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CSE 180 Homework #4

1.) $Pr[X = x_N] = Pr[X = x_R] = Pr[X = x_B] = \frac{1}{3}$

$\rightsquigarrow Pr[Z = R \mid X = x_R] = 0.8$

$\rightsquigarrow Pr[Z = N \mid X = x_B] = 0.2$

$Pr[Z_{RRB} \mid RRB] = ?$

use $P[Z_R \mid RRB] + P[Z_B \mid RRB] + P[Z_N \mid RRB]$

$P[Z_{RRB} \mid RRB] =$

① $\eta P(Z_R \mid X = R)^2 \cdot P(Z = R \mid X = B) \cdot (\frac{1}{3})$

② $+ \eta P(Z_B \mid X = R)^2 \cdot P(Z = B \mid X = B) \cdot (\frac{1}{3})$

③ $+ \eta P(Z_N \mid X = R)^2 \cdot P(Z = n \mid X = n) \cdot (\frac{1}{3})$

① $\eta\ (0.8)^2 \cdot (0.2) \cdot (\frac{1}{3})$ + factor out the 'η'

② $\eta\ (0.05)^2 \cdot (0.6) \cdot (\frac{1}{3})$ +

③ $\eta\ (0.15)^2 \cdot (0.2) \cdot (\frac{1}{3})$

$\eta\ ((0.0427) + (0.0005) + (0.0015))$

$\eta\ (0.0447) = P[Z_{RRB} \mid RRB]$

$\eta = \frac{P[Z_{RRB} \mid RRB]}{0.0447} = 1$

$\eta = \frac{1}{0.0447}$

$\eta = 22.37136465 32$

① $\eta\ (0.8)^2 \cdot (0.2) \cdot (\frac{1}{3}) = 0.9545115585$

② $\eta\ (0.05)^2 \cdot (0.6) \cdot (\frac{1}{3}) = 0.0111856823$

③ $\eta\ (0.15)^2 \cdot (0.2)\ (\frac{1}{3}) = 0.033557047$

$$\left[\begin{array}{ll} P[Z_R \mid RRB] & = 0.9545115585 \\ P[Z_B \mid RRB] & = 0.0111856823 \\ P[Z_N \mid RRB] & = 0.033557047 \end{array}\right]$$

2.) Transition & Sensor model :

$$x_t = x_{t-1} + 2u_t$$

$$1$$

$$z_t = 2x_t$$

Assume :

$x_0 \sim N(0,1)$; $R \sim N(0,1)$; $Q \sim N(0, 0.2)$

$u_t = 2$; $z_t = 5$

Prediction :

$$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

$$x_t = A_t x_{t-1} + B_t u_t + V_0$$

$$x_t = 1\, x_{t-1} + 2\, u_t + 0$$

$\rightsquigarrow A_t = 1$; $B_t = 2$; $V_0 = 0$

$$z_t = H_t x_t + W_t$$

$$z_t = 2 x_t + 0$$

$\rightsquigarrow H_t = 2$; $W_t = 0$ Standard deviation

$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

$$\bar{\Sigma}_t = (1)(1)(1) + (1)$$

$$\rightsquigarrow \bar{\Sigma}_t = 2$$

$$\rightsquigarrow \bar{\mu}_1 = 1(x_0) + (2)(2) = 4$$

UPDATE :

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$$

$$K_t = ((2)(2))((2)(2)(2) + (0.2))^{-1}$$

$$(4)(8.2)^{-1}$$

$$\rightsquigarrow K_t = 0.487804878$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - H_t \bar{\mu}_t)$$

$$\mu_t = 4 + K_t ((5) - (2)(4))$$

$$\mu_t = 4 + K_t (-3)$$

$$\rightsquigarrow \mu_t = 2.536585366$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$

$$\Sigma_t = (1 - K_t\, 2)\, 2$$

$$\rightsquigarrow \Sigma_t = 0.048780488$$

Return : $\mu_t = 2.536585366$; $\Sigma_t = 0.048780488$