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Math 141 Linear Analysis Homework #7

1.) Verify $L_A : \mathbb{R}^2 \to \mathbb{R}^2$. $L_A(\vec{x}) = A\vec{x}$, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is linear

① $L(\vec{x}+\vec{y}) = L(\vec{x}) + L(\vec{y})$

$$L\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix}\right) = L\left(\begin{bmatrix}1\\1\end{bmatrix}\right) + L\left(\begin{bmatrix}2\\2\end{bmatrix}\right)$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}$$

$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \checkmark \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$$

② $L(c\vec{x}) = cL(\vec{x})$

$$L\left(3\begin{bmatrix}1\\1\end{bmatrix}\right) = 3L\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$$

$$L\left(\begin{bmatrix}3\\3\end{bmatrix}\right) = 3\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\right)$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix} = \checkmark\ 3\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$$

2.) a.) $T: R \to R$. $T(x) = x^2$

① $L(x+y) = L(x) + L(y)$

$$L(1+2) = L(1) + L(2)$$

$$(3)^2 = 1 + 4$$

$$9 \neq 5 \quad \text{Non linear}$$

b.) $T: R \to R \quad T(x) = x + 3$

① $L(x+y) = L(x) + L(y)$

$$L(1+2) = L(1) + L(2)$$

$$3+3 = 4 + 5$$

$$6 \neq 9 \quad \text{Non linear}$$

c.) $T: R^2 \to R^2$, $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}3x_1\\x_1+2x_2\end{bmatrix}$

① $L(\vec{x}+\vec{y}) = L(\vec{x}) + L(\vec{y})$

$$L\left[\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix}\right] = L\left(\begin{bmatrix}1\\1\end{bmatrix}\right) + L\left(\begin{bmatrix}2\\2\end{bmatrix}\right)$$

$$\begin{bmatrix}3x_1\\x_1+2x_2\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}3x_1\\x_1+2x_2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}3x_1\\x_1+2x_2\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}$$

$$\begin{bmatrix}9x_1\\3x_1+6x_2\end{bmatrix} = \checkmark \begin{bmatrix}3x\\x_1+2x_2\end{bmatrix} + \begin{bmatrix}6x_1\\2x_1+4x_2\end{bmatrix}$$

$$\begin{bmatrix}9x_1\\3x_1+6x_2\end{bmatrix} = \begin{bmatrix}9x_1\\3x_1+6x_2\end{bmatrix}$$

② $L(c\vec{x}) = cL(\vec{x})$

$$L\left(2\begin{bmatrix}2\\2\end{bmatrix}\right) = 2L\left(\begin{bmatrix}2\\2\end{bmatrix}\right)$$

C cont.)

$$2\begin{bmatrix}3x_1\\x_1+2x_1\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}3x_1\\x_1+2x_1\end{bmatrix}\begin{bmatrix}4\\4\end{bmatrix}$$

$$2\begin{bmatrix}6x_1\\2x_1+4x_1\end{bmatrix} = \begin{bmatrix}12x_1\\4x_1+8x_2\end{bmatrix}$$

$$\begin{bmatrix}12x_1\\4x_1+8x_2\end{bmatrix} = \begin{bmatrix}12x_1\\4x_1+8x_2\end{bmatrix}$$ linear

d.)

$$T: R^2 \to R^2,\ T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1\\2x_2-1\end{bmatrix}$$

① $L(\vec{x}+\vec{y}) = L(\vec{x}) + L(\vec{y})$

$$L(2+3) = L(\vec{2}) + L(\vec{3})$$

$$\begin{bmatrix}x_1\\2x_2-1\end{bmatrix}\begin{bmatrix}5\\5\end{bmatrix} = \begin{bmatrix}x_1\\2x_2-1\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix} + \begin{bmatrix}x_1\\2x_2-1\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix}$$

$$\begin{bmatrix}5x_1\\10x_2-5\end{bmatrix} \overset{?}{=} \begin{bmatrix}2x_1\\4x_2-2\end{bmatrix} + \begin{bmatrix}3x_1\\6x_2-3\end{bmatrix}$$

$$\begin{bmatrix}5x_1\\10x_2-5\end{bmatrix} = \begin{bmatrix}5x_1\\10x_2-5\end{bmatrix}$$

② $L(c\vec{x}) = cL(\vec{x})$

$$L\left(2\begin{bmatrix}2\\2\end{bmatrix}\right) = 2L\left(\begin{bmatrix}2\\2\end{bmatrix}\right)$$

$$L\begin{bmatrix}4\\4\end{bmatrix} = 2L\left(\begin{bmatrix}2\\2\end{bmatrix}\right)$$

$$\begin{bmatrix}x_1\\2x_2-1\end{bmatrix}\begin{bmatrix}4\\4\end{bmatrix} = 2\begin{bmatrix}x_1\\2x_2-1\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}$$

$$\begin{bmatrix}4x_1\\8x_2-4\end{bmatrix} = 2\begin{bmatrix}2x_1\\4x_2-2\end{bmatrix}$$

$$\begin{bmatrix}4x_1\\8x_2-4\end{bmatrix} = \begin{bmatrix}4x_1\\8x_2-4\end{bmatrix}$$ Linear

3.) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad \vec{e}_1 = \begin{bmatrix}1\\0\end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix}0\\1\end{bmatrix} \quad T = L_A$

a.)

$$2\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} \ ; \ 3\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

$$A = \begin{bmatrix}2&0\\0&3\end{bmatrix}$$

b.)

$\vec{e}_1 = \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} \qquad A = \begin{bmatrix}0&1\\1&0\end{bmatrix}$

Reflection of $y=x$, x & y changes places

c.)

$$\vec{e}_1 = \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\\cos\theta\end{bmatrix}$$

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} = \begin{bmatrix}\cos(-\pi/4) & -\sin(-\pi/4)\\ \sin(-\pi/4) & \cos(-\pi/4)\end{bmatrix}$$

$$A = \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}$$

3.) d.)

$$\vec{e}_1 = \vec{e}_1 - 2\vec{e}_2 \qquad \vec{e}_2 = \vec{e}_2$$

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

4.) a.)

$L_D : R^3 \rightarrow R^3$; $L_D(\vec{x}) = D\vec{x}$

$L_E : R^2 \rightarrow R^3$; $L_E(\vec{x}) = E\vec{x}$

$L_F : R^3 \rightarrow R^2$; $L_F(\vec{x}) = F\vec{x}$

b.)

A transformation maps $\mathbb{R}^n$ to a $\mathbb{R}^n$ subspace in $\mathbb{R}^m$

c.)

- D is one-to-one since it is full rank and $L_D : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ it is one to one
- E is one-to-one because E is full rank
- F is not one to one

$$\vec{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{x}_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

d.)

- D is onto because it is full rank and Ker(D) only has the trivial Solution
- E is not onto because no Solution exist to find $\begin{bmatrix} 0 \\ 0 \end{bmatrix} n \in \mathbb{R}$
- F is onto because it is full rank and ker(F) only has trivial Solution.