fundamentals** (approximately 10 hours)
 - Stars
- **Phenomenology** (approximately 25 hours)
- **Dynamics of Non-collisional Systems**
 - Elliptical galaxies
 - Galaxy clusters

Reference Textbooks:

- **Introductory**
 - Karttunen et al., “Fundamental Astronomy”
 - Schneider, P. (2015), “Extragalactic Astronomy and Cosmology”
- **Advanced**
 - Binney, J. & Merrifield, M., “Galactic Astronomy”
 - Binney, J. & Tremaine, S., “Galactic Dynamics”

Contents

Introduction

When we observe the sky, we perceive it as a 2D surface, even though celestial objects actually exist in 3D space. To bridge this gap and measure distances in astronomy it is used a set of techniques known as the *distance ladder*. It consists of different methods, where each one relies on a specific physical phenomenon and is calibrated using the preceding method in the ladder. Only recently have precise instruments like ESA's Hipparcos (1989) and Gaia (2013) satellites enabled highly accurate stellar parallax measurements, with Gaia mapping distances to over a billion stars.

1.1 Reference Systems

To describe the position of an object in the sky, we need to define a reference system. For this purpose, astronomers often imagine all celestial objects as lying on a vast, imaginary *celestial sphere*, centered on the observer. Although this model has ancient origins, it remains extremely useful today. Since the celestial sphere is considered to have an infinite radius, we can ignore the small shifts caused by the Earth's rotation and orbit.

1.1.1 The Equatorial System

The equatorial coordinate system is defined by selecting a reference parallel and a reference meridian. The Earth's rotational axis remains nearly constant over time, so the equatorial plane (which is perpendicular to this axis) serves as a stable basis for a coordinate system that does not depend on the observer's location or the time of observation.

The celestial equator is the great circle where the celestial sphere meets the equatorial plane. The axis of this circle points toward the celestial poles. In the northern hemisphere, the north celestial pole is almost exactly aligned with the Earth's rotational axis and lies about one degree from Polaris. The angle between a star and the celestial equator (the equatorial plane) remains unchanged by the Earth's daily rotation. This angle is called the **declination** δ ($-90^\circ < \delta < +90^\circ$). For the second coordinate, we also need a fixed direction that is independent of the Earth's rotation. This direction is defined by the *vernal equinox* (Υ), which is the point on the celestial sphere where the Sun's path (the ecliptic) crosses the celestial equator at the moment of the spring equinox. The second coordinate is then defined as the angle measured eastward along the celestial equator from the vernal equinox. This angle is called the **right ascension** α (or R.A.), with values ranging from 0 to 24 hours.

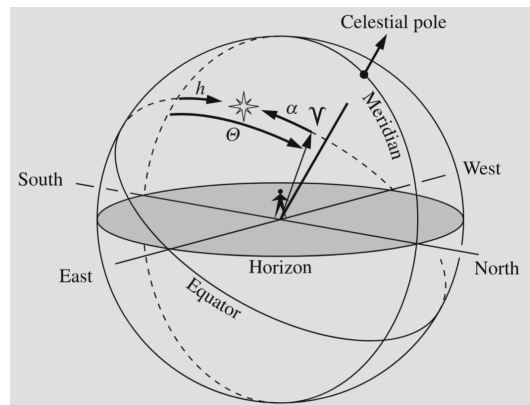


Figure 1.1: The Equatorial System [1]

The *sidereal time*, often denoted by Θ , measures the angle between the local meridian (The great circle on the celestial sphere that passes through both the celestial poles and the zenith of the observer) and the vernal equinox, increasing as the Earth rotates. For any celestial object, there is a simple and important relationship:

$$\Theta = h + \alpha$$

1.1.2 The Azimuthal System or Horizontal System

The azimuthal (or horizontal) system is defined relative to the observer's specific position on Earth. Its reference plane is the local *horizon* (the plane tangent to the Earth at the observer's location). Where this plane meets the celestial sphere forms the visible horizon. The point directly overhead is the *zenith*; the point directly beneath is the *nadir*.

Great circles passing through the zenith are called *verticals*, and each one meets the horizon at a right angle. As the Earth rotates, stars appear to rise in the east, reach their highest point (culminate) when they cross the *meridian* (the vertical circle connecting north, zenith, and south) and set in the west. The intersection points of the meridian with the horizon define the north and south directions.

In this system, one coordinate is the **altitude** (or elevation), a , the angle between the horizon and the object along its vertical circle. Altitude ranges from -90° to $+90^\circ$, and is positive above the horizon.

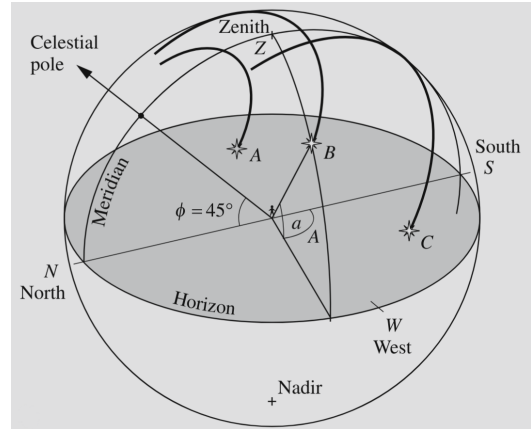


Figure 1.2: The Azimuthal System [1]

The second coordinate is the **azimuth**, A : the angle measured along the horizon from a fixed reference direction to the object's vertical circle. The reference is often north or south, and by convention the angle is measured clockwise (see tip below).

Since this system depends on both the observer's position and the time, the coordinates of the same star will be different for different observers and at different moments. For this reason, horizontal coordinates are not used in star catalogues.

💡 Tip: Azimuth direction

There are different conventions for the reference direction and sense of azimuth, so it is always important to verify which one is being used. Here, we measure azimuth clockwise from the south, as is common in astronomy.

1.1.3 The Galactic System

For studies of the Milky Way Galaxy, the most natural reference plane is the plane of the Milky Way itself. Because the Sun lies very close to this plane, it is convenient to place the origin of the galactic coordinate system at the Sun.

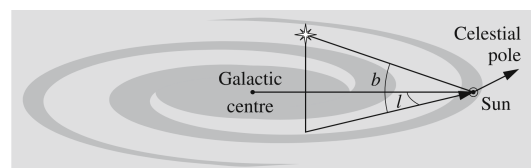


Figure 1.3: The Galactic System [1]

The **galactic longitude** l is measured counterclockwise (analogous to right ascension) along the galactic plane, starting from the direction of the center of the Milky Way, which lies in the constellation Sagittarius. The **galactic latitude** b is measured from the galactic plane: it is positive towards the north galactic pole and negative towards the south.

👁 Observation: Coordinate precision

If right ascension is given in hours, we need to provide one additional decimal place in seconds compared to the declination, to preserve equivalent angular accuracy. For example:

$$03^{\text{h}} 42^{\text{m}} 35.63^{\text{s}} \quad +42^\circ 32' 35.4''$$

1.2 Coordinate perturbations

Even for a star fixed relative to the Sun, its observed coordinates may shift due to various perturbing effects. While altitude and azimuth change with Earth's rotation, even right ascension and declination are subject to small variations over time.

1.2.1 Precession and nutation

The Earth's rotational axis is not fixed in space; instead, it traces out a slow circular motion around the north pole of the ecliptic. This slow motion, known as **precession**, causes the celestial poles and equator to shift over time, completing a full cycle roughly every 25,800 years. As a result, the coordinates of stars change slowly: star catalogues must specify the equinox, or reference epoch, to which their coordinates refer.

Superimposed upon precession is a smaller, periodic oscillation of the axis called **nutation**. It is primarily caused by the gravitational pull of the Moon (and, to a lesser extent, the Sun) on Earth's equatorial bulge. This results in a short-term "nodding" motion, with the main period being about 18.6 years, as the Moon's orbital plane precesses.

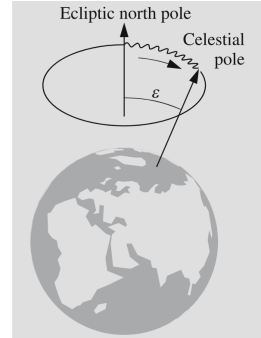


Figure 1.4:
Precession and
nutation [1]

Mathematically, the precessional motion can be described using the concept of torque:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

where \vec{L} is the angular momentum of the Earth, and $\vec{\tau}$ is the torque exerted mainly by the gravitational attraction of the Moon and Sun on the equatorial bulge. The change in angular momentum, $\Delta\vec{L}$, is perpendicular to \vec{L} , leading to a precession of the axis direction (rather than a change in tilt angle):

$$\Delta\vec{L} \perp \vec{L} \quad \text{and} \quad \vec{\tau} \perp \vec{L}$$

Both precession and nutation must be taken into account for precise astronomical coordinate systems, since they cause the celestial coordinate grid to shift over time.

👁 Observation: The vernal equinox point

The vernal equinox point (Υ) is not fixed in space. Due to the precession of Earth's axis, it gradually shifts westward along the ecliptic by approximately 50.25'' (arcseconds) per year. This slow drift means that the celestial coordinate system itself must be periodically updated to a reference epoch in star catalogs and astronomical calculations.

1.2.2 Aberration

Since the Earth is moving, the direction to a star appears to be shifted by a small angle due to the Earth's velocity. This effect is called **aberration**.

We can distinguish two types of aberration:

- **Annual aberration** is caused by the Earth's orbital motion around the Sun. This effect leads to a maximum apparent displacement of about 20.5'' (arcseconds) in the direction of Earth's motion.
- **Diurnal (daily) aberration** is caused by the Earth's rotation about its axis. This produces a much smaller maximum displacement, about 0.32''.

This phenomenon is usually already taken into account in the coordinates of the stars, so it is not necessary to correct for it.

1.2.3 Atmospheric refraction

Since light is refracted by the atmosphere, the direction of an object differs from the true direction by an amount depending on the atmospheric conditions along the line of sight.

If the object is not too far from the zenith, the atmosphere between the object and the observer can be approximated by a stack of parallel layers, each of which has a certain index of refraction n_i .

The zenith distance z of the object and the observed distance z_{obs} are related by the following equation:

$$n_0 \cdot \sin z_{obs} = n_1 \cdot \sin z_1 = \dots = 1 \cdot \sin z$$

where n_i are the indices of refraction of the different layers.

Let $R = z - z_{obs}$ be the *refraction angle*. It holds that:

$$\begin{aligned} n_0 \cdot \sin z_{obs} &= \sin z = \sin(z_{obs} + R) \\ &= \frac{\sin R \cos z_{obs}}{\sim R} + \frac{\cos R \sin z_{obs}}{\sim 1} \\ \text{(for small } R) &\approx \sin z_{obs} + R \cos z_{obs} \end{aligned}$$

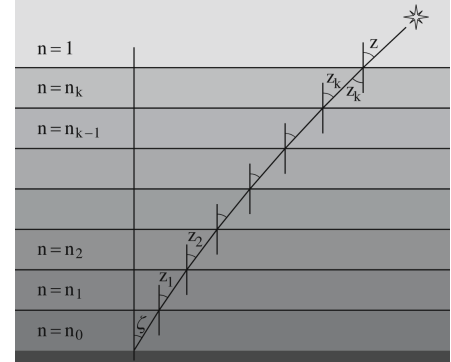


Figure 1.5: Refraction [1]

In addition to refraction, Earth's atmosphere also absorbs electromagnetic radiation, significantly impacting astronomical observations across various wavelengths:

- The **Troposphere** (0 – 10km), the lowest atmospheric layer, is composed primarily of H_2O , CO_2 , CO , N_2 , and O_2 . These molecules here strongly absorb light in the **infrared (IR)** region.
- The **Stratosphere** (10 – 80km) contains a significant concentration of ozone (O_3), which efficiently absorbs light in both the **ultraviolet (UV)** and **X-ray** regions.
- The **Ionosphere** (80 – 500km) is a region rich in ionized particles; it absorbs **radio waves**.

Beyond Earth's atmosphere, the **interstellar medium (ISM)**, composed of gas and dust, also absorbs and scatters electromagnetic radiation, particularly at **X-ray** and **UV** wavelengths.

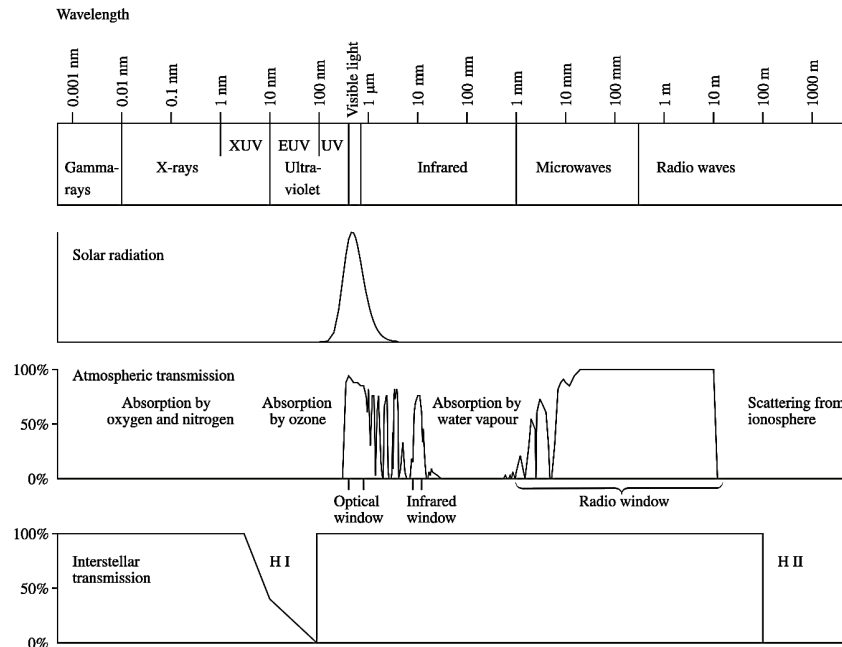


Figure 1.6: Atmospheric and interstellar absorption and transmission at different wavelengths: the top band shows the electromagnetic spectrum, followed by typical solar radiation at Earth; the third band displays atmospheric transmission with main absorption features (defining optical, infrared, and radio windows), while the bottom highlights interstellar absorption, especially by hydrogen, limiting UV and parts of the radio spectrum. [1]

👁 Observation: Zone of Avoidance

The **Zone of Avoidance** is the region near the Galactic plane ($|\alpha| \lesssim 10^\circ$) where absorption by dust and bright stars make optical observations of extragalactic objects very difficult.

1.2.4 Parallax

If we observe an object from different points, we see it in different directions. The difference of the observed directions is called the **parallax**. The parallax effect highly depends on the distance of the object. The closer the object, the greater the parallax.

Since the Earth is moving, if an observer observes a star after an interval of time, he will be looking at the object from a different angle. We can distinguish two kinds of parallax:

- **Diurnal (daily) parallax** is due to the change of direction due to the daily rotation of the Earth. The diurnal parallax also depends on the latitude of the observer; if the position is not specified, it is assumed to be at the equator.
- **Annual parallax** is due to the Earth's orbital motion around the Sun. The annual parallax is the maximum parallax effect and it is used to measure the distance of the stars.

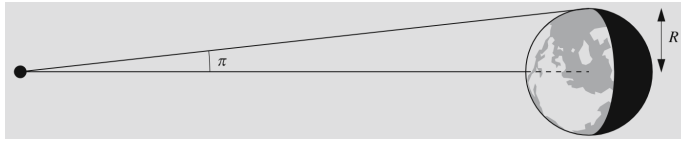


Figure 1.7: The parallax π is the angle subtended by the Earth's equatorial radius as seen from the object [1]

Warning: Parallax correction

Usually, parallax correction is not taken into account in the coordinates of the star catalogues.

Over time, "parallax" and "distance" have practically become synonymous in astronomy, especially in the context of photometric parallax. In fact, parallax serves as the foundation for one of the most widely used units for astronomical distances: the parsec.

A **parsec** (pc) is defined as the distance at which an astronomical object would exhibit a parallax angle of $1''$ (one arcsecond), when measured from two points separated by 1 astronomical unit, that is, from opposite sides of Earth's orbit around the Sun six months apart.

Numerically, one parsec is approximately 3.26 light-years, or about 3.086×10^{16} meters.

Over time, advancements in astronomical instrumentation have enabled increasingly precise measurements of stellar parallax, allowing us to probe greater distances:

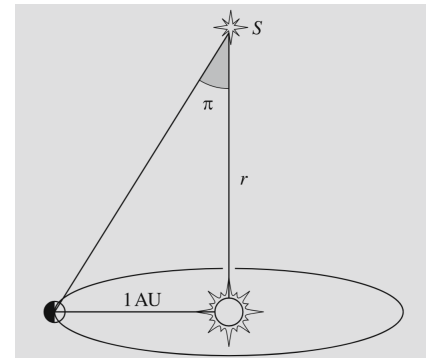


Figure 1.8: Parallax π of a star S is the angle subtended by the radius of the orbit of the Earth [1]

Method/Instrument	Parallax Precision	Distance Limit
From Earth	$\pi \approx 0.01''$	up to ~ 30 pc
Hipparcos satellite	$\pi \approx 0.001''$	up to ~ 1000 pc
Gaia mission	$\pi \approx 2 \cdot 10^{-4}''$	up to ~ 5000 pc (5 kpc)

Observation: Parallax measurement

The objects with the largest measured parallaxes are some of the stars nearest to the Sun. Some notable examples include:

- In 1838, Bessel measured the parallax of 61 Cygni: $\pi = 0.29''$
- Proxima Centauri, the closest star to the Sun: $\pi = 0.75''$

1.2.5 Observations from Satellites

Observing from space-based telescopes and satellites provides several significant advantages over ground-based astronomical observations:

- **Absence of Atmospheric Refraction:** Earth's atmosphere bends and distorts incoming starlight, limiting positional accuracy. Satellites, outside the atmosphere, avoid these effects entirely, resulting in sharper, more reliable measurements.
- **No Gravitational Flexure:** In orbit, instruments are effectively weightless, removing distortions caused by gravitational sagging that affect even the best ground-based observatories.
- **Sharper Images:** Space telescopes are unaffected by atmospheric blurring, so resolution is limited only by their optics (Airy disk), not by seeing.
- **Stable Observational Conditions:** Space offers a stable thermal and radiation environment, allowing for superb calibration and highly repeatable measurements across long timescales.

As a result, satellite missions such as *Hipparcos* and *Gaia* have revolutionized parallax measurements with unprecedented accuracy and reach.

Furthermore, since a star's observed flux (F) diminishes with the square of its distance (d) from us ($F \propto d^{-2}$), uncertainties in distance measurements are amplified in the derived fluxes. If the fractional uncertainty in distance is $\frac{\Delta d}{d}$, then the corresponding fractional uncertainty in the flux is:

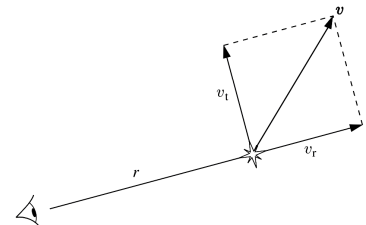
$$\frac{\Delta F}{F} \approx 2 \cdot \frac{\Delta d}{d}$$

E.g., if the parallax-based distance has a relative uncertainty of 20% ($\frac{\Delta d}{d} \sim 0.2$), the resulting uncertainty in the flux is $\frac{\Delta F}{F} \sim 40\%$, underlining the importance of minimizing distance errors.

1.3 Positional Astronomy

Any space motion can be decomposed into two components: the **radial velocity** (line-of-sight, v_r), directed along the observer's line of sight, and the **tangential velocity** (v_t), perpendicular to it.

$$v = \sqrt{v_t^2 + v_r^2}$$



As the universe is expanding, many extragalactic objects are moving away from us, and their light is observed to be shifted towards longer (redder) wavelengths. This is known as the **redshift** (z). This phenomenon is fundamentally similar to the Doppler effect for sound.

To understand the shift in wavelength, consider a source emitting electromagnetic waves with period T . If the source were at rest with respect to the observer, the emitted wavelength would be:

$$\lambda_0 = cT$$

where c is the speed of light.

If the source moves at velocity v relative to the observer (positive when receding, negative when approaching), during the same period T it covers a distance $s' = vT$ relative to the observer.

Thus, the observed wavelength λ becomes:

$$\lambda = cT + vT = (c + v)T$$

The change in wavelength is given by:

$$\Delta\lambda = \lambda - \lambda_0 = (c + v)T - cT = vT$$

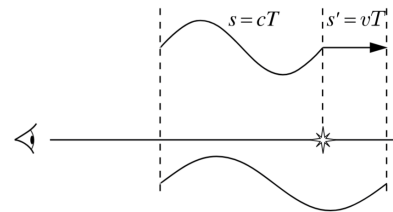


Figure 1.9: Wavelength shift [1]

1.3.1 Redshift

We define the **redshift** z as the fractional change:

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{vT}{cT} = \frac{v}{c}$$

This result applies when $v \ll c$ (non-relativistic limit).

For high velocities (e.g., distant galaxies or quasars), use the relativistic expression:

$$z = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} - 1$$

which for $v \ll c$ reduces to the previous, linear relation.

In most stellar and galactic contexts, velocities are much smaller than the speed of light ($v_{stars} = 2 - 400 \text{ km/s}$, $v_{galaxies} \sim 10^3 \text{ km/s}$, while $c = 3 \cdot 10^5 \text{ km/s}$), so the non-relativistic approximation is sufficient.

An observed redshift generally has two main contributions: a kinematic term from the galaxy's peculiar motion (z_{kin}) and a cosmological term from the expansion (z_{cosm}).

In a galaxy cluster, individual galaxies show slightly different observed redshifts z_i from both peculiar motions and expansion. The mean over N members defines the cluster redshift:

$$z_{cluster} = \frac{1}{N} \sum_{i=1}^N z_i$$

where N is the number of measured galaxies.

Since peculiar velocities roughly average to zero in the cluster frame, this identifies the cosmological redshift:

$$z_{cluster} = z_{cosm}$$

For each galaxy in the cluster, we can define its “rest-frame” (or peculiar) redshift:

$$z_{rf} = \frac{z_{obs} - z_{cluster}}{1 + z_{cluster}}$$

Here, z_{obs} is the total observed redshift for an individual galaxy, $z_{cluster}$ is the cosmological redshift of the cluster, and z_{rf} quantifies the galaxy's motion relative to the cluster frame.

1.3.2 Proper motion

Proper motion μ , $[\mu] = [\text{arcsec/year}]$, quantifies a star's apparent angular motion on the sky relative to the Sun, most evident for nearby or high-velocity objects.

A star's space velocity splits into a **radial velocity** (v_r) and a **tangential velocity** (v_t). The tangential velocity results in the proper motion, which can be measured by taking plates at intervals of several years or decades.

$$\tan \frac{\alpha}{2} \sim \frac{x/2}{d} \quad \Rightarrow \quad \mu \sim \frac{v_t}{d}$$

Thus, the tangential velocity follows from the proper motion and the distance:

$$v_t = 4.74 \mu d$$

where:

$$[v_t] = \text{km/s}$$

$$[d] = \text{pc}$$

$$[\mu] = \text{arcsec/year}$$

The proper motion μ is usually described in terms of two components: one along the declination direction, $\mu_\delta = \Delta\delta / 1 \text{ year}$, and one along the right ascension direction.

However, to properly express the motion in right ascension, we need to take into account that lines of right ascension (hour circles) get closer together as you move away from the celestial equator towards the poles. Therefore, the component in right ascension is written as $\mu_\alpha \cos \delta$, with $\mu_\alpha = \Delta\alpha / 1 \text{ year}$, where the factor of $\cos \delta$ adjusts for this effect. The total proper motion is then:

$$\mu = \sqrt{\mu_\delta^2 + \mu_\alpha^2 \cos^2 \delta}$$

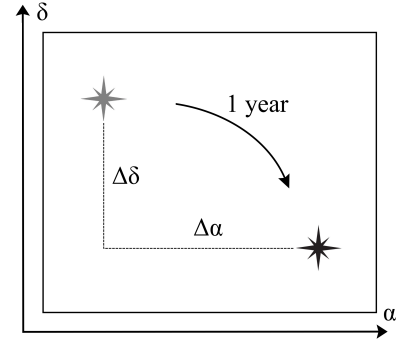


Figure 1.10: Proper motion [1]

👁 Observation: *Barnard's Star*

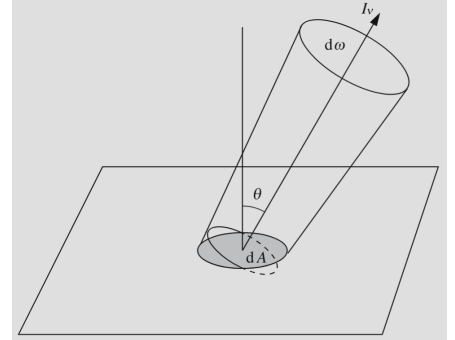
Barnard's Star exhibits the highest known proper motion of any star, moving across the sky at a remarkable rate of $\mu = 10.34 \text{ arcsec/year}$.

1.4 Magnitudes

Let us assume we have some radiation passing through a surface element dA . Some of the radiation will leave dA within a solid angle $d\omega$; the angle between $d\omega$ and the normal to the surface is denoted by θ . The amount of energy with frequency in the range $[\nu, \nu + d\nu]$ entering this solid angle in time dt is:

$$dE_\nu = I_\nu \cos \theta dA d\nu d\omega dt$$

Here, the coefficient I_ν is the specific intensity of the radiation at the frequency ν in the direction of the solid angle $d\omega$.



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Lecture 23/10/2025

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2.1 Spiral Galaxies

	S_a	S_b	S_c	S_d
M_B	$-17 - 23$	$-17 - 23$	$-16 - 22$	$-15 - 20$
M/M_\odot	$10^9 - 12$	$10^9 - 12$	$10^9 - 12$	$10^8 - 10$
V_{max}	$163 - 367$	$163 - 367$	$99 - 304$	$99 - 304$
L_{bulge}/L_{tot}	0.3	0.13	0.05	?
$B - V$	0.75	0.64	0.52	0.47
M_{gas}/M_B	0.04	0.08	0.16	0.25

Another important parameter is the **Opening angle** θ_0 , which is the angle between the major axis of the galaxy and the line of sight.

We can distinguish "grand design" and "flocculent" galaxies.

Surface	brightness	profile	M/L
Bulge	$2^{1/4}$	low	$\sim 5 \frac{M_d}{L_\odot}$
Disk	expo	low	$\sim 3 \frac{M_\odot}{L_\odot}$

There exists "Bulges" and "Pseudobulges".

We know very few pseudobulges.

Bulges have law of E

Pseudobulges have exponential profile \rightarrow rotation

Freeman Law:

$$\mu_0 \sim const$$

$$\mu_0 = 21.52 \pm 0.39 \quad (S_a \rightarrow S_c)$$

$$\mu_0 = 22.61 \pm 0.39 \quad (S_d)$$

TODO: How evolution can change the spiral:

Stellar Halo

$$MW, M31 \Rightarrow \rho \propto r^{-3}$$

If a galaxy is nearby and it is edge-on, we can see a thick disk.

the size of the disk is $\mu_B \sim 22.5 mag/arcsec^2$

Another important law is the **Law of Star Formation Rate** (Schmidt-Kennicutt Law):

$$\sum_{SFR} \propto \sum_{gas}^N$$

$$\frac{M_{\odot}}{y kpc^2}$$

Surface photometry bias

If we observe a portion of a galaxy, and we must "look through" a portion of the galaxy we will see a redder light than observing it from an angle where we don't have to "look through" any portion of the galaxy.

The same happens for the bulge.

Let's suppose a spiral with no dust: if we observe a "face-on" galaxy, we will see a brighter galaxy than observing the same galaxy "edge-on".

If we consider a very dusty spiral instead, we will notice that the galaxy appears with a similar brightness in both face-on and edge-on views, due to the dust.

Rotation curves

The rotation curve is the curve that shows the velocity of the stars in the galaxy as a function of the distance from the center of the galaxy.

...

A part of this velocity is related to the baryonic mass.

We can decompose the velocity:

DM Halo disk

If $\frac{M}{L} \sim \text{stars}$ We do not need to consider Dark Matter in the disk. If $\frac{M}{L} \gg \text{stars}$ We need to consider Dark Matter in the disk.

In general, the max velocity is higher if the luminosity is higher. If we pick galaxies with the same luminosity, the max velocity is higher for the galaxies with the higher mass.

arms:

grand design -> density moves (internal) S_c flocculent -> tidal interaction (external)

2.1.1 Virial Theorem

[karttunen demonstration]

If a system is virialized, the following equation holds:

$$2T + U = 0$$

where T is the kinetic energy and U is the potential energy.

Suppose we have a system of n point masses m_i with radius vectors r_i and velocities \dot{r}_i . We define a quantity A (the "virial" of the system) as follows:

$$A = \sum_{i=1}^n m_i \dot{r}_i \cdot r_i$$

Time derivative:

$$\dot{A} = \sum_{i=1}^n \left(\underbrace{m_i \dot{r}_i \cdot \dot{r}_i}_{2T} + \underbrace{m_i \ddot{r}_i \cdot r_i}_{F_i} \right)$$

$$\dot{A} = 2T + \sum_{i=1}^n n F_i \cdot r_i$$

Time average:

$$\langle \dot{A} \rangle = \frac{1}{\tau} \int_0^\tau \dot{A} dt = \langle 2T \rangle + \left\langle \sum_{i=1}^n F_i \cdot r_i \right\rangle$$

If the system remains bounded—that is, none of the particles escapes—all positions \vec{r}_i and velocities $\dot{\vec{r}}_i$ stay finite. In this case, the virial A does not grow without bound, and the integral in the previous equation also remains finite. When we consider an increasingly long timespan ($\tau \rightarrow \infty$), the time average $\langle \dot{A} \rangle$ approaches zero, and we obtain:

$$\langle 2T \rangle + \left\langle \sum_{i=1}^n F_i \cdot r_i \right\rangle = 0$$

where F_i is the gravitational force:

$$\vec{F}_i = -G m_i \sum_{j=1, j \neq i}^n m_j \frac{\vec{r}_i - \vec{r}_j}{r_{ij}^3}$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$.

The latter term of the virial theorem becomes:

$$\begin{aligned} \sum_{i=1}^n F_i \cdot r_i &= -G \sum_{i=1}^n \sum_{j=1, j \neq i}^n m_i m_j \frac{r_i - r_j}{r_{ij}^3} \cdot r_i \\ &= -G \sum_{i=1}^n \sum_{j=i+1}^n m_i m_j \frac{r_i - r_j}{r_{ij}^3} (r_i - r_j) \end{aligned}$$

Which is obtained combining:

$$\begin{aligned} (1) &= -G \sum_{i=1}^n \sum_{j=1, j \neq i}^n m_i m_j \frac{\vec{r}_i - \vec{r}_j}{r_{ij}^3} \cdot \vec{r}_i \\ (2) &= -G \sum_{j=1}^n \sum_{i=1, i \neq j}^n m_i m_j \frac{\vec{r}_j - \vec{r}_i}{r_{ji}^3} \cdot \vec{r}_j \\ (3) &= -G \sum \sum m_i m_j \frac{\vec{r}_i - \vec{r}_j}{r_{ij}^3} (-\vec{r}_j) \end{aligned}$$

...(maybe missing something)

we get:

$$\sum_{i=1}^n \vec{F}_i \vec{r}_i = -G \sum_{i=1}^n \sum_{j=i+1}^n \frac{m_i m_j}{r_{ij}} = U$$

From the virial theorem we want to derive **observational quantities**.

$$\sum_i m_i (v_i - \langle v \rangle)^2 - G \sum_{i>j} \frac{m_i m_j}{r_{ij}} = 0$$

$$\underbrace{\frac{\sum_i m_i (v_i - \langle v \rangle)^2}{\sum_i m_i}}_{\text{velocity dispersion}} - G \underbrace{\frac{\sum_{i>j} \frac{m_i m_j}{r_{ij}}}{(\sum_i m_i)(\sum_i m_i)}}_{\equiv \frac{1}{R_v} \text{ viral radius}} \sum_i m_i = 0$$

...

2D position, 1D observations

$$\sigma_v^2 - G \frac{M}{R_v} = 0$$

$$M = \frac{\sigma_v^2 R_v}{G}$$

If the system is spherical, we can use the projected radius to get the virial radius.

$$\sigma_v^2 = 3\sigma_{v,los}^2$$

Therefore, we have:

$$R_v = \frac{\pi}{2} R_{v,projected}$$

$$M = \frac{3\pi}{2} \frac{\sigma_{v,los}^2 R_{v,projected}}{G}$$

Segregation effect: Some objects behaves differently: not always mass is proportional to luminosity. Therefore we need to use the formula above.

Sometimes we will see "tensorial virial theorem", and "generalized virial theorem".

👁 Observation: *Virial Theorem validity*

The virial theorem is valid if the mass follows the same distribution of light, but this is not always the case.

The definition of the **virial radius** (or better, the *radius of the virial theorem*) is:

$$R_v = \frac{n^2}{\sum_{i>j} \frac{1}{r_{ij}}}$$

The **harmonic radius** is:

$$R_H = \frac{n(n-1)/2}{\sum_{i>j} \frac{1}{r_{ij}}}$$

Scaling relations - Spiral galaxies

The **Tully-Fisher relation** allows us to relate the luminosity of a galaxy to its velocity dispersion.

$$L \propto V_{max}^\alpha \quad \alpha \sim 4$$

Calibrated TF relation from nearby galaxies.

For distant galaxies, we can obtain the redshift from the spectrum, and the rotation velocity (and, in particular, V_{max}). Then we can calculate M , and then the distance from $m - M$.

- For nearby galaxies we can use cepheids to calculate the distance
- For very far galaxies we can use the Hubble law
- For the ones in the middle we can use the TF relation

VT + Spiral structure \rightarrow T-F relation

$$\mathcal{M} \propto \frac{V_{max}^2 R}{G} \left(\propto \frac{L}{\mathcal{M}} \right)$$

so we get:

$$L \propto \left(\frac{\mathcal{M}}{L} \right)^{-1} \frac{V_{max}^2 R}{G}$$

$$L \propto \left(\frac{\mathcal{M}}{L} \right)^{-2} \frac{R^2}{LG^2} v_{max}^4$$

$$L^2 \propto \left(\frac{\mathcal{M}}{L} \right)^{-2} \frac{R^2}{G^2} v_{max}^4$$

$$L \propto \left(\frac{\mathcal{M}}{L} \right)^{-2} \left(\frac{1}{\langle I \rangle G^2} \right) v_{max}^4$$

$$\frac{\mathcal{M}}{L} \sim const$$

The Faber-Jackson relation

The fundamental plane

$$(R_e, \sigma_0, \langle I \rangle_e)$$

$$R_e \propto \sigma_0^{1.4} \langle I \rangle_e^{-0.85}$$

Virial Theorem + Kormendy Relation \rightarrow Fundamental Plane

$$R_e \propto \langle I \rangle_e^{-0.82} + \frac{\mathcal{M}}{L} \propto \mathcal{M}^{0.2}$$

The $D_n - \sigma$ Relation

Be D_n the diameter of an ellipse within the average surface brightness I_n corresponds to a value of $20.75 \text{ mag/arcsec}^2$.

We have that:

$$D_n \propto \sigma_0^{1.33}$$

Spectrum $\rightarrow \sigma_0 \rightarrow D_n \rightarrow$ distance

Boh

1. Luminous galaxies \rightarrow Hubble types
2. Elliptical \neq Spiral galaxies
 - morphology
 - kinematics
 - gas content
 - SF (= color)
3. Kinematics
 - Spiral: ordered m^+ , V_{max}
 - Ellipticals: random m , σ_v

2.2 Lecture 28/10/2025

Galaxies' luminosity function

The luminosity function ($\Phi(L)$) is the number of galaxies per unit volume per unit luminosity.

$$v = \int_{-\infty}^{\infty} \Phi(M) dM = \int_0^{\infty} \Phi(L) dL$$

We encounter different problems:

- what's the distance of a cluster of galaxies?
- large scale structure
- Malquist bias - limited surveys in m

TODO: reproduce plot in the notebook

During the years, multiple attempts to solve the problem have been made:

- Press-Schechter (74) halos \rightarrow Mass function
- Schechter (79) \rightarrow Luminosity function

$$\Phi(L) = \left(\frac{\Phi^*}{L^*}\right) \left(\frac{L}{L^*}\right)^{\alpha} e^{-\left(\frac{L}{L^*}\right)}$$

where $\alpha \sim -1$

typical B-band:

$$\Phi^* = 1.6 \cdot 10^{-2} \{h^{+3}\}$$

$$M_B^* = -19.7 + 5 \log h \quad \Leftrightarrow \quad L_B^* = 1.2 \cdot 10^{10} h^{-2} L_{\odot, B}$$

Therefore, $\alpha = -1.07$

For K-band:

$$\Phi_K^* = 1.6 \cdot 10^{-2} Mpc^{-3}$$

$$M_K^* = -23.1$$

therefore, $\alpha = -0.9$

Observation: h^{+3}

h^{+3} is used to rescale the luminosity function. At first it was related to $\Omega_m \sim 1$, nowadays it is not so easy to determine, and it depends on H_0 .

$$L_{tot} = \int_0^{\infty} dL \Phi(L) L = \Phi^* L^* \Gamma(2 + \alpha)$$

which is finite for $\alpha \geq -2$

$$N_{tot} = \int_0^{\infty} \Phi(L) dL$$

which is finite for $\alpha > -1$

The 60% of L_{tot} is contained in the range $0.22L^* < L < 1.6L^*$

The 90% of L_{tot} is contained in the range $0.1L^* < L < 2.3L^*$

Typical luminosity of galaxies is $\Phi^* \sim 2 \cdot 10^{-2} Mpc^{-3}$, with an average separation of $4Mpc$.

In clusters, the density is way higher than in the field (the rest of the universe). In fact, the average distance between galaxies is $\sim 1 - 2Mpc$.

Specific Luminosity Function

LF for morphological types:

Field

TODO: put here plot (a) sheet G14 Fig 3.51

Clusters

TODO: put here plot (b) sheet G14 Fig 3.51

In general it is valid $L_{s0} > L_{sa}$ - Lenticular galaxies are brighter than spiral galaxies.

Effects due to evolution of galaxies:

A merge of spiral galaxies can result in an elliptical galaxy $S + S \rightarrow E$

Groups

- $N \lesssim 50$ galaxies
- $M \lesssim 3 \cdot 10^{13} M_{\odot}$
- $T_x \sim 1 \rightarrow 3 KeV$
- $\sigma_v \sim 200 \rightarrow 300 km/s$

Clusters

- $N \gtrsim 50$ galaxies
- $M \gtrsim 3 \cdot 10^{14} M_{\odot}$
- $T_x \sim 4 \rightarrow 10 KeV$
- $\sigma_v \sim 400 \rightarrow 1000 km/s$

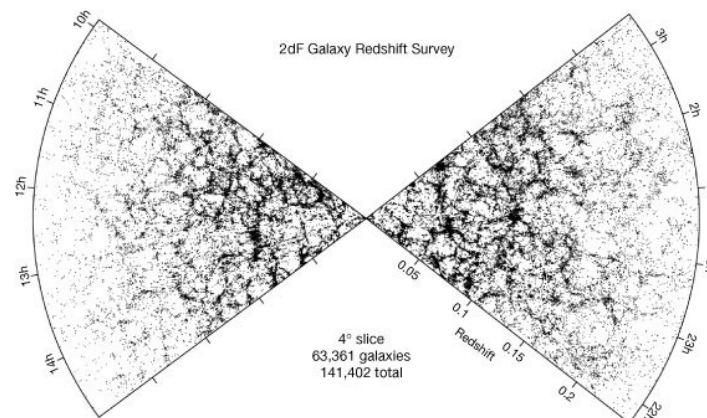
$$R_{Abell} = 1.5 h_{100}^{-1} Mpc \rightarrow 1.5 \cdot \frac{100}{70} h_{70}^{-1} M_r$$

Both are composed mainly by:

- Stars $\sim 3 - 5\%$
- Hot gas ($3 \cdot 10^7 K$) $\sim 15 - 20\%$
- Dark Matter $\sim 80\%$

Galaxy distribution

The following figure shows the distribution of galaxies in the universe.



Some structures are "real", such as the *Great Wall*, others are "apparent", such as the *Finger of God*.

💡 Tip: Galaxy distribution

In high redshift zones, the galaxies seem to be less dense than in low redshift zones. This is because the farthest galaxies have a lower apparent magnitude than the nearest ones.

- color bimodality: the second peak is given by the red color
-

Bibliography

- [1] Hannu Karttunen et al. *Fundamental astronomy*. English. 5th ed. United States: Springer, 2007.