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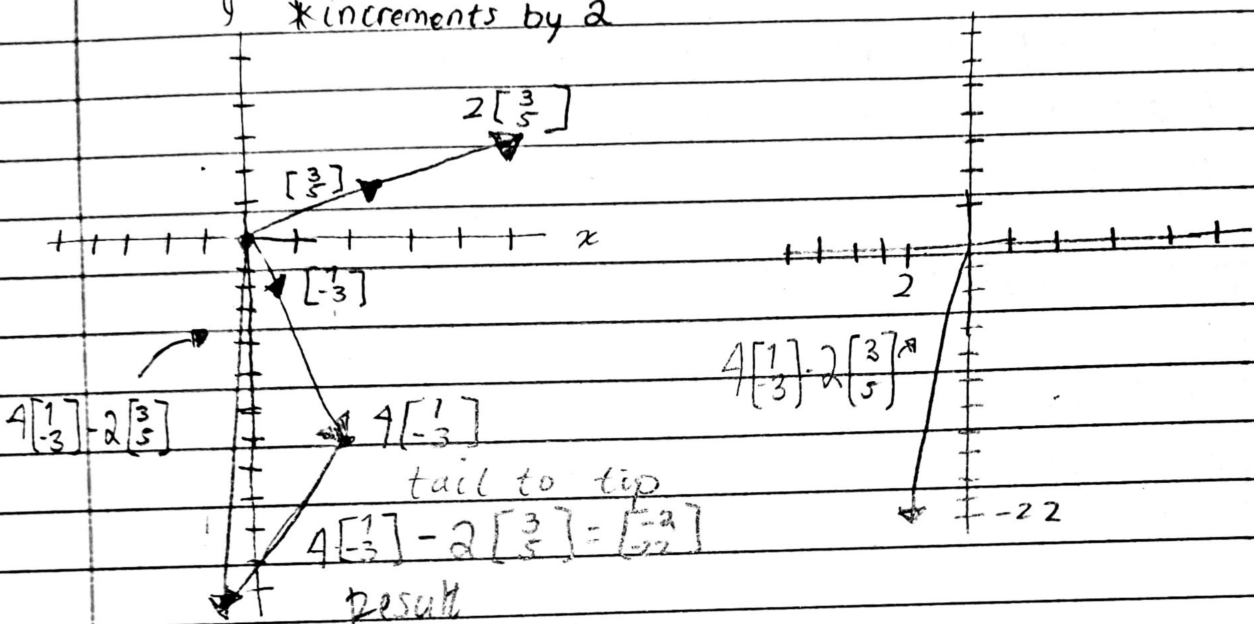
9/5/19
MATH 141

MATH 141 Linear Analysis Homework #2

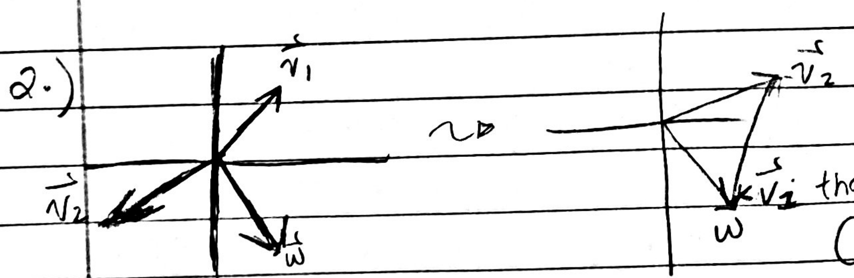
1.) Consider the Expression $4\begin{bmatrix}1\\-3\end{bmatrix} - 2\begin{bmatrix}3\\5\end{bmatrix}$

A.) Graph Representation Using Vectors $\begin{bmatrix}-2\\-22\end{bmatrix}$



B.) The Scalar's 4 & -2 Represents how many times the vectors will travel in their given distance

2.)



• Assuming $\vec{v}_2$ is two times greater in magnitude than $\vec{v}_1$, $\vec{w}$ is a linear Combination of $\vec{v}_1$ & $\vec{v}_2$.

• The estimated scalar values of $\vec{v}_1$ & $\vec{v}_2$ are -2 & -1 respectively.

3.) $x\begin{bmatrix}30\\1\end{bmatrix} + y\begin{bmatrix}21\\1\end{bmatrix} = \begin{bmatrix}16{,}200\\600\end{bmatrix}$ $\quad \left[\begin{array}{cc|c}30 & 21 & 16{,}200\\1 & 1 & 600\end{array}\right]$

$R_2 - \frac{1}{30}R1 = R2 \longrightarrow \left[\begin{array}{cc|c}30 & 21 & 16{,}200\\0 & \frac{3}{10} & 60\end{array}\right]$ $\quad 30x + 21y = 16200$, $\quad 3/10\,y = 60$

$y = 200$
$x = 400$

Car one used 400, Car two used 200.

$400\begin{bmatrix}30\\1\end{bmatrix} + 200\begin{bmatrix}21\\1\end{bmatrix} = \begin{bmatrix}16200\\600\end{bmatrix}$

$\nexists$ $\exists$ a soln x,

1) $3x = 107$ & 2) $x = 64$ /

$\Rightarrow$ 1) $\Rightarrow$ $x = \frac{107}{3}$ & $x = 64$ but $64 \neq \frac{107}{3}$

4.) a.) Suppose $x \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$ exist. $\begin{cases} 3x = 107 \\ x = 64 \end{cases}$ $x = 107/3$

Changing the vectors to a system of equations we see by contradiction that no solution exist because $x = 107/3$ and $x = 64$ but $107/3 \neq 64$.

b.) Suppose $x \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$ Exists $\begin{cases} x = 107 \\ 2x = 64 \end{cases} \rightarrow x = 32$

Changing the vectors into a systems of equation we see by contradiction that no solution exist becouse $x = 107$ and $x = 32$ but $107 \neq 32$

C.) $x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 3 & 6 & 107 \\ 1 & 2 & 64 \end{array}\right] \xrightarrow{R2 - \frac{1}{3}R1 = R2}$

$\left[\begin{array}{cc|c} 3 & 6 & 107 \\ 0 & 0 & 28.33 \end{array}\right]$ $3x + 6y = 107$ $0 \neq 28.33$ using Foward Gaussian elimination we see that there can be no solution for our equation because $0 \neq 28.33$.

D.) $x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 2 \end{bmatrix} + z \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 3 & 1 & 2 & 107 \\ 1 & 2 & 7 & 64 \end{array}\right] \xrightarrow{R2 - \frac{1}{3}R1 = R2}$

$A = \left[\begin{array}{ccc|c} 3 & 1 & 2 & 107 \\ 0 & 5/3 & 19/3 & 28.3\overline{3} \end{array}\right]$ $3x + y + 2z = 107$, $5/3 y + 19/3 z = 28.33$

Rank (A) = 2, pick $x \in \mathbb{R}^2$, $x \in$ Span (Col(A))

• When performing Foward Gaussian elemination and getting the Upper Echelon Form, we get the span of the matrix 2. Which means the set of vectors span all of $\mathbb{R}^2$, so when you pick an 'x' in $\mathbb{R}^2$, 'x' will be in the span-column of A. So there exists a solution to get to old man gauss's house.

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5.) ①

$$x\begin{bmatrix}1\\0\\0\end{bmatrix}+y\begin{bmatrix}-1\\1\\0\end{bmatrix}+z\begin{bmatrix}-1\\1\\1\end{bmatrix}=\begin{bmatrix}2\\-1\\3\end{bmatrix}$$

② $\left[\begin{array}{ccc|c}1&-1&-1&2\\0&1&1&-1\\0&0&1&3\end{array}\right]$ * Already in upper Echelon Form

$$x-y-z=2$$
$$y+z=-1$$
$$z=3$$

$$\begin{cases}z=3\\y=-4\\x=1\end{cases}$$

$$1\begin{bmatrix}1\\0\\0\end{bmatrix}-4\begin{bmatrix}-1\\1\\0\end{bmatrix}+3\begin{bmatrix}-1\\1\\1\end{bmatrix}=\begin{bmatrix}2\\-1\\3\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}-\begin{bmatrix}-4\\4\\0\end{bmatrix}+\begin{bmatrix}-3\\3\\3\end{bmatrix}\checkmark=\begin{bmatrix}2\\-1\\3\end{bmatrix}$$

- The vector equation above in matrix form is alredy given to us in upper Echelon form. Thus giving us the matrix rank of 3. This means the span of the set of vectors given is all of $\mathbb{R}^3$. Since $\begin{bmatrix}2\\-1\\3\end{bmatrix}$ is in $\mathbb{R}^3$, $\begin{bmatrix}2\\-1\\3\end{bmatrix}$ is a linear Combination of $\begin{bmatrix}1\\0\\0\end{bmatrix}$, $\begin{bmatrix}-1\\1\\0\end{bmatrix}$ & $\begin{bmatrix}-1\\1\\1\end{bmatrix}$.