Math 141 Linear Analysis Homework #6

1.) a.)
This is not true because the complete solution is $\vec{x}$ particular + $\vec{x}$ null space, not the linear combination of the two of them.

b.)
There can be multiple $\vec{x}$ particular for a given A matrix. Since $\vec{x}$ particular are just arbitrary constant

c.)
$$\left[\begin{array}{cc|c} 1 & 1 & a \\ 0 & 0 & b \end{array}\right] \quad \begin{array}{c} x+y=a \\ 0=b \end{array} \quad \begin{array}{c} y=0 \\ x=a \end{array} \quad \vec{x}_p = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

d.)
There will always exist a solution to the null space no matter what matrix A is. There is no possible way, no solution could be true.

2.) a.) i.)
This does not apply to C(A).
$N(A) = \{\vec{0}\}$ if null space is $\vec{0}$ then no the solution exist
ii.) This makes the matrix linearly independent if all possible pivot points exist for matrix A then $A\vec{x}=\vec{0}$ only has solution of $\vec{x}=\vec{0}$

iii.)
if the column vectors of A are all linearly independent then $A\vec{x}=\vec{0}$ only has solution of $\vec{x}=\vec{0}$, means only trivial soln exist for null space

iv.)
if uniqueness of solution exist for $A\vec{x}=\vec{b}$ for other $\vec{b}$. then $A\vec{x}=\vec{0}$ only has solution of $\vec{x}=\vec{0}$
uniqueness means linearly independent matrix which makes this statement true.

b.) i.)
Doesnot apply to N(A). if $C(A)=\mathbb{R}^2$ then $A\vec{x}=\vec{b}$ is soluable for any $\vec{b}$, because matrix A spans all of $\mathbb{R}^2$

ii.)
If all possible pivot points exist then $A\vec{x} = \vec{b}$ is solvable for all $\vec{b}$. This is true because if all possible pivot points means matrix A spans all of $\mathbb{R}^2$

iii.)
if column vectors of A are all linearly independent then $A\vec{x} = \vec{b}$ is solvable for all possible $\vec{b}$ because this means matrix A spans all of $\mathbb{R}^2$

3.) a.) #1 on worksheet.)
if all possible pivot points exist then this will make the two conditions true, because it will span all the matrixs dimension.

#2 on worksheet.)
if at least one pivot point exists then the conditions are true, since you can scalarly multiply $\vec{x}$ to find $\vec{b}$ many solutions.

#3 on worksheet.)
This is not possible b/c if it has a unique solution Then then it is full rank which contradicts the first condition

#4 on worksheet.)
All possible pivot points must exist to make the statements true, same as the first question of the worksheet.

b.) #1 on worksheet.)
for $A_{n \times m}$ all conditions are true if $r = n$

#2 on worksheet.)
for $A_{n \times m}$ all conditions are true if $r < n$

#3 on worksheet.)
for $A_{n \times m}$ all conditions cannot exist, Can't make true

#4 on worksheet.)
for $A_{m \times m}$ all conditions are true if $r = n$

4.) I think they are all linearly dependent because if one solution exist you can find a scalar multiple of that solution still making $A\vec{x}=\vec{0}$ true, which will make the set dependent.

5.) a.)

This is true, there are no scalar value that can make the equation: $a\begin{bmatrix}3\\0\\0\end{bmatrix}=\begin{bmatrix}0\\-7\\0\end{bmatrix}$ or $b\begin{bmatrix}6\\-7\\0\end{bmatrix}=\begin{bmatrix}3\\0\\0\end{bmatrix}$ true.

b.)

This is false, only the first column is a linear combination of the second, but 3^{rd} & 4^{th} are not linear combination of the 2^{nd} Column, b/c;

$a\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\-7\\0\end{bmatrix}$ or $a\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\0\end{bmatrix}$ no a exist to make this true.

c.)

This is true. Changing matrix A to upper echelon form doesn't change the linearly depedency on the Columns, so since they are independent in echelon form they are still independent in regular matrix form.

d.)

This is false, it is the same reason as part b. Changing a matrix A to upper echelon form doesn't change the matrix, only reduces the values

e.)

This is true, there is no difference in Column space when changing from matrix A to upper echelon form.

6.) a.)

$m = 3$ $n = 2$.

b.)

null space of a matrix; Solve Ker(LA)

$\rightsquigarrow$

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b cont.)

$$A = \left[\begin{array}{ccc|c} 1 & -5 & -7 & 0 \\ -3 & 7 & 5 & 0 \end{array}\right] \xrightarrow{R2+3R1=R2} \left[\begin{array}{ccc|c} 1 & -5 & -7 & 0 \\ 0 & -8 & -16 & 0 \end{array}\right]$$

Back substitution
$y=0 \quad z=1$
$y=1 \quad z=0$

$x - 5y - 7z = 0 \qquad x = 5 \,;\, x = 7$

$-8y - 16z = 0$

$$\ker(L_A) = \left\{ \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 1 \end{bmatrix} \right\}$$

c.)

span / column space of matrix A describe range (L_A)

range (L_A) is all possible solutions for:

$$x \begin{bmatrix} 1 \\ -3 \end{bmatrix} + y \begin{bmatrix} -5 \\ 7 \end{bmatrix} + z \begin{bmatrix} -7 \\ 5 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

for $x, y, z, a, b \in \mathbb{R}$

d.)

$$\text{image} = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ -5 & 7 \\ 7 & -5 \end{bmatrix}$$

all $\vec{x}$:

$$\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ -5 & 7 \\ 7 & -5 \end{bmatrix}$$

$$\begin{bmatrix} a & -3a \\ -5b & 7b \\ -7c & 5c \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ -5 & 7 \\ 7 & -5 \end{bmatrix}$$

$\hookrightarrow$ all possible $\vec{x}$

7.) a.)

covers R^m

b.)

only has the trivial solution

c.)

it is onto because there exist a solution to $\ker(L_A)$ that is not the trivial one, 8.) a.) i.)

For $A\vec{x} = \vec{b}$ has a unique solution for every $\vec{b}$

ii.)

null space only has the trival solution and Columns space is $\mathbb{R}^n$.

iii.)

Columns of A are lonearly independent

iv.)

There exist a pivet point in each column.

b.) i.)

No unique Solution exist for $\vec{b}$ multiple solutions exist

ii.)

There exist a solution to the nullspace that is not the trivial solution. Column space Span $\mathbb{R}^m$

iii.)

Column vectors are linearly dependent

(V.)

A has pivets in all rows