Adaptive Page Allocation: Cost and Replication Analysis

Maksymilian Neumann

May 27, 2025

Abstract

We analyze the performance of the COUNT replication algorithm under varying write probabilities p and threshold values D. We measure two primary metrics: the average cost per request and the average maximum number of replicas. Results are presented via bar charts and heatmaps to illustrate trends and trade-offs.

1 Introduction

In distributed shared memory systems, dynamic replication can reduce read and write costs by adaptively placing copies. The $COUNT$ algorithm tracks writes per node and replicates once a threshold D is exceeded. We evaluate how write intensity p and D affect (1) the average cost per request and (2) the maximum number of replicas.

2 Methodology

We simulate a 64-node clique. For each (D, p) pair, we perform 10 000 runs of 65 536 randomly chosen read/write requests (write with probability p, read with probability $1 - p$).

- **Read cost:** 0 if local replica exists, 1 otherwise.

- **Write cost:** $\#\text{replicas} - 1$ if requester holds a copy, else $\#\text{replicas}$.

- **Replication rule:** replicate to node when its local writes exceed D, incurring D in write cost.

Metrics are averaged across runs.

3 Results
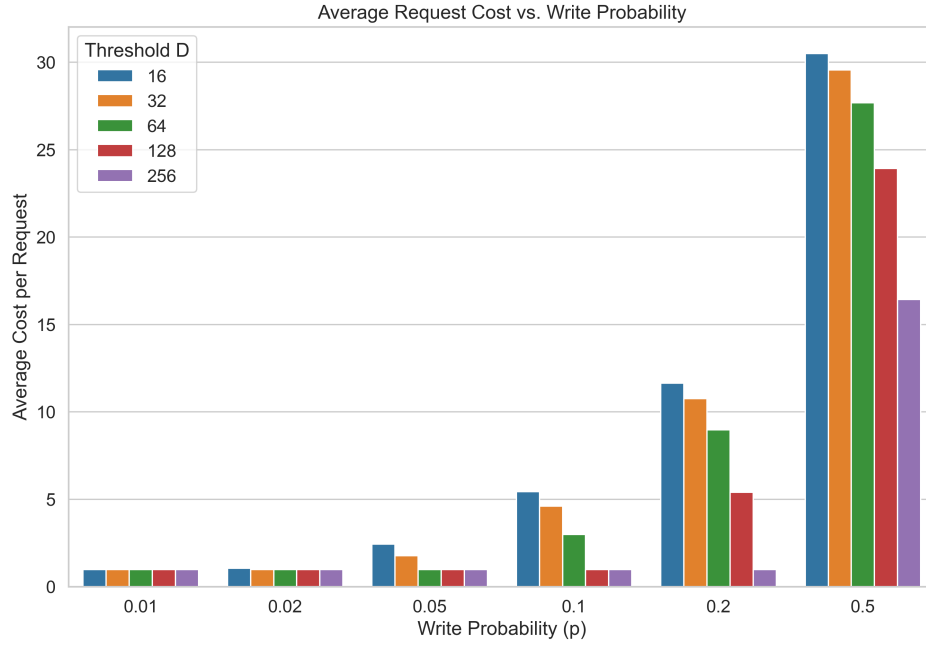
3.1 Average Cost per Request

Figure 1: Average request cost vs. write probability p for various thresholds D. Bars show mean cost over 10000 runs.

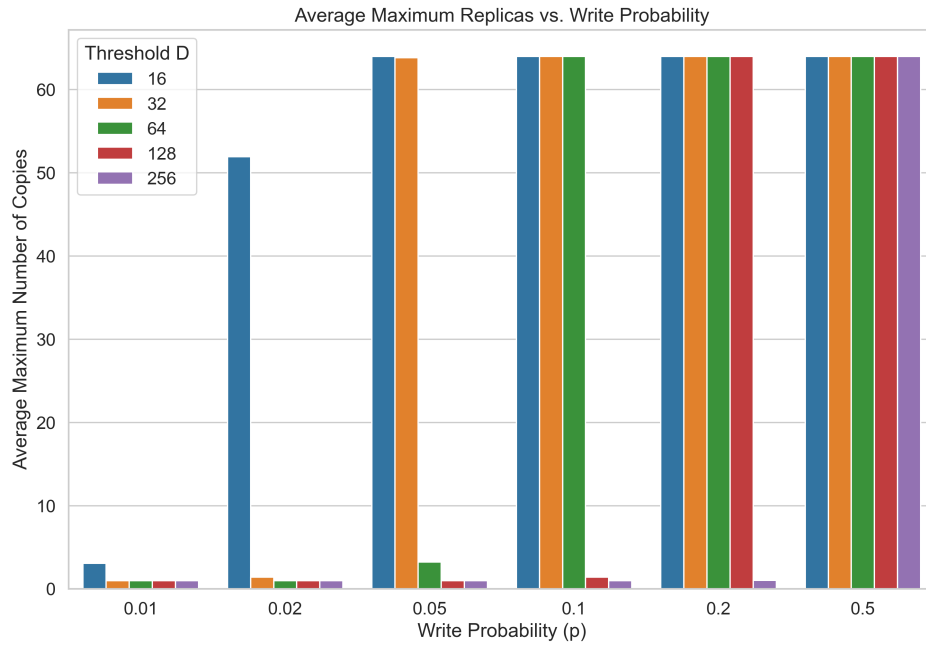## 3.2 Average Maximum Number of Replicas

Figure 2: Average maximum number of replicas vs. write probability p for various thresholds D.

3.3 Heatmap of Average Cost

Figure 3: Heatmap showing average cost per request as function of D (rows) and p (columns).
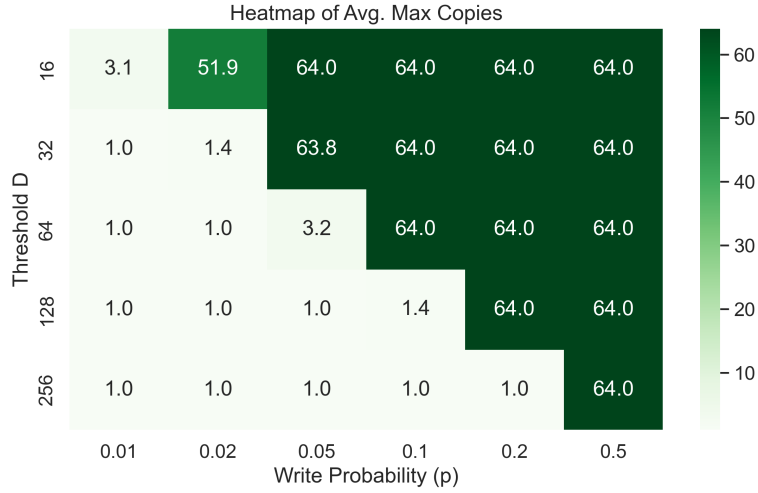
3.4 Heatmap of Average Max Copies

Figure 4: Heatmap showing average maximum replicas as function of D and p.

4 Discussion

- **Cost trends:** Lower D (e.g. 16) replicates aggressively, raising write overhead at high p but reducing read cost at low p. Higher D delays replication, which is beneficial when writes are rare.

- **Replication vs. writes:** For $p \geq 0.1$, most thresholds achieve full replication (64 copies), as shown by the plateau in Fig. 2 and heatmap (4). At moderate p, intermediate D values strike balance.

- **Design implication:** Choose D near the expected write count per node between reads to minimize total cost.