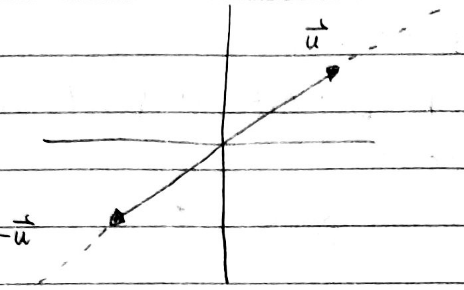
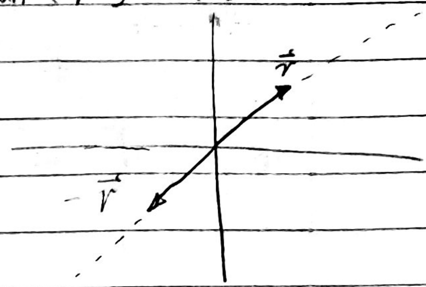
A.)

1.) Span$\{\vec{u}\} = \{c\vec{u} : c \in \mathbb{R}\}$



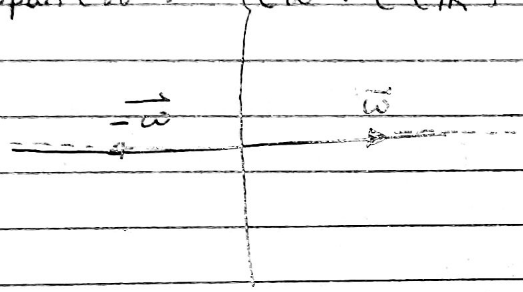
The vector only spans the line it Creates in $\mathbb{R}^2$

Span$\{\vec{v}\} = \{c\vec{v} : c \in \mathbb{R}\}$



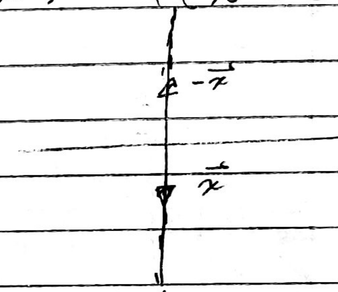
The vector only spans the line it Creates in $\mathbb{R}^2$

Span$\{\vec{w}\} = \{c\vec{w} : c \in \mathbb{R}\}$



The vector only spans the line it Creates in $\mathbb{R}^2$

Span$\{\vec{x}\} = \{c\vec{x} : c \in \mathbb{R}\}$



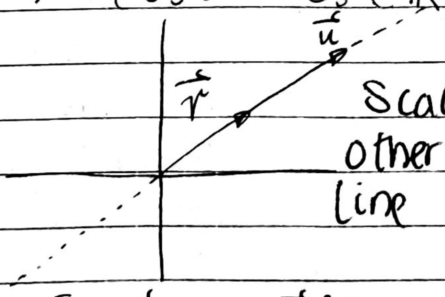
The vector only spans the line it creates in $\mathbb{R}^2$

B.)

Span$\{\vec{u}, \vec{v}\} = \{c_1\vec{u} + c_2\vec{v} : c \in \mathbb{R}\}$, Since they are scalar multiples we can say $\vec{v}$ is $\frac{1}{2}$ of $\vec{u}$. So now we get:

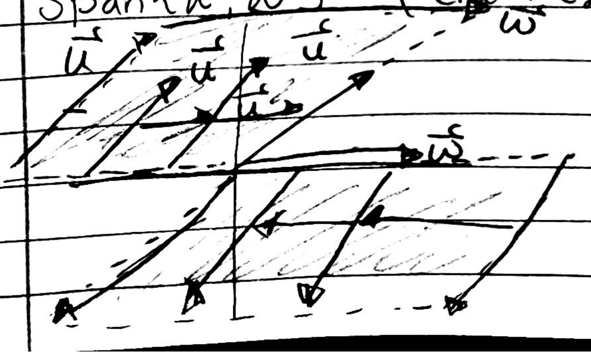
$\vec{v} = \frac{1}{2}\vec{u}$ $\rightsquigarrow$ $c_1\vec{u} + \frac{1}{2}c_2\vec{u} = (c_1 + \frac{1}{2}c_2)\vec{u} = c_3\vec{u}$ So,

Span$\{\vec{u}, \vec{v}\} = \{c_3\vec{u} : c_3 \in \mathbb{R}\}$



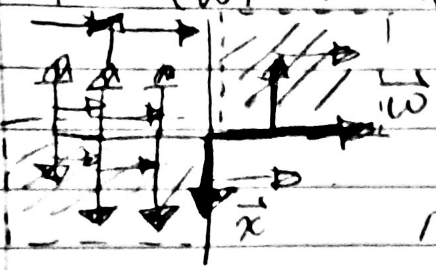
- Since the vectors are Scalar multiples of each other they only Span the line they Create.

Span$\{\vec{u}, \vec{w}\} = \{c_1\vec{u} + c_2\vec{w} : c \in \mathbb{R}\}$



- as represented in the mesh there are infinite many linear Combinations that vector $\vec{u}$ & $\vec{w}$ can make creating the span $\mathbb{R}^2$

$\text{span}\{\vec{w},\vec{x}\} = \{c_1\vec{w} + c_2\vec{x} ;\ c_1 \text{ \& } c_2 \in \mathbb{R}\}$



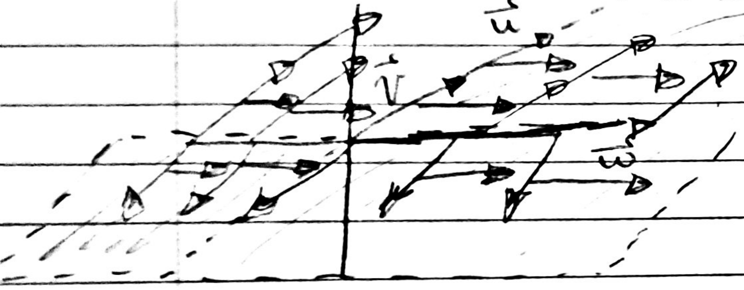
- As shown by the Mesh there are an infinite amount of linear Combinations that vectors $\vec{w},\vec{x}$ make that span $\mathbb{R}^2$

C.) $\text{span}\{\vec{u},\vec{v},\vec{w}\} = \{c_1\vec{u} + c_2\vec{v} + c_3\vec{w} \quad c_{1,2,3} \in \mathbb{R}\}$

since $\vec{u}$ is a scalar multiple of $\vec{v}$. $\vec{u} = 2\vec{v}$ so:

$2c_1\vec{v} + c_2\vec{v} + c_3\vec{w} \rightsquigarrow (2c_1 + c_2)\vec{v} + c_3\vec{w}$

$\rightsquigarrow \{c_4\vec{v} + c_3\vec{w} \quad c_4 \text{ \& } c_3 \in \mathbb{R}\} = \text{span}\{\vec{u},\vec{v},\vec{w}\}$

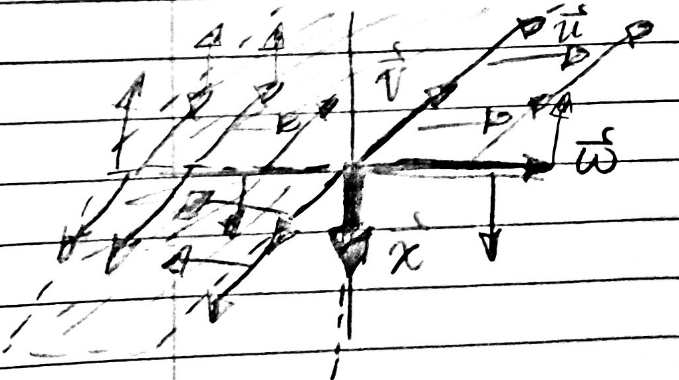


- As shown by the mesh, there are an infinite amount of linear combinations that vectors $\vec{u},\vec{v},\vec{w}$ make that span all of $\mathbb{R}^2$

$\text{span}\{\vec{u},\vec{v},\vec{w},\vec{x}\} = \{c_1\vec{u} + c_2\vec{v} + c_3\vec{w} + c_4\vec{x} ;\ c_1,c_2,c_3,c_4 \in \mathbb{R}\}$ since $\vec{u}$ is a scalar multiple of $\vec{v}$. $\vec{u} = 2\vec{v}$ so:

$2c_1\vec{v} + c_2\vec{v} + c_3\vec{w} + c_4\vec{x} \rightsquigarrow (2c_1 + c_2)\vec{v} + c_3\vec{w} + c_4\vec{x}$

$\rightsquigarrow \{c_5\vec{v} + c_3\vec{w} + c_4\vec{x} ;\ c_5, c_3, c_4 \in \mathbb{R}\} = \text{span}\{\vec{u},\vec{v},\vec{w},x\}$



- As shown by the mesh there are an infinite amount of linear Combination that vectors $\vec{u},\vec{v},\vec{w}, x$ make that span all of $\mathbb{R}^2$

2. A.) $x\begin{bmatrix}-1\\2\end{bmatrix} + y\begin{bmatrix}3\\4\end{bmatrix} + z\begin{bmatrix}5\\0\end{bmatrix} = \begin{bmatrix}3\\-6\end{bmatrix}$

B.) $\begin{cases} -x + 3y + 5z = 3 \\ 2x + 4y = -6 \end{cases}$

c.) There are more than one solution is the best description for this problem.

$$x\begin{bmatrix}-1\\2\end{bmatrix}+y\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}3\\-6\end{bmatrix} \rightsquigarrow \left[\begin{array}{cc|c}-1&3&3\\2&4&-6\end{array}\right] \xrightarrow{R_2+2R_1=R_2}$$

$$\left[\begin{array}{cc|c}-1&3&3\\0&10&0\end{array}\right] \quad \begin{aligned}-x+3y&=3\\10y&=0\end{aligned} \quad \begin{bmatrix}z=0\\x=-3\\y=0\end{bmatrix}$$

$$y\begin{bmatrix}3\\4\end{bmatrix}+z\begin{bmatrix}5\\0\end{bmatrix}=\begin{bmatrix}3\\-6\end{bmatrix} \rightsquigarrow \left[\begin{array}{cc|c}3&5&3\\4&0&-6\end{array}\right] \xrightarrow{3R_2-4R_1=R_2}$$

$$\left[\begin{array}{cc|c}3&5&3\\0&-20&-30\end{array}\right] \quad \begin{aligned}3y+5z&=3\\-20z&=-30\end{aligned} \quad z=3/2,\ y=\tfrac{-3}{2},\ x=0$$

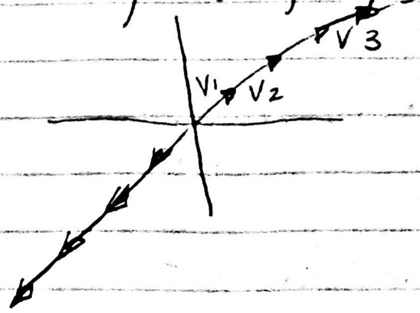
$$\begin{bmatrix}x=0\\y=-3/2\\z=3/2\end{bmatrix}$$

A.)

3.) • Set Contians 3 vectors and it spans all of $\mathbb{R}^2$ $\text{Span}\left\{\begin{bmatrix}3\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}1\\7\end{bmatrix}\right\}=\mathbb{R}^2$, using the example given in the class, vectors $\begin{bmatrix}3\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}$ span all of $\mathbb{R}^2$, and since $\begin{bmatrix}1\\7\end{bmatrix}$ is within the span of $\begin{bmatrix}3\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}$ adding vector $\begin{bmatrix}1\\7\end{bmatrix}$ does not Change the span.

B.)

• Set Contains 3 vectors but does not span all $\mathbb{R}^2$ $\text{Span}\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\2\end{bmatrix},\begin{bmatrix}3\\3\end{bmatrix}\right\}=\mathbb{R}$, these vectors do not span all of $\mathbb{R}^2$ because they are scalar multiples of themselves, so they only span the line the vector creates.



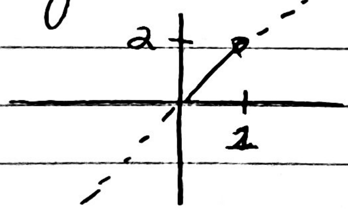
two vectors that span all of $\mathbb{R}^2$

C.) Span$\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\} = \mathbb{R}^2$, Using the example from Class, the vectors $\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}$ span all of $\mathbb{R}^2$,

D.) The set contains only one vector and its span is the entire $\mathbb{R}^2$.
There is no one vector that spans all of $\mathbb{R}^2$ because one vector can only span the line it creates with its scalar multiples. Ex: $c\vec{v}$ $c \in \mathbb{R}$

E.) The span of the set contains only one vector in $\mathbb{R}^2$

Span$\{\begin{bmatrix}1\\2\end{bmatrix}\}$, this is the only vector in $\mathbb{R}^2$, While still being able to increase its magnatude.

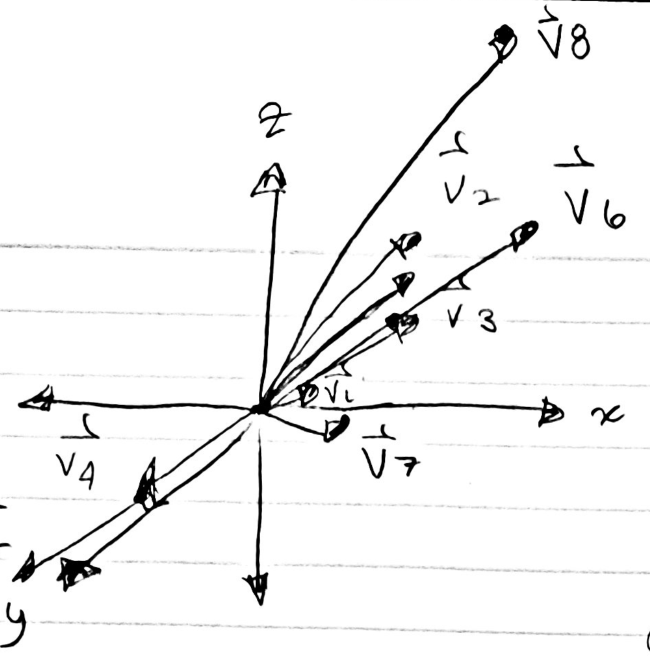


1.) A.) There has to be atleast three vectors in order to span all of $\mathbb{R}^2$ while being linearly dependant

B.) There can only be 1 vector in the set in order to not span all of $\mathbb{R}^2$, because having more than one vector that are linearly independent from each other increases the span.

C.) There can only be two vectors in order to span all of $\mathbb{R}^2$ and be linearly independant because if you add another vector that are linear independant from each other it will increase the span.

5.) A.)



$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ 2

$\vec{V}2 = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix}$ $V_5 = \begin{bmatrix} -6 \\ 3 \\ 3 \end{bmatrix}$

$V_3 = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$ 3 $V_6 = \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$

0 $V_4 = \begin{bmatrix} -4 \\ 2 \\ -2 \end{bmatrix}$ 4 $V_7 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

5 $V_8 = \begin{bmatrix} 11 \\ 5 \\ 15 \end{bmatrix}$

B.) • There is more than one way, when performing Foward Gausian elimination we get:

$$\left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 8 & 6 & 0 \end{array}\right] \xrightarrow{R2 - R1 = R2} \left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -3 & -3 & 0 \\ 1 & 8 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R3 - R1 = R3} \left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \xrightarrow{3R3 - 2R2 = R3} \left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 6y + 4z = 0 \rightsquigarrow x = -6y - 4z$

$-3y - 3z = 0 \rightsquigarrow y = -z \rightsquigarrow y = \eta$, $z = -\eta$ $\quad \eta \in \mathbb{R}$

$x = -6\eta + 4\eta \rightsquigarrow x = -2\eta$

For any value η there will exist multiple solutions to the problem,

• Yes, old man Gaus can hide in $\mathbb{R}^3$ since from the equation above the vectors given do not span all of $\mathbb{R}^3$.

6.) A) Name one vector in $\mathbb{R}^3$ in the span of $\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}$ is in the span $\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

because it is a scalar multiple of vector $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ So it is in its span.