Automata Theory and Computability

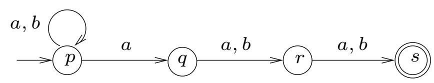
Assignment 1

(Total marks 90. Due in class on Tue 04th Feb 2025)

1. Give a DFA for the language of all strings over the alphabet $\{0, 1\}$ which contain an even number of 0's and least one 1. *(5)*

2. Show that the set of strings in $\{0, 1, 2\}^*$ which are base 3 representations of odd numbers, is regular. *(10)*

3. Consider the NFA below: *(15)*



   (a) Use the subset construction to obtain an equivalent DFA for the NFA below. Label each state of the DFA with the subset of states of the NFA that it corresponds to.

   (b) Give an 8 state DFA which accepts the same language.

4. Consider the language of nested C-style comments. The alphabet comprises characters "`/`", "`*`", and "`c`" (the latter symbol representing any ASCII character apart from "`/`" and "`*`"). The language allows all well-nested and complete comments. Thus strings like "`cc/*ccc*/c`" and "`cc/*cc/*ccc*/c*/ccc`" are in the language, but not "`cc/*c/*cc*/cc`". Is this language regular? Justify your answer. *(10)*

5. For a set of natural numbers X, define *binary*(X) to be the set of binary representations of numbers in X. Similarly define *unary*(X) to be the set of "unary" representations of numbers in X: *unary*$(X) = \{1^n \mid n \in X\}$. Thus for $X = \{2, 3, 6\}$, *binary*$(X) = \{10, 11, 110\}$ and *unary*$(X) = \{11, 111, 111111\}$.

   Consider the two propositions below:

   (a) For all X, if *binary*(X) is regular then so is *unary*(X).

   (b) For all X, if *unary*(X) is regular then so is *binary*(X).

   One of the statements above is true and the other is false. Which is which? Justify your answer. *(20)*

6. Give a language $L \subseteq \{a, b\}^*$ such that neither L nor $\{a, b\}^* - L$ contains an infinite regular set. *(10)*

7. Let L be an arbitrary subset of $\{a\}^*$. Prove that L^* is regular. *(10)*

8. Use the construction done in class to construct a regular expression corresponding to the language accepted by the DFA below (i.e. the expression corresponding to $L_{ss}^{\{s,p,a\}}$). *(10)*