

# Geometric Semantic Genetic Programming

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# Geometric Operators

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Crossover and mutation  
as operations on a metric space



# Metric Spaces

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## Distance

$$d : S \times S \rightarrow \mathbb{R}^+$$

Identity

$$d(x, y) = 0 \text{ iff } x = y$$

Symmetry

$$d(x, y) = d(y, x)$$

Triangular inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

## Segment

$$S(x, y) = \{z \mid d(x, z) + d(z, y) = d(x, y)\}$$

## Ball of radius $r$

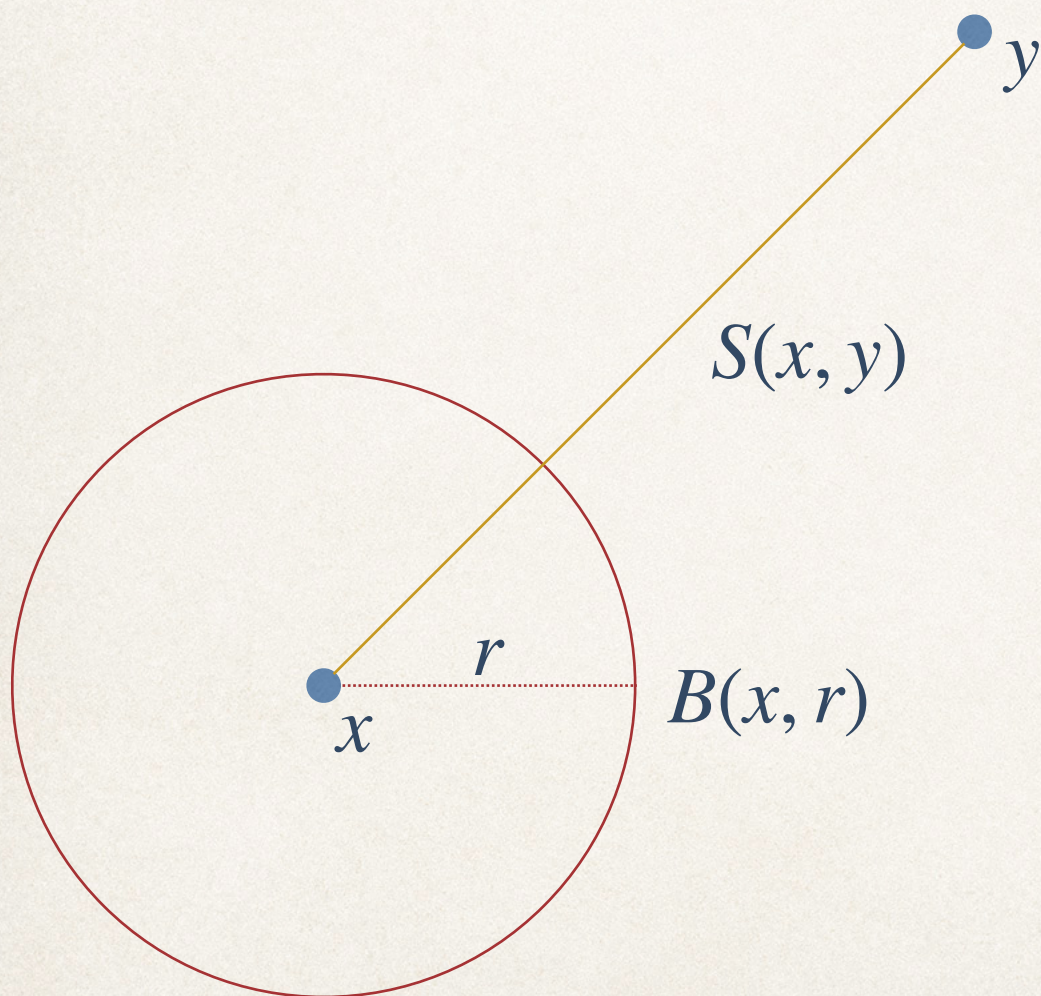
$$B(x, r) = \{y \mid d(x, y) \leq r\}$$



# Segments and Balls

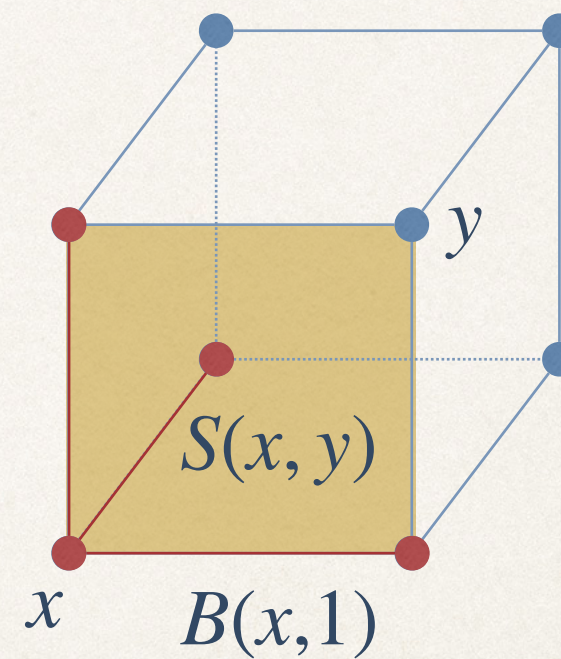
$\mathbb{R}^2$

Euclidean distance



$\{0,1\}^3$

Hamming distance





# Geometric Crossover

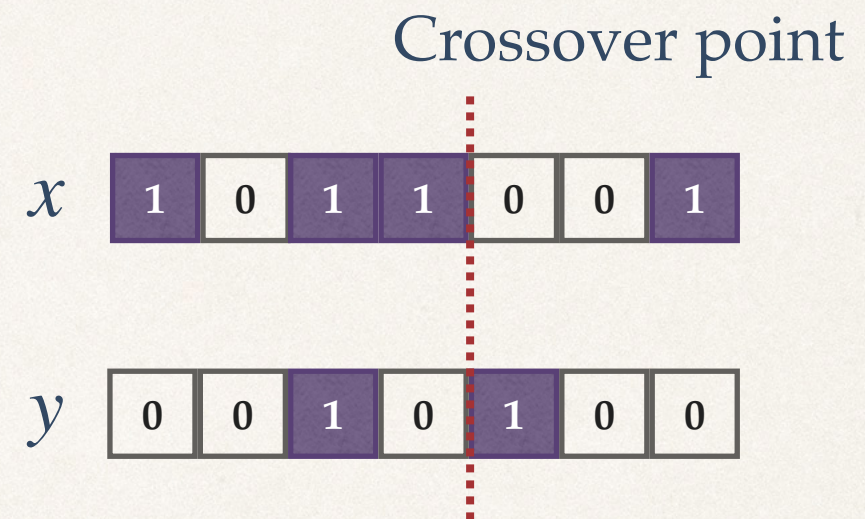
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## Geometric Crossover

For all  $x, y$

All possible results from crossover

Are inside  $S(x, y)$





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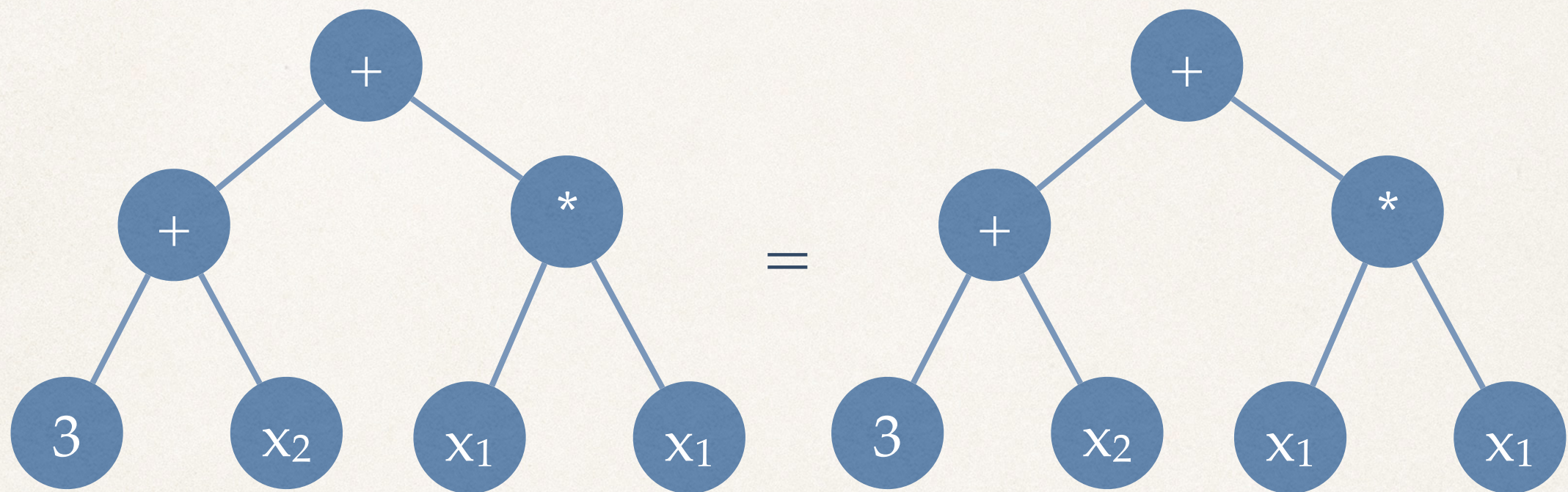
$$z_1 \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \in S(x, y)$$

$$z_2 \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \in S(x, y)$$



# Are All Crossovers Geometric?

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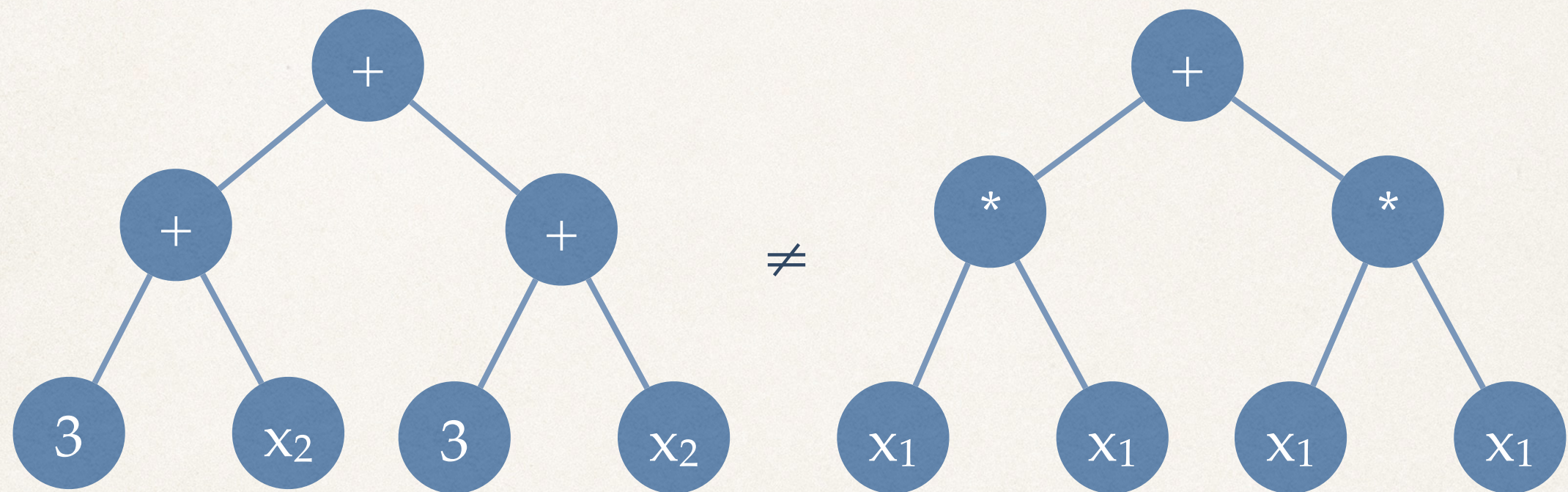
Crossover of a Tree with itself

$$S(x, x) = \{x\}$$



# Are All Crossovers Geometric?

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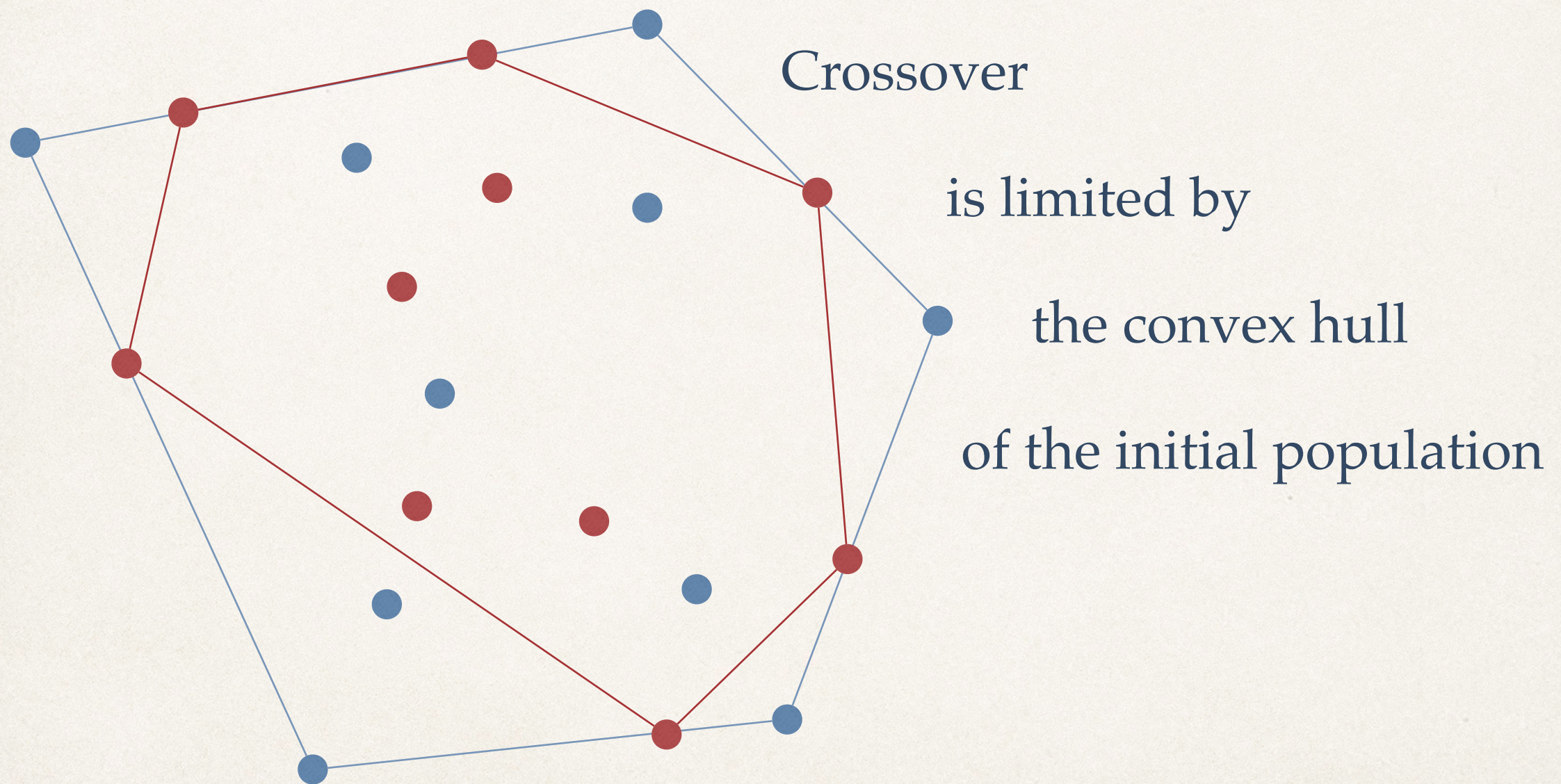


**Not a geometric crossover!**



# Consequences

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# Geometric Mutation

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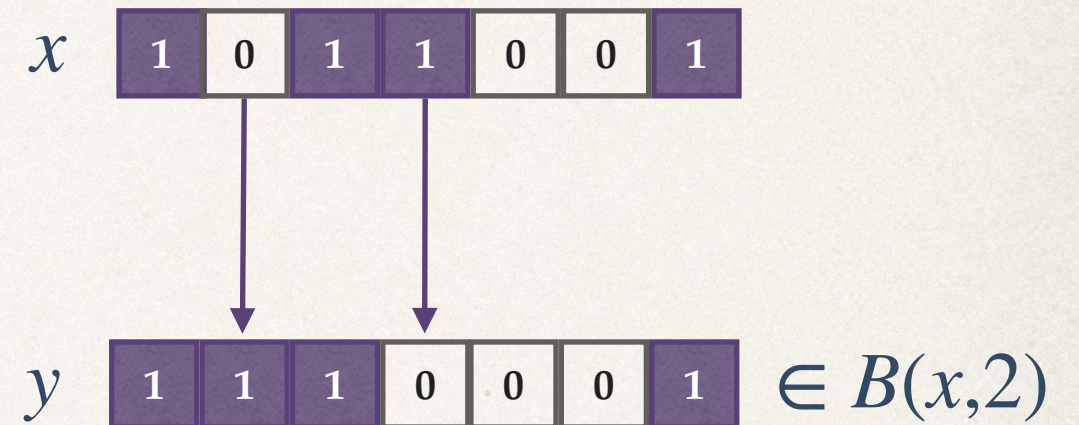
## Geometric Mutation (of radius $r$ )

For all  $x$

All possible results from the mutation

Are inside  $B(x, r)$

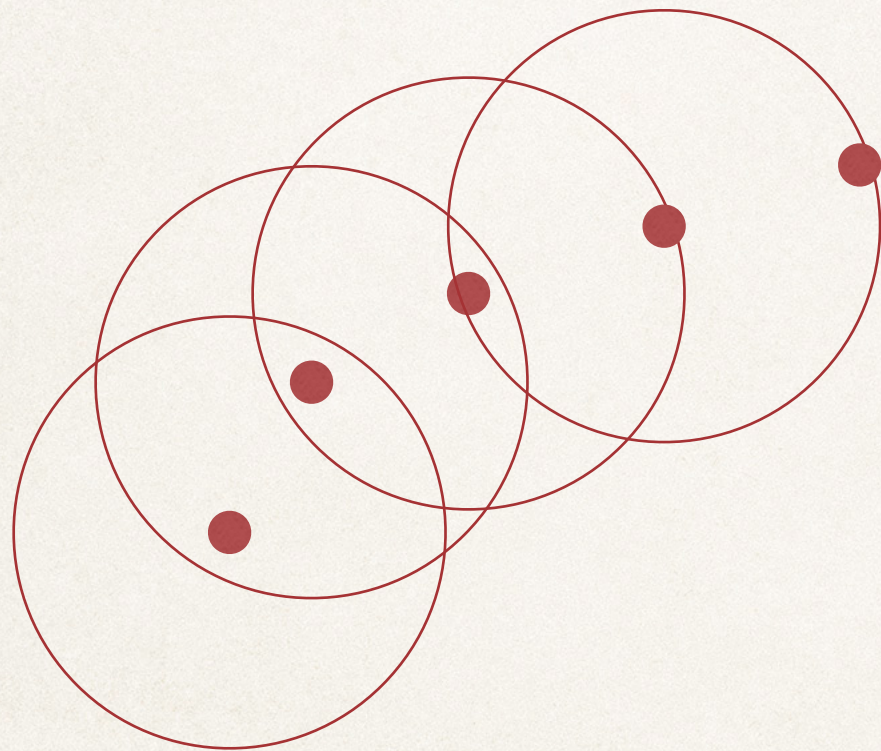
$$r = 2$$





# Mutation Can Take Us Everywhere\*

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## Exploration

Mutation enlarges the search space

Crossover restricts the search space

## Exploitation

\* Restrictions may apply for particular spaces.



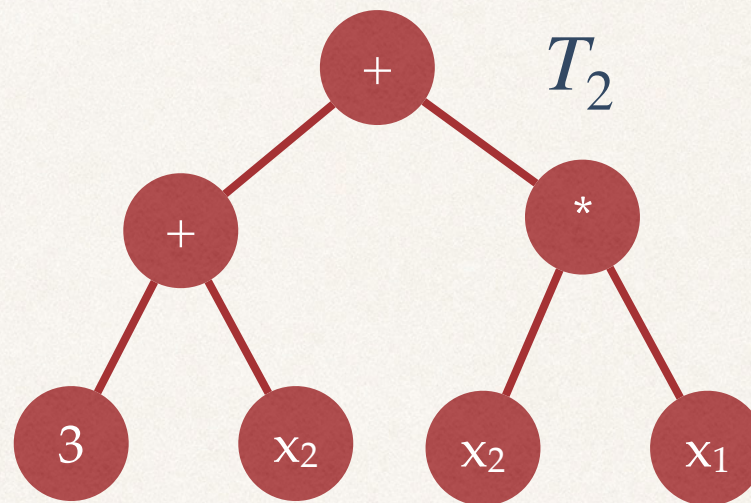
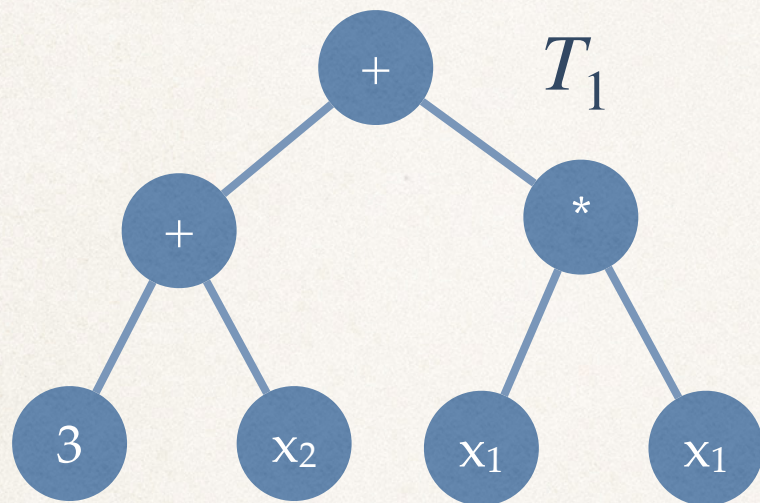
# Semantic Operators

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Controlling crossover and mutation effects  
on the *outputs* of an individual



# Syntax vs Semantics



Syntax

$x_1$	$x_2$	$T_1$	$T_2$
2	3	10	12
4	1	20	8
1	2	6	7

Semantic vectors

(10, 20, 6)

(12, 8, 7)

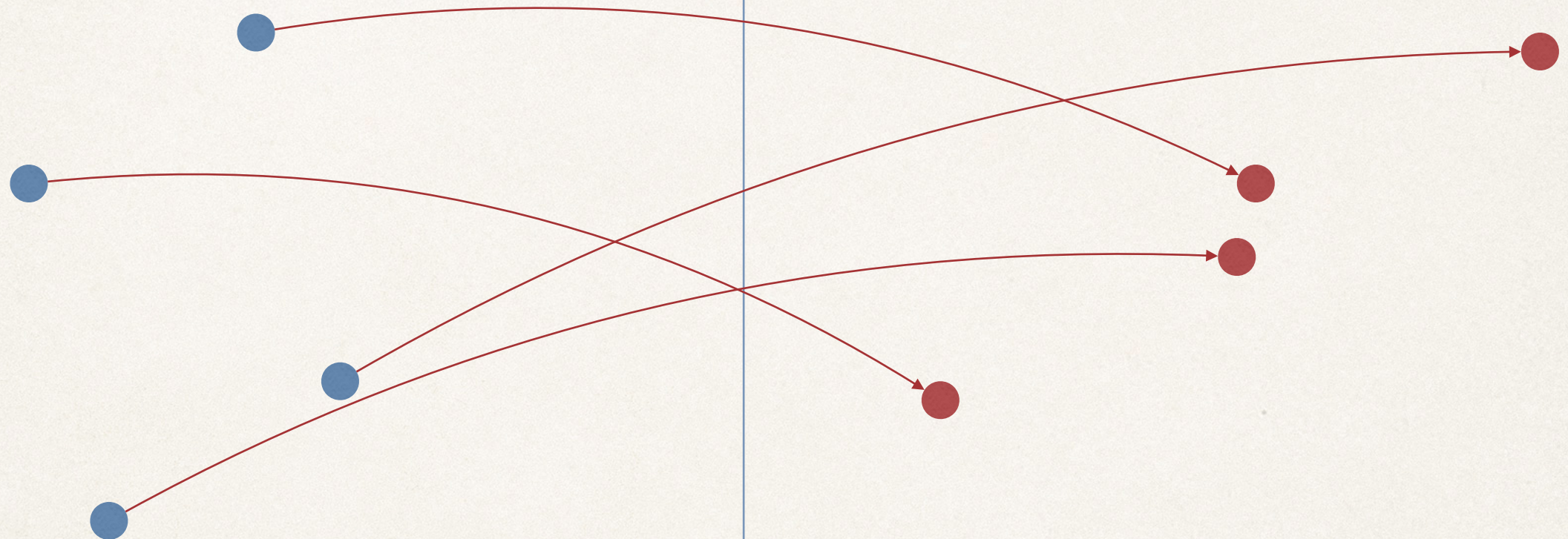
Semantics



# Syntax vs Semantics

Syntax Space

Semantic Space

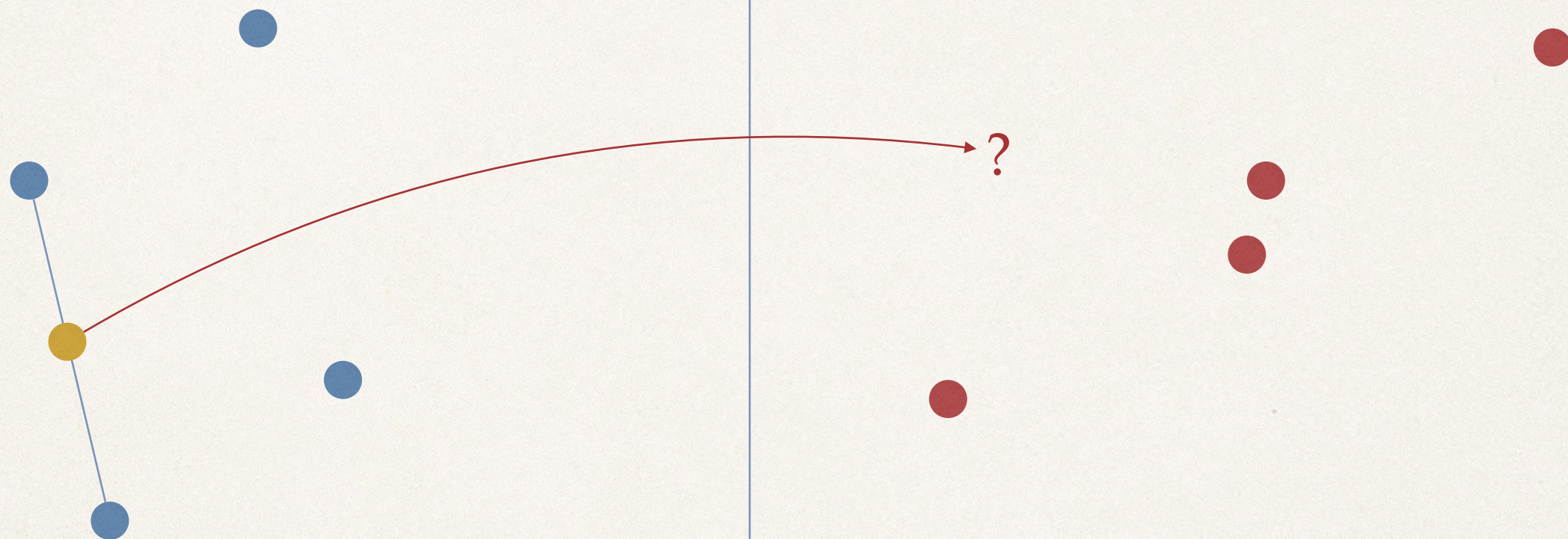




# Syntax vs Semantics

Syntax Space

Semantic Space

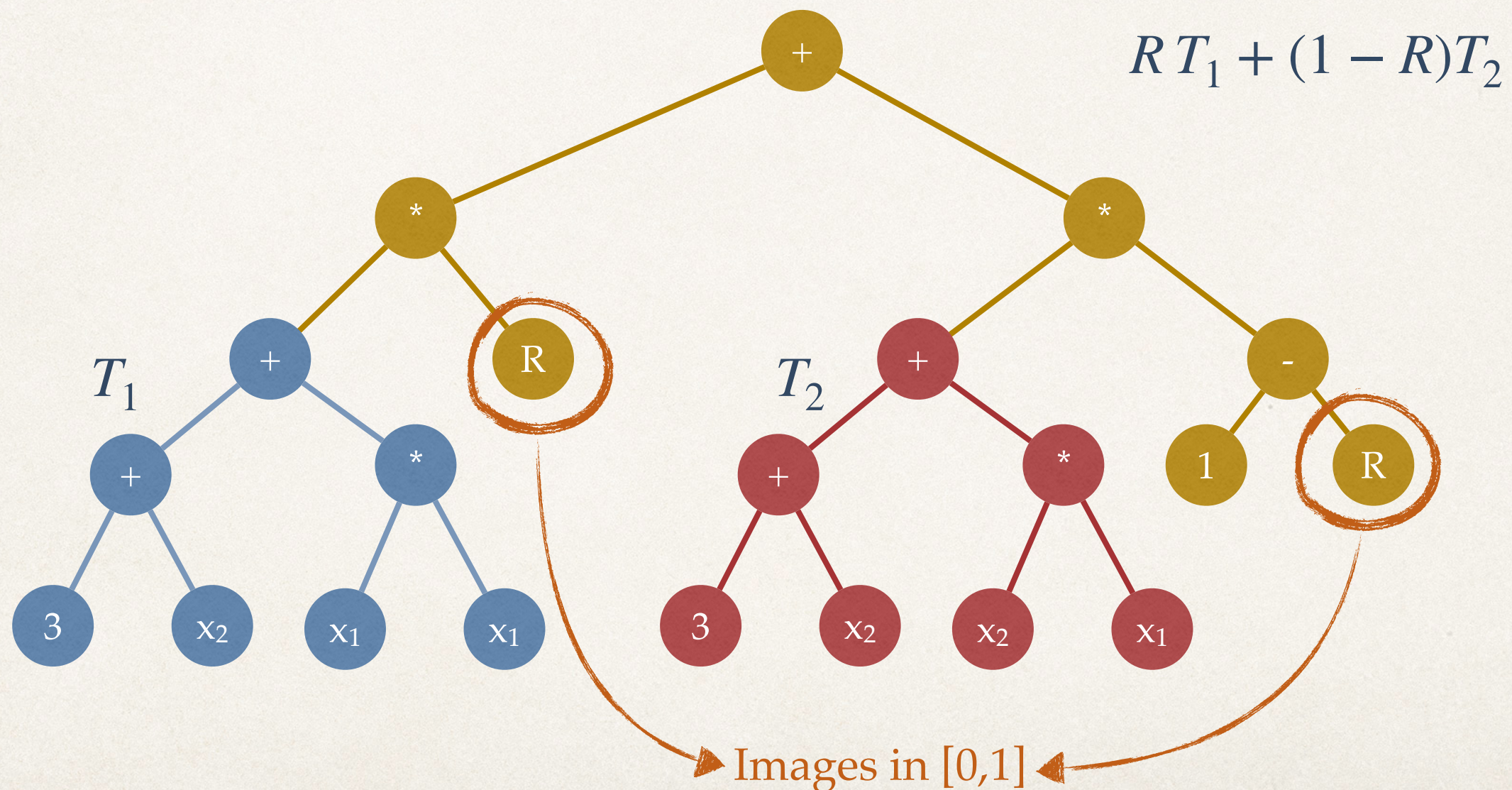




Can we make operators that are  
**geometric** in the **semantic** space?



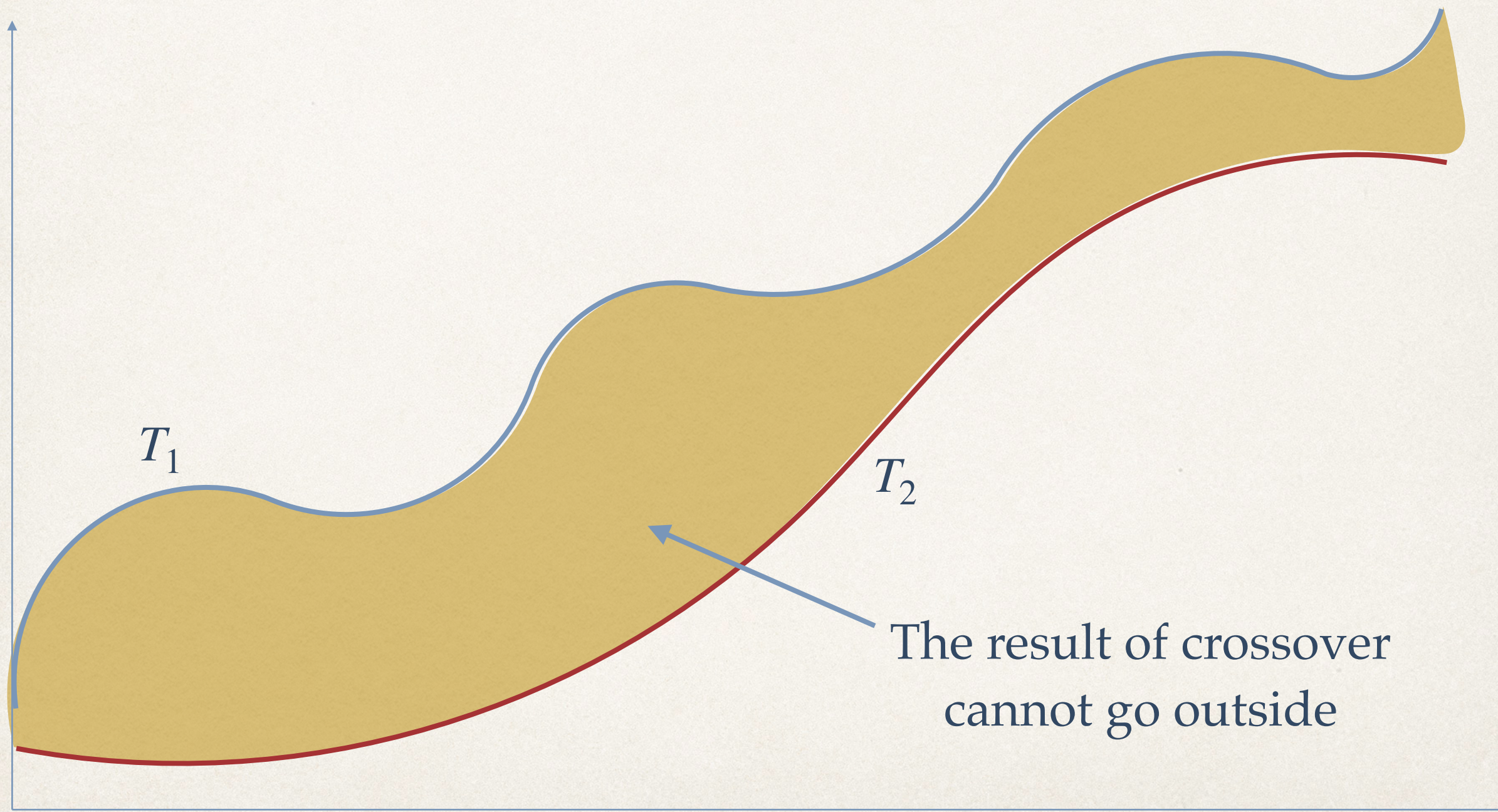
# Geometric Semantic Crossover





# Effect on the Semantics

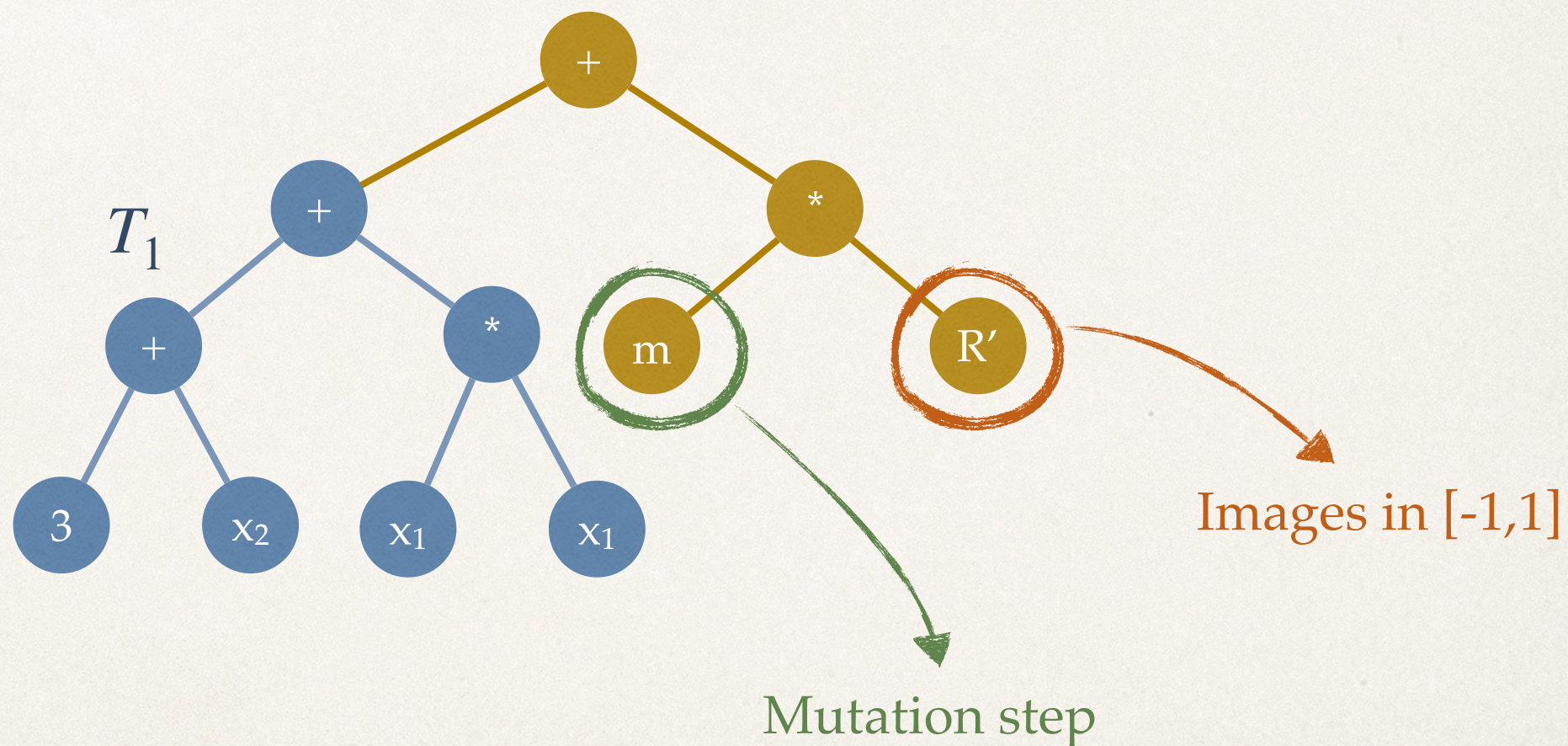
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# Geometric Semantic Mutation

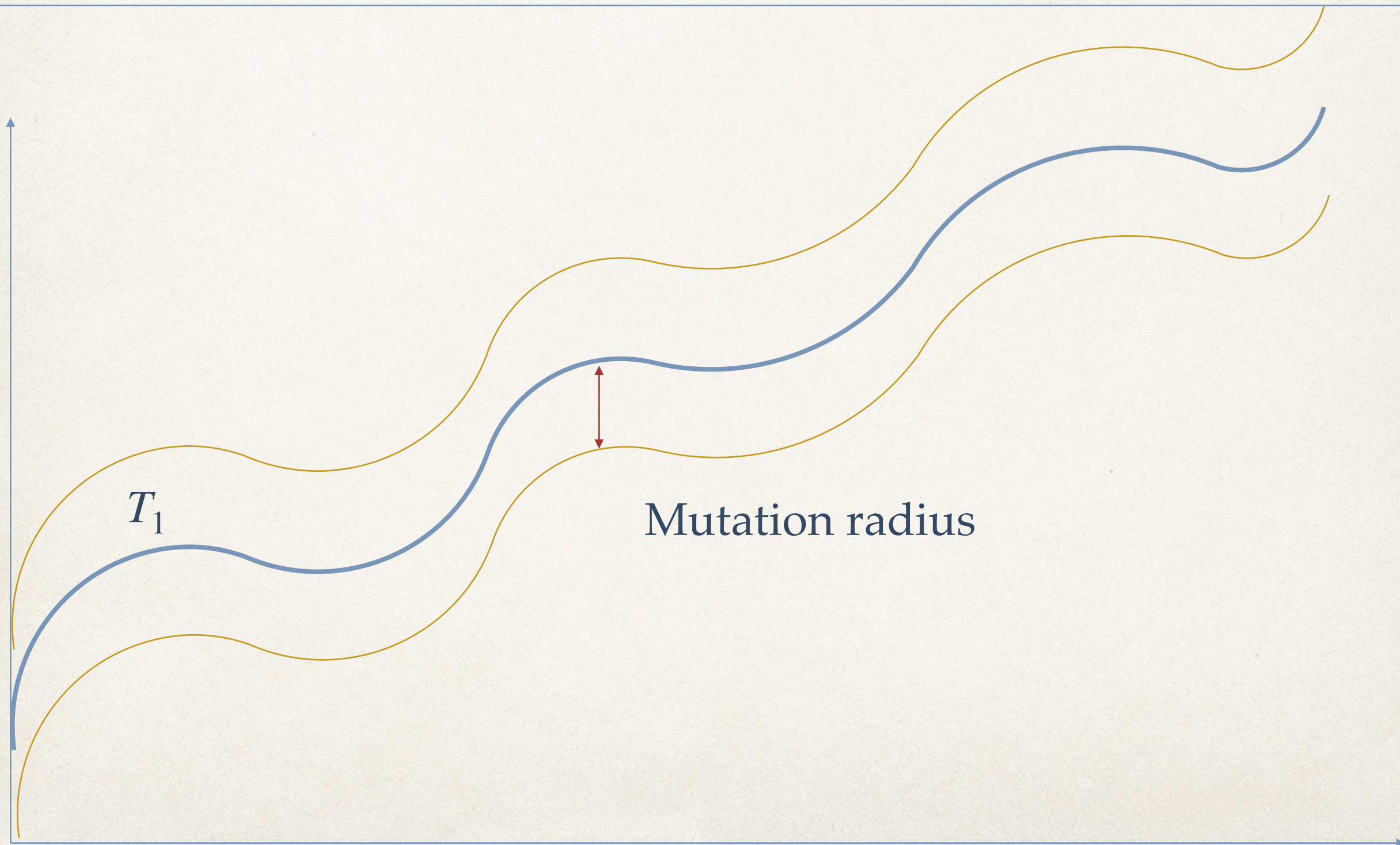
$$T_1 + m R'$$





# Effect on the Semantics

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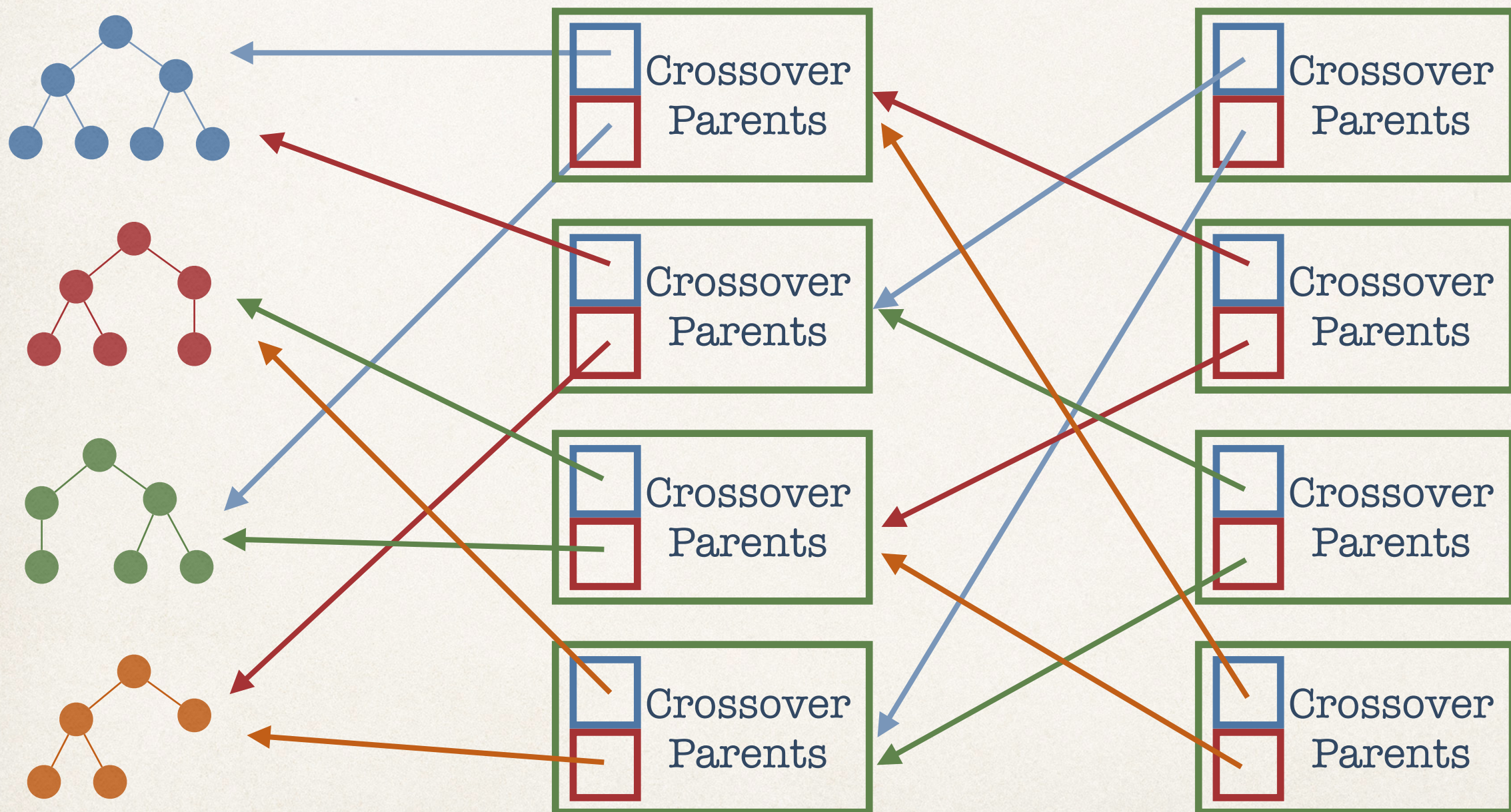
# Is this everything?

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- ❖ Different operators for different domains:  
Boolean, programs, etc.
- ❖ The tree-based representation is not the best one
  - ◆ Exponential growth wrt number of generations
  - ◆ Linear growth with subtree sharing.



# Fast GSGP





# An Example

