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MATH 141

MATH 141 Linear Analysis Homework #4

1.) F.)

This statement is false, there can exist a set of vectors that are all scalar multiples of each other resulting in the set of vectors not being linearly independent

Ex: Span$\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\2\end{bmatrix}\}$ if remove $\begin{bmatrix}2\\2\end{bmatrix}$ Span$\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\}$

$x\begin{bmatrix}0\\0\end{bmatrix}+y\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $y=0$ $x=\mathbb{R}$, they are dependent b/c there are more solution other than trivial one

G.) This statement is true, due to the definition of linearly independency the only solution to $x\vec{v}_1+y\vec{v}_2+\cdots+n\vec{v}_n=\vec{0}$; is the trivial solution. $n \in \mathbb{R}$ and $x=y=\cdots=n=0$,

Example for a $n \times n$ identity matrix:

$$\begin{bmatrix}1 & & 0\\ & \ddots & \\ 0 & & 1\end{bmatrix} \rightsquigarrow \begin{matrix}x=0\\y=0\\ \vdots\\ n=0\end{matrix}$$

$n \in \mathbb{R}$ Since only the diagonals in the identity matrix are 1 and the other spots in the matrix are equal to zero, The only solution to

$$x\begin{bmatrix}1\\0\\0\\ \vdots\\0\end{bmatrix}+y\begin{bmatrix}0\\1\\0\\ \vdots\\0\end{bmatrix}+\cdots+n\begin{bmatrix}0\\0\\ \vdots\\1\end{bmatrix}=\begin{bmatrix}0\\ \vdots\\0\end{bmatrix}$$

is the trivial identity matrix is always linearly independent

2.) a.)

Suppse $x\begin{bmatrix}a\\b\end{bmatrix}+y\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $a \neq 0$ Exist.

$$\left[\begin{array}{cc|c}a & c & 0\\ b & d & 0\end{array}\right] \xrightarrow{R2-\frac{b}{a}R1} \left[\begin{array}{cc|c}a & c & 0\\ 0 & d-\frac{bc}{a} & 0\end{array}\right] \quad \begin{matrix}ax+cy=0\\ 0x+\left(d-\frac{bc}{a}\right)y=0\end{matrix}$$

In order to satisfy $x=y=0$: $d-\frac{bc}{a}\neq 0 \longleftrightarrow ad-cb\neq 0$ making $y=0$, plugging back in $ax+c(0)=0 \rightsquigarrow ax=0$ $x=0$.

b.)

This leads to $x\begin{bmatrix}a\\b\end{bmatrix}+y\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}u\\v\end{bmatrix}$ always having a solution because it satifies the definition of $x\begin{bmatrix}a\\b\end{bmatrix}+y\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ only having the trivial solution making them linearly indipendent and the span of $\{\begin{bmatrix}a\\b\end{bmatrix},\begin{bmatrix}c\\d\end{bmatrix}\}=\mathbb{R}^2$

3.) c.)

System of equation
$$\begin{cases} x - y + z = 2 \\ 2x - 2y + 2z = 4 \\ -x + y - z = -2 \end{cases}$$

• Since the equations are Scalar multiples of each others and they equal as well as scalar multiples these equations are the same plane covering the same span

Gaussian example:

$$x\begin{bmatrix}1\\-1\\1\end{bmatrix} + y\begin{bmatrix}2\\-2\\2\end{bmatrix} + z\begin{bmatrix}-1\\1\\-1\end{bmatrix} \quad \begin{bmatrix}1\\-1\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\-2\\2\end{bmatrix} \;\&\; \begin{bmatrix}1\\-1\\1\end{bmatrix} = -\begin{bmatrix}-1\\1\\-1\end{bmatrix} \text{ so}$$

$$x\begin{bmatrix}1\\-1\\1\end{bmatrix} + \frac{1}{2}y\begin{bmatrix}1\\-1\\1\end{bmatrix} - z\begin{bmatrix}1\\-1\\1\end{bmatrix} \rightsquigarrow \left(x + \frac{1}{2}y - z\right)\begin{bmatrix}1\\-1\\1\end{bmatrix} \rightsquigarrow \left(x + \frac{1}{2}y - z\right) = \underbrace{C}_{\text{constant}}$$

$$C\begin{bmatrix}1\\-1\\1\end{bmatrix}$$

original matrix $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ -1 & -2 & 1 & 4 \\ 1 & 2 & -1 & 3 \end{array}\right]$

e.)

System of equation
$$\begin{cases} ① \; x + y + z = 2 \\ ② \; 2x + 2y + 2z = -3 \\ ③ \; x + 2y + 3z = 4 \end{cases}$$

• Since the equation 1 & 2 normal vectors are scalar multiples of each other the are parrellel to each other. Since equation 3 is not a scalar multiple of 1 & 2, equation 3 intersects equations 1 and 2. Matrix form:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & -3 \\ 1 & 2 & 3 & 4 \end{array}\right]$$

← as shown in the Matrix the first two rows are scalar multipless of each other showing they are parrellel to each other.

f.) System of equations $\begin{cases} x + y + z = 0 \\ 2x + 2y + 2z = -3 \\ 3x + 3y + 3z = 5 \end{cases}$

Since the normal vectors of the system of equations are scalar multiples of each other. This means they are parrellel to each other

matrix example:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & -3 \\ 3 & 3 & 3 & 5 \end{array}\right]$$

As seen in matrix form the columns are scalar multiples showing that they are parrellel to one another.