

inputs weights bias

↓ ↓ ↓

$$d = a @ b + c$$

We have $\frac{dL}{dd}$, we want both $\frac{dL}{da}$ and $\frac{dL}{db}$

$$\begin{bmatrix} d_{n1} & d_{n2} \\ d_{z1} & d_{z2} \end{bmatrix} = \begin{bmatrix} a_{n1} & a_{n2} \\ a_{z1} & a_{z2} \end{bmatrix} \begin{bmatrix} b_{n1} & b_{n2} \\ b_{z1} & b_{z2} \end{bmatrix} + \begin{bmatrix} c_1 & c_2 \\ c_1 & c_2 \end{bmatrix}$$

$$\begin{aligned} d_{n1} &= a_{n1} b_{n1} + a_{n2} b_{z1} + c_1 \\ d_{n2} &= a_{n1} b_{n2} + a_{n2} b_{z2} + c_2 \\ d_{z1} &= a_{z1} b_{n1} + a_{z2} b_{z1} + c_1 \\ d_{z2} &= a_{z1} b_{n2} + a_{z2} b_{z2} + c_2 \end{aligned}$$

Grad w.r.t. inputs to layer

$$\frac{dL}{da}$$

$$\frac{dL}{da_{n1}} = \frac{dL}{dd_{n1}} \cdot b_{n1} + \frac{dL}{dd_{n2}} \cdot b_{z1}$$

$$\frac{dL}{da_{n2}} = \frac{dL}{dd_{n1}} \cdot b_{n2} + \frac{dL}{dd_{n2}} \cdot b_{z2}$$

$$\frac{dL}{da_{z1}} = \frac{dL}{dd_{z1}} \cdot b_{n1} + \frac{dL}{dd_{z2}} \cdot b_{z1}$$

$$\frac{dL}{da_{z2}} = \frac{dL}{dd_{z1}} \cdot b_{n2} + \frac{dL}{dd_{z2}} \cdot b_{z2}$$

$$\begin{bmatrix} \frac{dL}{da_{n1}} & \frac{dL}{da_{n2}} \\ \frac{dL}{da_{z1}} & \frac{dL}{da_{z2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{dL}{dd_{n1}} & \frac{dL}{dd_{n2}} \\ \frac{dL}{dd_{z1}} & \frac{dL}{dd_{z2}} \end{bmatrix} \begin{bmatrix} b_{n1} & b_{n2} \\ b_{z1} & b_{z2} \end{bmatrix}$$

$$\Rightarrow \frac{dL}{da} = \frac{dL}{dd} @ b^T$$