Due before lecture on Monday, September 23, 2019

1. Here are two of a list of general statements about linear dependence/independence we made in lecture

   (f) Given a set of dependent vectors, by removing vectors from this set, we can always create a smaller but independent set while keeping the span the same as before.

   (g) The vectors of the identity matrix are independent. (Strategy: To form a linearly independent set, choose columns of the identity matrix.)

   For each of the above two statements,

   (1) determine it is true or false,

   (2) include an example to illustrate your claim.

   (3) show your detailed reasoning. *Recall that showing examples is NOT sufficient to argue that a statement is true while showing one counterexample is sufficient to argue that a statement is false.*

2. Prove the statement:

   Two linearly independent vectors in $\mathbb{R}^2$ span the entire $\mathbb{R}^2$.

   following these hints. (Quoting determinant will earn you 0 credits.)

   Use notations $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} c \\ d \end{bmatrix}$ for the two arbitrary vectors. Note that we are treating a, b, c, d as fixed numbers now. We assume that $a \neq 0$. To say that these two vectors are linearly independent is equivalent to say that

   $$x \begin{bmatrix} a \\ b \end{bmatrix} + y \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

   has only $x = y = 0$ as its solution. And to show that $x = y = 0$ is the only solution we need to perform Gaussian elimination.

   (a) Carry out the details of the Gaussian elimination using the letters a, b, c, d. What special conditions does the echelon form have to satisfy in order for $x = y = 0$ to be the only solution? Explain your reasoning.

   (b) Explain why those special conditions on the echelon form lead to that

   $$x \begin{bmatrix} a \\ b \end{bmatrix} + y \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

   always have a solution no matter what the right hand side u and v are.

3. In the exercise titled "Intersection Of Three Planes" done in lecture, we have determined that the intersection of the three planes in the following system

   $$\begin{aligned} x+y-2z&=3 \\ x-y+\ z&=2 \\ 2x\quad\ -\ z&=5 \end{aligned}$$

   cannot look like pictures (c), (e), or (f).

   For each of these three pictures , write down a 3×3 system (that is, a system of 3 equations with 3 unknowns) that corresponds to that picture. Explain why you think your system is valid. In other words, use sentences, math notations and calculations, etc. (whatever is necessary), to show that the picture representing the system you wrote down indeed looks like the given picture.