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CSE 180 Homework #1

1) Composite Rotation:

$$ {}^{A}_{B}R = R_4(\pi/3) \cdot R_2(\pi/2) \cdot R_1(\pi/4) \cdot R_3(\pi/6) \cdot R_5(\pi/3) $$

2) Transformation Matrices:

(1st) Rotation $\pi/2$ on x

Frame B: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/2) & -\sin(\pi/2) \\ 0 & \sin(\pi/2) & \cos(\pi/2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(2.) Add 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Rotation $\pi/2$ on z

$$\begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}_{B}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{B}_{A}T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) $q = a + bi + cj + dk$

$$R = \begin{bmatrix} 2(a^2+b^2)-1 & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & 2(a^2+c^2)-1 & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & 2(a^2+d^2)-1 \end{bmatrix}$$

$$RT = \begin{bmatrix} 2(a^2+b^2)-1 & 2(bc+ad) & 2(bd-ac) \\ 2(bc-ad) & 2(a^2+c^2)-1 & 2(cd+ab) \\ 2(bd+ac) & 2(cd-ab) & 2(a^2+d^2)-1 \end{bmatrix}$$

$RT \cdot R = I$

first Row using $a^2+b^2+c^2+d^2 = 1$

$2(a^2+b^2)-1)(2a^2+b^2)-1) + (2(bc+ad)(2(bc+ad)) + (2(bd+ac)(2(bd-ac))$

$(2(a^2+b^2)-1)(2b-ad) + (2(bc+ad)(2(a^2+c^2)-1) + (2(bd+ac)(2(cd+ab))$

$(2a^2+b^2)-1)(2(bd+ac)) + (2(bc+ad)(2(cd-ab)) + (2(bd+ac)(2(a^2+d^2)-1)$

first Row $= 1\ 0\ 0$

Second row

$(2(bc-ad)(2(a^2+b^2)-1) + (2(a^2+c^2)-1(2(bc+ad)) + (2(cd+ab)(2(bd-ac))$

$(2(bc-ad)(2(bc-ad)) + (2(a^2+c^2)-1(2(a^2+c^2)-1) + (2(cd+ab)(2(cd+ab))$

$(2(bc-ad)(2(bd+ac)) + (2(a^2+c^2)-1(2(cd-ab)) + (2(cd+ab(2(a^2+d^2)-1)$

Second row $= 0\ 1\ 0$

third Row

$(2(bd+ac)(2(a^2+b^2)-1) + 2(cd-ab)(2(bc-ad)) + (2(a^2+d^2)-1)(2(bd+ac))$

$(2(bd+ac)(2(bc+ad) + 2(cd-ab)(2(a^2+c^2)-1)) + (2(a^2+d^2)-1)(2(cd-ab))$

$(2(bd+ac)(2(bd-ac) + 2(cd-ab)(2(cd+ab)) + (2(a^2+d^2)-1)(2(a^2+d^2)-1)$

Second row $= 0\ 0\ 1$

final

$$R^T R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ thus making it a rotation Matrix

(4) Change of Coordinates

Assume known ${}^{B}_{A}T$, ${}^{C}_{W}T$, ${}^{B}_{C}T$, ${}^{W}_{B}T$

Robot A point of interest ${}^{A}P$

${}^{B}P = {}^{B}_{A}T\,{}^{A}P = P^{B}$

${}^{W}P = {}^{W}_{B}T\,{}^{B}_{A}T\,{}^{A}P = {}^{W}P$

${}^{C}P = {}^{C}_{W}T\,{}^{W}_{B}T\,{}^{B}_{A}T\,{}^{A}P = {}^{C}P$

You can determine all the points, All positive.

(5) Quaternions

$p = 1 + 2i - 3k$

$q = 5 + 4j + 2k$

(1) $(1 + 2i - 3k) \times (5 + 4j + 2k)$

$5 + 4j + 2k + 10i + 8ij + 4ik - 15k - 12kj - 6k^2$

$5 + 4j + 2k + 10i + 8k - 4j - 15k + 12i + 6$

$11 + 0j - 5k + 22i$

$\{pq = 11 + 22i - 5k\}$

(2) Norm pq

$|pq| = \sqrt{(11)^2 + (22i)^2 + (-5k)^2}$

$\sqrt{121 + 484 - 25}$

$\sqrt{630}$

$\sqrt{630} \approx 25.1$

$\left[\,|pq| \approx 25.1\,\right]$