11/17/19

MATH 181 Linear Analysis Homework #17

1.) a.) i.) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$ = eigen vectors

find $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \longrightarrow \left[\begin{array}{ccc|c} -1 & 0 & 1 & a \\ 0 & -1 & -1 & b \\ 2 & 5 & 9 & c \end{array}\right] \xrightarrow{R_3 + 2R_1 = R_3}$

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & a \\ 0 & -1 & -1 & b \\ 0 & 5 & 11 & C+2a \end{array}\right] \xrightarrow{R_3 + 5R_2 = R_3} \left[\begin{array}{ccc|c} -1 & 0 & 1 & a \\ 0 & -1 & 1 & b \\ 0 & 0 & 6 & C+2a+5b \end{array}\right]$$

We want $a\vec{x} + b\vec{y} + c\vec{z} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$

$$\begin{cases} -1x + z = a \\ -y + z = b \\ 6z = C + 2a + 5b \end{cases} \qquad \begin{aligned} x &= -1 \\ y &= 2 \\ z &= 2 \end{aligned}$$

Then

$$-1\, T\left(\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\right) + 2\, T\left(\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}\right) + 2\, T\left(\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}\right)$$

$$\longrightarrow (-1)(1/4)\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + \overset{-6}{(2)(-3)}\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix} + \overset{-6}{(2)(-3)}\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1/4 \\ 0 \\ -1/2 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ -30 \end{bmatrix} + \begin{bmatrix} -6 \\ 6 \\ -54 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -23/4 \\ 12 \\ -169/2 \end{bmatrix}}$$

ii) The Same process as the last part

$$a\vec{x} + b\vec{y} + c\vec{z} = \begin{bmatrix} 1/2 \\ -1/2 \\ 9/2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$-x + z = a$ $\quad$ $x = 0$

$-y + z = b$ $\quad$ $y = 1$

$6z = c + 2a + 5b$ $\quad$ $6z = \frac{6}{2}$ $\quad$ $z = 1/2$

Then

$$\cancel{0\,T\left(\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\right)} + 1\,T\left(\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}\right) + 1/2\,T\left(\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}\right)$$

$$-3T\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix} + -3/2\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 3 \\ -15 \end{bmatrix} + \begin{bmatrix} -3/2 \\ 3/2 \\ -3/2 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -3/2 \\ 9/2 \\ -57/2 \end{bmatrix}}$$

b.) Since $\begin{bmatrix} 3 \\ -5 \\ 37 \end{bmatrix}$ is in the span of the eingen vectors $\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$

it undergoes the eingenvalue of -3 so, $T(\vec{u}) \in \text{span}\left\{\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix} \& \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}\right\}$

$= -3\vec{u}$

1c.)

As solved in part A:

$$\begin{bmatrix} -1 \\ -2 \\ 12 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$-x + z = a$$
$$-y + z = b$$
$$6z = c + 2a + 5b$$

$$x = 1, \quad y = 2, \quad z = 0$$

$$1T\left(\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\right) + 2T\left(\begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}\right) + \cancel{0T\left(\begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}\right)} \rightarrow 0$$

$$1/4 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + -6 \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 6 \\ -59/2 \end{bmatrix}$$

2.) a.)

No, it does not, it only reflects the the line across the x-axis. This will be given by $T(x, y) = (x, -y)$ so the eigenvectors would be

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

b)

The Matrix would be $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Then you solve for $(A - \lambda I) = 0$

then $\begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix}$ $\quad (1-\lambda)(-1-\lambda) = 0$

$$\lambda = 1, -1$$

$\lambda = 1$

$$\rightarrow \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ -2v \end{bmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$\lambda = -1$ $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

3.) a.)

We Can See that Rank is 1 infinetly many Solutions to null space

We get $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = 0$ $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = 0$

Since the other Vectors get reduced to Zero when performing Gaussian elimination then the vectors ~~still~~ be Zero

b.) To Compute A^{100} we need to solve

$A = PDP^{-1}$

$P = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{\text{invert}} P^{-1} \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & -1/3 \\ -1/3 & 1/3 & 0 \end{bmatrix}$

In Class we proved for A^{100} we Can just do

$A^{100} = PD^{100}P^{-1}$ so

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 3^{100} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^{100} = \begin{bmatrix} 3^{99} & 3^{99} & 3^{99} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

i)

4.) a.) Solve for $A\vec{x}=0$ the Compare

$A\vec{x} = \lambda\vec{x}$

ii)

You need to solve the equation

$A - \lambda I = 0$ to find the associated eigen vetor

B.)

i.) $Ax = \lambda x$

$A(Ax) = A(\lambda x) = \lambda(Ax) = \lambda(\lambda x)$

$= \lambda^2(x) = A(Ax) = A^2x = \lambda^2 x$

(ii.)

$\cancel{(\lambda x^{-1}(A^{-1}A)Ax} = \cancel{\lambda x}\,(\cancel{\lambda^{-1}}x)(A^{-1}x)$

$\lambda^{-1}\vec{x} = (A^{-1}x)$

iii)

We have $Ax = \lambda x$ so

$(A+I)x = Ax + x = \lambda x + x$