Estimating gain and optimal strategy

Let A[i, j, e] be the expected gain of an optimal first-player strategy from coin i to coin j (exclusive) when the adversary is in the state $e \in {G, O}$. The adversary is in state G when he follows greedy strategy, and in the state O if he plays optimally.

The adversary (or alternatively, the second player) is modelized by a Markov chain.

The following dynamic program computes the array A for plays of even length, $ (j-i) = 2\N$:

$$ \begin{cases} A[i. i, e] = 0 & i = j \ A[i, j, e] = max \begin{cases} X_i + A[i + 1,j,e]\ X_{j-1} + A[i, j - 1, e] \end{cases} & i \lt j \end{cases} $$

When $j-i=2\N+1$, then the value of a game is calculated the following way:

$$ \begin{cases} A[i, j, G] = p \times \begin{cases} -X_{j-1} + A[i][j- 1][G] &greedy[i][j] = R\ -X_{i} + A[i + 1][j][G] &greedy[i][j] = L\ \end{cases}\ +(1-p) \times \begin{cases} -X_{j-1} + A[i][j- 1][O] &opt[i][j] = R\ -X_{i} + A[i + 1][j][O] &opt[i][j] = L\ \end{cases}\ \A[i, j, O] = q \times \begin{cases} -X_{j-1} + A[i][j- 1][O] &opt[i][j] = R\ -X_{i} + A[i + 1][j][O] &opt[i][j] = L \end{cases}\ +(1-q)\times \begin{cases} -X_{j-1} + A[i][j- 1][G] &opt[i][j] = R\ -X_{i} + A[i + 1][j][G] &opt[i][j] = L \end{cases}

\end{cases} $$