PHYS20762 Computational Physics

Project 2: Numerical Integration of Differential Equations
The Damped Harmonic Oscillator

2019/2020 Session, Semester 2

Project Description

For *your* given value of mass and spring constant (see the Class List), choose a time interval and damping term that illustrates the different accuracies of the proposed methods compared to the analytical solution for no external force (F=0). Investigate the effect of the choice of time interval (how small it is) on the accuracy of different methods.

- Write a program to calculate the solutions *x(t)* for each of the four proposed methods (Euler, improved Euler, Verlet and Euler-Cromer) and the analytical solution for e.g. 500-1000 steps for initial conditions x = 0 and x' = -1 m/s at t = 0 and zero external force. All four methods should be coded as functions that are called from the main program. The latter should be well laid out, structured and commented. The data should be written into a file, with proper checks that the file exists, etc. included in the code.
- Plot *x(t)* for a sufficient number of calculation steps to clearly show the differences between the four methods (to achieve this, choose suitable values of the damping term, b, and the time step, h); and with the analytical method. *Note: you will need to derive or look up the analytical solution for a damped simple harmonic oscillator*.
- Plot *x(t)* for a sufficient number of calculation steps to clearly show the effect of the smallness of the time step, h, on the accuracy of a chosen numerical method (Euler method, as the least accurate, will allow the best illustration of this effect); *hint: the energy is a good way to look at the accuracy.* For a given required accuracy (that you should choose for yourself) determine the step size/number of simulation steps required to achieve that accuracy; comment on this result in your code or by appropriately labelling graphs.
- Write code that reads in the data generated from your simulation of each method above, and plots out the phase space of the oscillator and the energy as a function of time.
- For the best method, plot solutions for damping term, b, (a) half of, (b) equal to and (c) double the critical value ($b_{cr} = 2\sqrt{k \cdot m}$). Add comments to your code that explain your results.
- For the best method show the effect of:

  (a) sudden application of an external force after a few oscillation periods (a 'push'). Explore different situations where the force has the same or opposite sign to the instantaneous velocity and is applied in different parts of a cycle. Comment on your findings in the code

  (b) forced oscillations with a sinusoidal external force with frequency different from the undamped natural frequency. Make sure your graph shows steady oscillations after the transient period. Compare with unforced oscillations (use the appropriate comments or plots).

Final Work: Using the best method, investigate the resonance, i.e. calculate and plot the amplitude of oscillations as a function of frequency for a range of frequencies above and below the natural frequency of the system.

21/01/2020 Dr Hywel Owen (adapted from previous course material)