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MATH 191 Linear Analysis Homework #8

1.)

	one-to-one but not onto	onto but not 1-1	both onto & 1-1
$R^2 \to R^2$	not possible, if one-to-one then it is onto	Not possible, if onto it means it is one-to-one	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$. full rank. Square matrixes are both
$R^3 \to R^3$	Not possible. Same def.	Not possible. Same def.	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ full rank. Square matrixes are both
$R^2 \to R^3$	$\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix} \to \begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$ one to one not onto	Not possible L.I but Rank(T)=3 But in 2D so canot be 3	Not possible $R^n \to R^m$ $n=m$ which is not true
$R^3 \to R^2$	Not possible, one-to-one when full rank, Rank(T) $\neq 3$	$\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}$ full rank, more than one soln.	Not possible $R^n \to R^m$ $n=m$ Which is not true

2.) a.)

it is a subspace ① $0 \in W$ $b_1 = 0$ $(0,0,0) \in W$

② $aw_1 + bw_2 \in W$

$a(0, w_{12}, w_{13}) + b(0, w_{22}, w_{23})$

$(0, aw_{12}, aw_{13}) + (0, bw_{22}, bw_{23})$

$(0, aw_{12} + bw_{22}, aw_{13} + bw_{23})$

since $b_1 = 0$ this $\in W$ so W is a subspace of $\mathbb{R}^3$

b.)

It is not a subspace of $\mathbb{R}^3$

① $0 \in W$ $b_1 = 1$ $0 \notin W$

c.)

It is not a subspace of $\mathbb{R}^3$

① $b_2 b_3 = 0$ $b_2 = 0 \,||\, b_3 = 0$ $0 \in W$ ✓

② $aw_1 + bw_1 \in W$

$a(2,0,4) + b(2,4,0)$

$(2a, 0, 4a) + (2b, 4b, 0)$

$(2a+2b, 4b, 4a) \notin W$ b/c $(4b)(4a) \neq 0$

d.)

Yes it is a subspace of $\mathbb{R}^3$

① $0 \in W$ $\quad 0\begin{pmatrix}1\\1\\0\end{pmatrix} + 0\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \in W$ ✓

② $a\begin{pmatrix}1\\1\\0\end{pmatrix} + b\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}a\\a\\0\end{pmatrix} + \begin{pmatrix}2b\\0\\b\end{pmatrix} = \begin{pmatrix}a+2b\\a\\b\end{pmatrix} \in W$

$a, b \in \mathbb{R}$, ✓ ok

e.)

Yes it is a subspace of $\mathbb{R}^3$

① $0 \in W$ $\quad b_3 - b_2 + 3b_1 = 0$ $\quad 0 - 0 + 3(0) = 0$ ✓

② $aw_1 + bw_2$

$a(b_3 - b_2 + 3b_1) + b(b_3 - b_2 + 3b_1) = 0$

$a(0) + b(0) = 0$ ✓

3.)

a.) True

① $0 \in W$ Clearly $\{0\}$ is $0 \in W$ ✓

for n amount of $\vec{0}$

② $a(0_1, \dots 0_n) + b(0_1, \dots, 0_n) \in W$

$a(\vec{0_n}) + b(\vec{0_n}) \in W$

$\vec{0} + \vec{0} \in W$

$\vec{0} \in W$ ✓

it is a subspace of $\mathbb{R}^n$

b.)

False

if $\{0\}$ is a subspace and $\mathbb{R}^2$ has to be a subspace then $0 \neq u \in \mathbb{R}^2$ so

$\text{Span}\{u\} = \{cu : c \in \mathbb{R}\}$ so

it has to cross the origin inorder to be a subspace cannot be any line

c.) True

A plane $ax + by + cz = 0$ $\quad a, b, c \in \mathbb{R}$

$A\vec{x} = \vec{0}$ $\quad A = [a, b, c]$

so true $\quad x = \begin{bmatrix}a\\b\\c\end{bmatrix}$ ND $(x \in \mathbb{R}^3 \mid A\vec{x} = \vec{0})$

4.) on the worksheet

5.) a.)

$m \times n$

3×4

$$\begin{bmatrix} 3 & 1 & 0 & -1 \\ 3 & 1 & -7 & 1 \\ 6 & 2 & 0 & -2 \end{bmatrix} \xrightarrow{R2 - R1 = R2} \begin{bmatrix} 3 & 1 & 0 & -1 \\ 0 & 0 & -7 & 2 \\ 6 & 2 & 0 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_1 = R3} \begin{bmatrix} \boxed{3} & 1 & 0 & -1 \\ 0 & 0 & \boxed{-7} & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim N(A) = n - r = 4 - 2 = 2$

$\dim Col(A) = r = 2$

b.)

$\dim N(A) = n - r =$ (# of colums) $-$ (rank)

↳ for a basis $N(A)$ each free variable corresponds to an element in the bases

$\dim Col(A) = r =$ rank

↳ A basis for $col(A)$ is given by the pivot columns

6.) postponed to next week

7.) a.)

$\{\vec{e}_1, \vec{e}_2, \dots \vec{e}_n\}$ is a basis for $\mathbb{R}^n$ so $\{\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n\}$ spans all of $\mathbb{R}^n$

so we can say

$C_1\vec{e}_1 + C_2\vec{e}_2 + \dots + C_n\vec{e}_n = \vec{x}$,

b.)

$C_1\vec{e}_1 + C_2\vec{e}_2 + \dots + C_n\vec{e}_n = \vec{x}$

$d_1\vec{e}_1 + d_2\vec{e}_2 + \dots + d_n\vec{e}_n = \vec{x}$

$(C_1 - d_1)\vec{e}_1 + (C_2 - d_2)\vec{e}_2 + \dots + (C_n - d_n)C_n = 0$

↳ Since they are L. I. this equation only has trivial solution. $C_n - d_n = 0$ so $C_n = d_n$

C.) Any vector in $\mathbb{R}^n$ can be written as a linear combination of $\vec{e}_1 \dots \vec{e}_n$ b/c $N(\vec{e}_n) = 0$ only has trivial solution and spans $\mathbb{R}^n$

3×4

$$A = \begin{array}{c} \\ m \end{array}\begin{array}{c} n \\ \begin{bmatrix} 3 & 1 & 0 & -1 \\ 3 & 1 & -7 & 1 \\ 6 & 2 & 0 & -2 \end{bmatrix} \end{array}$$ and its row echelon form is $U = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 0 & 0 & -7 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. If $N(A)$ is to be a subspace of some $\mathbb{R}^k$, what is k? Is $N(A)$ a subspace of your chosen $\mathbb{R}^k$? Why or why not?

$\vec{x}_{n1} \in N(A)$

$\vec{x}_{n2} \in N(A)$

$K = 2$

yes since $0 \in W$
AND
$a(w_1) + b(w_2) \in W$
since spans $\mathbb{R}^K$

$\vec{x}_{n1} + \vec{x}_{n2} \in N(A)$?

$A(\vec{x}_{n1} + \vec{x}_{n2}) = \vec{0}$?

$A\vec{x}_{n1} + A\vec{x}_{n2} = \vec{0}$?

$\vec{0} + \vec{0} = \vec{0}$

$c\vec{x}_{n1} \in N(A) \quad c \in \mathbb{R}$

$A(c\vec{x}_{n1}) = \vec{0}$?

$cA\vec{x}_{n1} = \vec{0} = c\vec{0} = \vec{0}$

2. True or false: $N(A) = N(U)$? Explain why.

yes, the N(u) describes the N(A). When turned to upper echelon form you don't change Null space or rank of Matrix A, inorder to find the N(A) you first need to find the upper echelon form of A

3. Find a basis for $N(U)$ and a basis for $N(A)$.

$3x + y - 1t$
$-7z + 2t$

$$x = \begin{bmatrix} \frac{1}{3}(-y + t) \\ -3x + t \\ \frac{2}{7}t \\ 7/2\, z \end{bmatrix} = x\begin{bmatrix} 0 \\ -3 \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ 0 \\ 0 \\ 7/2 \end{bmatrix} + t\begin{bmatrix} 1/3 \\ 1 \\ 2/7 \\ 0 \end{bmatrix}$$

Basis for N(u) & N(A) =

$$\begin{bmatrix} 0 \\ -3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7/2 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 1 \\ 2/7 \\ 0 \end{bmatrix}$$

Same $A = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 3 & 1 & -7 & 1 \\ 6 & 2 & 0 & -2 \end{bmatrix}$, and its row echelon form is $U = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 0 & 0 & -7 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. If $C(A)$ is to be a subspace of some $\mathbb{R}^k$, what is k? Is $C(A)$ a subspace of your chosen $\mathbb{R}^k$? Why or why not?

$K = 2$ Yes it is in the Subspace of $\mathbb{R}^2$

Since $0 \in W$ & $a(W_1) + b(W_2) \in W$

Since $C(A) = \mathbb{R}^K$

2. A revisit to Problem #5 of Homework 06.

(a) Observe that in matrix U, column-2 is a scalar multiple of column-1. Does the same relation occurs in matrix A, always, sometimes, never? Explain.

Yes it always occurs, becaus $a\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}3\\3\\6\end{bmatrix}$ $a = 3$

So $3\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}3\\3\\6\end{bmatrix}$

no matter what always will be Scalar multiple

(b) Also observe that in matrix U, column-4 is a linear combination of column-1 and column-3. Does the same relation occurs in matrix A, always, sometimes, never? Explain.

Always happens because U is a reduced form of A, So they are identical in what they will Solve or show.

(c) True or false: $C(A) = C(U)$? Explain why.

True in order to find C(A) you first need to find the upper echelon form of A which is U so $C(A) = C(U)$

3. Find a basis for $C(U)$ and a basis for $C(A)$.

Basis = pivot columns

so Basis of $C(U)$ & $C(A)$ =

$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 0 \end{bmatrix}$$