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1.7 The current $i = 24 \cos 4000t$ Amps.

20 points

Or $i = \frac{dq}{dt} = 24 \cos 4000t$

Hence, $dq = 24 \cos 4000t \, dt$

$$q = \int_0^t 24 \cos 4000t \, dt$$

$$= 24 \cdot \frac{\sin 4000t}{4000} \Big|_0^t$$

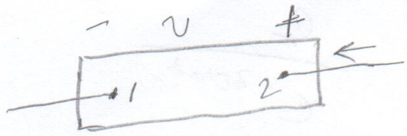
$$= \frac{24}{4000} \sin 4000t \quad (\text{since } \sin 0 = 0)$$

So $q(t) = 6 \times 10^{-3} \sin 4000t$

$= 6 \sin 4000t$ milli Coulombs.

1.15 figure 1.6 (d)

20 points



$v = -20V$

$i = 5A$

$p = vi$

7 a) $p = vi = -20 \times 5 = -100$ Watts
Since Power is -ve, it is being delivered by the box

7 b) Entering

6 c) Gain

1.18 $v = 75 - 75e^{-1000t}$ Volts

30 points $i = 50e^{-1000t}$ mAmps.

(a) $p = vi = (75-75e^{-1000t})(50e^{-1000t} \times 10^{-3})$

15 $= (75-75e^{-1000t})(0.05e^{-1000t})$

or $p = 3.75e^{-1000t} - 3.75e^{-2000t}$ Watts ← (A)

→ Maximum power happens at the 't' when $\frac{dp}{dt}$ is zero.

$$\frac{dp}{dt} = -3.75 \times 1000 \times e^{-1000t} - (-2000 \times 3.75 \times e^{-2000t})$$

$$= -3750e^{-1000t} + 7500e^{-2000t}$$

Make this equal 0.

So, $-3750e^{-1000t} + 7500e^{-2000t} = 0$

or $7500e^{-2000t} = 3750e^{-1000t}$

or $2e^{-2000t} = e^{-1000t}$

$$2 = \frac{e^{-1000t}}{e^{-2000t}}$$

$$2 = e^{1000t}$$

or $\ln 2 = 1000t$

Hence $t = \frac{\ln 2}{1000} = \frac{0.693147}{1000} = 693.147 \times 10^{-6}$

$= 693.147$ µSeconds

So, Maximum power is at $t = 693.147$ µSeconds

Since $p = 3.75e^{-1000t} - 3.75e^{-2000t}$ Watts

max p is at $t = 693.147\ \mu s$

$= 3.75 * e^{-1000 * 693.15 \times 10^{-6}} - 3.75 * e^{-2000 * 693.15 \times 10^{-6}}$

$= 3.75e^{-0.69315} - 3.75e^{-1.3863}$

$= 1.8749 - 0.9374 = 0.9375$

$= 937.5 \times 10^{-3}$

or $p_{max} = 937.5$ milli Watts

(b) Since $p = \frac{dw}{dt}$

15

Energy $w = \int p\, dt$

$= \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]dt$

$= \left. \frac{3.75}{-1000} e^{-1000t} - \frac{3.75}{-2000} e^{-2000t} \right]_0^\infty$

Since e^{∞} is 0 and e^0 is 1, this becomes

$w = \frac{3.75}{1000} - \frac{3.75}{2000} = 0.001875$

$= 1.875 \times 10^{-3}$

or Energy $= 1.875$ milli Joules

1.25 $\quad v = 100e^{-50t}\sin 150t$ Volts

30 points $\quad i = 20e^{-50t}\sin 150t$ Amps

a) 10 points $\quad$ power $p = vi$

$$= \left(100e^{-50t}\sin 150t\right) \times \left(20e^{-50t}\sin 150t\right)$$

or $p = 2000\, e^{-100t}\sin^2 150t$

at $t = 20\times10^{-3}$ seconds; $p = 2000\cdot e^{-100\times20\times10^{-3}}\cdot\sin^2 150\times20\times10^{-3}$

or $p = 2000\cdot e^{-2}\cdot\sin^2 3 = 2000 * 0.135335 * 0.002739$

$p = 0.74138$ Watts

(OR)

a) $\quad p = vi$

at $t = 20\times10^{-3}$; $v = 100 * e^{-50\times20\times10^{-3}} * \sin 150\times20\times10^{-3}$

$= 100 * e^{-1} * \sin 3$

$= 100 * 0.3679 * 0.05233$

$= 1.9254$

at $t = 20\times10^{-3}$, $i = 20e^{-50\times20\times10^{-3}} * \sin 150\times20\times10^{-3}$

$= 20e^{-1}\sin 3$

$= 0.3850$

Hence, $p = vi$

(at 20 ms) $= 0.7414$ Watts

b) $W = \int_0^\infty p\,dt$

20 points

$$W = \int_0^\infty \left(2000\, e^{-100t} \sin^2 150t\right) dt$$

We can eliminate the $\sin^2$ term by using:

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$W = 2000 \int_0^\infty e^{-100t} * \left(\frac{1}{2} - \frac{\cos 300t}{2}\right) dt$$

$$= 1000 \int_0^\infty e^{-100t}\, dt - 1000 \int_0^\infty e^{-100t} \cdot \cos 300t\, dt$$

A $-$ B

to solve Part B: $\int_0^\infty e^{-100t} \cos 300t\, dt$

Use the formula:

$$\int e^{ax} \cos bx\, dx$$

$$= \frac{1}{a^2 + b^2} e^{ax} (a \cos bx + b \sin bx)$$

$$w = \left. \frac{1000 e^{-100t}}{-100} \right|_0^\infty$$

$$- 1000 \left\{ \frac{e^{-100t}}{(100)^2 + (300)^2} \left[-100 \cos 300t + 300 \sin 300t \right] \right\} \Bigg|_0^\infty$$

Applying the limits $t = \infty$, Since e^{∞} is 0, all the factors become $\underline{0}$.

$t = 0$,

$$w = 10 - 1000 \left[\frac{+100}{10,000 + 90,000} \right]$$

$$\text{So, } w = 10 - \frac{1000 \times 100}{100,000}$$

$$= 10 - 1$$

$$\underline{w = 9 \text{ Joules}}$$