Due before 8:50am Tuesday, September 3, 2019

1. Perform our "standard" forward Gaussian elimination, as discussed in lecture, on matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   to reduce it to an (upper) echelon form.

2. (a) In one forward Gaussian elimination step, we need to calculate *(row-3)-5(row-2)*. The result of that calculation should be put into new row-2 or new row-3?

   (b) Explain why the following matrix is NOT in upper echelon form. Use forward-elimination to change it into a echelon form.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

3. A student is solving a linear system of equations, and the result after the forward elimination process looks like

$$\left[\begin{array}{ccc|c} -1 & 0 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{array}\right] \tag{1}$$

   (a) Write down a possible system that the student had in the beginning which is NOT

$$\begin{aligned} -x+0y+2z &= 0 \\ 2z &= 0 \end{aligned}$$

   (b) Is your system from part (a) solvable? If yes, find a solution. If not, why not?

   (c) You have a friend in this same class whose answers to both parts (a) and (b) are different from yours. Is it true that one of you must be mistaken? Why or why not?

4. (*Strang ed.4, §2.2 #43*) For what values of q does each of the following matrices have rank (a) 1, (b)2, (c) 3? Explain your reasoning in detail.

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$$

5. Consider the following vector equation

$$x \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} - y \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ h \end{bmatrix}$$

   where h is some real number.

   (a) Write down a linear system of equations that is equivalent to this vector equation.

(b) Write down a matrix on which forward Gaussian elimination is needed in order to find solutions. Use forward Gaussian elimination to reduce your matrix in (b) to an upper echelon form. Rewrite your echelon form back into a linear system of equations.

(c) Is there any solutions if $h = 4$? If yes, is the solution unique? If not, why not?

(d) Repeat part (c) if $h = 10$.

(e) Does there exist a value of h such that there is one and only one solution to the system? Explain you reason.