Due before lecture on Monday, November 18, 2019

1. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ has an eigenvector $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ associated with eigenvalue $1/4$ and two eigenvectors $\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$ both associated with eigenvalue -3. Answer all of the following questions ***without finding the matrix for*** T.
   (a) Identify the image of following vectors under the transformation T. Be sure to justify your conclusions.
   (i) $\begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$ (ii) $\begin{bmatrix} 1/2 \\ -1/2 \\ 9/2 \end{bmatrix}$
   (b) Explain why $T\left(\begin{bmatrix} 3 \\ -5 \\ 37 \end{bmatrix}\right) = -3\begin{bmatrix} 3 \\ -5 \\ 37 \end{bmatrix}$.
   (c) Calculate $T\left(\begin{bmatrix} -1 \\ -2 \\ 12 \end{bmatrix}\right)$.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects the entire $\mathbb{R}^2$ across the x-axis.
   (a) Without calculating a matrix A for the transformation T, determine what the eigenvectors and eigenvalues would be, if any. In other words, does the transformation have any stretch directions and associated stretch factors? Justify your answer.
   (b) Find a matrix A to represent the transformation T. Calculate its eigenvectors and associated eigenvalues for the matrix A, and verify your answers to part (a).

3. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
   (a) (*Strang* §*5.2 #3*) Without solving $\det(A - \lambda I) = 0$, use observation to find all eigenvalues of A and then find associated eigenvectors.
   Hint 1. What can you say about the rank of A and what does that tell you about the nullspace? What does nullspace have to do with eigen-theory?
   Hint 2. Note that the rows of A add up to the same number 3, which would lead you to another eigenvector-eigenvalue pair.
   (b) Compute A^{100} by diagonalizing A.

4. A is an n×n matrix.
   (a) (*Strang* §*5.1 #23*) Fill in the blanks.
   i. If you know $\vec{x}$ is an eigenvector of A, the way to find the associated eigenvalue is to ________________.
   ii. f you know λ is an eigenvalue of A, the way to find an associated eigenvector is to ________________.
   (b) (*Strang* §*5.1 #24*) Let λ be an eigenvalue of A with associated eigenvecter $\vec{x}$. That is, $A\vec{x} = \lambda\vec{x}$. Use part (a) as a hint to prove the following statements.
   i. λ^2 is an eigenvalue of A^2. (*Also review problem #6 of Homework 11.*)
   ii. If A is invertible, λ^{-1} is an eigenvalue of A^{-1}.
   iii. $\lambda + 1$ is an eigenvalue of $A + I$.