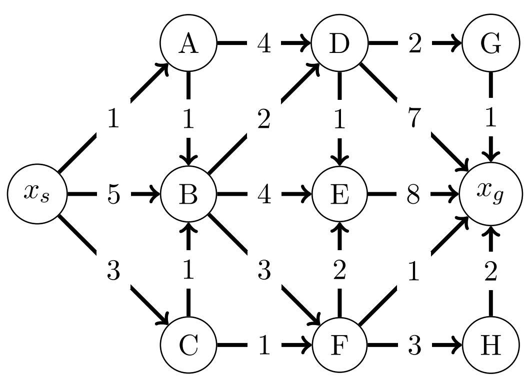
CSE 180 – Introduction to Robotics
Homework N. 2 – Spring 2019 SOLUTIONS

Important Information

1. This homework is due on April 4, 2019 at 4:15pm. The deadline is strict and will be enforced by CatCourses. **No exceptions.**
2. Your solution must be submitted in electronic form through CatCourses. Paper submissions will not be accepted. **No exceptions.**
3. If you write your solution by hand, write clearly. Unreadable documents will not be graded.
4. The method you follow to determine the solution is more important than the final results. Clearly illustrate and explain the intermediate steps. If you just write the final result you will get not credit.
5. **THIS HOMEWORK MUST BE SOLVED INDIVIDUALLY. NO EXCEPTIONS**.

1 Dijkstra's algorithm

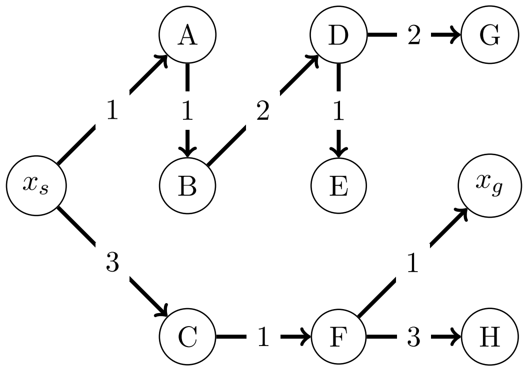
Consider the following graph. Let the start vertex be x_s and the goal vertex be x_g. Important: if a node has multiple outgoing edges, when the node is expanded the vertices adjacent to the node are processed in alphabetical order. Similarly, if multiple nodes in the $OPEN$ queue have the same priority value, sort them by alphabetical order.



1. Show how the algorithm plans a path from x_s to x_g. To display the various steps, follow exactly the same format used in the lecture notes (Example 4.4) and presented in class.
2. Show the tree produced by Dijkstra's algorithm at the end, together with the costs associated with each vertex.

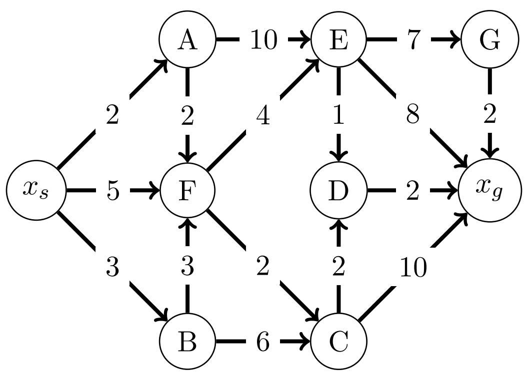
Answer:

Step	$OPEN$	x_s	A	B	C	D	E	F	G	H	x_g
0	$x_s/0$	$N/0$	N/∞	N/∞	N/∞	N/∞	N/∞	N/∞	N/∞	N/∞	N/∞
1	$A/1, C/3, B/5$	$N/0$	$x_s/1$	$x_s/5$	$x_s/3$	N/∞	N/∞	N/∞	N/∞	N/∞	N/∞
2	$B/2, C/3, D/5$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$A/5$	N/∞	N/∞	N/∞	N/∞	N/∞
3	$C/3, D/4, F/5, E/6$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$B/4$	$B/6$	$B/5$	N/∞	N/∞	N/∞
4	$D/4, F/4, E/6$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$B/4$	$B/6$	$C/4$	N/∞	N/∞	N/∞
5	$F/4, E/5, G/6, x_g/11$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$B/4$	$D/5$	$C/4$	$D/6$	N/∞	$D/11$
6	$E/5, x_g/5, G/6, H/7$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$B/4$	$D/5$	$C/4$	$D/6$	$F/7$	$F/5$
7	$x_g/5$	$N/0$	$x_s/1$	$A/2$	$x_s/3$	$B/4$	$D/5$	$C/4$	$D/6$	$F/7$	$F/5$



2 A*

Consider the directed weighted graph shown below.



Show how the algorithm $A*$ would determine the shortest path between x_s and x_g. Follow the same method used in class and in example 4.5 in the lecture notes.

Vertex	x_s	A	B	C	D	E	F	G	x_g
h	7	7	7	3	1	2	4	1	0

Answer:

$OPEN$	x_s	A	B	C	D	E	F	G	x_g
$x_s/0$	$N/0/7$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$
A, F, B	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$N/\infty/\infty$	$N/\infty/\infty$	$N/\infty/\infty$	$x_s/5/9$	$N/\infty/\infty$	$N/\infty/\infty$
F, B, E	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$N/\infty/\infty$	$N/\infty/\infty$	$A/12/14$	$A/4/8$	$N/\infty/\infty$	$N/\infty/\infty$
C, B, E	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$F/6/9$	$N/\infty/\infty$	$F/8/10$	$A/4/8$	$N/\infty/\infty$	$N/\infty/\infty$
D, B, E, x_g	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$F/6/9$	$C/8/9$	$F/8/10$	$A/4/8$	$N/\infty/\infty$	$C/16/16$
B, E, x_g	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$F/6/9$	$C/8/9$	$F/8/10$	$A/4/8$	$N/\infty/\infty$	$D/10/10$
E, x_g	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$F/6/9$	$C/8/9$	$F/8/10$	$A/4/8$	$N/\infty/\infty$	$D/10/10$
x_g, G	$N/0/7$	$x_s/2/9$	$x_s/3/10$	$F/6/9$	$C/8/9$	$F/8/10$	$A/4/8$	$E/15/16$	$D/10/10$

3 Navigation functions 1

Consider the following grid environment and let the cell marked as x_g be the goal cell. Assume the planning graph associated with the grid includes the standard four actions (up/down/left/right) similarly to what presented in the lecture notes, and let the black cells be untraversable. Define a navigation function $\psi : V \to \mathbb{R}_{\geq 0}$ for each cell/vertex in the grid. You can write the value of the value function directly in the grid.

Answer:

8	7	6	5	4	3	2	1	2	3
7	6	5	4	3	2	1	0	1	2
8	7	6	5	4	3	2	1	2	3
		7	6	5			2	3	4
10	9	8	7	6	5	4	3	4	5
11	10	9	8			5	4	5	6
12						6	5	6	7
13						7	6	7	8
14	13	12	11	10	9	8			9
15	14	13	12	11	10	9	10	11	10

4 Navigation functions 2

In the following grid, assume the goal cell is the one with the 0 value. As in the previous example assume that the set of actions is up/down/left/right. The assigned numeric values do not define a valid navigation function.

1. explain why the provided values do not define a valid navigation function;
2. show the changes that must be made to the given values to get a valid navigation function. Your answer should list the smallest number of changes.

Answer:

1. the wrong value is the 7 evidenced in the matrix. The reason is that from that cell it is possible to get to the goal (0), but all surrounding numerical values are higher than 7. Note that the 15 value in the lower left is not a problem because there are no constraints for the numerical values to be consecutive.
2. to fix the navigation function one just needs to switch the red value (7) to any integer number greater than or equal to 9.

6	5	4	3	2	1	2	3
7	6		2	1	0		
8	7		3	2	1	2	3
9	8			3	2	3	4
10	7		5	4	3	4	5
9	8		6	5	4	5	6
10	9	8	7	6	5		7
15	10	9	8	7	6	9	8