Due before lecture on Monday, October 21, 2019

1. Construct examples of linear transformation that satisfy the following requirements. If no such examples are possible, explain why. (Hint: Problems #6-8 of Homework 06 help you connect one-to-one or onto linear transformations to properties of matrices.)

	one-to-one but not onto	onto but not one-to-one	both one-to-one and onto
$\mathbb{R}^2 \to \mathbb{R}^2$			
$\mathbb{R}^3 \to \mathbb{R}^3$			
$\mathbb{R}^2 \to \mathbb{R}^3$			
$\mathbb{R}^3 \to \mathbb{R}^2$			

2. (*Strang §2.1 #2*) Which of the following subsets of $\mathbb{R}^3$ are actually subspaces? For each subspace you find, find a basis for that subspace. Describe your reasoning.

   (a) The plane of vectors $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ with first component $b_1 = 0$.

   (b) The plane of vectors $\vec{b}$ with first component $b_1 = 1$.

   (c) The vectors $\vec{b}$ with $b_2 b_3 = 0$ (notice that this is the union of two subspaces, the plane $b_2 = 0$ and the plane $b_3 = 0$).

   (d) All linear combinations of two given vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

   (e) The plane of vectors $\vec{b}$ that satisfy $b_3 - b_2 + 3b_1 = 0$.

3. Determine each of the following statements true or false. Explain your reasoning.

   (a) $\{\vec{0}\}$ is a vector subspace of any $\mathbb{R}^n$, where $\vec{0}$ has n zeroes as coordinates.

   (b) Any straight line in $\mathbb{R}^2$ is a vector subspace of $\mathbb{R}^2$.

   (c) Any two-dimensional plane going through the origin in $\mathbb{R}^3$ is a vector subspace of $\mathbb{R}^3$.

4. (*added on Wednesday*) Finish the worksheet in lecture titled "Basis for $N(A)$ and $C(A)$", a copy of which is posted on CatCourses. Turn in a digital copy of your solution together with the rest of this homework set, **and bring a hard copy of your solutions to class on Monday.**

5. (*revised on Wednesday*) ***The dimension of a vector subspace* $\mathbf{W}$*, denote by* $\dim \mathbf{W}$*, is defined to be the number of vectors in its basis.***

   (a) For the matrix in the worksheet, $A = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 3 & 1 & -7 & 1 \\ 6 & 2 & 0 & -2 \end{bmatrix}$. what is $\dim N(A)$? What is $\dim C(A)$?

   (b) If A is an m-by-n matrix with rank r. What is $\dim N(A)$? What is $\dim C(A)$. Explain your reasoning. (Hint: review the worksheet.)

6. (*postponed to next week*) $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation.

   (a) Is $\ker(T)$ a subspace of $\mathbb{R}^n$?. Explain your reasoning. If yes, how can you find a basis for $\ker(T)$?

   (b) Is $\text{range}(T)$ a subspace of $\mathbb{R}^m$?. Explain your reasoning. If yes, how can you find a basis for $\text{range}(T)$?

   (Hint: Connect $\ker(T)$ and $\text{range}(T)$ to column space and nullspace of some matrix.)

7. Follow the steps below to prove the theorem: If $\{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n\}$ is a basis for $\mathbb{R}^n$, then any vector $\vec{x}$ in $\mathbb{R}^n$ can be written as a linear combination of $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ in a unique way.

   (a) Which requirement for $\{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n\}$ to be a basis ensures that $\vec{x}$ can be written as *some* linear combination of $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$?

   (b) Suppose that $\vec{x}$ can be written as a linear combination of $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ in two different ways. That is,

   $$\vec{x} = c_1\vec{e}_1 + c_2\vec{e}_2 + \cdots + c_n\vec{e}_n, \quad \text{and} \quad \vec{x} = d_1\vec{e}_1 + d_2\vec{e}_2 + \cdots + d_n\vec{e}_n$$

   where all the c's are not the same as all the d's. By calculating $\vec{x} - \vec{x}$, show that one requirement for $\{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n\}$ to be a basis has been violated.

   (c) Explain briefly why putting parts (a) and (b) together leads to a proof of the theorem.