**Automata Theory and Computability**

*Assignment 2 (Myhill-Nerode and CFGs)*

(Total marks 80. Due on Thu 25th Feb 2025)

1. Let $L$ be a regular language over the alphabet $A$. Which of the following languages over $A$ are regular? Justify your answers. (*20*)

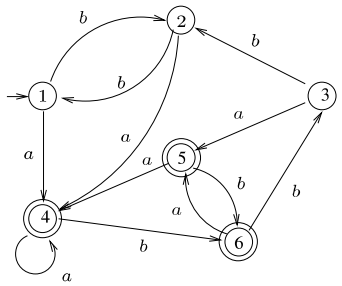
   (a) $\mathit{mid\text{-}thirds}(L) = \{v \mid \exists u, w : |u| = |v| = |w| \text{ and } uvw \in L\}$.

   (b) $\mathit{half}(L) = \{w \mid ww \in L\}$.

2. Describe the equivalence classes of the Myhill-Nerode relation $\equiv_L$ for the language $L = \{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. Depict the canonical DA for this language. (*10*)

3. Find a language $L$ for which the canonical Myhill-Nerode relation for $L$, $\equiv_L$, has all singleton equivalence classes. (*10*)

4. Minimize the DFA below using the algorithm done in class: (*10*)




5. Give a context-free grammar for the following language. Prove that your grammar is correct: (*10*)

   "Equal $a$'s and $b$'s" – i.e. $\{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. (Hint: give a grammar similar to the one for balanced parenthesis).

6. Give a context grammar for the language (*10*)

$$L_2 = a^*b^*c^* - \{a^n b^n c^n \mid n \geq 0\}.$$

7. Give an equivalent grammar in Chomsky Normal Form for the following CFG: (*10*)

$$\begin{aligned} S &\rightarrow aSbb \mid T, \\ T &\rightarrow bTaa \mid S \mid \epsilon. \end{aligned}$$