## A Multi-start Variable Neighborhood Tabu Search Algorithm for the Cyclic Bandwidth Problem - Supplementary Material

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## 1 Detailed Experimental Results

The results of our algorithm MVNTS and its competitors (ITPS and NILS) on 202 instances are presented in Tables 3 to 5. Columns |V| and |E| represent the number of vertices and edges in the graph. Column  $Cb_b$  denotes the best cyclic bandwidth found by corresponding algorithm, while column  $Cb_a$  indicates the average value obtained over 20 runs. Column time gives the average runtime over 20 runs for ITPS, NILS, and MVNTS. Instances with runtime less than 0.01 seconds are marked as "< 0.01". Notably, the best solution value among MVNTS and the reference algorithms is highlighted in bold. Note that the name of graphs from families caterpillars, 2D meshes, 3D meshes and hypercubes are abbreviated as "cp", "m2D", "m3D" and "hc", respectively.

**Table 1.** Detailed results for the small-scale instances.

				ITP	S	NII		S		MV	NTS					ITPS			NILS			MVNTS		
Graph	V	E	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	$Cb_{\ell}$	$Cb_b \ Cb_a \ time$		Graph	V	E	$Cb_b$	$Cb_a$ time		$Cb_b Cb_b$		time	$Cb_b Cb_a$		time	
ср3	9	8	3	3	< 0.01	3	3	< 0.01	3	3	< 0.01	jgl009	9	32	4	4	< 0.01	4	4	< 0.01	4	4	< 0.01	
rgg010	10	45	5	5	< 0.01	5	5	< 0.01	5	5	< 0.01	jgl011	11	49	5	5	< 0.01	5	5	< 0.01	5	5	< 0.01	
cp4	14	13	3	3	< 0.01	3	3	< 0.01	3	3	< 0.01	cp5	20	19	4	4	< 0.01	4	4	< 0.01	4	4	< 0.01	
cycle20	20	20	1	1	0.05	1	1	< 0.01	1	1	< 0.01	m2D5x4	20	31	4	4	0.06	4	4	< 0.01	4	4	< 0.01	
path20	20	19	1	1	0.06	1	1	< 0.01	1	1	< 0.01	$can_24$	24	68	5	5	0.42	5	5	< 0.01	5	5	< 0.01	
cycle25	25	25	1	1	0.11	1	1	0.06	1	1	< 0.01	m2D5x5	25	40	5	5	0.06	5	5	< 0.01	5	5	< 0.01	
path25	25	24	1	1	0.19	1	1	< 0.01	1	1	< 0.01	cp6	27	26	5	5	< 0.01	5	5	< 0.01	5	5	< 0.01	
cycle30	30	30	1	1	0.26	1	1	0.04	1	1	< 0.01	m2D5x6	30	49	5	5	0.11	5	5	< 0.01	5	5	< 0.01	
path30	30	29	1	1	0.44	1	1	< 0.01	1	1	< 0.01	pores_1	30	103	7	7	0.02	7	7	< 0.01	7	7	< 0.01	
tree2x4	31	30	4	4	< 0.01	4	4	< 0.01	4	4	< 0.01	ibm32	32	90	9	9	0.02	9	9	< 0.01	9	9	< 0.01	
cp7	35	34	6	6	< 0.01	6	6	< 0.01	6	6	< 0.01	cycle35	35	35	1	1	0.53	1	1	0.22	1	1	< 0.01	
m2D5x7	35	58	5	5	0.23	5	5	< 0.01	5	5	< 0.01	path35	35	34	1	1	0.72	1	1	0.02	1	1	< 0.01	
bcspwr01	39	46	4	4	3.69	4	4	0.18	4	4	0.04	cycle40	40	40	1	1	1.0	1	1	0.28	1	1	< 0.01	
m2D5x8	40	67	5	5.1	56.35	5	5	< 0.01	5	5	< 0.01	path40	40	39	1	1	1.53	1	1	0.08	1	1	< 0.01	
tree3x3	40	39	7	7	< 0.01	7	7	< 0.01	7	7	< 0.01	bcsstk01	48	176	12	12	0.05	12	12	0.01	12	12	0.01	
bcspwr02	49	59	7	7	0.02	7	7	< 0.01	7	7	< 0.01	curtis54	54	124	8	8	0.39	8	8	0.04	8	8	0.05	
will57	57	127	6	6	0.14	6	6	0.17	6	6	0.09	$dwt_59$	59	104	6	6	0.21	6	6	0.1	6	6	0.06	
impcol_b	59	281	17	17	0.03	17	17	0.01	17	17	< 0.01	can_61	61	248	13	13	0.02	13	13	< 0.01	13	13	< 0.01	
$can_62$	62	78	6	6	0.41	6	6	0.46	6	6	0.15	m3D4	64	144	14	14	12.89	14	14	0.19	14	14	0.5	
$dwt_66$	66	127	3	3	0.32	3	3	0.09	3	3	0.02	$dwt_72$	72	75	5	5	0.68	5	5	0.25	5	5	0.13	
$can_73$	73	152	15	15	0.43	15	15	0.03	15	15	0.02	ash85	85	219	9	9	0.26	9	9	0.09	9	9	0.06	
$dwt_87$	87	227	10	10	0.25	10	10	0.34	10	10	0.13	$can_96$	96	336	7	7.05	198.05	7	7	0.53	7	7	0.89	
cycle100	100	100	1	1.1	22.42	1	1	1.13	1	1	< 0.01	m2D10x10	100	180	11	11	0.46	10	10	0.02	10	10	< 0.01	
nos4	100	247	10	10	0.38	10	10	0.02	10	10	< 0.01	path100	100	99	1	1	22.4	1	1	0.44	1	1	< 0.01	
cp13	104	103	10	10	1.98	10	10	0.05	10	10	0.08	tree10x2	111	110	28	28	< 0.01	28	28	< 0.01	28	28	< 0.01	
$dwt_234$	117	162	11	11	5.18	11	11	0.22	11	11	0.18	bcspwr03	118	179	10	10	2.24	10	10	1.56	10	10	0.31	
cp14	119	118	11	11	0.58	11	11	0.13	11	11	0.07	tree3x4	121	120	15	15	0.63	15	15	0.01	15	15	< 0.01	
cycle125	125	125	1	2.1	45.89	1	1	0.88	1	1	< 0.01	m2D5x25	125	220	6	6	0.73	5	5	0.37	5	5	0.74	
m3D5		300			138.69		21	0.17	21			path125		124	1	1	25.55	1	1	0.59	1	1	< 0.01	
bcsstk04	132	1758	37	37	0.46	37	37	0.13	37	37	0.18	$can_144$	144	576	7	8.5	162.45	7	7	1.36	7	7	1.6	
$lund_a$	147	1151	23	$^{23}$	0.92	23	$^{23}$	0.19	23	23		lund_b	147	1147	23	23	0.69	23	23	0.15	23	23	0.28	
cycle150		150	1	2.75	25.47	1	1	1.6	1	1	< 0.01	m2D10x15			11	11	31.46	10	10	0.09	10	10	0.03	
path150	150	149	1	1	84.01	1	1	0.63	1	1	< 0.01	cp16	152	151	13	13	0.36	13	13	0.32	13	13	0.26	
		1135		20	1.19	20	20	0.59	20			tree5x3		155	26	26	0.06	26	26	< 0.01		26	< 0.01	
$can_161$			19	19	10.75	18	18	2.53	18			$dwt_162$	162		9	9	84.43	9	9	2.83	9	9	0.78	
cp17		169	14	14	0.35	14	14	0.45	14		0.31	cycle175	175	175	1	2.85	99.42	1	1	1.83	1	1	< 0.01	
m2D7x25			8	8	55.51	7	7	0.23	7	7	0.46	path175		174	1		131.27		1	0.7	1	1	< 0.01	
tree13x2			46	46	< 0.01	46	46	< 0.01		46		$can_187$	187	652	8	10.1	4.88	8	8	1.99	8	8	21.85	
$\mathrm{dwt}\_193$				32	2.74	32	32	0.68	32			cycle200	200		1	2.5	24.21	1	1	2.23	1	1	< 0.01	
m2D8x25	200	367	9	9.25	65.95	8	8	0.37	8	8	0.3	path200	200	199	1	1.15	101.06	1	1	1.33	1	1	< 0.01	

 ${\bf Table~2.~Detailed~results~for~the~medium-scale~instances.}$ 

	ITPS			5		NIL	S	MVNTS							ITP	S	NILS			MVNTS			
Graph	V	E	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	Graph	V	E	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time
cp19	209	208	15	15	70.37	15	15	1.44	15	15	0.93	$dwt_209$	209	767	23	23	89.44	23	23	3.01	23	23	0.94
m3D6	216	540	30	30	150.94	30	30	0.87	30	30	0.36	$dwt_221$	221	704	13	14.4	152.47	13	13	2.23	13	13	4.2
$can_229$	229	774	28	28.45	160.31	28	28.3	218.61	28	28	1.09	$dwt_245$	245	608	21	21	78.16	21	21	5.68	21	21	2.92
tree2x7	255	254	19	19	0.89	19	19	0.24	19	19	0.22	$can_256$	256	1330	59	59	92.38	59	59	21.17	59	59	8.8
$lshp_265$	265	744		17			17	0.33	17	17	0.09	$can_268$	268	1407	52	52	32.09		52	2.14	52	52	0.93
bcspwr04	274	669	24	24	47.82		24	5.29	24	24	2.81	ash292	292	958		19.75	12.97	19	19	5.22	19	19	5.05
$can_292$	292	1124	38	39.65	181.6	38	38	120.09	38	38	6.22	cp23	299	298	19	19	5.85	19	19	2.71	19	19	2.51
cycle300	300		1	3.8	34.74	1	1	3.29	1	1	< 0.01	m2D15x20				17	39.83		15	1.34	15		0.64
path300	300	299	1	1.3	171.1	1	1	1.78	1	1	< 0.01	$dwt_307$		1108		29.35		26	26	7.63	26		19.44
tree17x2	307	306	77	77	0.05	77	77	0.02	77	77	< 0.01	$dwt_310$	310	1069	12	12.35	45.49	12	12	1.78	12	12	0.86
m3D7	343	882	41		240.51		40	2.76	40	40	1.3	$dwt_361$		1296		15	56.13	14	14	2.89	14		0.07
plat362		2712			104.92		34	29.69	34	34	8.0	plskz362		880		18	54.47		18	3.65	18		2.75
$lshp_406$		1155			64.77		21	1.66	21	21	0.73	$dwt\_419$		1572		42.1	246.78			310.51			33.12
bcsstk06		3720		45	172.12		45	27.71		45	32.61	bcsstk07		3720		45	171.05		45	27.64			32.36
bcsstm07		3416		45	143.84		45	30.28	45	45	33.2	$impcol_d$		1267			136.64		35	27.69			25.14
bcspwr05	443	590			323.91		27	37.83		27	18.29	$can_445$		1682			211.64		46	16.76			12.79
tree21x2	463	462			0.12					116	< 0.01	cp29	464	463		24	39.54		$^{24}$	10.13			6.45
nos5		2352		64	114.52				63	63	20.8	cycle475	475	475	1		181.01		1	5.36	1	1	< 0.01
m2D19x25	475	906			49.21		19	2.93	19	19	2.88	path475	475	474		2.05	237.51		1	3.52	1	1	< 0.01
$494$ _bus			36		220.99		29	181.01		28.05		$dwt_{503}$		2762		62.1	86.25		41	10.24			195.9
m3D8					85.42		52	9.06	52	52	2.5	sherman4		1341			112.35		27	4.32	27		2.73
$lshp_577$		1656			236.2		25	5.4	25	25	1.71	$dwt_592$		2256			333.25		29	14.54			12.41
$can_634$					101.3			221.69		72	98.25	cycle650				5.9	348.8		1	8.79	1	1	< 0.01
m2D25x26		1249			179.67		25	14.36		25	8.97	path650	650		1		450.56		1	6.19	1	1	< 0.01
tree25x2				163	0.29			0.14			< 0.01	$662$ _bus	662	906			326.56			283.31		38	56.39
cp35	665	664			167.6		29			29	8.42	nos6		1290			170.89		16	37.82			27.42
685_bus		1282			308.66		32	47.02		32	35.51	can_715					359.06		60	76.28			64.92
m3D9				153.95		65	65	26.67		65	4.04	nos7		1944			280.71		65	17.95			8.83
dwt_758		2618			447.46		20	16.72	20	20	3.33	lshp_778					201.28		29	12.33		29	5.39
tree5x4	781	780		98	2.1	98	98	0.45	98	98	0.13	bcsstk19					244.13		14	24.38			0.5
cp39	819		33		163.54		33	33.37	33	33	12.75	cycle825	825	825	6		460.64		1	12.1	1	1	< 0.01
path825	825		3	9.4	490.31		1	8.97	1	1	< 0.01	$can_838$					400.12						59.49
		1622			262.13		28	31.08	28	28	31.64	young1c		1624			307.54		29	12.15			4.25
young3c		1671			407.02		30	59.16	30		188.49	$dwt_869$		3208			191.14						
$dwt_878$		3285			298.85		24	49.4	24	24	7.54	gr_30_30		3422			301.91						5.19
$dwt_918$		3233			149.72					32	10.75	jagmesh1		2664			311.24		20	21.39			36.77
nos3		7442			413.67		43	106.7		43	13.58	dwt_992		7876			377.42		35	52.55			12.36
		1000			533.51		1	17.02	1	1	< 0.01	m2D20x50					208.69		20	21.34			13.42
m3D10	1000	2700	82	192.1	130.74	80	80	104.55	80	80	11.68	path1000	1000	999	7	14.65	542.93	1	1	14.14	1	1	< 0.01

 ${\bf Table~3.~Detailed~results~for~the~large-scale~instances.}$ 

				ITPS			NILS			MVNT	ΓS					ITPS			NILS		MVNTS		
Graph	V	E	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	Graph	V	E	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time	$Cb_b$	$Cb_a$	time
dwt_1005	1005	3808	47	59.75	272.44	47	47	107.81	47	47	7.63	dwt_1007	1007	3784	28	42.25	305.46	28	28	43.38	28	28	1.02
jagmesh2	1009	2928	34	37.9	367.55	33	33	26.11	33	33	33.45	tree2x9	1023	1022	58	58	41.13	57	57	10.51	57	57	2.77
cp44	1034	1033	38	42.8	248.96	37	37	49.31	37	37	16.68	can_1054	1054	5571	80	144.5	324.41	79	79	93.29	79	79	20.91
can_1072	1072	5686	117	263.85	217.9	115	115.65	294.5	115	115	128.24	bcsstk09	1083	8677	66	68.3	398.39	62	62	74.09	61	61	62.27
jagmesh3	1089	3136	35	39.55	391.31	33	33	31.68	33	33	3.54	1138_bus	1138	1458	134	159	256.85	49	50.5	309.48	49	49.95	198.82
jagmesh7			30	46	418.02		25.15	212.47		25.35	226.43	jagmesh8			48	65	336.59	32	32.2	234.02		32.4	183.91
jagmesh5			25	37.65	488.89		20.1	203.16		22.95	296.28	bcsstk27		27451	51	150.95	342.9	45	45.3	251.32		49.25	150.32
$dwt_{1242}$			105		104.53		56.65	293.29		54.05	209.22	lshp1270	1270		40	128.1	431.94		37	64.12	37	37	75.1
m3D11			100		232.59		97.65	255.29		96	14.18	jagmesh9			31	57.4	504.08		25.15	300.31		25.7	156.54
jagmesh6			32		535.12		18.9	159.81		18	194.94	jagmesh4			38		532.95		20	143.59		20	91.51
bcspwr06							49	101.97		49	200.68	lshp1561		4560	66		312.21		41	148.09		41.6	76.69
bcspwr07					302.63		51	78.19	51	51	93.16			2213	232	265.9	341.2			373.72		61.05	308.59
bcspwr09						63	65.7	294.31			313.82	m3D12		4752		433	226.76		124.35			114	27.84
lshp1882		5511		600.95		45	45.65	230.7	45	45.1	100.29	plat1919		15240		589.8	342.29			341.34			183.19
plsk1919		4831		480	167.75		36.7	261.77		37.5	177.57	bcsstk26		14207		463.3		122		383.79			264.75
bcsstk13		40940			332.53		491.68				312.97	hc11		11264		557.2	430.7	535		352.35			343.19
m3D13				550.95	337.5	136	278.05			133	41.93	lshp2233		6552	745	745	20.93	49	50.95	279.39		49.05	12.69
lshp2614		7683		872	35.61	53		302.55		53	21.53	dwt_2680				894	104.25		55.7	345.94		53.05	81.09
lshp3025		8904		1009	54.53	57		353.06		58.85	62.62	besstk23		21022		918.1	551.3			453.93			302.02
lshp3466		10215		1156	81.66	62	453.25			61.5	100.05	bcsstk24				1195.1					169		441.55
hc12							1106.45					bcsstk28				2112.25							314.63
bcspwr10	5300	8271	1114	1284.1	414.72	153	393.45	512.53	142	170.95	426.05	hc13	8192	53248	3828	3842.7	598.41	2819	2838.25	589.48	2127	2178.75	538.19