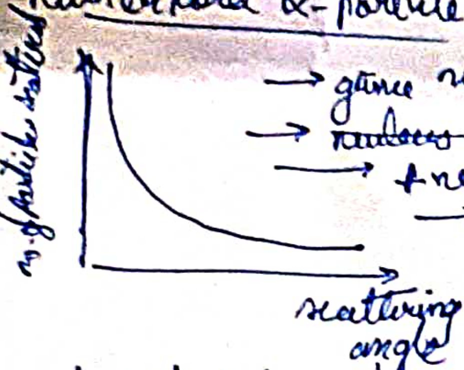
Rutherford α-particle exp.



→ gives nucleus exist
→ ~~nucleus spin~~ electron spinning
→ +ve charge nucleus
→ electron spirals inside (problem)

closest distance of approach (of α)

$K.E_\alpha = P.E_\alpha$

$E_i = E_f$

$\Rightarrow \frac{1}{2} m v_0^2 = \frac{K(2e)(Ze)}{r_0}$

impact parameter

$$b = \frac{KZe^2 \cot(\theta/2)}{\frac{1}{2} m v_0^2 \text{ (or K.E)}}$$

$b \propto \frac{1}{\theta}$

Conductor electric field

High probability in numerical questions

Bohr's model

Postulate

$F_c = \frac{dU}{dr}$

↪ 1. $\frac{mv^2}{r} = F_c$ — (I)

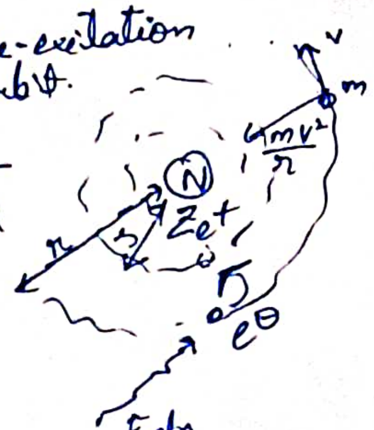
$\frac{KZe^2}{r^2} = \frac{mv^2}{r}$

$L = mvR = nh/2\pi$ — (II)

L = Angular momentum

→ Excitation or de-excitation is fixed for orbit. — (III)

$E_{\lambda} = \frac{hc}{\lambda}$



radius of nth orbit

$r_n = \frac{a_0 n^2}{Z}$

$a_0 = 0.529 \mathring{A}$

velocity of nth orbit

$v_n = v_0 \frac{Z}{n}$

$v_0 = 2.2 \times 10^6 \text{ m/s}$

Time period of nth orbit

$T \propto \frac{n^3}{Z^2}$

$B = \frac{\mu_0 i}{2r_n}$

Energy in nth orbit

$E_n = \frac{-2\pi^2 m e^4 K^2 Z^2}{n^2 h^2}$

$= \frac{-13.6 Z^2}{n^2} (eV)$

Current in nth orbit

$I = q/t$

$= qf$

$\Rightarrow I = \frac{qZ^2}{n^3}$

Magnetic moment

$(M) = I.A$ A = Area (model is 2D)

$= \frac{Z^2}{n^3} \cdot \frac{n^4}{Z^2} = n$

Induction of Z = n

Spectral series

(cold gas) Absorption :

(hot gas) Emission :

continuous (hot gas) :

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

n_1 ⇝ deexcitation shell

$R = 109677 \text{ cm}^{-1}$ (Rydberg constant)

Spectral series		Region	n_1	n_2
λ_{max} = min energy	Lymen	→ UV	1	2,3,4..
	Balmer	→ Vis light	2	3,4,5 ...
	Paschen	→ IR	3	4,5,6...
	Brackett	→ IR	4	5,6,7 ..
	Pfund	→ IR	5	6,7,8 ...
	Humphrey	→ far IR	6	7,8,9...

α, β, γ, δ line

given n_1

$\alpha = n_1 + 1 = n_2$

$\beta = n_1 + 2 = n_2$

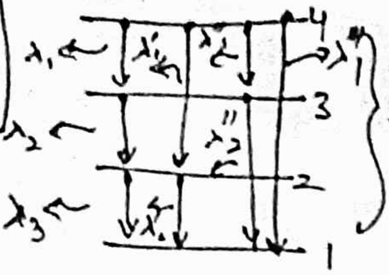
$\gamma = n_1 + 3 \to n_2$

$\delta = n_1 + 4 \to n_2$

Rutherford / Bohr

number of spectral lines emitted

$$= \left[\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}\right]$$



} numerous ways of dexcitation

n_1 = de-excitation shell

n_2 = excitation shell.

→ drawbacks
→ Splitting of spectral lines under Electric & magnetic field. (Zeeman's & Stark effect)
→ valid for 1e⁻ system (H, He^{+}, Li^{2+}, Be^{3+} ...)
→ 2D, based on circular orbit
→ could not explain variation in spectral line intensity
→ only considered particle nature.

Radius of nucleus

$R = R_0 A^{1/3}$

$R_0 = 1.2\,fm = 1.2 \times 10^{-15} m$

Nuclear force
→ Range ($10^{-15} m$)
→ attractive, repulsive
→ strength $F_a \propto \frac{1}{r^7}$ } attraction
$F_r \propto \frac{1}{r^9}$ } repulsion

→ charge independent
→ spin dependent ↑↓ ↑↑ (same spin attract)

Nuclear stability

$Z < 20$ (light weight) } — ①

$\frac{N}{Z} = 1$

$20 < Z < 60$ (medium) } — ②

$\frac{N}{Z} = 1.~~2~~7$

Q - factor

$= (M_{reactant} - M_{prod})\,c^2$

$Q = (KE)' \propto \left[\frac{A}{A-4}\right]$

mass defect

Difference in separate mass from whole mass due to mass converted to binding energy.

Binding energy (E)

= mass defect $(\Delta m)\,c^2$

$\frac{\text{binding energy}}{\text{nucleon}} \propto$ stability (nuclear)

Radioactive decay law

Radioactive disintegration ∝ no. of undecayed nuclei

$-\frac{dN}{dt} \propto N$ (λ = disintegration constant)

$\therefore -\frac{dN}{dt} = \lambda N$

$\Rightarrow N = N_0 e^{-\lambda t}$ (1st order Rxn)

(N: Initial no. of nuclei)

or

$A = A_0 e^{-\lambda t}$ (A = Activity)

$N_{decay} = N_0(1 - e^{-\lambda t})$ (No need to learn)

$60 < Z$ (yeah buddy) } — ③

$\frac{N}{Z} = 1.57$

mean life of nuclei

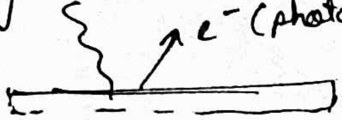
$= \frac{1}{k} = \frac{1}{\lambda}$

$0.693 = \ln(2)$

Threshold frequency

Min frequency

work function

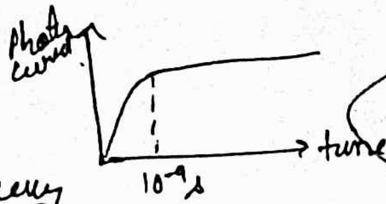


↳ rate of flow = photo current.

$\Rightarrow \phi = h\nu_0 = \frac{hc}{\lambda_0}$

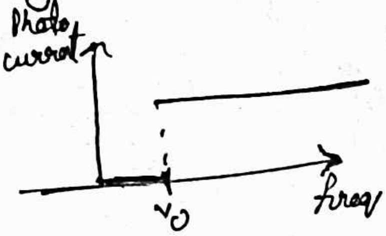
$h = 6.626 \times 10^{-34}$ J/s

λ_0 = threshold frequency (λ_{max})



frequency never decides current.

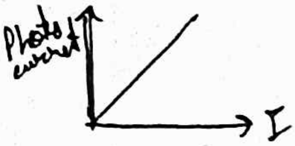
Intensity never decides stopping potential

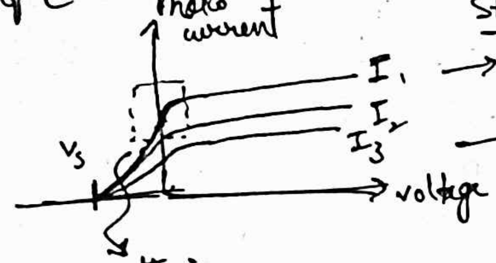


Intensity

$I \propto e^{\ominus}$ photo electrons

$\nu_0 \rightarrow$ K.E of $e^{\ominus}$





scattering of electron at start

Stopping potential

→ v where most energetic electrons stops

→ v₀ Same for all intensities

De Broglie Hypothesis

$\lambda = \frac{h}{m v_{rel}} = \frac{h}{p}$ } for radiation particles.

$\Rightarrow P = \frac{h}{\lambda}$, $E = \frac{hc}{\lambda} \Rightarrow E = Pc$

Einstein explanation

→ Radiation in quanta (packets)

Energy of photon $= h\nu\,(n)$, n = no. of photons

→ 1 photon interacts with $1 e^{\ominus}$

→ intensity ∝ no. of photons (obviously)

$E_{light} = \phi + K.E_{e^{\ominus}}$ — Max K.E

$\Rightarrow h\nu = h\nu_0 + \frac{1}{2}mv^2$ —— (i)

$\Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2}mv^2$

$\Rightarrow h\nu = h\nu_0 + e^{\ominus} V_s$ —— (ii)

V_s → potential difference (work done against electric field)

$\lambda_{electron}$: $\frac{12.2}{\sqrt{V}}$ Å

λ_{proton} : $\frac{6.86}{\sqrt{V}}$ Å

$\lambda_{deutron}$: $\frac{0.202}{\sqrt{V}}$ Å

λ_{α} : $\frac{0.101}{\sqrt{V}}$ Å

V = potential different

λ = de-broglie wavelength.

Photoelectric efficiency

Total photons / Total photoelectrons

$P.E = \frac{\text{no. of photoelectrons}}{\text{no. of photons}}$ (but reageating)

$v_1 = \frac{(m_1 - e m_2) u_1 + (1+e) m_2 u_2}{m_1 + m_2}$

$v_2 = \frac{(m_2 - e m_1) u_2 + (1+e) m_1 u_1}{m_1 + m_2}$

e = coeff. of rest.

Power of photon

$$P = \frac{E}{t} = \frac{n h\nu_0}{t} = \left(\frac{n}{t}\right)(h\nu_0)$$

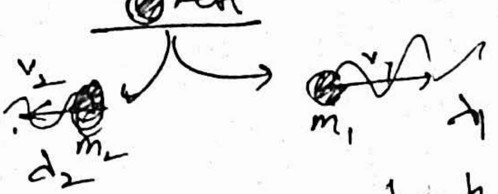
$\left(\frac{n}{t}\right)$ → no. of photons per second.

$(h\nu_0)$ → Energy of 1-photon

$$\left(\frac{n}{t}\right) = \frac{P}{h\nu_0}$$

$\left(\frac{n}{t}\right)$ → rate of photon emission

Explosion



$$\lambda_1 = \frac{h}{m_1 v_1}$$

$$\lambda_2 = \frac{h}{m_2 v_2}$$

In here,

$F_{ext} = 0$

$\Rightarrow P_i = P_f$ (conservation)

$\Rightarrow m_1 v_1 = m_2 v_2$

$\Rightarrow \lambda_1 = \lambda_2$

Ⓐ

Kinetic theory of gas

degree of freedom $= (3A - R)$

A = atoms

R = bonds

mono 3

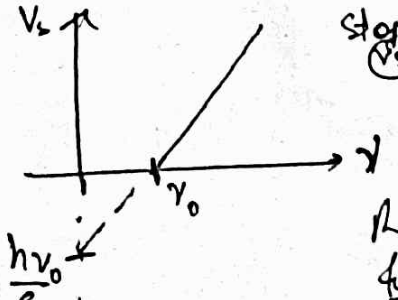
diatomic 5

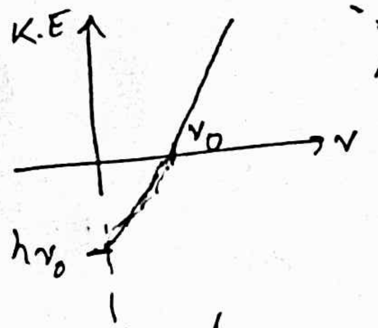
triatomic 7

Equation (Einstein's)

$$h\nu = h\nu_0 + eV_s$$

Stopping potential (V_s) vs frequency.





Law of equipartition of energy

$E_{DoF} = \frac{1}{2}KT$ for 1 DoF

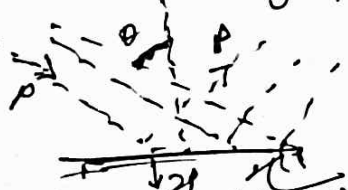
$\Rightarrow E_{net} = \frac{f}{2}K_BT$, f = DoF

$$\lambda = \frac{h}{\sqrt{2m(K.E)}} = \frac{h}{\sqrt{fmKT}}$$

$K_B = 1.38 \times 10^{-23}$ j/K for 1 molecule (Boltzmann)

$R = 8.31$ j/mol/K

Radiation force ① (crash & jump)



$$F_{plate} = \frac{2IA_s\cos^2\theta}{c} \quad (100\% \text{ reflection})$$

A_s = Area of surface (of projection) where light falls

θ = ∠ of incidence

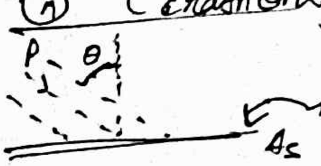
I = Intensity

c = speed of light.

Radiation pressure

$= \frac{F}{A}$ (Ⓜ)

Radiation force ② (crash only)



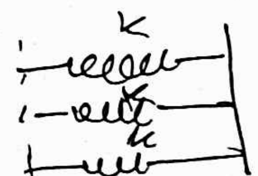
$$F_x = \frac{IA_s\sin\theta\cos\theta}{c}$$

$$F_y = \frac{IA_s\cos^2\theta}{c}$$

(100% absorbed)

Ⓐ for sphere,

$F \Rightarrow$ (crash only) for both case



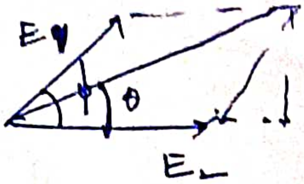
spring combination

≡ capacitor combination

≢ resistance combination.

Wave optics

Superposition



$\tan\theta = \dfrac{E_1 \sin\phi}{E_2 + (E_1 \cos\phi)}$

$E_1 = E_{0_1} \sin(\omega t)$

$E_2 = E_{0_2} \sin(\omega t + \phi)$

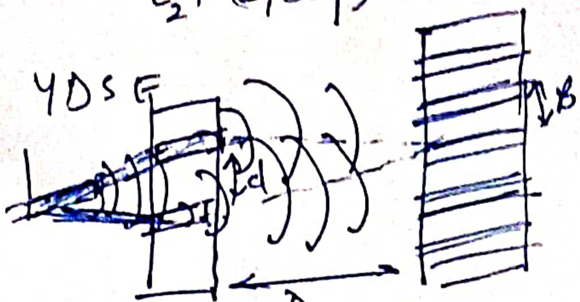
$\Delta\phi = \phi$

$E_{net} = \sqrt{E_{0_1}^2 + E_{0_2}^2 + 2E_{0_1}E_{0_2}\cos\phi}$ — case (i) (coherence)

$= E_0$

$\Rightarrow E_0 \sin(\omega t + \theta)$

YDSE



fringe width (β)

$\Rightarrow \beta = \dfrac{D\lambda}{d}$

$E_{net} = \sqrt{E_{0_1}^2 + E_{0_2}^2 + 2E_{0_1}E_{0_2}\cos((\omega_2 - \omega_1)t + (\theta_2 - \theta_1))}$

case (ii) (incoherence)

Angular fringe width (α)

$\alpha = \dfrac{\beta}{D}$

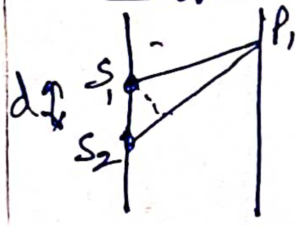
Maximum never forms at ∞.

contrast (max-min interference ratio) $= \left\{\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right\}^2$

order of maxima (no. except CM) $= \left[\dfrac{d}{\lambda}\right]$

order of minima $= \left[\dfrac{d}{\lambda} + \dfrac{1}{2}\right]$

Path difference (Δx)



$\Delta x = S_2P_1 - S_1P_1 = d\sin\theta$

maxima case

$\Delta x = n\lambda$

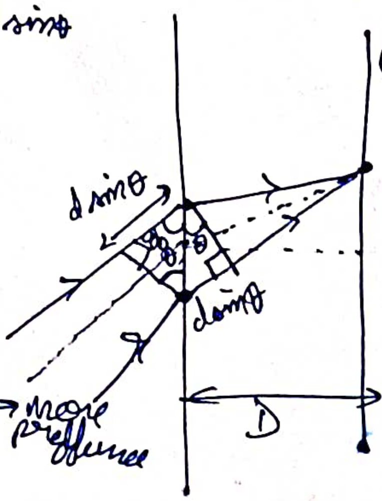
minima case

$\Delta x = (2n+1)\dfrac{\lambda}{2}$

or

$(2n-1)\dfrac{\lambda}{2}$ → more preference

$\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$



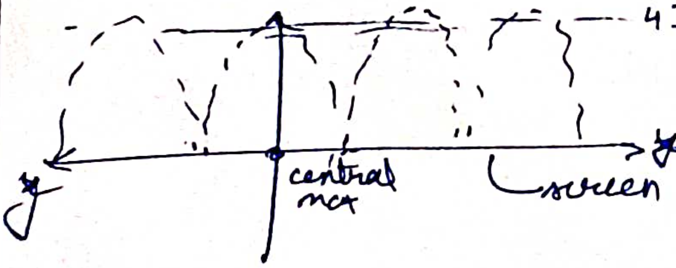
Ⓐ calculate path difference (here 'd's)

$\Rightarrow \Delta x = d\sin\theta - d\sin\theta_0$

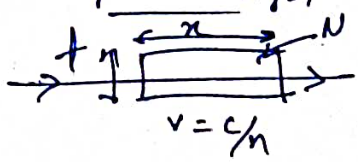
$\Rightarrow \Delta x = \dfrac{dy}{D} - d\sin\theta_0$

θ_0 is small

Intensity pattern



optical length



x = geometrical len.

YDSE in transparent medium

β in air $= \dfrac{\lambda_0 D}{d} = \beta_0$

$\lambda = v/f$, $v = \dfrac{c}{n}$

$\Rightarrow$ $\lambda_0 = c/f$

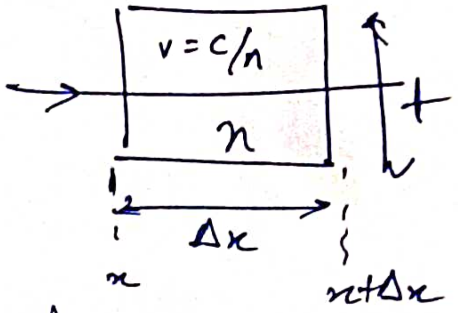
$\Rightarrow \lambda = \dfrac{c}{nf} = \left(\dfrac{c}{f}\right)\left(\dfrac{1}{n}\right)$

$\Rightarrow \lambda = \dfrac{\lambda_0}{n}$

$\Rightarrow n\lambda = \lambda_0$

$$\boxed{\beta_{med} = \dfrac{\beta_{air}}{n}}$$

optical length



Δx = Geometrical length

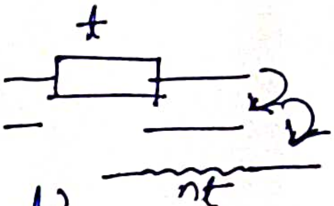
$\Delta\phi = \dfrac{\omega n \Delta x}{c}$

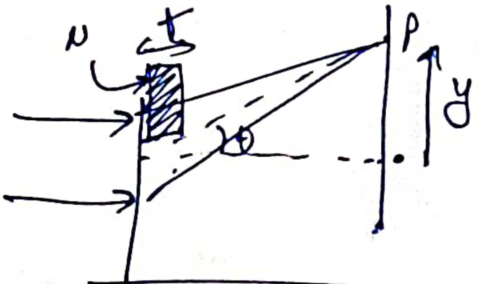
if $E_p = E_0 \sin(\omega t - kx)$

$E_2 = E_0 \sin \omega\left(t - \dfrac{x}{v}\right)$

$n\Delta x \rightarrow$ optical length (always consider only this length for air)

Insertion of parallel slab





$\Delta x = S_2P - S_1P$

$= S_2P - \{S_1P - t + Nt\}$

$= (S_2P - S_1P) - (N-1)t$

$= d\sin\theta - (N-1)t$

$= d\tan\theta - (N-1)t$ (if $\theta \approx$ small)

$$\boxed{\Rightarrow \Delta x = \dfrac{dy}{D} - (N-1)t}$$

shape of interference pattern

fringes : Hyperbolic

maxima case

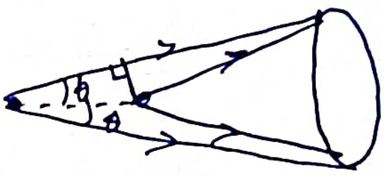
$y = \dfrac{Dt}{d}(N-1) + n\beta$

minima case $\rightarrow$ distraction

$y = \dfrac{Dt}{d}(N-1) + \beta(n - 0.5)$

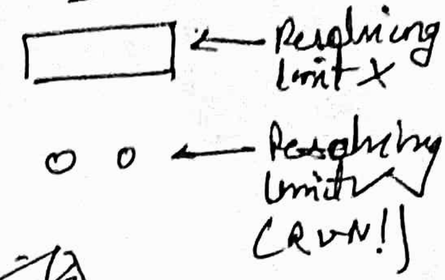
$\hookrightarrow$ distraction

Pinhole arrangement



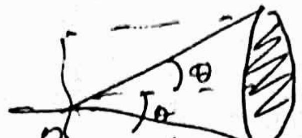
Resolving limit & resolving power

Resolving power $\propto \frac{1}{\text{Resolving Limit}}$

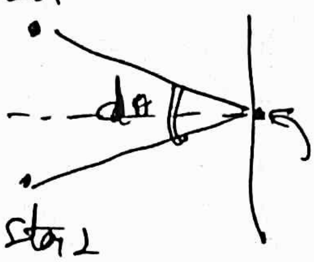


Resolving limit ⟶ distance after which distinguishing between two sources is impossible

Resolving limit $= \frac{\lambda}{2\mu \sin\theta}$, $R.P = \frac{1}{R.L}$

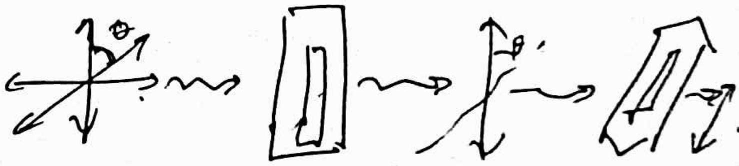


$R.L = \lambda\theta = \frac{1.22}{a}\lambda$.



Polarizer (eats up components)

~~..~~ variation of intensity ~~of~~ ~~..~~ with polarisation



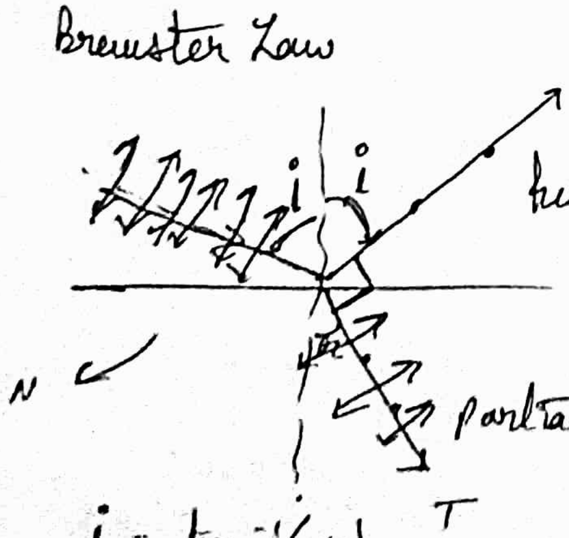
Process: Dichroism

Polarizer ⟶ made of polymer
⟶ Aligned
⟶ Nicole prism

$I \propto E_0^2$

$I = kE_0^2\cos^2\theta$ (Malus law)
↓
½ if initially unpolarized.

Brewster Law



$i + 90 + r = 180°$

~~$\lambda r = 180°$~~

$\Rightarrow$ if $x = 180 - (i+r) = 90°$

$i \to$ incident

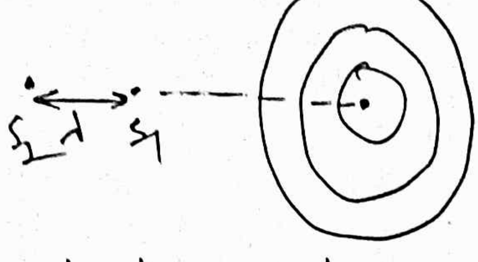
$r \to$ refracted

~~..~~

then Brewster law

$i = \tan^{-1}(\mu)$

Freunhoffer diffraction



circular fringes

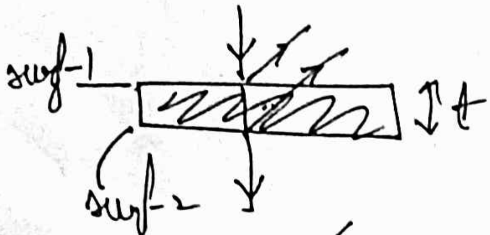
$\sin\theta_n = \frac{n\lambda}{a}$ } minima

$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$ } maxima

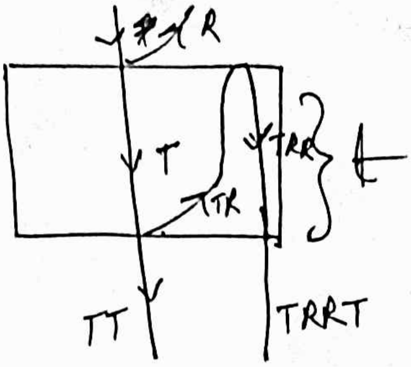
(divide point sources by 2, 4, 6 in minima and by 3, 5, 7 in maxima)

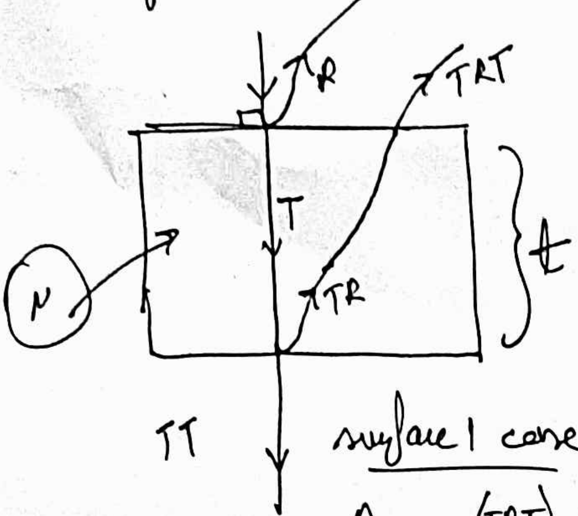
Thin film interference

(special case YDSE)



surface 2





surface 1 case

$\Delta x = (TRT) - (R)$
$= 2\mu t$

$\Rightarrow \phi = \frac{2\pi}{\lambda}(2\mu t)$

now, in both cases,

$(2n+1)\pi = \frac{2\pi}{\lambda}(2\mu t)$

$\Rightarrow \lambda_1 = \frac{4\mu t}{(2n+1)}$

$(2n-1)\pi = \frac{2\pi}{\lambda}(2\mu t)$

$\Rightarrow \lambda_2 = \frac{2\mu t}{n}$ } max (opp convention)

since $\lambda_1 < \lambda_2$

λ_2 will glow

surface 2 case

$\Delta x = 2\mu t$

$\phi = \frac{2\pi}{\lambda}(2\mu t)$

$= \frac{4\mu t\pi}{\lambda}$

now, in both cases

$\Delta\phi = (2n+1)\pi$

$\Rightarrow \lambda_1 = \frac{4\mu t}{2n+1}$

$\Delta\phi = 2n\pi$

$\Rightarrow \lambda_2 = \frac{2\mu t}{2n}$ } max

λ_2 will glow

diffraction by pinhole

$\sin\theta = \frac{1.22\lambda}{b}$

$b \to$ aperture (diameter)

$R = D\theta$

Reynolds number (R_e)

$$R_e = \frac{\rho v d}{\eta}$$

$R_e < 1000$ (streamline)
$R_e > 2000$ (Turbulent)
$1000 < R_e < 2000$ (unsteady)

$\rho \rightarrow$ density
$v \rightarrow$ speed
$d \rightarrow$ pipe cross sectional diameter
$\eta \rightarrow$ coeff of viscosity

P-V relation (horizontal flow)

$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Loss of Pressure energy = gain of volumetric kinetic energy.

Equation of continuity

$\hookrightarrow$ volume of rate flow is const (only for ideal fluid)

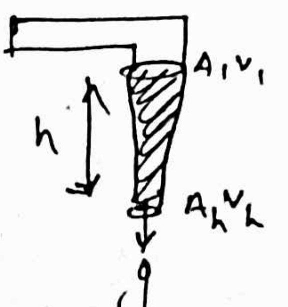
$\frac{dV}{dt}$ = const. (everywhere)

$V \rightarrow$ volume

$\Rightarrow A\frac{dx}{dt} \Rightarrow Av = \text{const.}$

(v = velocity)

$A_1 v_1 = A_2 v_2$



$A_1 v_1 = A_h v_h$

free fall case

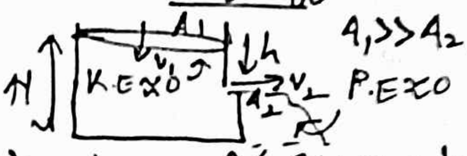
$\Rightarrow 2g(h) = v_h^2 - v_1^2$

$\Rightarrow v_h^2 = 2gh + v_1^2$

$\Rightarrow v_h = \sqrt{2gh + v_1}$

$$A_h = \frac{A_1 v_1}{\sqrt{2gh + v_1}}$$

speed of efflux



$A_1 >> A_2$, $v_2 >> v_1$

$\cancel{P_0} + h\rho g = \cancel{P_0} + \frac{1}{2}\rho v_2^2 + 0$

$\Rightarrow v_2 = \sqrt{2hg}$

(2D kinematics)

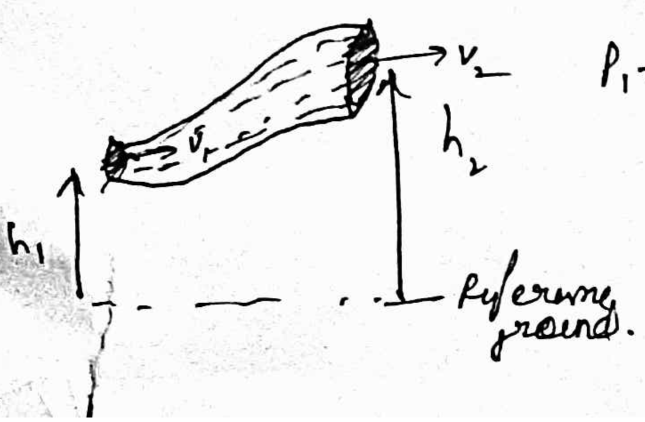
Range $= v_2 \sqrt{\frac{2(H-h)}{g}}$

for max range $\Rightarrow \frac{dR}{dh} = 0$

Bernoulli's eqn

Pressure energy + KE/vol + P.E/vol = const.

$\Rightarrow P + \frac{1}{2}\rho v^2 + \rho g h$ = const. (everywhere)



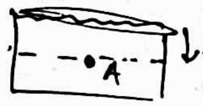
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Relative density (specific gravity) = $\frac{\rho_{fluid}}{\rho_{water\ at\ 4^\circ C}}$

$\rho_{Hg} = 13.6 \times 10^3\ kg/m^3$

$\rho_w = 10^3\ kg/m^3$

Pressure variation (vessel at rest)

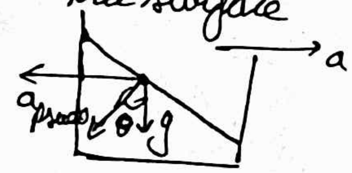


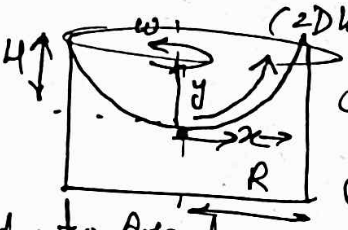
$P_A = \rho g h + P_0$

$\rho g h$ → Gauge pressure; P_0 → Atmos

P_A is independent of vessel shape

free surface





$y = \frac{\omega^2 x^2}{2g}$

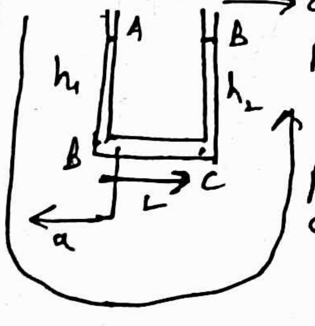
$y = \frac{\frac{v^2}{x^2} \cdot x^2}{2g} = \frac{v^2}{2g}$

on comparing

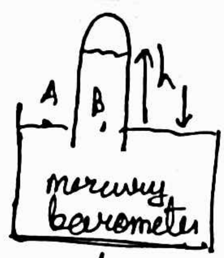
$\Rightarrow s = \frac{v^2}{2a}$ (3rd eq. of motion)

$g_{eff} = \sqrt{a^2+g^2}$ (due to pseudo force)

$\tan\theta = \frac{a}{g}$



$P_A + \rho g h_1 - \rho a L - \rho g h - P_D = 0$



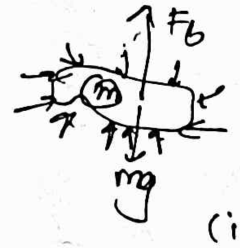
$P_A = P_B$

$P_0 = 1.01 \times 10^5\ Pa$

$\Rightarrow P_0 = h\rho g$ (inside tube)

$\Rightarrow$ 1 atm $= 760$ mmHg

Archimedes' principle / Floatation



$F_b = \rho_L V_d g$

V_d = volume of displaced liquid.

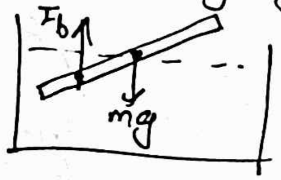
(i) Body sinks, $\rho_s > \rho_L$

(ii) Body float $\rho_s \leq \rho_L$

$\rho_s < \rho_L$ some portion outside

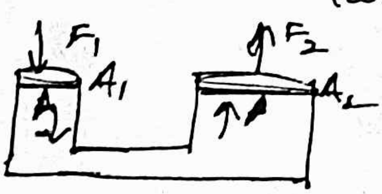
$\rho_s = \rho_L$ completely submerged.

centre of buoyancy



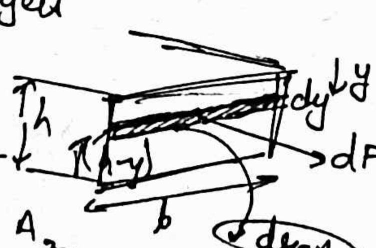
centre of buoyancy = centre of mass of displaced liquid.

Pascal's law: external pressure gets distrib'd in all directions



$F_2 = PA_2$

$= \frac{F_1}{A_1} A_2$



$dF = \Rightarrow dF = y\rho h \times b\,dy$

(F×A)

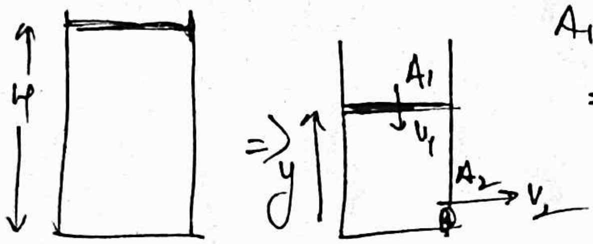
$d\tau = dF(h-y)$

Apparent weight:

$W_{apparent} = mg - F_b$

$= mg - \rho_L V_d g = mg - \rho_L \frac{m}{\rho_s}(g) = m\left(1 - \frac{\rho_L}{\rho_s}\right)$

vessel emptying time



$A_1 v_1 = A_2 v_2$

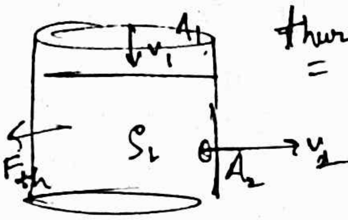
$\Rightarrow A\left(\frac{-dy}{dt}\right) = A_2\sqrt{2gy}$

integrate $(H \to y,\ 0 \to t)$

$\Rightarrow t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{y}\right)$

$\Rightarrow t_0 = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}$

Thrust of vessel



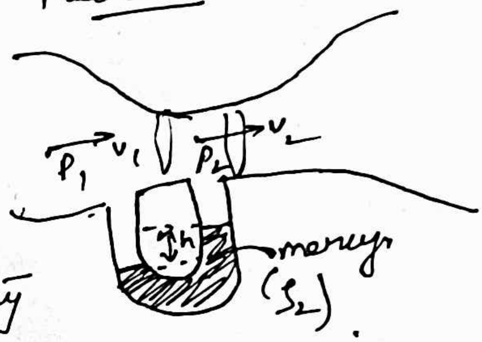
thrust $= \rho_L \cdot (A_2 v_2^2)$

since $v \to$ velocity of efflux

$v^2 = (\sqrt{2gH})^2$

$\Rightarrow \rho_L A_2 (2gH) = $ Thrust

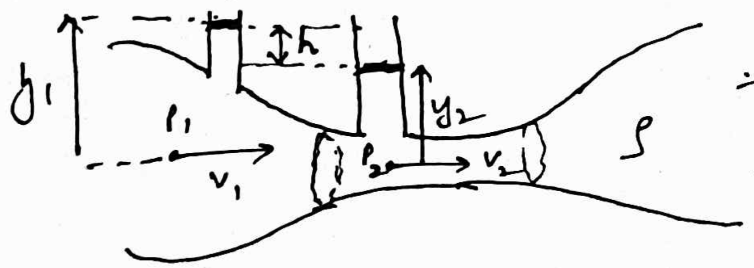
model - 2



Venturimeter

(device to measure flow speed)

working principle: Bernoulli's + Eqn of continuity



$A_1 v_1 = A_2 v_2$

$\Rightarrow v_1 = \frac{A_2 v_2}{A_1}$

$\Rightarrow v_2 = \frac{A_1 v_1}{A_2}$

replace ρ in P.E's with ρ_{Hg}. (slight diff. in formula)

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$ {Pressure difference is cause of height difference}

$\Rightarrow \cancel{\rho} g y_1 - \cancel{\rho} g y_2 = \frac{1}{2}\cancel{\rho}(v_2^2 - v_1^2)$

$\Rightarrow 2g(y_1 - y_2) = v_2^2 - v_1^2$

$\Rightarrow 2gh = v_2^2 - v_1^2$

$\Rightarrow 2gh = \frac{A_1^2 v_1^2}{A_2^2} - v_1^2$

$\Rightarrow A_2^2\, 2gh = (A_1^2 - A_2^2) v_1^2$

$\Rightarrow v_1^2 = \left(\frac{2gh\, A_2^2}{A_1^2 - A_2^2}\right)$

$\Rightarrow v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$

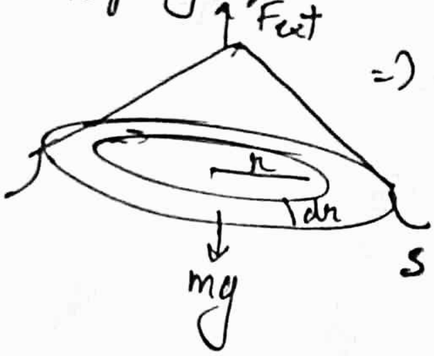
Surface tension

1. Cohesive and adhesive
(self-attracting) (else-attracting)

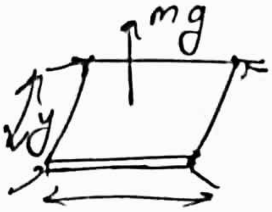
2. Surface tension
(i) surface wants mini. area
(ii) Acts as stretched membrane

$S = \frac{F}{L}$; F is tangential to surface
; L = linear length

3. Lifting of bodies



$\Rightarrow F_{ext} = mg + S(2)(2\pi r)$



$F_{ext} = mg + S(2(x+y))$

4. surface energy
caused by intermolecular interaction

$S = E/A$; A = Area
E = Energy
S = tension

$E = S\pi r^2 4$

$E = 8\pi r^2 S$
(inner + outer)

(thin wall)

coalescence

$O + O + \overset{S}{O} \rightarrow$ (n)
radius: r (lower surface area)

$E_i = nS(4\pi r^2)$

$E_f = S(4\pi R^2)$

$\Delta E = \cancel{S4\pi R^2(1-n)}$
$S4\pi(R^2 - nr^2)$

volume same (relation of R & r)

$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

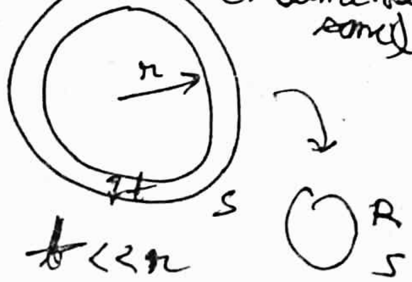
$\Rightarrow R = rn^{1/3}$

$\Rightarrow \Delta E = 4\pi S r n^{1/3}(r - n^{2/3}r)$

$= mS\Delta\theta$

; m → mass
S → specific heat capacity
$\Delta\theta$ → temperature drop/rise of system

Bubble collapse
(volume stays same)



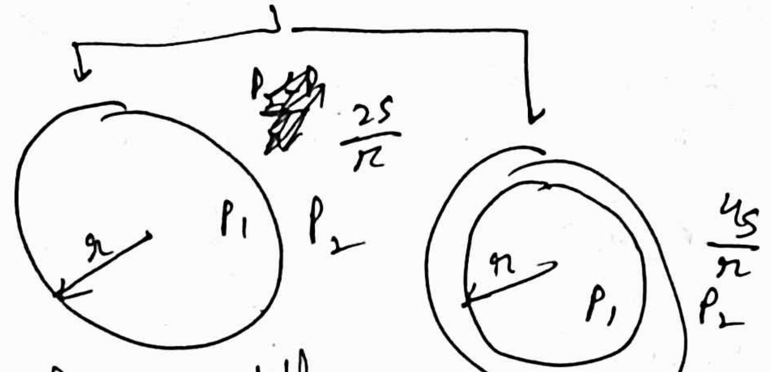
$t << r$

$\Rightarrow \frac{4}{3}\pi r^2 t = \frac{4}{3}\pi R^3$

$\Rightarrow R = (r^2 t)^{1/3}$

$\Delta E = \pi(r^2 t)^{2/3} - (S)(2)(4\pi r^2)$

6. Excess pressure in Liquid drop and Bubble

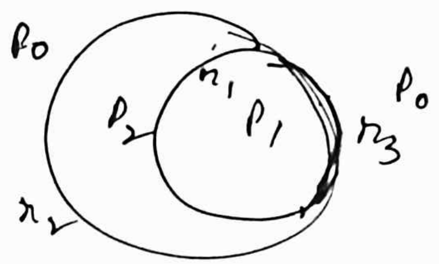


Pressure difference ∝ 1/radius of curvature

Bubble inside liquid (quarium) has only 1 surface. (Air drop)

$P = \frac{2S}{r}$

(ii) Internal contact (special case)



high p — low p

$P_1 - P_0 = \frac{2S}{r_3}$ — (i)

$P_1 - P_2 = \frac{2S}{r_1}$

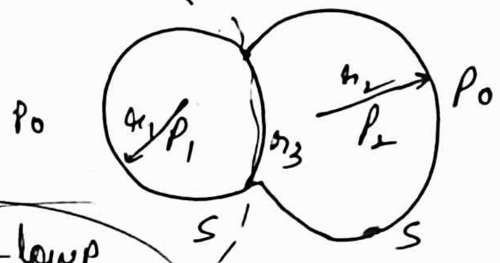
$P_2 - P_0 = \frac{2S}{r_2}$

$\Rightarrow P_1 - P_0 = \frac{2S}{r_2} + \frac{2S}{r_1}$ — (ii)

$\Rightarrow \frac{2S}{r_3} = \frac{2S}{r_2} + \frac{2S}{r_1}$

$\Rightarrow r_3 = \frac{r_2 r_1}{r_2 + r_1}$

7. two soap bubbles in contact



$P_2 - P_0 = \frac{2S}{r_2}$

$P_1 - P_0 = \frac{2S}{r_1}$

$P_2 - P_1 = \frac{2S}{r_3}$

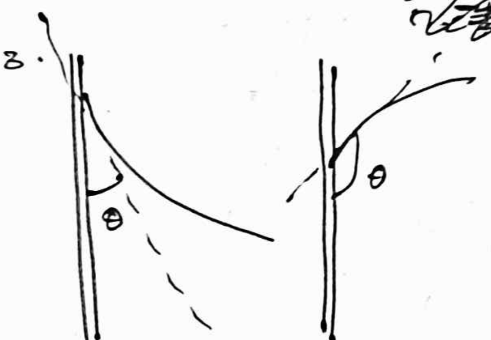
Also,

$P_2 - P_1 = \frac{2S}{r_2} - \frac{2S}{r_1} + \cancel{P_0} - \cancel{P_0}$

$\Rightarrow \frac{1}{r_2} - \frac{1}{r_1} = \frac{1}{r_3}$

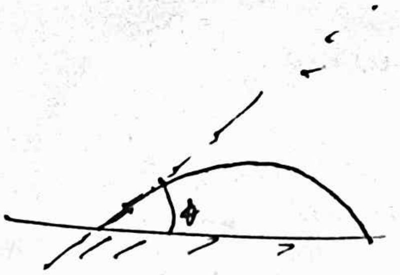
$\Rightarrow r_3 = \frac{r_2 r_1}{r_2 - r_1}$

8.



θ < 90°   θ > 90°

ex: water   ex: Hg (mercury)



$\theta < 90^\circ$

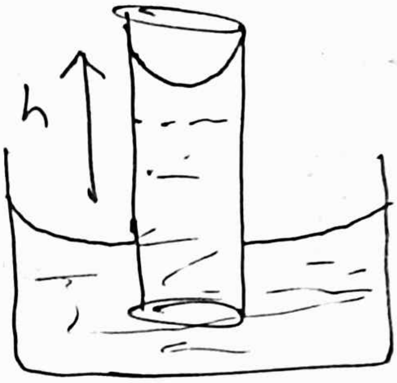
Liquid spreads (wets the surface)

$\theta > 90^\circ$

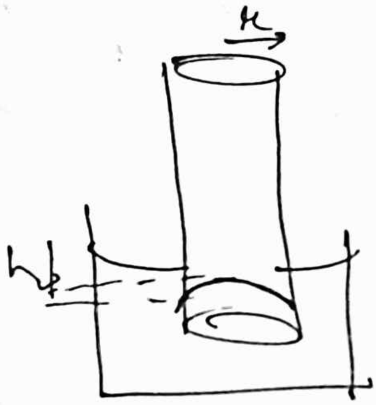
Generally θ ~~for~~ distilled H_2O is 0°

9. Capillary action

(i) thin tube



$\theta < 90^\circ$



$\theta > 90^\circ$

$h = \dfrac{2s\cos\theta}{r\rho g}$ (ideally) or $\dfrac{2s}{R\rho g}$; $s \rightarrow$ tension

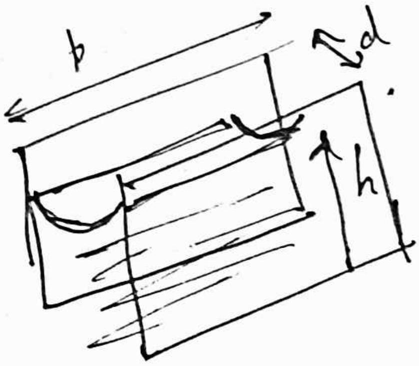
θ : Angle of contact

ρ : density

g : acc. due to gravity

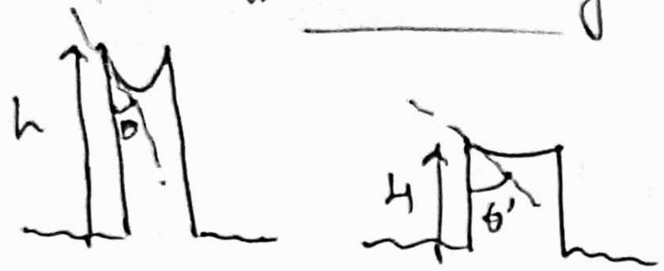
R : meniscus radius

(b) Plastic plates



$h = \dfrac{2s\cos\theta}{d\rho g}$, generally if it's H_2O, $\theta = 0$ (ideally)

10. insufficient tube length ($H < h$)



$h = \frac{2S\cos\theta}{\rho g}$, but $H < h$

→ in this case liquid doesn't overflow rather it reaches top and adjusts contact angle to θ'

($\theta' > \theta$)

11. Force to separate glass plates

A: Liquid cross sectional area.

$F = \frac{2SA\cos\theta}{d}$; S → tension

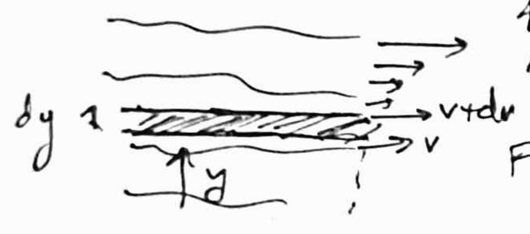
$\cos\theta = 1$ if H_2O

A = cross section

θ = Angle of contact

d = distance between plates

12. understanding viscous force (like friction)



fluid oppose relative motion between different layers.

$F \propto A \frac{dv}{dy}$

$\Rightarrow F = -\eta A \frac{dv}{dy}$ (opposing force)

$F_{ext} = F_v = \eta A \frac{dv}{dy}$

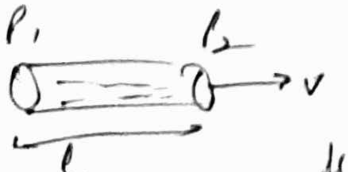
$= \eta A \frac{v}{t}$

$[\eta] = ML^{-1}T^{-1}$

unit : Poiseuille

CGS : 1 Poiseuille = 10 poise

(PI)



Poiseuille's eqn for rate flow

flow rate $\Rightarrow a \frac{\pi r^4}{8\eta L}(P_1 - P_2)$

Stokes Law & Terminal velocity

As $v\uparrow$, $F_v\uparrow$

and at certain speed

$F_b + F_v = mg \Rightarrow a = 0,\ v = \text{const}$

Terminal velocity

Stokes law

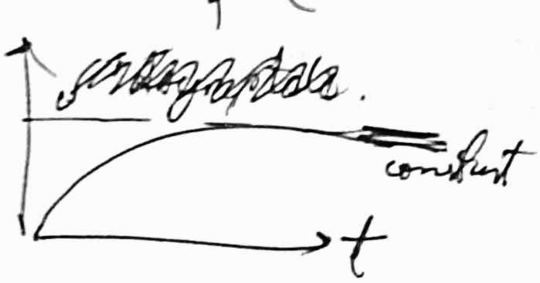
$f = 6\pi\eta r v$

$$\rho_L \frac{4}{3}\pi r^3 g + 6\pi\eta r v = \rho_s \frac{4}{3}\pi r^3 g$$

$$\Rightarrow v = \frac{\rho_s \frac{4}{3}\pi r^3 g - \rho_L \frac{4}{3}\pi r^3 g}{6\pi\eta r}$$

$$\Rightarrow v = \frac{\frac{4}{3}\pi r^3 g(\rho_s - \rho_L)}{6\pi\eta r} = \frac{4r^2 g(\rho_s - \rho_L)}{18\eta}$$

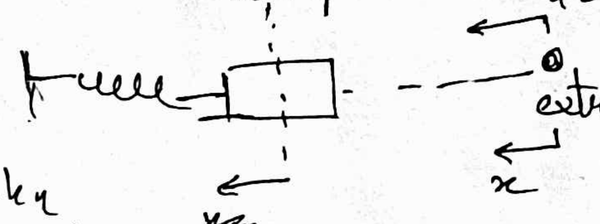
$$\Rightarrow v = \frac{2r^2 g(\rho_s - \rho_L)}{9\eta}$$



SHM

$F \propto -x$

$F = -kx$



$\Rightarrow ma = -kx$

$\Rightarrow a = \left(-\frac{k}{m}\right)x$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{kx}{m} = 0$

$\Rightarrow x = A\sin(\omega t + \phi)$

$\omega^2 = \frac{k}{m} \Rightarrow \omega = \pm\sqrt{\frac{k}{m}} = 2\pi f = \frac{v}{\lambda}$

$T = \frac{2\pi}{\omega}$

$v = A\omega\cos(\omega t + \phi)$ → initial phase

$a = A\omega^2 \sin(\omega t + \phi + \pi/2)$

$v(x) = \pm\left(\sqrt{A^2 - x^2}\right)\omega$

$K.E = \frac{1}{2}m\omega^2(A^2 - x^2)$

$P.E = \frac{1}{2}m\omega^2 x^2$

~~spring comb~~ spring combination

= oppt. of resistance (inverse) (ulta)

special arrangement (parallel)

$\boxed{k_{eq} = k_1 + k_2}$

Linear SHM

→ displacement from mean

(2) find $\vec{a}$

(3) $a = -\underbrace{(\text{some const.})}_{\omega^2} \times x$

(4) $\Rightarrow T = 2\pi/\omega$

Angular SHM

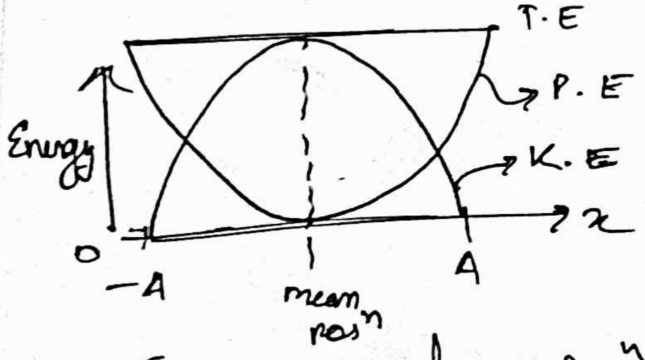
(1) Angle (θ) (small) from mean

(2) $\tau = I\alpha$ (find α)

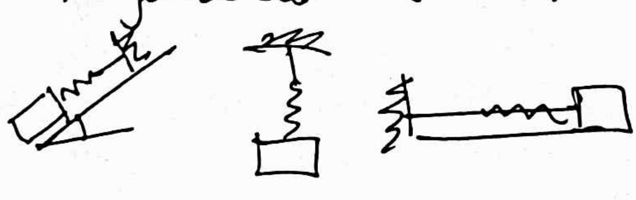
$= F(x)$

(3) $\alpha = -(\text{some const.}) \times \theta$

$T = 2\pi/\omega$



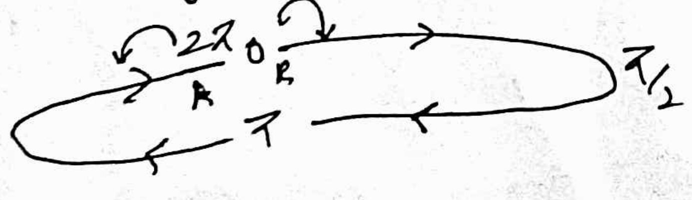
Spring block (Any arrangement)



$T = 2\pi\sqrt{\frac{m}{k}}$ (remains same)

maybe gap if mean posn potential energy $\neq 0$

Phase changes



Physical pendulum

spring cut (k, l = const.)

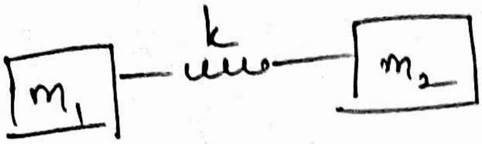
k, l = k_1, l_1 + k_2, l_2

$\Rightarrow kl = k_1 l_1 = k_2 l_2$

(product remains same)

~~Damped oscillations~~

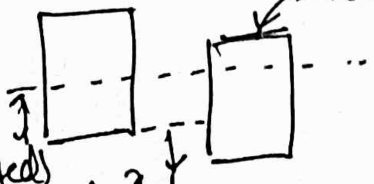
Two block system



$$\left\{ m_{reduced} = \frac{m_1 m_2}{m_1 + m_2} \right\}$$

$$T = 2\pi \sqrt{\frac{m_{reduced}}{k}}$$

Block in liquid.



(ρ density)

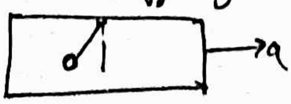
$$T = 2\pi \sqrt{\frac{h}{g}}$$ → can be solved by buoyant force

or

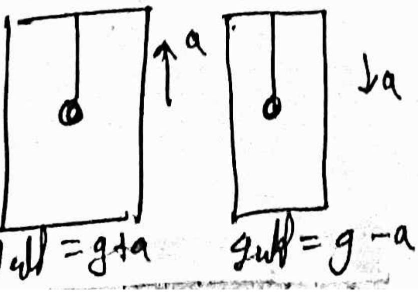
$$T = 2\pi \sqrt{\frac{m}{A \rho_L g}}$$

T of simple pendulum

$$T = 2\pi \sqrt{\frac{L}{g_{eff}}}$$



$g_{eff} = \sqrt{g^2 + a^2}$

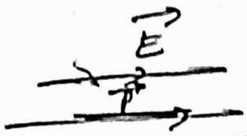


$g_{eff} = g + a$ $\qquad$ $g_{eff} = g - a$

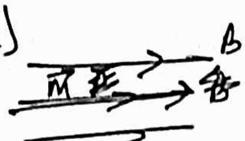
$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

I = MOI of body about (point of suspension)

d : Distance between O & C (centre of mass)

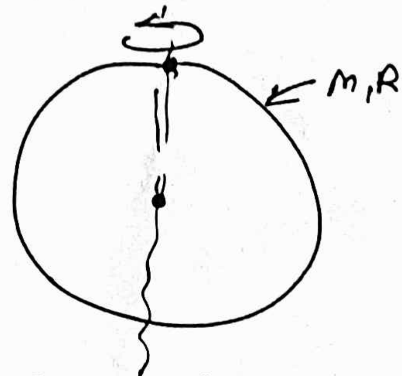


$$T = 2\pi \sqrt{\frac{I}{PE}}$$



$$T = 2\pi \sqrt{\frac{I}{MB}}$$

Ex:



$I = 2MR^2$

$$\therefore T = 2\pi \sqrt{\frac{2MR^2}{mgR}}$$

Torsional pendulum

$\tau = -c\theta$

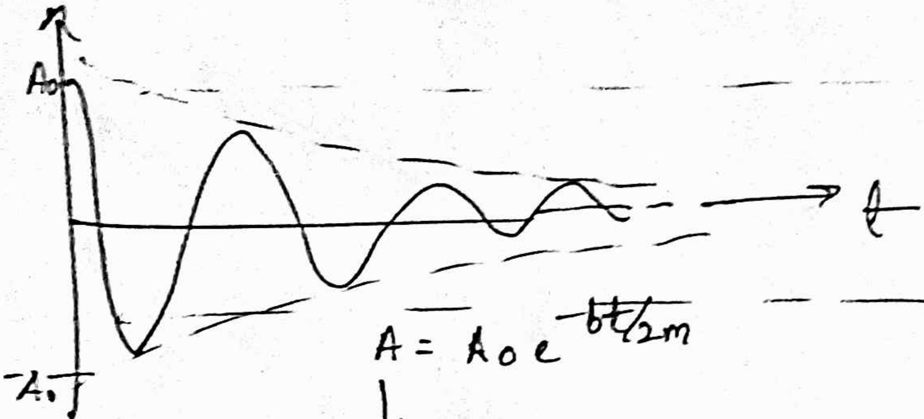
$I\alpha = -c\theta$

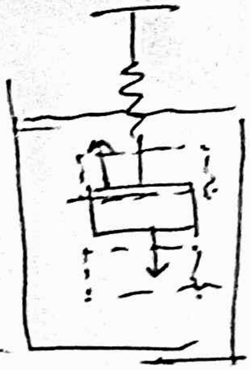
$\Rightarrow \alpha = \frac{-c\theta}{I}$

$\Rightarrow \alpha = -\left(\frac{c}{I}\right)\theta$, $\frac{c}{I} \to \omega^2$

$\Rightarrow T = 2\pi \sqrt{\frac{I}{c}}$

Damped oscillations





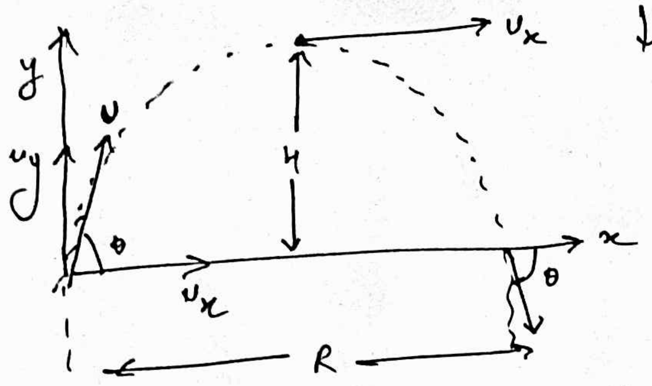
$A = A_0 e^{-bt/2m}$

Amplitude

b = damping const. (dependent on medium submerged)

$y = A(t)\cos(\omega' t + \theta)$

A → Amplitude
t → time
ω' → special Angular freq. $= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
m → mass
t → time
θ → initial phase
b → damping const.

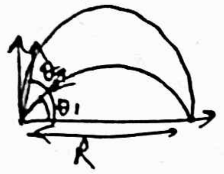


Angle of elevation = Angle on drop

(iv) $a_x = 0 \Rightarrow \boxed{v_x = u\cos\theta \text{ always const}}$

(v.) if $\theta_1 + \theta_2 = 90°$, (complimentary case)

$\Rightarrow R_1 = R_2$ (same range)



(A) except in min. trolley case.

Standard Equation of trajectory

At any time t,

$x = u\cos\theta \cdot t$

$y = t u\sin\theta - \frac{1}{2}gt^2$

~~Substitute~~ make x eq. subject in θ,

$\Rightarrow t = \frac{x}{u\cos\theta}$

replace t in y.

$\Rightarrow y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$

$\equiv ax - bx^2 \rightarrow$ Quadratic

Range $= \frac{a}{b}$ } $\rightarrow$ Ratio (lightning formula ⚡)

for

(i) Time of flight

$T = \frac{2u_y}{g} = \frac{2u_y}{g}$

(ii) max. height

$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

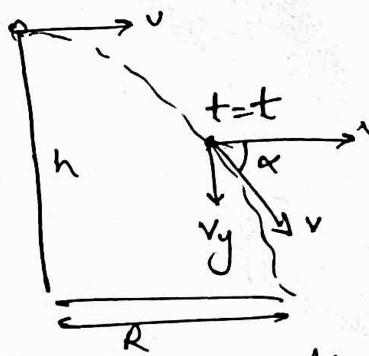
(iii) Range

$R = \frac{2u_x u_y}{g} = \frac{u^2\sin(2\theta)}{g}$

or

$u_x \cdot T$ ↳ time of flight

model 2



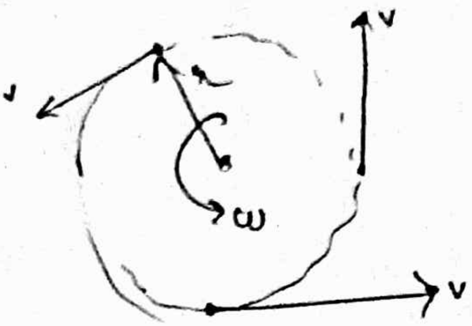
(i) Time of flight

$T = \sqrt{2h/g}$

(ii) $R = u\sqrt{2h/g} = u.T$

(iii) $v_y = gt$

In inclined model ~~in~~ tilt the system lmao!



Angular displacement : θ

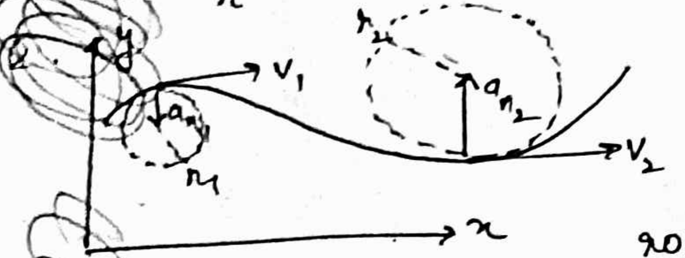
Angular velocity : $\frac{d\theta}{dt}$

Angular acceleration : $\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

$\omega = \frac{v}{r}$

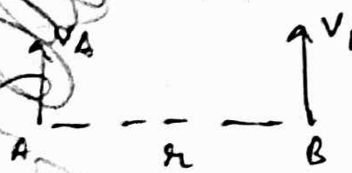
$\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow a_t = r\alpha$ (tangential accⁿ)

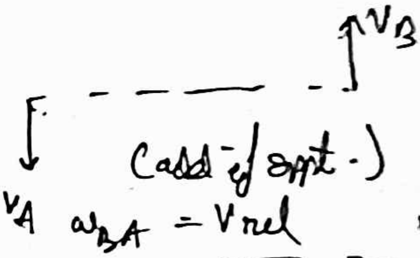
$a_n = \frac{v^2}{r} = \omega^2 r$ (centripetal or normal accⁿ)



$r_1 = \frac{v_1^2}{a_{n_1}}$ $r_2 = \frac{v_2^2}{a_{n_2}}$

roc = $\frac{\text{tangential velocity}}{\text{centripetal acceleration}}$

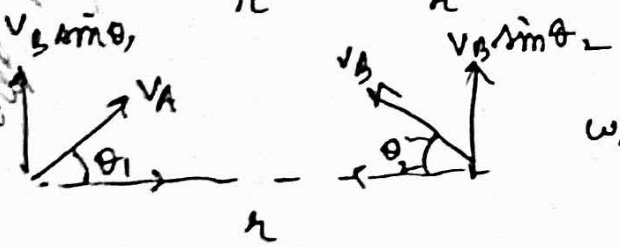




(case of oppt.)

$\omega_{BA} = \frac{V_{rel}}{r} = \frac{V_B - V_A}{r}$

$\omega_{BA} = \frac{V_{rel}}{r} = \frac{V_B + V_A}{r}$



$\omega_{AB} = \frac{V_B \sin\theta_2 - V_A \sin\theta_1}{r}$

uniform circular motion

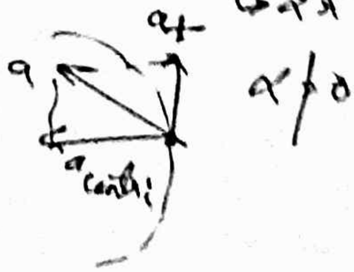
$\alpha = 0$, $a_t = 0$

$a_{centri} = \frac{v^2}{r} = \omega^2 r$

non-uniform motion

$a = \sqrt{a_t^2 + a_n^2}$

$a_t \hookrightarrow r\alpha$ $\quad a_n \hookrightarrow \frac{v^2}{r}$

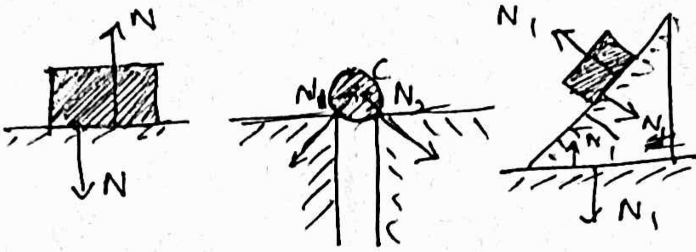


$\alpha \neq 0$

Static Equilibrium

$\sum \vec{F} = 0$

$\Rightarrow \sum m\vec{a} = 0$



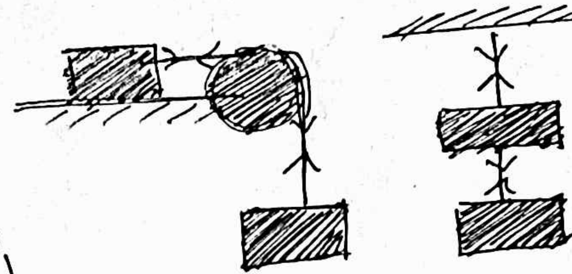
Common forces acting on body

↳ weight (mg)

↳ Normal (N) (pushing)

↳ Tension (T) (pulling)

↳ friction (f)

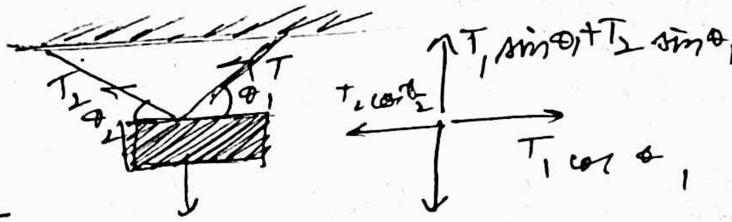


variable force

$F(t) = m\frac{dv}{dt}$

$F(x) = mv\frac{dv}{dx}$

$F(v) = m\frac{dv}{dt}$ or $mv\frac{dv}{dx}$



All forces are resolved into x & y.

Lami's theorem $F \rightarrow \bullet \xrightarrow{a}$

→ rest at body

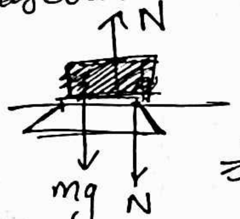
→ precisely 3 forces

$\left[\frac{\text{force}}{\text{oppt. angle}}\right] = \text{const}$

$\Rightarrow \frac{T_1}{90+\theta_2} = \frac{T_2}{90+\theta_1} = \frac{mg}{\theta}$

concept of weighing machine

measures normal force

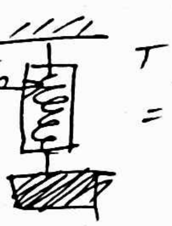


$N = mg$

Reading $\Rightarrow \frac{N}{g} = m$

FBD Law

Pulley block system

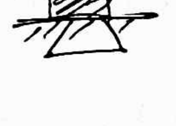


$\rightarrow m = 0$

$\sum \vec{F} = 0$

(resolve in x & y)

2 bodies considered same object if acceleration const (same) for both

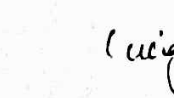


$N - mg = ma$

$\Rightarrow N = ma + mg$

$= m(a+g)$

(heaviness)



$mg - N = ma$

$\Rightarrow N = m(g-a)$

(lightness)

if $a = g$,

$N = 0$

(weightlessness free fall)

concept of spring balance

$T = mg$

$\Rightarrow$ Reading $= T/g = m$

AYUSH DUTTA

Work - Energy power

work

Force is constant

$W = \vec{F}\cdot\vec{S}$

$= \vec{F}\,(\vec{r_2} - \vec{r_1})$

Force is variable $f(x)$

$W = \int_{x_1}^{x_2} f(x)\,dx$

or Area of F Vs x graph

Force is variable $f(t)$

$W = \int_{t_1}^{t_2} ma(t)\ \boxed{\times f(t)\,dt}$

or

$a(t) = \frac{F(t)}{m}$

$v = \int \frac{f(t)}{m}\,dt$

$\boxed{dx = \int \frac{f(t)}{m}\,dt\,dt}$

Potential energy

Gravitational $\longrightarrow mgh$

$\longrightarrow \frac{-Gm_1m_2}{r}$

Spring $\longrightarrow \frac{1}{2}kx^2$

Electrostatics $\longrightarrow \frac{kq_1q_2}{r}$ (charge)

$\longrightarrow -\vec{p}\cdot\vec{E}$

magnets $\longrightarrow -\vec{M}\cdot\vec{B}$

Elasticity $\longrightarrow \frac{1}{2}$ (stress)(strain)(volume)

work energy theorem

$W_{external} + W_{conserv} + W_{non\text{-}conser} = \Delta K.E$

if $W_c = \Delta K$ or $\Delta U + \Delta K = 0$

then

$M.E = P.E + K.E$ (law of energy conserv)

conservative force, $f = \frac{-dU}{dx}$

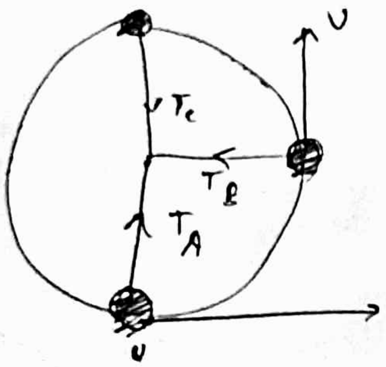
$\longrightarrow$ stable eq. $\frac{d^2U}{dx} > 0$ (can do SHM)

$\longrightarrow$ unstable eq. $\frac{d^2U}{dx} < 0$

$\longrightarrow$ neutral $\frac{d^2U}{dx} = 0$

Potential Energy

- elastic → $U = \frac{1}{2} \times$ stress $\times$ strain
- gravit → $U = mgh$
  - → $U = -Gm_1m_2/r$ (only if r is very big) used
- spring → $\frac{1}{2}kx^2$
- magnetism → $U = -\vec{M}\cdot\vec{B}$
- Electrostatics → $U = kq_1q_2/r$
- Dipole → $U = -\vec{p}\cdot\vec{E}$



vertical circular motion

$v \geq \sqrt{5gr}$ (complete circle)

both cases

1. string + bob
2. A particle inside a hollow sphere

$v \geq \sqrt{4gr}$ (complete circle)

2. mass attached to light rod.

if $v > \sqrt{5gr}$

$T_A - T_B = 3mg$
$T_A - T_C = 6mg$ } found by applying ~~Loss in K.E~~ Gain in G.E = Loss in K.E

Power (unit: Watt)

$P_{av} = \frac{\Delta W}{\Delta t}$

or

$P_{inst} = \frac{dw}{dt} = \frac{d(F.l)}{dt} = \frac{F\,dF}{dt} + \frac{l\,dl}{dt}$ ~ only if F & l are not const.

$= F\frac{d\vec{l}}{dt}$ → only if F is const.

AYUSH DUTTA

1. charge flow

$$q_{flown} = \int_{t_1}^{t_2} i(t)\, dt$$

0 1 2 3 4 5 6 7 8 9
B B R O Y G B V G W

Black Green
Brown Blue
Red Violet
orange Gray
yellow white

lighter → colourful
↓
monochrome

Colour code formula

$[(D_1 \times 10) + D_2] + 10^{D_3} \pm$ Tolerance

Golden is always weight less 5%.
Silver is always heavier 10%.

Import. current parameters

$v_d = \frac{eE\tau}{m}$ ← avg. time between collisions between two. (relaxation time)

$I = neAv_d$

n = no. of free electrons per volume (electronic density, volumetric)

Mobility $(\mu) = \frac{v_d}{E}$

current density, areal $(j) = I/A$

$I = \vec{J} \cdot \vec{A}$

$E = \rho J$ (ρ → resistivity)

$\Rightarrow \rho = \frac{m v_d (A)}{\tau . ne^2 A v_d} = \frac{m}{\tau ne^2}$

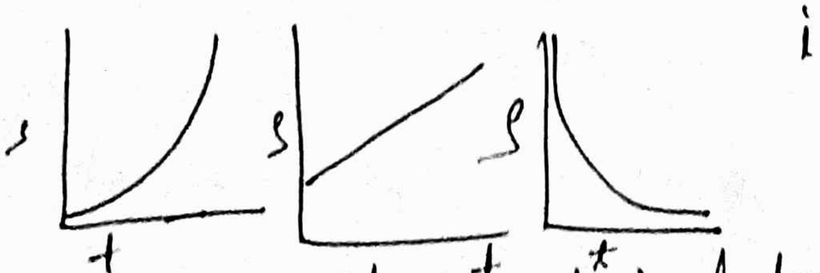
variation with temperature (T)

$T\uparrow, \tau\downarrow$

$\Rightarrow \rho \propto \frac{1}{\tau}$, hence $\rho\uparrow$

For small variation in temp

$$\frac{R_{T_2}}{R_{T_1}} = (1 + \alpha \Delta T)$$



copper — nichrome, manganin, constantan — semiconductor

Combination of cell

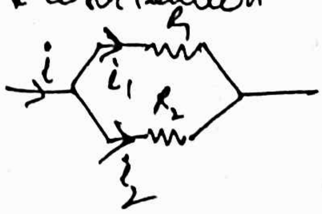
Series: $E_{eq} = E_1 + E_2 + \dots$

parallel: $E_{eq} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \dots$

r → internal resistance

Circuit Analysis techniques

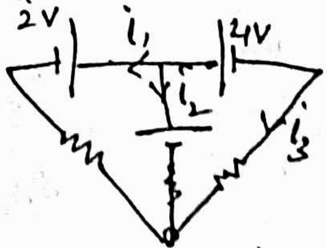
i-distribution



(applicable in series) (same i)

$i_1 = \frac{i R_2}{R_1 + R_2}$, $i_2 = \frac{i R_1}{R_1 + R_2}$

Point potential method



$i_1 + i_2 + i_3 = 0$

$q = ne^-$

$e = 1.6 \times 10^{-19} C$

$m_e = 9.3 \times 10^{-31} kg$

Colulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = k \approx 9 \times 10^9 \ Nm^2/C^2$$

ϵ_0 : permittivity of free space

$\hookrightarrow 8.85 \times 10^{-12} \ C^2/Nm^2$

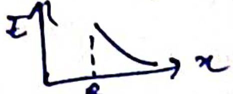
(order 12)

Field due to charged sphere

solid/Hollow (case 1) : conductor

$x < R$; $E = 0$ (equipotential)

$x \geq R$, $E = \frac{kQ}{x^2}$



solid (case 2) : insulator

$x < R$; $E = \frac{kQx}{R^3}$

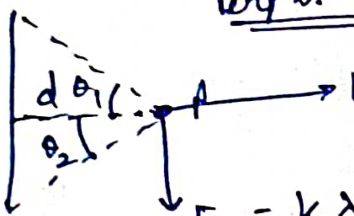
or

$$E = \frac{\rho x}{3\epsilon_0}$$

$x \geq R$; $E = \frac{kQ}{x^2}$ (same outside)

Feild due to line charge

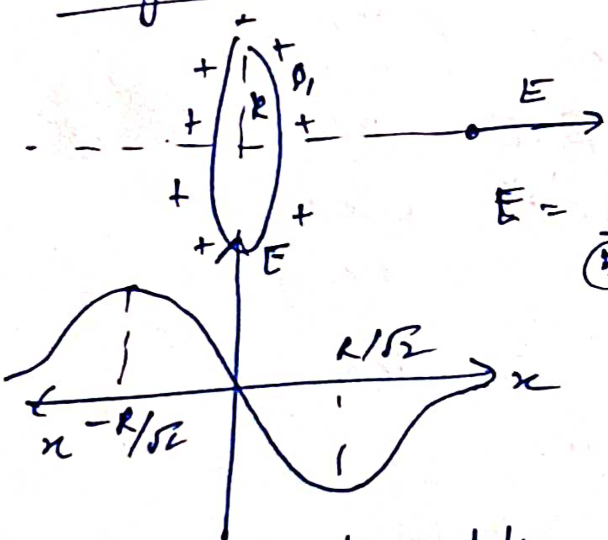
Perp is sign (+) , para is cos (-)



$$E_\perp = \frac{k\lambda}{d}(\sin\theta_2 + \sin\theta_1)$$

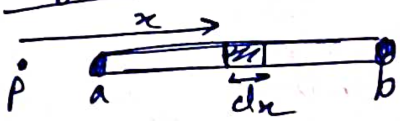
$$E_\parallel = \frac{k\lambda}{d}(\cos\theta_2 - \cos\theta_1)$$

Feild due to charged ring



$$E = \frac{kQx}{(x^2+R^2)^{3/2}}$$

Field in Non-uniform charge distribution (case-1 rod)



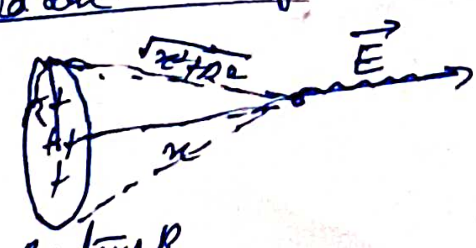
Linear charge density $\rightarrow \lambda(x)$ (fun. of x)

$$dq = \lambda(x) dx$$

$$E_p = \frac{k(dq)}{x^2}$$

$$\Rightarrow E_p = \int_a^b \frac{k(\lambda(x) \cdot dx)}{x^2}$$

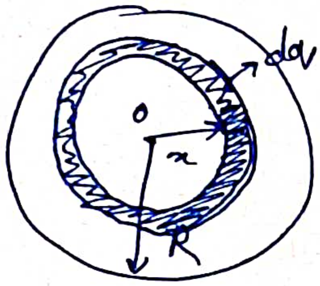
Field due to charged disc



Radius R

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

Field in Non-uniform charge distrias (case-2 Areal sphere)



$E = \frac{kQ_{in}}{R^2}$

$Q_{in} = \int_0^R \rho(x) \cdot 4\pi x^2$

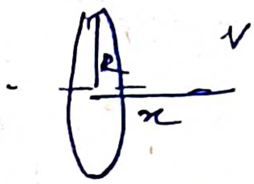
$dq = \rho(x) \cdot 4\pi x^2$

↓

Areal charge density

Electrostatic potential

case-1 (ring)



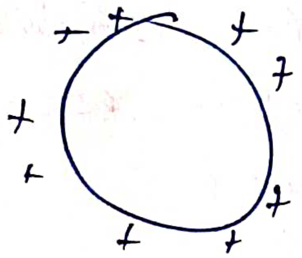
$V = \frac{kQ}{\sqrt{x^2+R^2}}$

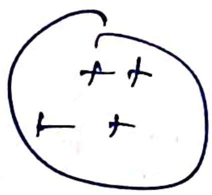
Self energy

case-1 (hollow)

case-2 (solid)

case-3 (discrete system)





q_1

r

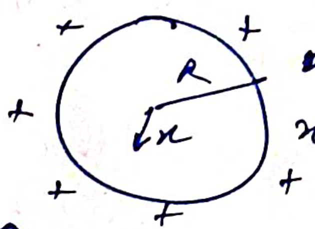
q_2

$U = \frac{kq_1q_2}{r}$

$U = \frac{kQ^2}{2R}$

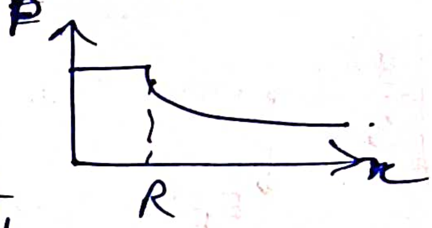
$U = \frac{3kQ^2}{5R}$

case-2 (shell) (conductor)



$x < R, V = \frac{kQ}{R}$

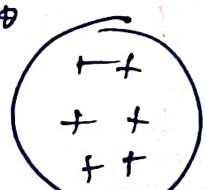
$x > R; V = \frac{kQ}{x}$



dipole field at general point

$E_{net} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

case-3 (insulator)



$x < r; V = \frac{kQ}{2R^3}(3R^2-x^2)$

$x > r; V = \frac{kQ}{x}$

E-V relation

$E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{dy}\hat{j} - \frac{\partial V}{dz}\hat{k}$

$\Delta V = -\int \vec{E} \cdot \vec{dr}$

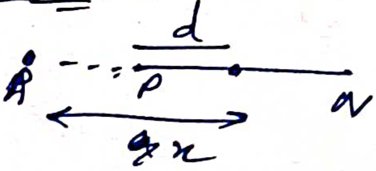
Dipole

Torque : $\vec{p} \times \vec{E}$

potential energy : $-\vec{p} \cdot \vec{E}$

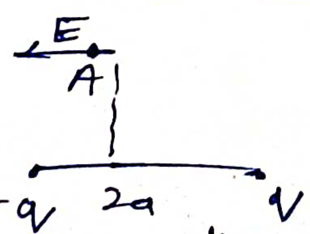
Field of a dipole

case-1 Axial



$E = \frac{2kp}{r^3}$

case-2 Equitorial

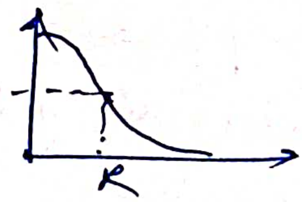


$E = \frac{kp}{r^3}$

Gauss law

$\oint \vec{E} \cdot dA = \frac{q_{in}}{\epsilon_0}$

$= kQ_{in}\Omega$



$\Omega = 2\pi(1-\cos\phi)$

Earthing

v of body remains 0.

Doppler's effect

Red shift (increase in frequency)
Blue shift (increase in frequency)

distance ↑ frequency ↓
⇒ distance $\propto \frac{1}{\text{frequency}}$

formula

$$\frac{\Delta f}{f} = \frac{-V_{radial}}{c} \quad (\text{only if } V_{rad} << c)$$

Δf = change in frequency
= $f_{apparent} - f_{source}$
= $f_{final} - f_{initial}$

V_{rad} = radial component of velocity

$\Rightarrow f\lambda = c$

$$\frac{\Delta f}{f} = \frac{-\Delta\lambda}{\lambda}$$

$$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{V_{radial}}{c} \quad \left\{ \text{used to find radial velocity of distant object} \right\}$$

Q. With what speed should a galaxy move outward with respect to Earth so that the sodium-D line at wavelength 5890Å is observed at 5896Å

$\Rightarrow \Delta\lambda = 6$Å
$\lambda = 5890$Å

$$V_{rad} = -\left(-\frac{6}{5890}\right)(-3\times10^8) = \left(\frac{18000\times10^5}{5890}\right)$$

$= 3.05\ m/s \times 10^5\ m/s$

(i) $= \frac{3.06\times10^5}{10^3} = 306\ km/s$

5.056
5890) 18000
-17670
33000
-29450
35500
-35340
...