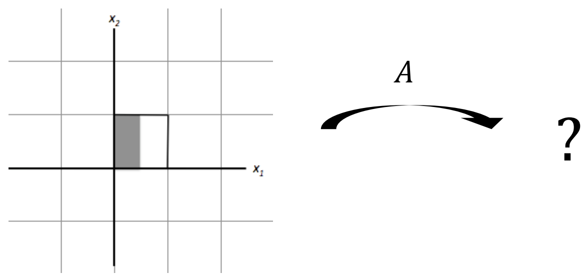
Due before lecture on Monday, October 14, 2019

1. Verify that functions defined by a matrix is always linear. More precisely, verify that $L_A : \mathbb{R}^2 \to \mathbb{R}^2$, $L_A(\vec{x}) = A\vec{x}$, with $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, is linear.

2. Determine whether each of the following functions is linear or not. Explain your reasoning.
   (a) $T : \mathbb{R} \to \mathbb{R}$, $T(x) = x^2$.
   (b) $T : \mathbb{R} \to \mathbb{R}$, $T(x) = x + 3$.
   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 \\ x_1 + 2x_2 \end{bmatrix}$
   (d) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ 2x_2 - 1 \end{bmatrix}$

3. Assume that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. Let $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Draw the image of the "half-shaded unit square" (shown below) under the given transformation T , and find the matrix A such that $T = L_A$.



   (a) T stretches by a factor of 2 in the x-direction and by a factor of 3 in the y-direction.
   (b) T is a reflection across the line $y = x$.
   (c) T is a rotation (about the origin) through $-\pi/4$ radians.
   (d) T is a vertical shear that maps $\vec{e}_1$ into $\vec{e}_1 - \vec{e}_2$ but leaves the vector $\vec{e}_2$ unchanged.

4. For any given m×n matrix A, we are going to use the notation L_A to denote the linear transformation that A defines, i.e., $L_A : \mathbb{R}^n \to \mathbb{R}^m : L_A(\vec{x}) = A\vec{x}$. For each given matrix, answer the following questions.

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \qquad E = \begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 0 & 2 \end{bmatrix} \qquad F = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

   (a) Rewrite $L_D : \mathbb{R}^n \to \mathbb{R}^m$ with correct numbers for m and n filled in for each matrix. Repeat for L_E and L_F.
   (b) Find some way to explain in words and/or graphically what this transformation does in taking vectors from $\mathbb{R}^n$ to $\mathbb{R}^m$. You might find it helpful to try out a few input vectors and see what their image is under the transformation.
   (c) Is this transformation one-to-one? (Hint: Review problem#6 of Homework 06.)
      i. If so, explain which properties of the matrix make the transformation one-to-one.

ii. If not, given an example of two different input vectors having the same image.

(d) Is this transformation onto? (Hint: Review problem#6 of Homework 06.)

i. If so, explain which properties of the matrix make the transformation onto.

ii. If not, given an example of a vector in $\mathbb{R}^m$ that is not the image of any vector in $\mathbb{R}^n$.