MATH141 Linear Analysis Homework # 11

1) Did it in the last homework

2) a.)

$F_1 = \det([1]) = 1$

$F_2 = \det\left(\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\right) = 2$

b.)

$F_n = 1 \cdot F_{n-1} + 1 \cdot \det(\text{altered matrix})$

$F_n = 1 \cdot F_{n-1} + 1 (\cdot 1 F_{n-2} \cancel{+1 \cdot 0}$

$F_n = F_{n-1} + F_{n-2}$

3.) Same as part 'b' but a sign change since the inside sign is different.

a.)

$D_n = D_{n-1} - D_{n-2}$

$D_n = 1 \cdot D_{n-1} - 1 \cdot \det(\text{altered matrix})$

$D_n = 1 \cdot D_{n-1} - 1 \cdot 1 \cdot D_{n-2} - 1 \cdot 0$

$D_n = D_{n-1} - D_{n-2}$

b.)

We have $D_n = D_{n-1} - D_{n-2}$

$D_1 = 1$

$D_2 = 0$

$D_3 = D_{3-1} - D_{3-2} = 0 - 1 = -1$

$D_4 = D_{4-1} - D_{4-2} = -1 - 0 = -1$

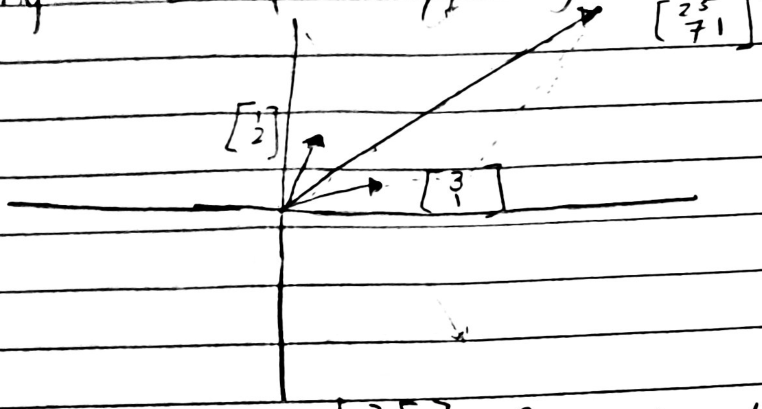
$D_5 = D_{5-1} - D_{5-2} = -1 - (-1) = 0$

$D_6 = 1$

$D_7 = 1$

$D_8 = 0$ => We see an alternation of signs. Since it wont land on 0, we can predict D_{1000} to be -1 due to this alternation.

4.) Drawing the following graph for β



a.) The vector $\begin{bmatrix}25\\71\end{bmatrix}$ Can be located by finding scalars a & b for the equation

$$a\begin{bmatrix}3\\1\end{bmatrix} + b\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}25\\71\end{bmatrix}$$

This will allow us to find a Coordinate pair (a,b) that will take us to $\begin{bmatrix}25\\71\end{bmatrix}$ in the coordinate system β

b.) To find the vector you need to find the Coordinate pair to go to $\begin{bmatrix}8\\3\end{bmatrix}$ then you plug them into the above equation and youll find them in α coordinate system

c.) $$a\begin{bmatrix}3\\1\end{bmatrix} + b\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}7\\-1\end{bmatrix} \,\&\, \begin{bmatrix}6\\3\end{bmatrix} \quad (3, -2)$$

$$\beta : \begin{bmatrix}3\\-2\end{bmatrix} \,\&\, \begin{bmatrix}-16/5\\18/5\end{bmatrix}$$

d.) $$7\begin{bmatrix}3\\1\end{bmatrix} + -1\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}20\\5\end{bmatrix}$$

$$-6\begin{bmatrix}3\\1\end{bmatrix} + 3\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}15\\-4\end{bmatrix}$$

$$\alpha : \begin{bmatrix}20\\5\end{bmatrix} \,\&\, \begin{bmatrix}15\\-4\end{bmatrix}$$

5.) a.)

Red: $A=(0,1)$ $B=(1,2)$ $C=(-3,-2)$

$d=(1,0)$ $e=(3,-1/2)$ $f=(-2\,1/2,-1)$

Black: $A=(1,4)$ $B=(4,7)$ $C=(-8,-5)$

$d=(2,-1)$ $e: (5\tfrac{1}{2},-5)$ $f=(6,-1\tfrac{1}{2})$

b.) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} a+2b \\ c+2d \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a+4b \\ c+4d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2a-b \\ 2c-d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$Q = \begin{bmatrix} 4/9 & -1/9 \\ 1/9 & 2/9 \end{bmatrix}$

Relatinship: $\dfrac{1}{\det(P)} \cdot P = Q$

$P^{-1} = Q$

c.)