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MATH 1A1 Linear Analysis Homework #10

1.) a.) For $A_{m \times n}$ to be invertable:

- $A\vec{x} = \vec{b}$ is solvable for any RHS $\vec{b}$ and the solution is always unique
- Null space $(A) = \vec{0}$ AND Column space of $A = \mathbb{R}^m$
- Column vectors must be linearly independent
- All pivot points must exist in each column & row

b.) In order for a matrix to be onto & one to one the matrix must be a square matrix by defenition.

2.) Completed on the last homework

3.) Completed on the last homework

4.) a.)

If A is invertable then $AA^{-1} = I$ so $(AB = AC)$ the $B = C$ because:

$$A^{-1}(AB) = (AC)A^{-1} \Rightarrow AA^{-1}B = AA^{-1}C$$

$$\Rightarrow IB = IC \rightarrow B = C$$

b.) $A \neq 0, B, \& , C$ satisfy $AB = AC$ but $b \neq C$

This is true for any A that has a column or row of zeroes. Example:

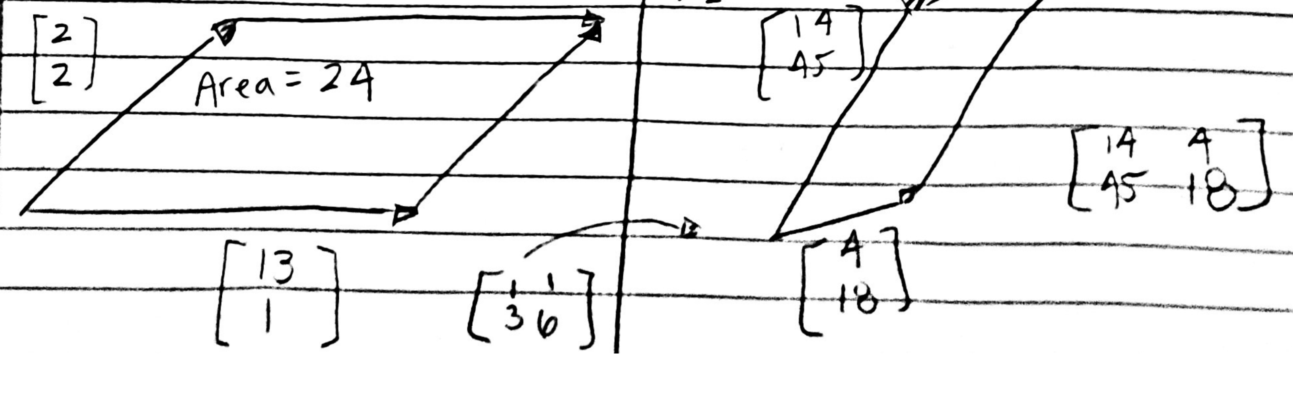
$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$ $AB = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$ $AC = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

$AB \stackrel{\checkmark}{=} AC$ $\quad B \neq C \quad A \neq 0$

$A = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ $AB = \begin{bmatrix} 0 & 0 \\ 5 & 5 \end{bmatrix}$ $AC = \begin{bmatrix} 0 & 0 \\ 5 & 5 \end{bmatrix}$

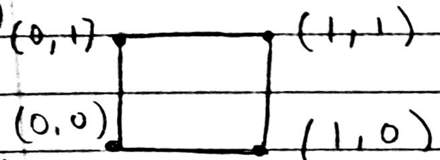
5.) optional so won't do it! OwO

6.) P_1 $\quad P_2$



9.) The area will be ad - bc because you Create a rectangle over the parrellelagram and find the area of each piece That is not the parrallelogram and substract from the total area of The rectangle. ad - bc = 72

b.) drawn in the page before and I am Correct on the area of P_2, area = 72.

c.) 

$B: \mathbb{R}^2 \to \mathbb{R}^2$

d.) det(B) = 1   det (AB) = 72

det of a matrix is just finding the area so the area of B = 1 and A = 72 multiply both and you get det(AB)

e.) $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 14 & 4 \\ 45 & 18 \end{bmatrix}$ $AB = \begin{bmatrix} 14 & 4 \\ 45 & 18 \end{bmatrix}$

det (A) = 72   det(B) = 1   det (AB) = 72

7.) a.)

The relationship of det (AB), det(A), & det(B) is det (AB) = det (A) · det(B)

b.) The relationship of det(A) & det (A^{-1}) is

$\det(A^{-1}) = \dfrac{1}{\det(A)}$ b/c if $\det(A^{-1}) = 0$ then the inverse does not exist and making the LHS undefined.

8.) a.) $\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$ det $I_{n \times n} = 1$

b.) Changes from negative to positive or positive to −

$\det\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$ $\det\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1$

C.) det (A) = 0

$\det\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 1 - 1 = 0$ $\det\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} = 12 - 12 = 0$

8 cont.)

d.) the det is the same

$\det\begin{bmatrix}1&2\\3&4\end{bmatrix} = -2 \xrightarrow{\quad} \det\begin{bmatrix}1&2\\1&0\end{bmatrix} = -2$

$R_3 - 2R_1 = R_3$

e.) $\det(A) = 0$

$\det\begin{bmatrix}1&1\\0&0\end{bmatrix} = 0 \qquad \det\begin{bmatrix}1&3&5\\2&4&6\\0&0&0\end{bmatrix} = 0$

f.) $\det(k\,A_{m\times n}) = k^n$

$\det 2\begin{bmatrix}1&0\\0&1\end{bmatrix} = \det\begin{bmatrix}2&0\\0&2\end{bmatrix} = 4 \qquad 2^2 = 4$

g.) product of diagonals

$\det\begin{bmatrix}2&0&0\\0&3&0\\0&0&2\end{bmatrix} = \begin{matrix}2(6) - 0 - 0 = 12\\ 2\cdot 3\cdot 2 = 12\end{matrix}$

h.) product of diagonals

$\det\begin{bmatrix}2&1&1\\0&3&2\\0&0&2\end{bmatrix} = \begin{matrix}2(6) - 0 - 0 = 12\\ 2\cdot 3\cdot 2 = 12\end{matrix}$

i.) $\det(A) = 0$; $\det(A) \neq 0$

$\begin{bmatrix}1&1\\2&2\end{bmatrix} \longrightarrow \begin{bmatrix}1&1\\0&0\end{bmatrix} \quad \det\begin{bmatrix}1&1\\0&0\end{bmatrix} = 0 \quad \det\begin{bmatrix}1&0\\0&1\end{bmatrix} = 1$

j.) $\det(AB) = \det(A)\det(B) \qquad \det(A^{-1}) = \dfrac{1}{\det(A)}$

$A = \begin{bmatrix}1&0\\0&1\end{bmatrix} \quad B = \begin{bmatrix}1&3\\2&4\end{bmatrix} \quad \det(A) = 1$

$\det(B) = -2 \qquad 1\cdot -2 = -2$

$AB = \begin{bmatrix}1&3\\2&4\end{bmatrix} \quad \det(AB) = -2$

$A = \begin{bmatrix}1&2\\3&4\end{bmatrix} \quad A^{-1} = \begin{bmatrix}-2&1\\3/2&-1/2\end{bmatrix} \quad \begin{matrix}\det(A) = -2\\ \det(A^{-1}) = -1/2\end{matrix}$

$\dfrac{1}{\det(A)} = \dfrac{1}{-2}$

K.) $\det(A^T) = \det(A)$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$\det(A) = -2$ $\quad \det(A^T) = -2$

9.)

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix} \xrightarrow{R_2 - \frac{1}{4}R_1 = R_2} \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & -1 & 0 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 - \frac{1}{2}R_1 = R_3} \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix} \xrightarrow{R_4 - \frac{1}{4}R_1 = R_4}$$

$$\rightarrow \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_3 + 2R_2 = R_3} \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_4 - \frac{1}{3}R_3 = R_4}$$

$$\det \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 4 \cdot 1 \cdot -3 \cdot 1 = \boxed{-12}$$