1a. Solve: 1 - P[n + 1] = (1 - P[n])*(1 – p) with P[1] = p for P[n]. P[n]=1 - $(1 – p)^n$ which is the pmf of geometric distribution.
1b. Solve *-F'(x) == -f0 *(1 - F(x)), F(0) == 0. F(x) = 1 – exp(- f0 x)* which is the cdf of the exponential distribution.
1c. sigma^2

2. x = sqrt(3*u – floor(3*u)) + floor(3*u), u drawn from U[0,1]; maximize f(x)/g(x) = (3/2)*(x – floor(x)) to obtain a C so that 1/C is maximized. Thus the acceptance probability is 1/C = 2/3. If v is drawn from U[0,1] accept x generated as above if v less than or equal to (x – floor(x)).

3. a^n, a^n mod m

4.

(i) Assuming that the two counters operate independently of each other determine the expected number of waiting customers and their mean waiting time at each counter.

	Commercial	Personal	
λ	6/h	12/h	
μ	12/h	24/h	
$\rho = \frac{\lambda}{\mu}$	0.5	0.5	
$L_q = \frac{\rho^2}{1-\rho}$	0.5	0.5	**Answer.**
$W_q = \frac{\rho}{\mu(1-\rho)}$	5 min	2.5 min	**Answer.**

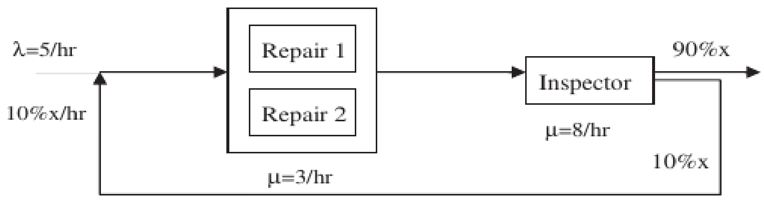
(ii) What is the effect of operating the two queues as a two-server queue with arrival rate 18/h and service rate 18/h? What conclusion can you draw from this operation?

	Two-server queue	
λ	18/h	
μ	18/h	
Number of servers (s)	2	
$\rho = \frac{\lambda}{s\mu}$	0.5	
$\alpha = \frac{\lambda}{\mu}$	1	
$p_0 = [\sum_0^1 \frac{\alpha^r}{r!} + \frac{\alpha^2}{2(1-\rho)}]^{-1}$	0.33	
$L_q = \frac{\rho\alpha^2 p_0}{2(1-\rho)^2}$	0.33	**Answer.**
$W_q = \frac{\alpha^2 p_0}{(2)\ 2\mu(1-\rho)^2}$	1.33 min	**Answer.**

Conclusion: The two-server queue operation is more efficient than the two single-server operations.

5. |x/a – 1| is sufficiently small

6.



$x = \frac{\lambda}{1-10\%} = 5.556/hr$

At the repair station: $w = \frac{1}{\mu(1-\rho^2)} = \frac{1}{3(1-(\frac{5.556}{(2)(3)})^2)} = 2.34hr$

At the inspection station: $w = \frac{1}{8(1-\frac{5.556}{8})} = 0.41hr$

The maximum arrival rate the system can handle without adding personnel is: $\lambda = (2)(3)(90\%) = 5.4/hr$ because the utilization at the repair stations are much higher than that at the inspection station, which indicates the repair stations are the bottleneck of the system.

7. c = 1. LQ = 2.82, lambda_e = 0.311, L = 3.75, cost 43.5
c = 2. LQ = 0.42, L = 1.66, cost = 28.6 improvement 14.9

8.(a) F(y) = 0, y < -1/2; F(y) = (2/3)*(y+1/2)^(3/2), y in [-½, ½);
F(y) = y + 1/6 – (2/3)*(y-1/2)^(3/2), y in [½, 3/2); F(y) = 1, y > 3/2
(b) Cov = 0 ; apply E(XY) = E(E(XY|X)= E(X E(Y|X)) = 0 [odd function].

9. 7*(m-1)/144 + 2*(m-2)/48 = (13*m-19)/144