DS290 Modelling and Simulation
Aug–Dec 2025 Semester

Full Marks: 25.
Nominal Time Allowed to Work on the Paper: 90 Minutes.

October 21, 2025

Important:

- Please indicate approximately what percentage of lectures you could attend from the beginning of the course.
- Please make sure to write your name and SR no. at the beginning of your answer script.
- The tables for t-distribution and the standard normal distribution are attached at the end of the question paper.

Q1. An electronic component with mean to failure of x years can be purchased for $2x$ thousand Rupees (thus, more reliable the component, the more expensive it is). The value of x is restricted to be between 1 and 10 years and the actual time to failure is modeled as exponentially distributed. The mission for which the component is used lasts for 1 year; if the component fails in less than one year, then there is a cost of 20,000 Rupees for early failure. What value of x should be chosen to minimize the expected total cost (purchase plus early failure)? (4 marks)

Q2. Let $W_{n+1} = \max(W_n + S_n - T_n, 0)$ be the recursive state update of an M/G/1 queue with infinite calling population and infinite system capacity, such that W_n is the waiting time at the n^{th} arrival, S_n the service time and T_n the inter-arrival time at the n^{th} arrival. Assume service times and inter-arrival times are iid. Given the correlation between any W_n and W_{n+1} is ρ_W, and W, the waiting time, has a long run average of w. Show that the above recursion generates an $AR(1)$ time series with suitable assumptions (state clearly) consistent with M/G/1 queue. Derive the convergence of the AR(1) process to the long run average and the long run variance of the waiting times W. (4 marks)

Q3. Let $\overline{y} := \dfrac{\sum_{j=1}^{n} y_j}{n}$ and let

$$\hat{y}_i = \overline{y} + \sum_{j=1}^{K} \hat{\beta}_j \left(x_{ji} - \frac{\sum_{i=1}^{n} x_{ji}}{n} \right)$$

be the predicted value of y via the fitted multiple linear regression model

$$Y_i = \beta_0 + \sum_{j=1}^{K} \beta_j x_{ji} + \epsilon_i,$$

with $x_1, x_2, \ldots, x_K$ as the independent variables. Denote the prediction error $e_i := y_i - \hat{y}_i$. Derive an unbiased estimator of $\mathrm{Var}(\epsilon_i) = \sigma^2$. (4 marks)

Q4. The following table shows the analysis of the average cost output from 10 replications of 5 models that are being considered for the simulation of a system. The objective is to select the model that outputs the least average cost. Calculate whether the following table has enough number of replications for selecting the model that outputs the least average cost with a 96% confidence level. Also, compute if the number of replications is adequate for the above selection if the practically significant difference is 3. (4 marks)

Table 1: Replication Data

Replication	Avg Cost 1	Avg Cost 2	Avg Cost 3	Avg Cost 4	Avg Cost 5
1	7.6373	5.2587	5.8389	8.2064	9.7678
2	6.5244	4.2275	4.8392	7.2047	8.8036
3	13.1426	15.3749	13.7859	16.1918	17.8318
4	12.9925	10.6375	11.2746	13.7282	15.3572
5	6.0970	3.9108	4.5280	6.8655	8.5000
6	6.2691	3.8416	4.3754	6.6956	8.1976
7	2.0640	4.6737	2.5447	4.9386	6.3736
8	6.8304	4.1287	4.7100	7.2374	8.7528
9	16.7394	14.2877	14.9347	17.4566	19.0692
10	6.3254	3.9487	4.5558	6.9718	8.5595
average	**6.5448**	**8.9464**	**7.1387**	**9.5497**	**11.1213**

Q5. Let $\overline{y} := (\sum_{j=1}^{n} y_j)/n$, and

$$\hat{y}_i = \overline{y} + \hat{\beta}_1 \Big(x_i - \frac{\sum_{j=1}^{n} x_j}{n} \Big)$$

be the predicted value of y via fitted simple linear regression with x as the independent variable. Let $e_i := y_i - \hat{y}_i$ be the prediction error, and $r_i := \hat{y}_i - \overline{y}$ be the regression component. Show that

$$\sum_{i=1}^{n} (y_i - \overline{y})^2 = \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} r_i^2.$$

(3 marks)

(b) Let $y_i, \ i = 1, \ldots, n$ be the time series outputs along the same sample path from the simulation of the same model. Assume the y_i outputs to be identically distributed with variance σ^2 and mean μ. Assume ρ_1 be the stationary lag-k auto-correlation. Find the expectation:

$$E\left[\frac{\sum_{i=1}^{n} y_i y_{i-1}}{\sum_{i=1}^{n} y_i^2}\right]$$

(2 marks)

Q6. Let Z_i be the actual average number of jobs in a shop for the i^{th} period. Let Y_i be the average number of jobs predicted by a simulation model for the same i^{th} period. For 4 periods, the data are as follows:

Table 2: Job Data

	1	**2**	**3**	**4**
$\boldsymbol{Z_i}$	21.7	19.2	22.8	19.4
$\boldsymbol{Y_i}$	24.6	21.1	19.7	24.9

(i) Conduct a statistical test to check the consistency of the actual system output and the simulation output. Use a 5% level of significance.

(ii) If the difference of 2 jobs is regarded as important to detect, what sample size (that is, how many periods of observation) is required to guarantee a probability of at least 0.8 for detecting the difference if it indeed exists. The level of significance may be taken as 5%.

(4 marks)

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599

Standard Normal Cumulative Probability Table

Cumulative probabilities for NEGATIVE z-values are shown in the following table.

Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003