Ch 6 Homework Problems (100 marks) (25 each)

Inductors & Capacitors

Pr 6.1
25 marks

The current in a $50 \mu H$ inductor is known to be

$$i_L = 18te^{-10t} \text{ A for } t \geq 0.$$

a). find voltage, v_L.

$$v_L = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = 18\left[t(-10e^{-10t}) + e^{-10t}\right]$$

$$= 18e^{-10t}(1 - 10t).$$

$$\text{So } v_L = 50 \times 10^{-6} * 18e^{-10t}(1 - 10t)$$

$$= 0.9e^{-10t}(1 - 10t) \text{ millivolts, for } t > 0.$$

b). power at 200 milliseconds

$$p = vi$$

$$= \left(0.9e^{-10t}(1 - 10t) * 10^{-3}\right) * \left(18te^{-10t}\right)$$

$$p(200ms) = (0.9e^{-10*200\times10^{-3}}(1 - 10*200\times10^{-3}) * 10^{-3}) * (18te^{-10*200\times10^{-3}})$$

$$= \underset{v}{(-0.12 \times 10^{-3})} * \underset{i}{(0.487)}$$

$$= -58.4 \times 10^{-6} \text{ Watts}$$

c). Since $p_{(200ms)}$ is $-ve$, the inductor is Delivering!

d) Energy $w = \frac{1}{2} L i^2$

$$w_{(200ms)} = \frac{1}{2} * (50 \times 10^{-6}) * (.487)^2$$

$$w_{(200ms)} = 5.93 \times 10^{-6} \text{ Joules}$$

e). Max energy: Energy in inductor is maximum when current is maximum.

$$i_L = 18te^{-10t} \text{ Amps}$$

$$\frac{di_L}{dt} = 18e^{-10t}(1 - 10t)$$

This is zero at max i_L.

So $18e^{-10t}(1-10t) = 0$

Thus, $1 - 10t = 0$, or $t_{max} = \frac{1}{10} = 0.1$ sec

→ Max i_L happens at $t = 0.1$ seconds.

Also, $i_{L(max)} = 18 * 0.1 * e^{-10*0.1}$

$= 662 \times 10^{-3}$ Amps.

So, $w_{max} = \frac{1}{2} L i^2_{max} = \frac{1}{2} * 50 \times 10^{-6} * (662 \times 10^{-3})^2$

$= 10.96 \times 10^{-6}$ or 10.96 micro Joules

Pr 6.13 Voltage across a 5 μF capacitor is

25

$$v_c = 500t\,e^{-2500t}\ \text{V} \quad \text{for } t \geq 0$$

a) find current i_c for $t > 0$.

$$i = C\frac{dv}{dt}$$

$$= (5\times10^{-6})\left[500\left(-2500t\,e^{-2500t} + e^{-2500t}\right)\right]$$

$$= 2.5\,e^{-2500t}(1-2500t) \times 10^{-3}\ \text{Amps.}$$

$$i = 2.5\,e^{-2500t}(1-2500t)\ \text{mAmps.}$$

b). power at $t = 100\ \mu S$.

$$p = vi$$

$$v_{100\mu s} = 500 * 100\times10^{-6} * e^{-2500 * 100\times10^{-6}}$$

$$= 0.05 * 0.7788$$

$$= 38.94\times10^{-3}\ \text{Volts.}$$

$$i_{100\mu s} = 2.5\,e^{-2500\times100\times10^{-6}}(1-2500\times100\times10^{-6})$$

$$= 2.5 * e^{-0.25} * (1-0.25)\ \text{mAmps}$$

$$= 1.46\ \text{mA}$$

So $p_{100\mu s} = vi = (38.94\times10^{-3}) * (1.46\times10^{-3}) = 56.86\ \mu W$

c) Since p is positive, the Capacitor is Absorbing power.

d) Energy $w = \frac{1}{2} C v^2$

at $100\mu s$, $v_{100\mu s} = 38.94 \times 10^{-3}$ Volts

Hence $W_{100\mu s} = \frac{1}{2} \times (5 \times 10^{-6}) \times (38.94 \times 10^{-3})^2$

$= 3.79 \times 10^{-9}$

$= \underline{3.79 \text{ n Joules}}$

e) In a Capacitor, max Energy is at max Voltage.

$v_c = 500t\,e^{-2500t}$ Volts

$\frac{dv_c}{dt} = 500\,e^{-2500t}(1 - 2500t)$. This is 0, when $t = \frac{1}{2500}$ seconds

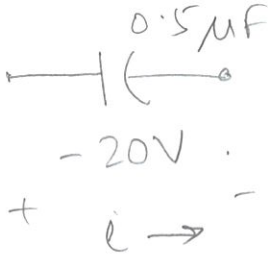
So $v_{max} = 500 \times \frac{1}{2500} \times e^{-1} = 73.57$ mV.

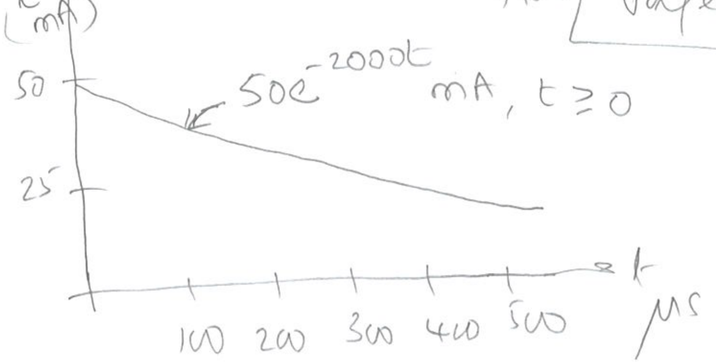
Hence $p_{max} = p_{\frac{1}{2500}\text{ seconds}} = \frac{1}{2} C v_{max}^2$

$= \frac{1}{2} \times (5 \times 10^{-6}) \times (73.57 \times 10^{-3})^2 = \underline{\underline{13.53 \text{ nJ}}}$

6.19
25

HW/ Page 5





Given $v_c(0) = -20V$

a) Find energy at $t = 500\mu s$

$$i_c = 50e^{-2000t} \times 10^{-3} \text{ Amps}$$

So $v_c(t) = \frac{1}{C}\int_0^t i_\tau d\tau + v(0)$

or $v_c(t) = \frac{1}{0.5\times10^{-6}}\int_0^t 50\times10^{-3} e^{-2000\tau} d\tau - 20$

At t = 500μs, $v = \frac{1}{0.5\times10^{-6}} * 50\times10^{-3} * \left(\frac{1}{-2000}\times e^{-2000t}\right)\Big|_0^{500\mu s} - 20$

$$= \frac{100,000}{2000}(e^{-1} + e^{0}) - 20 = 50(1-e^{-1}) - 20$$

$$= 11.61 \text{ volts}$$

$$w = 1/2\, Cv^2 = \frac{1}{2}\times(0.5\times10^{-6})\times 11.61^2$$

So $w_{500\mu s} = 33.7\ \mu J$

b). Energy at ∞: $v_\infty = 50*(-e^{-\infty} + e^{0}) - 20$. (from above eqn for v)

$$= 30 \text{ volts}$$

$$w_\infty = \frac{1}{2} * 0.5\mu F * 30 = 225\ \mu J$$

Pr 6.24

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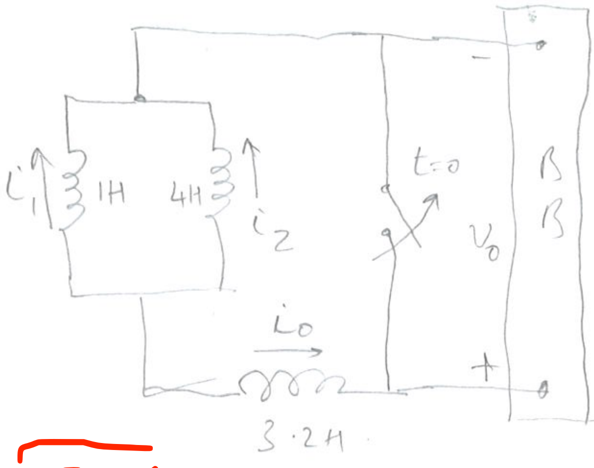
$v_o = 2000e^{-100t}$

$i_1(0) = -6A$

$i_2(0) = 1A$

The switch is opened at $t=0$

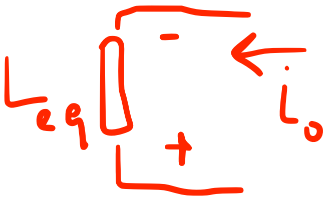


a) $i_o = -(i_1 + i_2)$

$i_o(0) = -[i_1(0) + i_2(0)]$

$= -[-6+1]$

$= 5$ Amps



$L_{eq} = 1H \| 4H + 3.2H$

$= 4H$ ✓

b) $v_o(t) = 2000e^{-100t}$

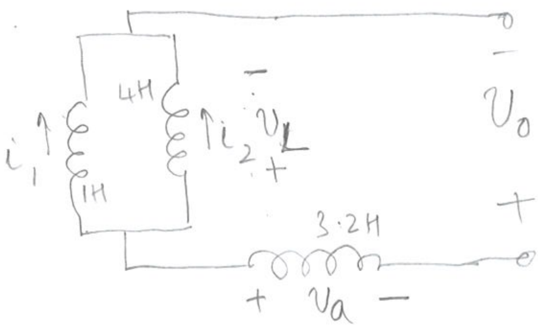
hence $i_o = \frac{1}{L_{eq}}\int v\,dt = -\frac{1}{4}\int_0^t 2000e^{-100\tau}d\tau + i_o(0)$

$= -\frac{1}{4} * 2000 * \frac{1}{-100} e^{-100\tau}\Big|_0^t + 5$

$= 5(e^{-100t} - 1) + 5$

$i_o(t) = 5e^{-100t}$ Amps for $t \geq 0$

c) ~~A~~ to find i_1 & i_2, we should first find the voltage across them



$v_o = 2000e^{-100t}$

KVL $+v_L - v_o - v_a = 0$

or $v_L = v_o + v_a$

$$v_L = v_0 + v_a$$

$$v_L = 2000e^{-100t} + L\frac{di_0}{dt}$$

$$= 2000e^{-100t} + 3.2\frac{d}{dt}(5e^{-100t})$$

$$= 2000e^{-100t} + 3.2*(-500e^{-100t})$$

$$= 2000e^{-100t} - 1600e^{-100t}$$

So $v_L = 400e^{-100t}$ Volts

Now $i_1 = \frac{1}{L}\int_0^t v_c\,d\tau + i_1(0)$ & $i_2 = \frac{1}{L}\int_0^t v_c\,d\tau + i_2(0)$

$i_1 = \frac{1}{1}\int_0^t 400e^{-100\tau}d\tau - 6$ & $i_2 = \frac{1}{4}\int_0^t 400e^{-100\tau}d\tau + 1$

So: $i_1 = \left.\frac{400e^{-100t}}{-100}\right|_0^t - 6$ & $i_2 = \frac{1}{4} * \left.\frac{400e^{-100t}}{-100}\right|_0^t + 1$

$i_1 = \frac{1}{1}(-4e^{-100t} + 4) - 6$ & $i_2 = \frac{1}{4}(-4e^{-100t} + 4) + 1$

or $i_1 = -4e^{-100t} - 2$ Amps & $i_2 = -e^{-100t} + 2$ Amps (for $t \geq 0$).

c). Initial energy in the inductors ; $w(0) = w_1(0) + w_4(0) + w_{3.2}(0)$

$$w = \frac{1}{2}Li^2$$

So: $w(0) = \frac{1}{2} * 1 * (-6)^2 + \frac{1}{2} * 4 * (2)^2 + \frac{1}{2} * 3.2 * 5^2$

$= 60$ Joules.

f). W delivered to the Black box is

$$w = \frac{1}{2} L_{eq} i_{\infty}^2 = \frac{1}{2} * 4 * (5 * e^{-100 * \infty})^2$$

$\hookrightarrow$ since $i_0 = 5e^{-100t}$

$$= \frac{1}{2} \times 4 \times 5^2$$

$$w = 50J$$

g). Energy trapped in the inductors =

Initial energy − delivered energy

$= 60 - 50$

$= 10$ Joules