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Math 141 Linear Analysis Homework #1

1.)
$$A=\begin{bmatrix}1&1&1&-2\\3&3&-1&6\\1&-1&2&-1\\0&0&1&0\end{bmatrix}\xrightarrow{R_2-3R1}\begin{bmatrix}1&1&1&-2\\0&0&-4&12\\1&-1&2&-1\\0&0&1&0\end{bmatrix}$$

$$\xrightarrow{R3-R1}\begin{bmatrix}1&1&1&2\\0&0&-4&12\\0&-2&1&8\\0&0&1&0\end{bmatrix}\quad R3\leftrightarrow R2\begin{bmatrix}1&1&1&2\\0&-2&1&8\\0&0&-4&12\\0&0&1&0\end{bmatrix}$$

$$\xrightarrow{R4+R3\div 4}\begin{bmatrix}\boxed{1}&1&1&2\\0&\boxed{-2}&1&8\\0&0&\boxed{-4}&12\\0&0&0&\boxed{3}\end{bmatrix}$$ UPPER Echelon Form

4 pivot points

2.) A.) When calculating R3 − 5R2 the resulting should be put into R3, because we are reducing R3 by 5R2.

B.) $\begin{bmatrix}0&1&0&0&0\\0&0&1&7&8\\0&0&2&0&0\end{bmatrix}$ is not in upper echelon because the sub matrix of $\begin{bmatrix}1&7&8\\2&0&0\end{bmatrix}$ can still be further reduce. This is due to the fact that there is still a non zero number under the pivot point 1.

3.) A.) A possible system would be:

$$\left[\begin{array}{ccc|c}-1&0&2&1\\-1&0&4&1\end{array}\right]\quad R2-R1$$

B.) My solution is solvable, using the system

① $-x+0y+2z=1$

② $-x+0y+4z=1$

$2z-1=x$ substitude x in ②

$-(2z-1)+4z=1$

$1+2z=1$ $[z=0,\ x=-1]$

C.) There is no mistake, there are multiple solutions to solve this problem.

4.) A.) Rank 1

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}$$

• No matter what q is, there cannot be a value of q in which the rank of the matrix will be 1.

$$A = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$$

• When q is equal to 6, the rank of the matrix becomes 1 this is due to the fact that row 2 will all be zeroes after Gaussian Elemination.

B.) Rank 2

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}$$

• When q is equal to zero the rank of the matrix will be 2. This is due to the fact that when Gaussian elemination is performed the last number will be last pivot point but since we let q be zero, zero cannot be a pivot point.

$$A = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$$

• When q is equal to any real number except 6, the rank of the matrix is 2. This is due to the fact that if q is 6, it will make the two pivot points zero making the rank equal to 1.

C.) Rank 3

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}$$

• When q is equal to any real number except 0, the rank will be 3. This is due to the fact that if q is zero the last available pivot point is not existent because pivot points cannot be zero

$$A = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$$

• There are no possible values of q in which the rank of the matrix is 3 There has to be at least threw rows to have a max of 3 ranks.

5.)

$$x \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} - y \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ h \end{bmatrix}$$

A.)

$$3x - 2y = 5$$
$$3x - y = 7$$
$$6x - 4y = h$$

B.)

$$\left[\begin{array}{cc|c} 3 & 2 & 5 \\ 3 & 1 & 7 \\ 6 & 4 & h \end{array}\right] \xrightarrow{R2 - R1 = R2} \left[\begin{array}{cc|c} 3 & 2 & 5 \\ 0 & -1 & 2 \\ 6 & 4 & h \end{array}\right]$$

$$\xrightarrow{R3 - 2R1 = R3} \left[\begin{array}{cc|c} 3 & 2 & 5 \\ 0 & -1 & 2 \\ 0 & 0 & h-10 \end{array}\right] \qquad \begin{array}{l} 3x - 2y = 5 \\ -y = 2 \\ 0 = h - 10 \end{array}$$

Upper echelon form

C.) There is no solution for $h = 4$. This is because $0 = h - 10$ when $h = 4$, $0 \neq -6$. This will make -6 equal to zero which is not true.

D.) When h is equal to 10, the solution exist. The solution is unique.

E.) There only exist one value of h, if the value of h is anything but 10, there will be no solution.