

1.) (Ω, A, P) is not a probability space since its current A is given as $A = \{0, \Omega, \{1,2\}, \{3,4,5,6\}\}$ which is only four possible outcomes. With the equation $A = 2^n$ with 'n' being the possible outcomes in the probability space. We get six as 'n' since $\Omega = \{1,2,3,4,5,6\}$, substituting six in for 'n' we get $A = 2^6 = 64$. Since the A given does not satisfy this property, (Ω, A, P) is not a probability space, the correct A will be $A = \{0, \Omega, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\}, \dots \{1,2,3,4,5,6\}\}$.

2.) a.) Probability the outcome is divisible by three, given that it is black.

The black colored numbers are : 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35. This gives us a probability of $16/37$. If we divide these numbers that do not give a remainder, we are left with 3, 9, 15, 21, 27, 33

Which gives us answer of $\frac{6}{37}$.

b.) Probability that the outcome is divisible by three or four, given that a number is in the range 1 to 12.

This range gives us the number: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Then we divide all these numbers with three and four, and if these numbers are divided with no remainder then it complies with these rules. This gives us : 3, 4, 6, 8, 9, 12.

Which gives us the answer of a probability of $\frac{6}{37}$.

3.) We are looking for $P(A|B)$ using the Bayes rules we get $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$, we don't have $P(B|A)$, or $P(B)$. $P(A)$ and $P(A^C)$ is given with both equaling 0.5, since the door can either be closed or not. Using the theorem to find $P(B|A)$, since we have $P(B^C | A)$, $P(B|A) = 1 - P(B^C | A)$. Substituting the values we get $P(B|A) = 0.9$. To solve $P(B)$ we will use the theorem $P(B) = P(B|A) * P(A) + P(B|A^C) * P(A^C)$ which will result in $P(B) = 0.55$. Having all the values to solve for $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$.

The final probability is $P(A|B) = 0.8181818182$.