11/24/19

MATH141 Linear Analysis HW 13

1.) $|A - \lambda I| = \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 4-\lambda & 0 \\ 12 & 2 & 2-\lambda \end{bmatrix}$ $(1-\lambda)(4-\lambda)(2-\lambda)$

$= (4 - \lambda - 4\lambda + \lambda^2)(2-\lambda)$

$= 8 \cancel{-10\lambda} \cancel{+2\lambda^2} \cancel{-4\lambda} \cancel{+5\lambda^2} \cancel{-\lambda^3} - 48$

$\cancel{-12\lambda}$

$= -\lambda^3 + 7\lambda^2 - 2\lambda - 40 \rightarrow (-\lambda-2)(\lambda^2 - 9\lambda + 20)$

$= (-\lambda-2)(\lambda-4)(\lambda-5) = \boxed{\lambda = -2, 4, 5}$

$\lambda = -2$ $\begin{bmatrix} 3 & 0 & 1 \\ 0 & 6 & 0 \\ 12 & 2 & 4 \end{bmatrix} \xrightarrow{R_3 - 4R_1 = R_3}$

$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 6 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ $R3 - 1/3 R_2 = R3$ $\left[\begin{array}{ccc|c} 3 & 0 & 1 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} 3x_1 + x_3 = 0 \\ 6x_2 = 0 \end{cases}$ $x_1 = -\frac{1}{3}x_3$

$x_2 = 0$

$x_3 = x_3$

eigen vector $= \begin{bmatrix} -1/3\, x_3 \\ 0 \\ x_3 \end{bmatrix}$ $x_3 = 1$ $\begin{bmatrix} -1/3 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 4$ $\begin{bmatrix} -3 & 0 & 1 \\ 0 & 0 & 0 \\ 12 & 2 & -2 \end{bmatrix} \xrightarrow{R3 + 4R_1 = R3} \begin{bmatrix} -3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$

$\begin{cases} -3x_1 + x_3 = 0 \\ 2x_2 + 2x_3 = 0 \end{cases}$ $x_1 = 1/3\, x_3$

$x_2 = -x_3$

$x_3 = x_3$

eigen Vector $= \begin{bmatrix} 1/3\, x_3 \\ -x_3 \\ x_3 \end{bmatrix}$ $x_3 = 1$ $\begin{bmatrix} 1/3 \\ -1 \\ 1 \end{bmatrix}$

$\lambda = 5$ $\begin{bmatrix} -4 & 0 & 1 \\ 0 & -1 & 0 \\ 12 & 2 & -3 \end{bmatrix} \xrightarrow{R3 + 3R_1 = R3}$

$\begin{bmatrix} -4 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \xrightarrow{R_3 + 2R_2 = R3}$ $\left[\begin{array}{ccc|c} -4 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Cont (1)

$$\begin{cases} -4x_1 + x_3 = 0 \\ -x_2 = 0 \end{cases} \qquad \begin{aligned} x_1 &= 1/4\, x_3 \\ x_2 &= 0 \\ x_3 &= x_3 \end{aligned}$$

eigen vector $= \begin{bmatrix} 1/4\, x_3 \\ 0 \\ x_3 \end{bmatrix}$ $\quad \lambda_3 = 1 \quad \begin{bmatrix} 1/4 \\ 0 \\ 1 \end{bmatrix}$

The eigen values tells us that a matrix will be stretching by a factor of $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and in the direction of the general eigen vector.

(2) a.) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ det = 1

b.) when eigen value = 0

$\begin{bmatrix} 3 & 12 \\ 1 & 4 \end{bmatrix} \longrightarrow$ det = 0
not invertable but

$\begin{bmatrix} 3-\lambda & 12 \\ 1 & 4-\lambda \end{bmatrix}$ $\quad (3-\lambda)(4-\lambda) - 12$

$\cancel{12} - 7\lambda + \lambda^2 \cancel{-12}$

$(\lambda^2 - 7\lambda) \rightarrow \lambda(\lambda - 7)$

$\lambda = 0, 7$ $\quad \begin{bmatrix} 0 & 0 \\ 0 & 7 \end{bmatrix}$

(3) a.) True, as long as eigen value is not zero it is invertoble

b.) False, $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

c.) False $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4.) a.) False, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ det $= 1 - 0 - 0$

det $= 1$

inverse exists

b.) True, Since the only eigenvalue is 1, there can be a possibility that there are repeating eigen values as seen in the example above.

c.) True, since you need at least $\mathbb{R}^n$, n# of eigen values, and we only have 1.

5.) a.) Linearly independent

b.) null(A) $= (A - 0I) = \text{Span}\{\vec{u}\}$

Col(A) $= \text{span}\{\vec{v}, \vec{w}\}$ non zero eigen value.

c.) $A\vec{x} = \vec{v} + \vec{w} = (\frac{1}{3}\vec{v} + \frac{1}{5}\vec{w} \mid A$

one soln $= A(\frac{v}{3} + \frac{w}{5})$

general $= \vec{v}/3 + \vec{w}/5 + c\vec{u}$

d.)

$A\vec{x} = \vec{u}$ does not have a soln because $\vec{u} \notin \text{Span}\{\vec{v}, \vec{w}\} =$

$\vec{u} \notin \text{Col}\{\vec{v}, \vec{w}\} = \vec{u} \notin \text{Col}(A)$

e.) Not invertable because there exist an eigen value $= 0$

6.) $A_{n \times n}$ $A\vec{u} = 2\vec{u}$ & $A\vec{v} = 5\vec{v}$ $\vec{u}$ & $\vec{v} \neq 0$

① $x\vec{u} + y\vec{v} = \vec{0}$ $x = a$ $b = y$

$(a\vec{u} + b\vec{v} = \vec{0}) A$

$aA\vec{u} + bA\vec{v} = \vec{0}$

② $2a\vec{u} + 5b\vec{v} = \vec{0}$

$2a\vec{u} + 5b\vec{v} + x\vec{u} + y\vec{v} = 0$

$2a\vec{u} + 5b\vec{v} + 2a\vec{u} + 5\vec{v} = 0$

$(a + b)(2\vec{u} + 5\vec{v} + 2\vec{u} + 5\vec{v}) = 0$

$(a+b)(4\vec{u} + 10\vec{v}) = 0$

(6) Since we know $\vec{u}$ & $\vec{v} \neq 0$ then $(a+b) = 0$ in order for the equation to be true so the only value where this can be true is a & $b = 0$

(7) a.) by making $\lambda = 0$ we get ever λ_n = a positive integer which is equal to doing $\lambda = \lambda_1$, you are just left with the positive integer and we get the $\det(A)$ if A is an upper triangular form = to the product of the eigen values.

b.)