

## COMS10014 Worksheet: A-level revision

These questions are taken or adapted from the University of Bristol, School of Mathematics workbook of A-level material that students should be fluent in when they begin their studies.

### Polynomials

- Factorise  $x^2 - 1$ ,  $a^2 + 4b^2 - 4ab$  and  $t^3 - 7t + 6$ . For the last one, note the root at 1.
- Find the roots of  $x^2 - 5x + 6 = 0$ ,  $x^2 - x - 1 = 0$  and  $x^4 - 3x^2 = 0$ .
- Convert  $x^2 - 2x + 6$  into the form  $(ax + b)^2 + c$ .
- Divide  $x^3 + 5x^2 - 2x - 24$  by  $(x + 4)$  with remainder.
- Express the following as partial fractions:  

$$\frac{2}{x^2 - 1}, \quad \frac{4x + 1}{(x + 1)^2(x - 2)}, \quad \frac{x + 13}{(x + 1)(x - 2)(x + 3)}$$
- Expand  $(2 + \frac{3}{x})^5$  using binomial expansion.

### Calculus

- Analyse the following functions to determine stationary points and extrema (local minima and maxima):  $x^2 + 2$ ,  $x^3 + 3x + 3$ ,  $x^3 - 3x^2 + 3x$ .
- Consider  $f(x) = x^2$ . Sketch the following in different colours in the same graph:  
 $f(x)$ ,  $2f(x)$ ,  $2f(x) + 3$ ,  $f(x - 2)$
- Find the derivatives of the following with respect to  $x$ :  
 $\sin(x^2)$ ,  $a^x$ ,  $\ln(x^a + x^{-a})$ ,  $(\sin(x^2 + 1))^2 + 3\sin(x^2 - 1)$
- Integrate the following. For the second one, assume  $x \in (0, 1)$ :  

$$\int \frac{1}{2 + x^2} dx, \quad \int \frac{1}{x\sqrt{1-x}} dx, \quad \int_0^\infty x e^{-x} dx, \quad \int_0^1 \frac{x^2 + 1}{x^3 + 3x + 2} dx, \quad \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Note: for the last one, where else have you seen this expression?